OPTIMIZATION IN PUBLIC TRANSPORTATION

Stop Location, Delay Management and Tariff Zone Design in a Public Transportation Network
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Aims and Scope
Optimization has been expanding in all directions at an astonishing rate during the last few decades. New algorithmic and theoretical techniques have been developed, the diffusion into other disciplines has proceeded at a rapid pace, and our knowledge of all aspects of the field has grown even more profound. At the same time, one of the most striking trends in optimization is the constantly increasing emphasis on the interdisciplinary nature of the field. Optimization has been a basic tool in all areas of applied mathematics, engineering, medicine, economics and other sciences.

The series Springer Optimization and Its Applications publishes undergraduate and graduate textbooks, monographs and state-of-the-art expository works that focus on algorithms for solving optimization problems and also study applications involving such problems. Some of the topics covered include nonlinear optimization (convex and nonconvex), network flow problems, stochastic optimization, optimal control, discrete optimization, multi-objective programming, description of software packages, approximation techniques and heuristic approaches.
OPTIMIZATION IN PUBLIC TRANSPORTATION

Stop Location, Delay Management and Tariff Zone Design in a Public Transportation Network

By

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Springer
To my parents
Helga and Volker Schumacher
Public transportation plays an important role in most populated areas. Especially in metropolitan regions public transportation systems are widely used. But unfortunately, public transportation is often a subject of complaints. Customers are annoyed about “unfair prices”, about “bad service” and in particular get upset in case of delays. Such complaints are understandable, but for the public transportation companies it is often impossible to provide a better service without increasing the costs. The reason for these difficulties is the complexity and the size of the planning problems arising.

The theory of optimization provides a sound methodology for finding good solutions, if a mathematical model of the respective problem is known. Moreover, due to the availability of fast computers many problems that seemed to be intractable some years ago can nowadays be solved.

This work provides suitable models for planning public transportation systems from a customer-oriented point of view, but taking into account the limited budget public transportation companies have to respect. In particular, we develop and analyze optimization models for the following three problems:

Part I: Stop location. Here we deal with the location of stops along bus routes, or of stations along railway tracks. As objective functions we consider the number of customers living close to a station and the additional travel time arising by the stopping activities of the trains or buses. In particular, we discuss how to find the minimal number of stops to cover
a given set of demand points or demand regions, how to cover as many customers as possible with a given budget and both problems together in a bicriteria setting.

Part II: Delay management. If a vehicle arrives at a station with a delay, passengers who wish to change into another vehicle, say a bus, may miss their connection, if this bus departs on time. Such wait-depart decisions and their impact on the whole transportation system are investigated from the customers’ perspective. As objective functions we hence discuss the sum of all delays over all passengers, the number of missed connections, and the sum of all delays over all vehicles. The latter two objectives are treated as a bicriteria optimization problem.

Part III: Zone planning. In order to design a zone tariff system, the complete transportation area has to be partitioned into zones, and prices for traveling through 1,2,3,... zones have to be defined in such a way that the current income of the public transportation company does not decrease too much. As objective function we consider the deviations between the new prices and some given reference prices. These deviations can be interpreted as the fairness of the new tariff system or as the changes to the current ticket prices.

All three problems were brought to my attention within real-world projects, and some of the obtained results have already been implemented and applied in practice. Nevertheless, the main focus of this work is to develop a consistent mathematical theory and to present basic results within all three fields.

- The stop location problem is treated using the concept of gauges and ideas of continuous location theory. A finite dominating set of possible new stops can be derived. This allows us to formulate the stop location problem as a set covering problem. By using the special structure of the covering matrix which is due to the geometrical properties of the stop location problem, efficient solution methods for this type of set covering problems are developed.

- For the delay management problem three different, but equivalent mixed integer programming formulations are presented. By combining these models many structural results for the delay management problem are obtained. In particular, it is possible to identify cases in which the problem is solvable efficiently. Furthermore, methods of project planning are applied to determine Pareto solutions.

- Finally, the design of zone tariff systems in public transportation is modeled by methods of graph theory. The obtained theoretical results together with ideas of clustering theory are utilized for deriving solution approaches.

