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Edited by
ANDRÉS GOMBEROFF
Centro de Estudios Científicos (CECS)
DONALD MAROLF
University of California, Santa Barbara (UCSB)

Springer

Andres Gomberoff
Centro de Estudios Cientificos
Avenida Arturo Prat 514
Valdivia, Chile

Donald Marolf
Physics Department
University of California
Santa Barbara, CA 93106

Series Editor:
Claudio Teitelboim
Centro de Estudios Cient´ıficos
Valdivia, Chile

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Foreword

The Centro de Estudios Cientificos (CECS) began a new phase of its existence at the end of 1999, when it moved to the city of Valdivia, 800 kilometers South of the capital of Chile, Santiago. The letter “S", which stood for Santiago in the original acronym has been maintained to provide a sense of historical continuity, and it is now - when necessary - explained as arising from the plural in the word Científicos.

Valdivia used to be part of the “frontier" in the early days of the country and one still breathes frontier air in it. This frontier air has inspired the Center to undertake new and bolder challenges in science and exploration, such as an unprecedented airborne exploration of the Amundsen Sea in West Antarctica and the development of a state of the art Transgenic Facility.

However, in the midst of all this excitement and frenetic activity, we were distinctly reminded by the physicists who came to Valdivia from many countries to take part in the School of Quantum Gravity that, as Richard Feynman used to say: “there is nothing better in life than eating cookies and talking about Physics".

Claudio Teitelboim
Director, Centro de Estudios Científicos
Valdivia, April 2004.

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Contributing Authors

Thibault Damour Institut des Hautes Études Scientifiques (IHÉS).

Andrés Gomberoff Centro de Estudios Científicos (CECS).

Marc Henneaux Université Libre de Bruxelles.

Ted Jacobson University of Maryland.

Juan Maldacena Institute for Advanced Study.

Robert C. Myers Perimeter Institute for Theoretical Physics, University of Waterloo and McGill University.

Hermann Nicolai Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut.

Rafael D. Sorkin Syracuse University.

Washington Taylor Center for Theoretical Physics, Massachusetts Institute of Technology (MIT).

Claudio Teitelboim Centro de Estudios Científicos (CECS).

Robert M. Wald University of Chicago.

Frank Wilczek Center for Theoretical Physics, Massachusetts Institute of Technology (MIT).

Preface

The 2002 Pan-American Advanced Studies Institute *School on Quantum Gravity* was held at the Centro de Estudios Cientificos (CECS),Valdivia, Chile, January 4-14, 2002. The school featured lectures by ten speakers, and was attended by nearly 70 students from over 14 countries. A primary goal was to foster interaction and communication between participants from different cultures, both in the layman's sense of the term and in terms of approaches to quantum gravity. We hope that the links formed by students and the school will persist throughout their professional lives, continuing to promote interaction and the essential exchange of ideas that drives research forward.

This volume contains improved and updated versions of the lectures given at the School. It has been prepared both as a reminder for the participants, and so that these pedagogical introductions can be made available to others who were unable to attend. We expect them to serve students of all ages well.

ANDRES GOMBEROFF AND DONALD MAROLF

Acknowledgments

We are greatly indebted to all of the many people and organizations who made the school such a success. Key support provided by the US National Science Foundation and the US Department of Energy via a grant to Syracuse University through the Pan-American Advanced Studies Institutes program is gratefully acknowledged. We also thank the Millennium Science Initiative (Chile), Fundación Andes, the Tinker Foundation, and Empresas CMPC for their support through CECS. We hope that our school has lived up to the standards of all of these fine programs and that this proceedings will help to encourage the future growth of both PASI and CECS.

We thank the administrative staff and colleagues at CECS for their hard work and for providing a marvelous environment for the school.

Last but not least, we would like to express our thanks to Syracuse University and its excellent staff for their support and assistance in all phases of the project. We thank Penny Davis in particular for her many hours of effort. Although it was often behind the scenes, coordination with participants and North American funding agencies by Penny and other staff members was crucial for making this school a reality.

THE THERMODYNAMICS OF BLACK HOLES

Robert M. Wald
Enrico Fermi Institute and Department of Physics
University of Chicago
5640 S. Ellis Avenue
Chicago, Illinois 60637-1433
rmwa@midway.uchicago.edu

Abstract

We review the present status of black hole thermodynamics. Our review includes discussion of classical black hole thermodynamics, Hawking radiation from black holes, the generalized second law, and the issue of entropy bounds. A brief survey also is given of approaches to the calculation of black hole entropy. We conclude with a discussion of some unresolved open issues.

This article is based upon an article of the same title published in Living Reviews in Relativity, http://www.livingreviews.org.

1. Introduction

During the past 30 years, research in the theory of black holes in general relativity has brought to light strong hints of a very deep and fundamental relationship between gravitation, thermodynamics, and quantum theory. The cornerstone of this relationship is black hole thermodynamics, where it appears that certain laws of black hole mechanics are, in fact, simply the ordinary laws of thermodynamics applied to a system containing a black hole. Indeed, the discovery of the thermodynamic behavior of black holes – achieved primarily by classical and semiclassical analyses – has given rise to most of our present physical insights into the nature of quantum phenomena occurring in strong gravitational fields.