The theory presented in this text and the obtained results open a wide field for further developments and implementations of the suggested approaches. The algorithms that have already been tested on our real-world data confirm the practical usefulness of the models and show their potential for future applications.
Before concluding the preface I wish to add several acknowledgments. First of all, I thank Horst W. Hamacher for his support, for the pleasant and constructive work together with him, and for his helpful advice in any question I had.

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Contents

1 Customer-oriented Traffic Planning ......................... 1
  1.1 Customer-oriented Transportation .......................... 1
  1.2 Public Transportation Network and Customer Data ........ 5

Part I Stop Location

2 Introduction ..................................................... 11
  2.1 Application ................................................ 13
  2.2 Literature Review ......................................... 14
  2.3 A Model for Continuous Stop Location .................... 15

3 Covering All Demand Points .............................. 21
  3.1 Feasibility and Complexity of Complete Cover ............ 22
  3.2 A Finite Dominating Set .................................. 24
  3.3 Complete Cover Along a Polygonal Line .................. 29
  3.4 Set Covering With Consecutive Ones Property .......... 32
  3.5 Complete Cover in a Realistic Network .................. 40
  3.6 Set Covering With Almost Consecutive Ones Property ... 46

4 Bicriteria Stop Location ................................... 59
  4.1 Constraint Problems and Lexicographic Minimality ....... 60
  4.2 Integer Programming Formulations ....................... 62
  4.3 Bicriteria Set Covering With Consecutive Ones Property . 65
  4.4 Varying the Radius ...................................... 71

5 Extensions ....................................................... 75
  5.1 Covering Demand Regions ................................. 76
  5.2 Minimizing the Total Door-to-door Travel Time .......... 85
Part II Delay Management

6 Introduction .......................................................... 95
   6.1 Application ..................................................... 97
   6.2 Related Literature ........................................... 98
   6.3 A Model for the Delay Management Problem ............... 100
   6.4 Event-activity Networks in Delay Management ............ 104

7 Delay Management With Fixed Connections .................... 109
   7.1 Linear Programming Approach ............................... 110
   7.2 Relation to the Critical Path Method ..................... 111
   7.3 Relation to the Feasible Differential Problem ............. 115

8 Minimizing the Sum of All Delays ............................. 119
   8.1 A Linear Model ............................................. 121
   8.2 Activity-based Model ..................................... 125
   8.3 Constant Weights and the Never-meet Property ........... 133
   8.4 A Simple Special Case .................................. 145
   8.5 Solving the model with constant weights ................. 147
   8.6 Solving the Total Delay Management Problem ............. 163

9 The Bicriteria Delay Management Problem .................... 175
   9.1 A First Analysis .......................................... 176
   9.2 Integer Programming Formulation ........................ 179
   9.3 Lexicographic and Supported Efficient Solutions .......... 180
   9.4 Finding All Efficient Solutions .......................... 182

10 Extensions .......................................................... 195
   10.1 The General Delay Management Problem .................. 195
   10.2 Railway and Bus Specific Requirements .................. 201

Part III Tariff Planning

11 Introduction .......................................................... 207
   11.1 Frequently Used Tariff Systems .......................... 208
   11.2 Application ................................................... 212
   11.3 Literature Review ......................................... 213
   11.4 A Model for the Zone Design Problem .................... 213

12 Finding Zones and Zone Prices ................................. 219
   12.1 The Fare Problem .......................................... 220
   12.2 The Maximum Deviation Zone Design Problem ............ 224
   12.3 Extensions for Real-world Problems ...................... 232
Contents XIII

A  Integer Programming ........................................ 237
B  Bicriteria Optimization ................................... 239
C  Gauges as Distance Measures ............................... 243

Frequently Used Notation ........................................ 247
List of the Main Problems ........................................ 251
References ......................................................... 253
Index ............................................................. 265
Customer-oriented Traffic Planning

1.1 Customer-oriented Transportation

Although public passenger transportation plays an important role especially in large metropolitan areas, it also has to be carefully planned and organized in a rural environment. There are economical, environmental, and social reasons for considering the needs of customers when planning public transportation.

- First of all, if a public transportation company attracts more customers then it will sell more tickets and hence its income will (usually) increase.
- An environmental aim is to decrease the amount of individual traffic (mainly in large cities) and thus reduce its negative effects such as pollution, noise, and congestion. This is sometimes accomplished by imposing restrictions or fines through high parking fees, tolls, closure of roads, or car-free days. A way of avoiding this would be to offer such a good alternative that (at least some) people voluntarily decide to use public transportation instead of their cars.
- In areas with few inhabitants, congestion usually is no problem. The challenge here is to offer an affordable transportation mode for people who do not have the opportunity to travel by car, e.g., children, elderly people, or citizens without a driving license.

We now briefly introduce in an informal way some of the problems occurring in public transportation. Three of them – locating stops, delay management, and tariff planning – will be discussed in detail in subsequent chapters. An overview of the problems considered in this text and their relation to other customer-related steps in the planning phase and at the operational level is given in Figure 1.1. Note that sometimes the same model can be used for online decisions and for long-term decisions at the same time. This is for example the case for the model that we will present for the delay management problem.
Network planning

Network planning includes the design of the transportation network, i.e., sit-
ing the stations and the bus routes or train tracks. The outcome of the pro-
cess of network planning is the public transportation network (PTN). Net-
work planning problems have been treated in the general context of net-
work flow problems. In the public transportation literature we refer, e.g., to 
[CW86, BM95, CG02] and to the references given therein. However, in real 
life a PTN is usually not designed from scratch, but only modifications of an 
existing PTN are considered, such as

- finding new stations in a railway or bus network,
- closing existing stations, or
- finding a subnetwork for opening rapid transit lines.

For these problems literature is rather sparse. Locating stops or stations in 
the PTN will be discussed in Part I of this text. For finding subnetworks for 
operating an underground system or a rapid transit line, hub location models 
have been developed by [NSS01].
1.1 Customer-oriented Transportation

**Line planning**

Line planning concerns the definition of paths in the PTN on which service should be offered, i.e., the routes of the bus or railway lines. The line planning problem has been well studied in the literature. For an early contribution we refer to [Die78]. In [BKZ96, Bus97] the goal is to maximize the number of passengers with direct connections under the constraint that all passengers can be transported. The solution methods proposed use advanced integer programming techniques. Under a similar constraint, the goal in [CvDZ96] is minimization of costs for the public transportation company. Line planning problems considering different types of vehicles simultaneously were studied in [GvHK04, GvHK02]. Various models and algorithms are discussed in [Goo04].

A new approach is to take into account that the behavior of the customers depends on the design of the lines. A first model including such demand changes was treated with simulated annealing in cooperation with *Deutsche Bahn*, see [Kli00b, Sch01a]. Moreover, the choice of the routes of customers depends on the (unknown) line plan. Finding a line plan together with optimal routes for the customers has recently been considered in [SS05, Sch05b, BGP04a, BGP04b, Sch05a, LMMO06]. In these approaches, the goal is to design lines in such a way that the traveling time of the customers is minimized. The first two of these publications also include the number of transfers of customers in the objective function. The special case of locating one single line so as to maximize the number of passengers is treated in [LMO05].

**Timetabling**

Timetabling determines the departure and arrival times for all trips at all stations. Here two cases are distinguished.

Case 1: All rides within the same line start in *periodic* time intervals, e.g., at 7:03, 7:33, 8:03, 8:33, 9:03 and so on.

Case 2: The timetable of the rides is *non-periodic*.

Many papers and theses deal with problems related to timetabling. An overview of the literature in this area will be given in Section 6.2 (see page 98) within the context of delay management.

**Tariff planning**

Tariff planning concerns the determination of fares for the customers. Different systems are possible.

- **distance tariff**: The price depends on the length of the journey.
- **unit tariff**: Each journey costs the same.
zone tariff: The complete area is partitioned into zones, and the prices depend on the number of passed zones, from the origin to the destination of the journey.

In tariff planning the problem is to design a new tariff system along with its prices. A common requirement is that the new income of the public transportation company should not decrease compared to its current income. On the other hand, the customers should find the new system acceptable. Designing zone tariffs under such criteria will be treated in Part III.