The purpose of this article is to provide a review of the following aspects of black hole thermodynamics:

- At the purely classical level, black holes in general relativity (as well as in other diffeomorphism covariant theories of gravity) obey certain laws which bear a remarkable mathematical resemblance to the ordinary laws of thermodynamics. The derivation of these laws of classical black hole mechanics is reviewed in section 2.

- Classically, black holes are perfect absorbers but do not emit anything; their physical temperature is absolute zero. However, in quantum theory black holes emit Hawking radiation with a perfect thermal spectrum. This allows a consistent interpretation of the laws of black hole mechanics as physically corresponding to the ordinary laws of thermodynamics. The status of the derivation of Hawking radiation is reviewed in section 3.

- The *generalized second law* (GSL) directly links the laws of black hole mechanics to the ordinary laws of thermodynamics. The arguments in favor of the GSL are reviewed in section 4. A discussion of entropy bounds is also included in this section.

- The classical laws of black hole mechanics together with the formula for the temperature of Hawking radiation allow one to identify a quantity associated with black holes – namely $A/4$ in general relativity – as playing the mathematical role of entropy. The apparent validity of the GSL provides strong evidence that this quantity truly is the physical entropy of a black hole. A major goal of research in quantum gravity is to provide an explanation for – and direct derivation of – the formula for the entropy of a black hole. A brief survey of work along these lines is provided in section 5.

- Although much progress has been made in our understanding of black hole thermodynamics, many important issues remain unresolved. Primary among these are the "black hole information paradox" and issues related to the degrees of freedom responsible for the entropy of a black hole. These unresolved issues are briefly discussed in section 6.

Throughout this article, we shall set $G = \hbar = c = k = 1$, and we shall follow the sign and notational conventions of [1]. Although I have attempted to make this review be reasonably comprehensive and balanced, it should be understood that my choices of topics and emphasis naturally reflect my own personal viewpoints, expertise, and biases.

2. Classical Black Hole Thermodynamics

In this section, I will give a brief review of the laws of classical black hole mechanics.

In physical terms, a black hole is a region where gravity is so strong that nothing can escape. In order to make this notion precise, one must have in mind a region of spacetime to which one can contemplate escaping. For an asymptotically flat spacetime (M, g_{ab}) (representing an isolated system), the asymptotic portion of the spacetime "near infinity" is such a region. The *black hole* region, $\mathcal{B}$, of an asymptotically flat spacetime, (M, g_{ab}), is defined as

$$\mathcal{B} \equiv M - I^-(\mathcal{I}^+), \tag{1}$$

where $\mathcal{I}^+$ denotes future null infinity and I^- denotes the chronological past. Similar definitions of a black hole can be given in other contexts (such as asymptotically anti-deSitter spacetimes) where there is a well defined asymptotic region.

The *event horizon*, $\mathcal{H}$, of a black hole is defined to be the boundary of $\mathcal{B}$. Thus, $\mathcal{H}$ is the boundary of the past of $\mathcal{I}^+$. Consequently, $\mathcal{H}$ automatically satisfies all of the properties possessed by past boundaries (see, e.g., [2] or [1] for further discussion). In particular, $\mathcal{H}$ is a null hypersurface which is composed of future inextendible null geodesics without caustics, i.e., the expansion, θ, of the null geodesics comprising the horizon cannot become negatively infinite. Note that the entire future history of the spacetime must be known before the location of $\mathcal{H}$ can be determined, i.e., $\mathcal{H}$ possesses no distinguished local significance.

If Einstein's equation holds with matter satisfying the null energy condition (i.e., if $T_{ab}k^a k^b \geq 0$ for all null k^a), then it follows immediately from the Raychauduri equation (see, e.g., [1]) that if the expansion, θ, of any null geodesic congruence ever became negative, then θ would become infinite within a finite affine parameter, provided, of course, that the geodesic can be extended that far. If the black hole is *strongly asymptotically predictable* – i.e., if there is a globally hyperbolic region containing $I^-(\mathcal{I}^+) \cup \mathcal{H}$ – it can be shown that this implies that $\theta \geq 0$ everywhere on $\mathcal{H}$ (see, e.g., [2, 1]). It then follows that the surface area, A, of the event horizon of a black hole can never decrease with time, as discovered by Hawking [4].

It is worth remarking that since $\mathcal{H}$ is a past boundary, it automatically must be a C^0 embedded submanifold (see, e.g., [1]), but it need not be C^1. However, essentially all discussions and analyses of black hole event horizons implicitly assume C^1 or higher order differentiability of $\mathcal{H}$. Recently, this higher order differentiability assumption has been eliminated for the proof of the area theorem [3].

The area increase law bears a resemblance to the second law of thermodynamics in that both laws assert that a certain quantity has the property of never decreasing with time. It might seem that this resemblance is a very superficial one, since the area law is a theorem in differential geometry whereas the second law of thermodynamics is understood to have a statistical origin. Nevertheless, this resemblance together with the idea that information is irretrievably lost when a body falls into a black hole led Bekenstein to propose [5, 6] that a suitable multiple of the area of the event horizon of a black hole should be interpreted as its entropy, and that a *generalized second law* (GSL) should hold: The sum of the ordinary entropy of matter outside of a black hole plus a suitable multiple of the area of a black hole never decreases. We will discuss this law in detail in section 4.