Summarizing, line planning, timetabling and network design problems have been well studied in the literature so far. There are other problems belonging to the strategic planning process in public passenger transportation, like

- rolling stock circulation,
- vehicle scheduling,
- shunting,
- crew management,
- crew rostering,
- maintenance issues.

Since these problems have no direct effects on the customers they will not be considered in this text. Various models and solution approaches for these problems exist. For references the reader is referred, e.g., to the proceedings of the CASPT meetings which are mentioned below.

We finally list two operational problems, which have to be solved on-line in case of disturbances.

*Delay management:*

Suppose that a vehicle arrives at a station with a delay. Should a connecting vehicle wait for passengers who wish to change or should it depart on time? The goal is to minimize the inconvenience caused by delays from the customers’ point of view. The delay management problem will be treated in Part II.

*Re-scheduling of vehicles:*

Especially in rail transportation, construction sites, delays, or any other disturbance make a re-scheduling of trains necessary. This is a difficult problem since many constraints have to be taken into account. The main requirement in many railway companies is that no two trains are allowed to occupy the same segment of a track (called block) at the same time. The goal may be to return to the original schedule as quickly as possible, or to minimize the additional delays of the trains. The problem has mainly been considered in transportation and engineering sciences, and is practically often solved based on priority rules. Operations research models
can be found in [AFT02, BHK99, ADGGT99, Kro97, Zwa96, AD96], while there are also many other successful approaches from various areas, including [Tör05b, TJ05, WS05, PMP04, Jac04, vE01, Fay00, HKF96, PT82]. A recent overview with many references is given in [Tör05a].

Other operational problems include the re-scheduling of crew in case of unexpected absent drivers, re-planning of rosters, or maintenance re-scheduling.

For more details about the mathematical models used in the planning process in public transportation we refer to the basic rail transportation models of [Ass80] and to the survey of Bussieck, Winter and Zimmermann [BWZ97]. Another overview is given by Borndörfer, Grötschel and Löbel [BGL98]. Patriksson and Labbé [PL02] collected articles about the state-of-the-art in the field of transportation planning. The survey of Cordeau, Toth and Vigo [CTV98] focuses in particular on routing and scheduling in rail transportation. Railway planning problems are also addressed in the surveys of [Wag03, GJP04] and in the forthcoming collection [GKS06]. Moreover, we refer to the conference proceedings of the CASPT (Computer-Aided Scheduling of Public Transport) meetings [Wre81, Rou85, DW88, DR92, DBP95, Wil99, VD01], to the TRISTAN (Triennial Symposium on Transportation Analysis) [BT96] conferences, and to the proceedings of the ATMOS workshops [Zar01, Wag02, Ger04, GKS06].

1.2 Public Transportation Network and Customer Data

We start with a formal definition of a public transportation network, a simple example of which is depicted in Figure 1.2.

Definition 1.1. A public transportation network is a finite, undirected graph PTN = (V, E) with

- a node set V representing stops or stations, and
- an edge set E, where each edge e = \{u, v\} indicates that there exists a direct ride from station u to station v (i.e., a ride that does not pass any other station in between).

In public transportation, an ordered pair of stops (or stations) u, v is often called a relation.

Within the network design step, the PTN is constructed, or modifications of an already existing PTN such as adding new stops or closing existing ones are planned. However, for all other purposes, like line planning, timetabling, delay management, or tariff planning, we assume the PTN as given and fixed. It may happen that the set E of direct rides in the PTN is not given, but a
timetable is at hand. Possibilities to construct $E$ in such a case are discussed in [Lie01].

Since we mainly deal with optimization problems from the customers’ point of view, we now discuss the data about the customers needed for our models.

**OD-matrix**: The origin-destination matrix (OD-matrix) $W = (W_{uv})$ is a $|V| \times |V|$ matrix containing the number of customers who wish to travel from station $u$ to station $v$ for all relations $(u, v)$ in the PTN. Instead of the number of customers, the number of sold tickets can be given. The latter is in particular needed for tariff planning.

**Traffic load**: The traffic load is defined by the number of customers traveling along an edge $e \in E$ or through a node $v \in V$ in the PTN, and is denoted by $c_e$ or $c_v$, respectively. The traffic load can be given as number of customers per hour, per day, per week, or per year. The traffic load will be used for the stop location problem (Part I), and as an approximation in the delay management problem (in Part II).

We now summarize some notation that will be used throughout the text.

**Notation 1.2.** Let $\text{PTN} = (V, E)$ and let $I$ be a fixed time interval.

- For all $u, v \in V$ let $W_{uv}$ denote the demand of relation $(u, v)$, i.e., the number of passengers who wish to travel from station $u$ to station $v$ within the time interval $I$. The matrix $W = (w_{uv})_{u,v \in V}$ is called the origin-destination matrix or, shorter, the OD-matrix.
- For all $e \in E$ let $c_e$ denote the traffic load of an edge $e$, i.e., the number of customers using edge $e$ within the time interval $I$. 
Moreover, for \( v \in V \) let \( c_v \) denote the traffic load of station \( v \), i.e., the number of customers traveling through station \( v \) within the time interval \( I \).

Note that \( I \) needs to be chosen appropriately for the respective application one has in mind. For example, in tariff planning, \( I \) usually refers to a long period such as a whole year, hence \( W_{uv} \) can be used to calculate the annual income on the relation from \( u \) to \( v \). For line planning, however, the traffic load is important to make sure that all passengers can be transported. Here \( I \) is usually a short interval, like the morning traffic period (e.g., from 6 to 8 a.m.), and \( c_e \) is used to calculate the number of vehicles needed along edge \( e \) within this period. Apart from these widely used data we sometimes need the following more detailed information about the customers.

Demand within a point or region (needed in Part I): When dealing with the location of stops close to customers, we assume that a set of demand points or demand regions is given. The number of (potential) customers within a demand point \( d \) is denoted by \( w_d \). Alternatively, the number of (potential) customers within a demand region \( D \) is called \( w_D \).

Paths of the customers (needed in Part II): For calculating the delay of a customer, it is not enough to know where his journey has started and to which station he wishes to travel. Also of interest are the starting time and the stations in which a transfer to other vehicles occurs, i.e., the path followed within the transportation network, as well as the vehicles used. For a path \( p \) let \( w_p \) denote the number of customers using this path. Since the information about such paths is often not available we will also present models which do not rely on this specific information.

Number of changing passengers (needed in Part II): For calculating how many passengers miss a connection \( a \) from some vehicle \( g \) to another vehicle \( h \) at a station \( v \) we need the number of transfer passengers \( w_a \) who plan to use connection \( a \) to change between the respective trains.

Destinations of the customers (needed in Part II): In delay management, it is also convenient to use the number of customers \( C^g_v \) who reach their final destination \( v \) traveling in some vehicle \( g \).

Note that the latter two data sets can be easily obtained if the paths of the customers are known. If only the OD-matrix \( W \) is known, it is possible to approximate the traffic loads by finding a set of reasonable paths from \( u \) to \( v \) for each relation \( u,v \) and dividing \( W_{uv} \) among these paths. Doing this for all relations and then adding for each edge \( e \in E \) the weights of all paths containing \( e \) gives an approximation of the traffic load of \( e \). Formally, this is stated next.
Algorithm 1: Approximating traffic loads

Input: PTN and OD-matrix $W = (W_{uv})_{u,v \in V}$.
Output: Traffic load $c_e$ for each edge $e \in E$.

Step 1. For each pair $u,v \in V$ with $W_{uv} > 0$ determine a set of ‚reasonable’ paths from $u$ to $v$
$P_{1}^{uv}, P_{2}^{uv}, \ldots, P_{k}^{uv}$,
and assign weights $w_{P_{1}^{uv}}, w_{P_{2}^{uv}}, \ldots, w_{P_{k}^{uv}}$ to these paths in such a way that
\[\sum_{i=1}^{k} w_{P_{i}^{uv}} = W_{uv}.\]

Step 2. For all $e \in E$ set
\[c_{e} = \sum_{u,v \in V} \sum_{i=1, \ldots, k} \sum_{e \in P_{i}^{uv}} w_{P_{i}^{uv}}.\]