The remaining laws of thermodynamics deal with equilibrium and quasi-equilibrium processes. At nearly the same time as Bekenstein proposed a relationship between the area theorem and the second law of thermodynamics, Bardeen, Carter, and Hawking [7] provided a general proof of certain laws of "black hole mechanics" which are direct mathematical analogs of the zeroth and first laws of thermodynamics. These laws of black hole mechanics apply to stationary black holes (although a formulation of these laws in terms of isolated horizons will be briefly described at the end of this section).

In order to discuss the zeroth and first laws of black hole mechanics, we must introduce the notions of stationary, static, and axisymmetric black holes as well as the notion of a Killing horizon. If an asymptotically flat spacetime (M, g_{ab}) contains a black hole, $\mathcal{B}$, then $\mathcal{B}$ is said to be *stationary* if there exists a one-parameter group of isometries on (M, g_{ab}) generated by a Killing field t^a which is unit timelike at infinity. The black hole is said to be *static* if it is stationary and if, in addition, t^a is hypersurface orthogonal. The black hole is said to be *axisymmetric* if there exists a one parameter group of isometries which correspond to rotations at infinity. A stationary, axisymmetric black hole is said to possess the "t–ϕ orthogonality property" if the 2-planes spanned by t^a and the rotational Killing field ϕ^a are orthogonal to a family of 2-dimensional surfaces. The t–ϕ orthogonality property holds for all stationary-axisymmetric black hole solutions to the vacuum Einstein or Einstein-Maxwell equations (see, e.g., [8]).

A null surface, $\mathcal{K}$, whose null generators coincide with the orbits of a one-parameter group of isometries (so that there is a Killing field ξ^a normal to $\mathcal{K}$) is called a *Killing horizon*. There are two independent results (usually referred to as "rigidity theorems") that show that in a wide variety of cases of interest, the event horizon, $\mathcal{H}$, of a stationary black hole must be a Killing horizon. The first, due to Carter [9], states that for a static black hole, the static Killing field t^a must be normal to the horizon, whereas for a stationary-axisymmetric black hole with the t–ϕ orthogonality property there exists a Killing field ξ^a of the form

$$\xi^a = t^a + \Omega \phi^a \tag{2}$$

which is normal to the event horizon. The constant Ω defined by Eq. (2) is called the *angular velocity of the horizon*. Carter's result does not rely on any field equations, but leaves open the possibility that there could exist stationary black holes without the above symmetries whose event horizons are not Killing horizons. The second result, due to Hawking [2] (see also [10]), directly proves that in vacuum or electrovac general relativity, the event horizon of any stationary black hole must be a Killing horizon. Consequently, if t^a fails to be normal to the horizon, then there must exist an additional Killing field, ξ^a, which is normal to the horizon, i.e., a stationary black hole must be nonrotating (from which staticity follows [11, 12, 13]) or axisymmetric (though not necessarily with the t–ϕ orthogonality property). Note that Hawking's theorem makes no assumptions of symmetries beyond stationarity, but it does rely on the properties of the field equations of general relativity.

Now, let $\mathcal{K}$ be any Killing horizon (not necessarily required to be the event horizon, $\mathcal{H}$, of a black hole), with normal Killing field ξ^a. Since $\nabla^a(\xi^b\xi_b)$ also is normal to $\mathcal{K}$, these vectors must be proportional at every point on $\mathcal{K}$. Hence, there exists a function, κ, on $\mathcal{K}$, known as the *surface gravity* of $\mathcal{K}$, which is defined by the equation

$$\nabla^a(\xi^b\xi_b) = -2\kappa\xi^a. \tag{3}$$

It follows immediately that κ must be constant along each null geodesic generator of $\mathcal{K}$, but, in general, κ can vary from generator to generator. It is not difficult to show (see, e.g., [1]) that

$$\kappa = \lim(Va), \tag{4}$$

where a is the magnitude of the acceleration of the orbits of ξ^a in the region off of $\mathcal{K}$ where they are timelike, $V \equiv (-\xi^a\xi_a)^{1/2}$ is the "redshift factor" of ξ^a, and the limit as one approaches $\mathcal{K}$ is taken. Equation (4) motivates the terminology "surface gravity". Note that the surface gravity of a black hole is defined only when it is "in equilibrium", i.e., stationary, so that its event horizon is a Killing horizon. There is no notion of the surface gravity of a general, non-stationary black hole, although the definition of surface gravity can be extended to isolated horizons (see below).

In parallel with the two independent "rigidity theorems" mentioned above, there are two independent versions of the zeroth law of black hole mechanics. The first, due to Carter [9] (see also [14]), states that for any black hole which is static or is stationary-axisymmetric with the t–ϕ orthogonality property, the surface gravity κ, must be constant over its event horizon $\mathcal{H}$. This result is purely geometrical, i.e., it involves no use of any field equations. The second, due to Bardeen, Carter, and Hawking [7] states that if Einstein's equation holds with the matter stress-energy tensor satisfying the dominant energy condition, then κ must be constant on any Killing horizon. Thus, in the second version of the zeroth law, the hypothesis that the t–ϕ orthogonality property holds is eliminated, but use is made of the field equations of general relativity.