Note that the difficulty of the algorithm above is to express the customers’ behavior by an appropriate set of weighted paths, i.e., the skills are more of a practical nature rather than of mathematical hardness. For simplicity, $k$ is set to 1 in many applications, and the only path $P_{1}^{uv}$ for the relation from $u$ to $v$ is chosen as a shortest path. In this case, Algorithm 1 simplifies to computing the following expression for all $e \in E$:
\[c_{e} = \sum_{u,v \in V: e \in P_{1}^{uv}} W_{uv}.\]
Part I

Stop Location
Establishing stops (or stations) within a transportation network is fundamental for offering public transportation service, since stops are an important part of the PTN. But it is not clear in advance, how many stops are reasonable, and where they should be built. Let us consider the effects of stops on the customers:

- On the one hand, many stops are advantageous from the customers’ point of view, since they increase the accessibility of the trains or buses. Establishing a new stop may hence attract new customers and increase the demand. In bus transportation, the covering radius is often assumed to be 400 m, meaning that a customer will think about using a bus, only if the next bus stop is within a distance of at most 400 m. In rail transportation, the covering radius is larger, and is usually assumed to be 2 km.
- On the other hand, each additional stop increases the transportation time (e.g., by two minutes in rail transportation) for all trains or buses stopping there. This makes the transportation service unattractive to customers.

Moreover, this additional running time of trains (or buses) is costly for the transportation company, and also fixed costs arise for establishing a new stop.

In the continuous stop location problem we deal with the location of new stops along a given track system. This means, we assume that the tracks for the trains are already built, or the routes for the buses are already fixed. For the sake of simplicity we will use the wordings “stops” and “tracks” in the following, but keep in mind that the models and algorithms presented can also be applied for bus transportation.

We further assume a (possibly empty) set of already existing stops or stations. As input data we also need the locations of the potential customers, given as points or as regions in the plane, and the traffic load along the edges of the given tracks. An example for a set of demand points is depicted in Figure 2.1. Our goal is to locate additional stops along the tracks such that
as many (potential) customers as possible live closer than a given radius \( r \) to their nearest stop, and such that
- the increase of travel time caused by the new stops is as small as possible.

\[ \text{demand point} \]
\[ \text{given tracks} \]

\textbf{Fig. 2.1.} The set of tracks \( T \) and a set of demand points \( D \) in the plane.

The result we obtain by solving the continuous stop location problem defines the PTN which is the basis for many subsequent optimization models in public transportation planning. Establishing no stop at all means that the additional travel time is minimal, but for none of the customers does the accessibility increase. The other extreme is to open stops until the complete demand is covered. The following optimization problems will be treated in this chapter.

- In the \textit{complete cover stop location problem (CSL)} we want to cover all potential customers with as few stops as possible, or with as few costs as possible. The problem will be treated in Chapter 3 for the case that the demand is given at points and in Section 5.1 for the case of demand regions.
- The \textit{bicriteria stop location problem (BSL)} focuses on minimizing the additional travel time and on maximizing the covered demand simultaneously. This provides solutions between the two extremes of covering the complete demand and of establishing no (additional) stop at all. (BSL) is discussed in Chapter 4.
- In the \textit{door-to-door travel time stop location problem (DSL)} we investigate the door-to-door travel time over all customers. The door-to-door travel time for a customer is given by the time he needs to get to the first station of his trip plus the time of the trip itself plus the time he needs to reach his final destination after leaving the public transportation system. (DSL) will be considered in Section 5.2.
Chapter 2 is structured as follows: We start by presenting the application which motivated us to deal with continuous stop location problems. A literature review on stop location is given next. Then we present a model for the continuous stop location problem, enabling us to evaluate the interesting objective functions.

2.1 Application

When comparing railway systems all over Europe, it turns out that Switzerland has a higher amount of rail transportation than other countries. Among others, one reason could be that in Switzerland the number of stops compared to the overall length of the track system is significantly higher than in other countries. The interesting question arising by this observation is, if it is an advantage or a disadvantage to have many stops. To come to an answer, we consider a customer-oriented point of view. A quality criterion for the customers which is influenced by the number of stops is the door-to-door travel time of their journeys, including the time they need to get from home to their departure stations and the time they need to reach their final destinations. A priori it is not clear if this time will increase or decrease by opening new stops along the track system.