A *bifurcate Killing horizon* is a pair of null surfaces, $\mathcal{K}_A$ and $\mathcal{K}_B$, which intersect on a spacelike 2-surface, $\mathcal{C}$ (called the "bifurcation surface"), such that $\mathcal{K}_A$ and $\mathcal{K}_B$ are each Killing horizons with respect to the same Killing field ξ^a. It follows that ξ^a must vanish on $\mathcal{C}$; conversely, if a Killing field, ξ^a, vanishes on a two-dimensional spacelike surface, $\mathcal{C}$, then $\mathcal{C}$ will be the bifurcation surface of a bifurcate Killing horizon associated with ξ^a (see [15] for further discussion). An important consequence of the zeroth law is that if $\kappa \neq 0$, then in the "maximally extended" spacetime representing a stationary black hole, the event horizon, $\mathcal{H}$, comprises a branch of a bifurcate Killing horizon [14]. This result is purely geometrical – involving no use of any field equations. As a consequence, the study of stationary black holes which satisfy the zeroth law divides into two cases: "extremal" black holes (for which, by definition, $\kappa = 0$), and black holes with bifurcate horizons.

The first law of black hole mechanics is simply an identity relating the changes in mass, M, angular momentum, J, and horizon area, A, of a stationary black hole when it is perturbed. To first order, the variations of these quantities in the vacuum case always satisfy

$$\delta M = \frac{1}{8\pi}\kappa\delta A + \Omega\delta J. \tag{5}$$

In the original derivation of this law [7], it was required that the perturbation be stationary. Furthermore, the original derivation made use of the detailed form of Einstein's equation. Subsequently, the derivation has been generalized to hold for non-stationary perturbations [11, 16], provided that the change in area is evaluated at the bifurcation surface, $\mathcal{C}$, of the unperturbed black hole (see, however, [17] for a derivation of the first law for non-stationary perturbations that does not require evaluation at the bifurcation surface). More significantly, it has been shown [16] that the validity of this law depends only on very general properties of the field equations. Specifically, a version of this law holds for any field equations derived from a diffeomorphism covariant Lagrangian, L. Such a Lagrangian can always be written in the form

$$L = L\left(g_{ab}; R_{abcd}, \nabla_a R_{bcde}, ...; \psi, \nabla_a \psi, ...\right), \tag{6}$$

where ∇_a denotes the derivative operator associated with g_{ab}, R_{abcd} denotes the Riemann curvature tensor of g_{ab}, and ψ denotes the collection of all matter fields of the theory (with indices suppressed). An arbitrary (but finite) number of derivatives of R_{abcd} and ψ are permitted to appear in L. In this more general context, the first law of black hole mechanics is seen to be a direct consequence of an identity holding for the variation of the Noether current. The general form of the first law takes the form

$$\delta M = \frac{\kappa}{2\pi} \delta S_{\rm bh} + \Omega \delta J + ..., \tag{7}$$

where the "..." denote possible additional contributions from long range matter fields, and where

$$S_{\rm bh} \equiv -2\pi \int_{\mathcal{C}} \frac{\delta L}{\delta R_{abcd}} n_{ab} n_{cd}. \tag{8}$$

Here n_{ab} is the binormal to the bifurcation surface $\mathcal{C}$ (normalized so that $n_{ab}n^{ab} = -2$), and the functional derivative is taken by formally viewing the Riemann tensor as a field which is independent of the metric in Eq. (6). For the case of vacuum general relativity, where $L = R\sqrt{-g}$, a simple calculation yields

$$S_{\rm bh} = A/4, \tag{9}$$

and Eq. (7) reduces to Eq. (5).

The close mathematical analogy of the zeroth, first, and second laws of thermodynamics to corresponding laws of classical black hole mechanics is broken by the Planck-Nernst form of the third law of thermodynamics, which states that $S \to 0$ (or a "universal constant") as $T \to 0$. The analog of this law fails in black hole mechanics – although analogs of alternative formulations of the third law do appear to hold for black holes [18] – since there exist extremal black holes (i.e., black holes with $\kappa = 0$) with finite A. However, there is good reason to believe that the "Planck-Nernst theorem" should not be viewed as a fundamental law of thermodynamics [19] but rather as a property of the density of states near the ground state in the thermodynamic limit, which happens to be valid for commonly studied materials. Indeed, examples can be given of ordinary quantum systems that violate the Planck-Nernst form of the third law in a manner very similar to the violations of the analog of this law that occur for black holes [20].

As discussed above, the zeroth and first laws of black hole mechanics have been formulated in the mathematical setting of stationary black holes whose event horizons are Killing horizons. The requirement of stationarity applies to the entire spacetime and, indeed, for the first law, stationarity of the entire spacetime is essential in order to relate variations of quantities defined at the horizon (like A) to variations of quantities defined at infinity (like M and J). However, it would seem reasonable to expect that the equilibrium thermodynamic behavior of a black hole would require only a form of local stationarity at the event horizon. For the formulation of the first law of black hole mechanics, one would also then need local definitions of quantities like M and J at the horizon. Such an approach toward the formulation of the laws of black hole mechanics has recently been taken via the notion of an *isolated horizon*, defined as a null hypersurface with vanishing shear and expansion satisfying the additional properties stated in [21]. (This definition supersedes the more restrictive definitions given, e.g., in [22, 23, 24].) The presence of an isolated horizon does not require the entire spacetime to be stationary [25]. A direct analog of the zeroth law for stationary event horizons can be shown to hold for isolated horizons [26]. In the Einstein-Maxwell case, one can demand (via a choice of scaling of the normal to the isolated horizon as well as a choice of gauge for the Maxwell field) that the surface gravity and electrostatic potential of the isolated horizon be functions of only its area and charge. The requirement that time evolution be symplectic then leads to a version of the first law of black hole mechanics as well as a (in general, non-unique) local notion of the energy of the isolated horizon [26]. These results also have been generalized to allow dilaton couplings [24] and Yang-Mills fields [27, 26].