Note that by a stop we do not mean a fully equipped station, but just a stopping point for the trains, which is relatively cheap for the railway company. Our results and some of our algorithmic approaches have been implemented and tested using data of the largest German railway company, Deutsche Bahn. Here we located new stops along the track system, relevant for regional trains, i.e., all regional trains are supposed to stop while the fast long-distance trains pass through. Our real-world data is described next.

- We use 30,637 demand regions, given as polygons with an average of 45 nodes per polygon. These polygons are not identical with the borders of the communities and also do not form a partition of Germany. They represent the population distribution better than community borders since green land is excluded. This means that most of the data is very accurate; even relatively small towns are given as a set of more than 10 different demand regions.
- The PTN we used represents the network of Deutsche Bahn. It has a size of 6,828 stations and 8,724 edges.
- For each demand region we furthermore know the number of inhabitants, and for each edge we got an approximation of the traffic load, i.e., the number of customers using the edge.

Moreover, Deutsche Bahn specified some of the necessary parameters for our models. The time needed for an additional stopping activity of a regional train was estimated as two minutes. For the covering radius, a distance of 2 km is often used in rail transportation.
2.2 Literature Review

The importance of planning stops carefully and different customer-oriented criteria for bus stop location were already discussed in the case study of Demetsky et al, see [DAL82]. Among the many possible objective functions one goal is to establish as few stops as possible in such a way that all customers are covered. This was done in [Gle75] and in [MDSF98, Mur01a, Mur01b]. In the latter papers, the public transportation network in Brisbane, Australia was analyzed in detail and it turned out that 84.5 % of the stops are not necessary in terms of covering a set of given demand points within a Euclidean distance of 400 m, i.e., closing them would not decrease the actual number of covered customers. The stop location problem was treated in a discrete setting in these papers, i.e., the authors either considered only the actual stops, or they assumed that a finite candidate set of new stops is given. This leads to an unweighted set covering problem, also called location set covering problem which was introduced in [TSRB71, TR73]. In the context of stop location this problem has been solved by [Mur01a] using the Lagrangian-based set covering heuristic of [CFT99]. A new discrete stop location model was developed by La- porte et al. [LMO02]. They investigate which candidate stops along one given line in Seville should be opened, taking into account demand regions and constraints on the inter-station space. The coverage of a new stop is determined using a gravitation model. Finally, they solved the problem by a longest path algorithm in an acyclic graph. Their model resembles the maximum coverage location problem originally presented in [CR74, WC74].

The difference between the continuous stop location problem considered here and most papers published so far is that in the continuous stop location problem we do not choose the stops from a known set of possible candidates, but allow establishing a new stop anywhere along the given railway tracks (or along the given bus routes). The covering information can hence not be given explicitly but must be calculated by some (geometric) formula. The first approaches dealing with a continuous candidate set were given in [HLS01] and [SHLW02]. They are described in more detail in Section 5.2 and in Chapter 3. The results of [RS04, Sch05c, SS03] are based on these two papers and can be found in Section 3.6, Chapter 9, and Section 5.1. The research of [KPS03] was also motivated by this research. They deal with a variant of the continuous stop location problem, aiming to cover as much demand as possible with a given number of new stops, see Section 4.1. In [MMW04] the stop location problem has been investigated and solved for the case of two intersecting lines. Solving the stop location problem by data reduction of the underlying covering problem has been studied in [Mec03] and in [MW04].
2.3 A Model for Continuous Stop Location

Let $G = (V, E)$ be a finite, planar graph with straight-line embedding in the plane. In real-world data sets, the nodes of $G$ represent either existing stations or important breakpoints. We identify each edge $e \in E$ by a line segment in the plane. Moreover,

- $c_e$ is the traffic load along edge $e \in E$, i.e., the number of customers using edge $e$, and
- $c_v$ is the traffic load through station $v \in V$, i.e., the number of customers passing through station $v$ (and not getting on or off there).

Both parameters can be given, for example, in customers per day.