In comparing the laws of black hole mechanics in classical general relativity with the laws of thermodynamics, it should first be noted that the black hole uniqueness theorems (see, e.g., [8]) establish that stationary black holes – i.e., black holes "in equilibrium" – are characterized by a small number of parameters, analogous to the "state parameters" of ordinary thermodynamics. In the corresponding laws, the role of energy, E, is played by the mass, M, of the black hole; the role of temperature, T, is played by a constant times the surface gravity, κ, of the black hole; and the role of entropy, S, is played by a constant times the area, A, of the black hole. The fact that E and M represent the same physical quantity provides a strong hint that the mathematical analogy between the laws of black hole mechanics and the laws of thermodynamics might be of physical significance. However, as argued in [7], this cannot be the case in classical general relativity. The physical temperature of a black hole is absolute zero (see subsection 4.1 below), so there can be no physical relationship between T and κ. Consequently, it also would be inconsistent to assume a physical relationship between S and A. As we shall now see, this situation changes dramatically when quantum effects are taken into account.

3. Hawking Radiation

In 1974, Hawking [28] made the startling discovery that the physical temperature of a black hole is not absolute zero: As a result of quantum particle creation effects, a black hole radiates to infinity all species of particles with a perfect black body spectrum, at temperature (in units with $G = c = \hbar = k = 1$)

$$T = \frac{\kappa}{2\pi}. \quad (10)$$

Thus, $\kappa/2\pi$ truly is the *physical* temperature of a black hole, not merely a quantity playing a role mathematically analogous to temperature in the laws of black hole mechanics. In this section, we review the status of the derivation of the Hawking effect and also discuss the closely related Unruh effect.

The original derivation of the Hawking effect [28] made direct use of the formalism for calculating particle creation in a curved spacetime that had been developed by Parker [29] and others. Hawking considered a classical spacetime (M, g_{ab}) describing gravitational collapse to a Schwarzschild black hole. He then considered a free (i.e., linear) quantum field propagating in this background spacetime, which is initially in its vacuum state prior to the collapse, and he computed the particle content of the field at infinity at late times. This calculation involves taking the positive frequency mode function corresponding to a particle state at late times, propagating it backwards in time, and determining its positive and negative frequency parts in the asymptotic past. His calculation revealed that at late times, the expected number of particles at infinity corresponds to emission from a perfect black body (of finite size) at the Hawking temperature (Eq. (10)). It should be noted that this result relies only on the analysis of quantum fields in the region exterior to the black hole, and it does not make use of any gravitational field equations.

The original Hawking calculation can be straightforwardly generalized and extended in the following ways. First, one may consider a spacetime representing an arbitrary gravitational collapse to a black hole such that the black hole "settles down" to a stationary final state satisfying the zeroth law of black hole mechanics (so that the surface gravity, κ, of the black hole final state is constant over its event horizon). The initial state of the quantum field may be taken to be any nonsingular state (i.e., any Hadamard state – see, e.g., [15]) rather than the initial vacuum state. Finally, it can be shown [30] that all aspects of the final state at late times (i.e., not merely the expected number of particles in each mode) correspond to black body[1] thermal radiation emanating from the black hole at temperature (Eq. (10)).

It should be noted that no infinities arise in the calculation of the Hawking effect for a free field, so the results are mathematically well defined, without any need for regularization or renormalization. The original derivations [28, 30] made use of notions of "particles propagating into the black hole", but the results for what an observer sees at infinity were shown to be independent of the ambiguities inherent in such notions and, indeed, a derivation of the Hawking effect has been given [31] which entirely avoids the introduction of any notion of "particles". However, there remains one significant difficultly with the Hawking derivation: In the calculation of the backward-in-time propagation of a mode, it is found that the mode undergoes a large blueshift as it propagates near the event horizon, but there is no correspondingly large redshift as the mode propagates back through the collapsing matter into the asymptotic past. Indeed, the net blueshift factor of the mode is proportional to $\exp(\kappa t)$, where t is the time that the mode would reach an observer at infinity. Thus, within a time of order $1/\kappa$ of the formation of a black hole (i.e., $\sim 10^{-5}$ seconds for a one solar mass Schwarzschild black hole), the Hawking derivation involves (in its intermediate steps) the propagation of modes of frequency much higher than the Planck frequency. In this regime, it is difficult to believe in the accuracy of free field theory – or any other theory known to mankind.

An approach to investigating this issue was first suggested by Unruh [32], who noted that a close analog of the Hawking effect occurs for quantized sound waves in a fluid undergoing supersonic flow. A similar blueshifting of the modes quickly brings one into a regime well outside the domain of validity of the continuum fluid equations. Unruh suggested replacing the continuum fluid equations with a more realistic model at high frequencies to see if the fluid analog of the Hawking effect would still occur. More recently, Unruh investigated models where the dispersion relation is altered at ultra-high frequencies, and he found no deviation from the Hawking prediction [33]. A variety of alternative models have been considered by other researchers [34, 35, 36, 37, 38, 39, 40]. Again, agreement with the Hawking effect prediction was found in all cases, despite significant modifications of the theory at high frequencies.

The robustness of the Hawking effect with respect to modifications of the theory at ultra-high frequency probably can be understood on the following grounds. One may view the backward-in-time propagation of modes as consisting of two stages: a first stage where the blueshifting of the mode brings it into a WKB regime but the frequencies remain well below the Planck scale, and a second stage where the continued blueshifting takes one to the Planck scale and beyond. In the first stage, the usual field theory calculations should be reliable. On the other hand, after the mode has entered a WKB regime, it seems plausible that the kinds of modifications to its propagation laws considered in [33, 34, 35, 36, 37, 38, 39, 40] should not affect its essential properties, in particular the magnitude of its negative frequency part.

Indeed, an issue closely related to the validity of the original Hawking derivation arises if one asks how a uniformly accelerating observer in Minkowski spacetime perceives the ordinary (inertial) vacuum state (see below). The outgoing modes of a given frequency ω as seen by the accelerating observer at proper time τ along his worldline correspond to modes of frequency $\sim \omega \exp(a\tau)$ in a fixed inertial frame. Therefore, at time $\tau \gg 1/a$ one might worry about field-theoretic derivations of what the accelerating observer would see. However, in this case one can appeal to Lorentz invariance to argue that what the accelerating observer sees cannot change with time. It seems likely that one could similarly argue that the Hawking effect cannot be altered by modifications of the theory at ultra-high frequencies, provided that these modifications preserve an appropriate "local Lorentz invariance" of the theory. Thus, there appears to be strong reasons for believing in the validity of the Hawking effect despite the occurrence of ultra-high-frequency modes in the derivation.

There is a second, logically independent result – namely, the Unruh effect [41] and its generalization to curved spacetime – which also gives rise to the formula (10). Although the Unruh effect is mathematically very closely related to the Hawking effect, it is important to distinguish clearly between them. In its most general form, the Unruh effect may be stated as follows (see [42, 15] for further discussion): Consider a classical spacetime (M, g_{ab}) that contains a bifurcate Killing horizon, $\mathcal{K} = \mathcal{K}_A \cup \mathcal{K}_B$, so that there is a one-parameter group of isometries whose associated Killing field, ξ^a, is normal to $\mathcal{K}$. Consider a free quantum field on this spacetime. Then there exists at most one globally nonsingular state of the field which is invariant under the isometries. Furthermore, in the "wedges" of the spacetime where the isometries have timelike orbits, this state (if it exists) is a KMS (i.e., thermal equilibrium) state at temperature (10) with respect to the isometries.

Note that in Minkowski spacetime, any one-parameter group of Lorentz boosts has an associated bifurcate Killing horizon, comprised by two intersecting null planes. The unique, globally nonsingular state which is invariant under these isometries is simply the usual ("inertial") vacuum state, $|0\rangle$. In the "right and left wedges" of Minkowski spacetime defined by the Killing horizon, the orbits of the Lorentz boost isometries are timelike, and, indeed, these orbits correspond to worldlines of uniformly accelerating observers. If we normalize the boost Killing field, b^a, so that Killing time equals proper time on an orbit with acceleration a, then the surface gravity of the Killing horizon is $\kappa = a$. An observer following this orbit would naturally use b^a to define a notion of "time translation symmetry". Consequently, by the above general result, when the field is in the inertial vacuum state, a uniformly accelerating observer would describe the field as being in a thermal equilibrium state at temperature

$$T = \frac{a}{2\pi} \tag{11}$$

as originally discovered by Unruh [41]. A mathematically rigorous proof of the Unruh effect in Minkowski spacetime was given by Bisognano and Wichmann [43] in work motivated by entirely different considerations (and done independently of and nearly simultaneously with the work of Unruh). Furthermore, the Bisognano-Wichmann theorem is formulated in the general context of axiomatic quantum field theory, thus establishing that the Unruh effect is not limited to free field theory.

Although there is a close mathematical relationship between the Unruh effect and the Hawking effect, it should be emphasized that these results refer to *different* states of the quantum field. We can divide the late time modes of the quantum field in the following manner, according to the properties that they would have in the analytically continued spacetime [14] representing the asymptotic final stationary state of the black hole: We refer to modes that would have emanated from the white hole region of the analytically continued spacetime as "UP modes" and those that would have originated from infinity as "IN modes". In the Hawking effect, the asymptotic final state of the quantum field is a state in which the UP modes of the quantum field are thermally populated at temperature (10), but the IN modes are unpopulated. This state (usually referred to as the "Unruh vacuum") would be singular on the white hole horizon in the analytically continued spacetime. On the other hand, in the Unruh effect and its generalization to curved spacetimes, the state in question (usually referred to as the "Hartle-Hawking vacuum" [44]) is globally nonsingular, and *all* modes of the quantum field in the "left and right wedges" are thermally populated.[2]

The differences between the Unruh and Hawking effects can be seen dramatically in the case of a Kerr black hole. For the Kerr black hole, it can be shown [42] that there does not exist any globally nonsingular state of the field which is invariant under the isometries associated with the Killing horizon, i.e., there does not exist a "Hartle-Hawking vacuum state" on Kerr spacetime. However, there is no difficultly with the derivation of the Hawking effect for Kerr black holes, i.e., the "Unruh vacuum state" does exist.

It should be emphasized that in the Hawking effect, the temperature (10) represents the temperature as measured by an observer near infinity. For any observer following an orbit of the Killing field, ξ^a, normal to the horizon, the locally measured temperature of the UP modes is given by

$$T = \frac{\kappa}{2\pi V} \, , \tag{12}$$

where $V = (-\xi^a \xi_a)^{1/2}$. In other words, the locally measured temperature of the Hawking radiation follows the Tolman law. Now, as one approaches the horizon of the black hole, the UP modes dominate over the IN modes. Taking Eq. (4) into account, we see that $T \to a/2\pi$ as the black hole horizon, $\mathcal{H}$, is approached, i.e., in this limit Eq. (12) corresponds to the flat spacetime Unruh effect.

Equation (12) shows that when quantum effects are taken into account, a black hole is surrounded by a "thermal atmosphere" whose local temperature as measured by observers following orbits of ξ^a becomes divergent as one approaches the horizon. As we shall see in the next section, this thermal atmosphere produces important physical effects on quasi-stationary bodies near the black hole. On the other hand, it should be emphasized that for a macroscopic black hole, observers who freely fall into the black hole would not notice any important quantum effects as they approach and cross the horizon.

4. The Generalized Second Law (GSL)

In this section, we shall review some arguments for the validity of the generalized second law (GSL). We also shall review the status of several proposed entropy bounds on matter that have played a role in discussions and analyses of the GSL.

4.1 Arguments for the validity of the GSL

Even in classical general relativity, there is a serious difficulty with the ordinary second law of thermodynamics when a black hole is present, as originally emphasized by J.A. Wheeler: One can simply take some ordinary matter and drop it into a black hole, where, according to classical general relativity, it will disappear into a spacetime singularity. In this process, one loses the entropy initially present in the matter, and no compensating gain of ordinary entropy occurs, so the total entropy, S, of matter in the universe decreases. One could attempt to salvage the ordinary second law by invoking the bookkeeping rule that one must continue to count the entropy of matter dropped into a black hole as still contributing to the total entropy of the universe. However, the second law would then have the status of being observationally unverifiable.

As already mentioned in section 2, after the area theorem was proven, Bekenstein [5, 6] proposed a way out of this difficulty: Assign an entropy, $S_{\rm bh}$, to a black hole given by a numerical factor of order unity times the area, A, of the black hole in Planck units. Define the *generalized entropy*, S', to be the sum of the ordinary entropy, S, of matter outside of a black hole plus the black hole entropy

$$S' \equiv S + S_{\rm bh}. \tag{13}$$

Finally, replace the ordinary second law of thermodynamics by the *generalized second law* (GSL): The total generalized entropy of the universe never decreases with time,

$$\Delta S' \geq 0. \tag{14}$$

Although the ordinary second law will fail when matter is dropped into a black hole, such a process will tend to increase the area of the black hole, so there is a possibility that the GSL will hold.

Bekenstein's proposal of the GSL was made prior to the discovery of Hawking radiation. When Hawking radiation is taken into account, a serious problem also arises with the second law of black hole mechanics (i.e., the area theorem): Conservation of energy requires that an isolated black hole must lose mass in order to compensate for the energy radiated to infinity by the Hawking process. Indeed, if one equates the rate of mass loss of the black hole to the energy flux at infinity due to particle creation, one arrives at the startling conclusion that an isolated black hole will radiate away all of its mass within a finite time. During this process of black hole "evaporation", A will decrease. Such an area decrease can occur because the expected stress-energy tensor of quantum matter does not satisfy the null energy condition – even for matter for which this condition holds classically – in violation of a key hypothesis of the area theorem.

However, although the second law of black hole mechanics fails during the black hole evaporation process, if we adjust the numerical factor in the definition of $S_{\rm bh}$ to correspond to the identification of $\kappa/2\pi$ as temperature in the first law of black hole mechanics – so that, as in Eq. (9) above, we have $S_{\rm bh} = A/4$ in Planck units – then the GSL continues to hold: Although A decreases, there is at least as much ordinary entropy generated outside the black hole by the Hawking process. Thus, although the ordinary second law fails in the presence of black holes and the second law of black hole mechanics fails when quantum effects are taken into account, there is a possibility that the GSL may always hold. If the GSL does hold, it seems clear that we must interpret $S_{\rm bh}$ as representing the *physical* entropy of a black hole, and that the laws of black hole mechanics must truly represent the ordinary laws of thermodynamics as applied to black holes. Thus, a central issue in black hole thermodynamics is whether the GSL holds in all processes.

It was immediately recognized by Bekenstein [5] (see also [7]) that there is a serious difficulty with the GSL if one considers a process wherein one carefully lowers a box containing matter with entropy S and energy E very close to the horizon of a black hole before dropping it in. Classically, if one could lower the box arbitrarily close to the horizon before dropping it in, one would recover all of the energy originally in the box as "work" at infinity. No energy would be delivered to the black hole, so by the first law of black hole mechanics, Eq. (7), the black hole area, A, would not increase. However, one would still get rid of all of the entropy, S, originally in the box, in violation of the GSL.

Indeed, this process makes manifest the fact that in classical general relativity, the physical temperature of a black hole is absolute zero: The above process is, in effect, a Carnot cycle which converts "heat" into "work" with 100% efficiency [45]. The difficulty with the GSL in the above process can be viewed as stemming from an inconsistency of this fact with the mathematical assignment of a finite (non-zero) temperature to the black hole required by the first law of black hole mechanics if one assigns a finite (non-infinite) entropy to the black hole.

Bekenstein proposed a resolution of the above difficulty with the GSL in a quasi-static lowering process by arguing [5, 6] that it would not be possible to lower a box containing physically reasonable matter close enough to the horizon of the black hole to violate the GSL. As will be discussed further in the next sub-section, this proposed resolution was later refined by postulating a universal bound on the entropy of systems with a given energy and size [46]. However, an alternate resolution was proposed in [47], based upon the idea that, when quantum effects are taken into account, the physical temperature of a black hole is no longer absolute zero, but rather is the Hawking temperature, $\kappa/2\pi$. Since the Hawking temperature goes to zero in the limit of a large black hole, it might appear that quantum effects could not be of much relevance in this case. However, despite the fact that Hawking radiation at infinity is indeed negligible for large black holes, the effects of the quantum "thermal atmosphere" surrounding the black hole are not negligible on bodies that are quasi-statically lowered toward the black hole. The temperature gradient in the thermal atmosphere (see Eq. (12)) implies that there is a pressure gradient and, consequently, a buoyancy force on the box. This buoyancy force becomes infinitely large in the limit as the box is lowered to the horizon. As a result of this buoyancy force, the optimal place to drop the box into the black hole is no longer the horizon but rather the "floating point" of the box, where its weight is equal to the weight of the displaced thermal atmosphere. The minimum area increase given to the black hole in the process is no longer zero, but rather turns out to be an amount just sufficient to prevent any violation of the GSL from occurring in this process [47].

The analysis of [47] considered only a particular class of gedankenexperiments for violating the GSL involving the quasi-static lowering of a box near a black hole. Of course, since one does not have a general proof of the ordinary second law of thermodynamics – and, indeed, for finite systems, there should always be a nonvanishing probability of violating the ordinary second law – it would not be reasonable to expect to obtain a completely general proof of the GSL. However, general arguments within the semiclassical approximation for the validity of the GSL for arbitrary infinitesimal quasi-static processes have been given in [48, 49, 15]. These arguments crucially rely on the presence of the thermal atmosphere surrounding the black hole. Related arguments for the validity of the GSL have been given in [50, 51]. In [50], it is assumed that the incoming state is a product state of radiation originating from infinity (i.e., IN modes) and radiation that would appear to emanate from the white hole region of the analytically continued spacetime (i.e., UP modes), and it is argued that the generalized entropy must increase under unitary evolution. In [51], it is argued on quite general grounds that the (generalized) entropy of the state of the region exterior to the black hole must increase under the assumption that it undergoes autonomous evolution.

Indeed, it should be noted that if one could violate the GSL for an infinitesimal quasi-static process in a regime where the black hole can be treated semiclassically, then it also should be possible to violate the ordinary second law for a corresponding process involving a self-gravitating body. Namely, suppose that the GSL could be violated for an infinitesimal quasi-static process involving, say, a Schwarzschild black hole of mass M (with M much larger than the Planck mass). This process might involve lowering matter towards the black hole and possibly dropping the matter into it. However, an observer doing this lowering or dropping can "probe" only the region outside of the black hole, so there will be some $r_0 > 2M$ such that the detailed structure of the black hole will directly enter the analysis of the process only for $r > r_0$. Now replace the black hole by a shell of matter of mass M and radius r_0, and surround this shell with a "*real*" atmosphere of radiation in thermal equilibrium at the Hawking temperature (10) as measured by an observer at infinity. Then the ordinary second law should be violated when one performs the same process to the shell surrounded by the ("real") thermal atmosphere as one performs to violate the GSL when t

he black hole is present. Indeed, the arguments of [48, 49, 15] do not distinguish between infinitesimal quasi-static processes involving a black hole as compared with a shell surrounded by a ("real") thermal atmosphere at the Hawking temperature.

In summary, there appear to be strong grounds for believing in the validity of the GSL.

4.2 Entropy bounds

As discussed in the previous subsection, for a classical black hole the GSL would be violated if one could lower a box containing matter sufficiently close to the black hole before dropping it in. Indeed, for a Schwarzschild black hole, a simple calculation reveals that if the size of the box can be neglected, then the GSL would be violated if one lowered a box containing energy E and entropy S to within a proper distance D of the bifurcation surface of the event horizon before dropping it in, where

$$D < \frac{S}{(2\pi E)}. \tag{15}$$

(This formula holds independently of the mass, M, of the black hole.) However, it is far from clear that the finite size of the box can be neglected if one lowers a box containing physically reasonable matter this close to the black hole. If it cannot be neglected, then this proposed counterexample to the GSL would be invalidated.

As already mentioned in the previous subsection, these considerations led Bekenstein [46] to propose a universal bound on the entropy-to-energy ratio of bounded matter, given by

$$S/E \leq 2\pi R, \tag{16}$$

where R denotes the "circumscribing radius" of the body. Here "E" is normally interpreted as the energy above the ground state; otherwise, Eq. (16) would be trivially violated in cases where the Casimir energy is negative [52] – although in such cases in may still be possible to rescue Eq. (16) by postulating a suitable minimum energy of the box walls [53].