**Definition 2.1.** Given $G = (V, E)$ define the track system

$$T = \bigcup_{e \in E} e = \{x \in \mathbb{R}^2 : x \in e \text{ for some } e \in E\} \subseteq \mathbb{R}^2$$

as the set of points on edges of the planar embedding of $G$.

Our goal is to establish stops (or stations), which are represented by points in $T$. The evaluation of a set $S \subseteq T$ is described next.

**Additional Travel Time**

To calculate the additional travel time induced by some set of stations $S \subseteq T$ we take the number of customers affected by the additional stopping activities and multiply them by the time $t_{\text{stop}}$ which is needed for an additional stop. According to Deutsche Bahn, $t_{\text{stop}}$ can be assumed to be two minutes, independent of the location of the stop. This is specified in the following notation:

**Definition 2.2.** Given $s \in T$ let

$$g(s) = \begin{cases} s & \text{if } s \in V \\ e & \text{if } s \in e, s \notin V. \end{cases}$$

Furthermore, given a finite set $S \subseteq T$ we define

$$f_{\text{time}}(S) = \sum_{s \in S} t_{\text{stop}} c_{g(s)}.$$  

For an infinite set $S$ we define $f_{\text{time}}(S) = \infty$.

Since $t_{\text{stop}}$ is a constant, e.g., two minutes in rail transportation, it can be neglected for the optimization process. Furthermore, note that $f_{\text{time}}(S) = |S|$ if all traffic loads $c_{g(s)} = 1$, i.e., if we assume that each edge is used by exactly one customer. Hence, we will refer to the unweighted problem if we deal with the special case of minimizing the number of stations.
The Cover of a Set of Stops

To deal with the accessibility of potential customers, we next assume that $\mathcal{D} \subseteq \mathbb{R}^2$ is a finite set of either

- demand points, or of
- pairwise disjoint demand regions

representing important points or regions such as settlements, industrial areas, shopping centers, or leisure parks.

**Notation 2.3.** For $\mathcal{D}$ let

$$D_{total} = \bigcup_{D \in \mathcal{D}} D$$

be the demand set. Note that $D_{total} = \mathcal{D}$ if $\mathcal{D}$ consists of demand points.

We now introduce the notion of covering with respect to a distance measure $\gamma$. We may specify different distance measures for each of the elements of $\mathcal{D}$, i.e., for each of the demand points or regions. As distance measure $\gamma_D$ we allow any norm or gauge (see Appendix C); readers who are not familiar with gauges may simply imagine $\gamma_D$ as the Euclidean distance. For $d \in \mathbb{R}^2$, $S \subseteq \mathbb{R}^2$, let (as usual)

$$\gamma_d(d, S) = \min_{s \in S} \gamma_d(d, s).$$

**Notation 2.4.** Let $d \in D_{total}$. Then $\gamma_d$ denotes the distance measure associated with $d$.

If $\mathcal{D}$ consists of demand regions, and $D \in \mathcal{D}$, then we require for all points $d_1, d_2 \in D$

$$\gamma_{d_1} = \gamma_{d_2} = \gamma_D.$$

A demand point is covered, if the distance to its closest station is smaller than or equal to a given radius $r$, where the used distance need not be the same for all demand points. Formally, this is specified below.

**Definition 2.5.** Given $r > 0$, and $S \subseteq \mathcal{T}$.

1. A point $d \in D_{total}$ is covered by $S$ if $\gamma_d(d, S) \leq r$.
2. Furthermore, the cover is $S$ is $\text{cover}_D(S) = \{d \in D_{total} : d \text{ is covered by } S\}$.

If it is clear to which set $\mathcal{D}$ we refer, we just write cover$(S)$. Furthermore, for $s \in S$ we use cover$(s)$ for cover$(\{s\})$. Note that if $\gamma_d = \gamma$ for all $d \in D_{total}$ we obtain

$$\text{cover}_D(S) = \{d \in \mathbb{R}^2 : \gamma(d, S) \leq r\} \cap D_{total}.$$

The cover of a point is illustrated in Figure 2.2. The small rectangles in parts (a) and (b) represent the demand points $d_1, \ldots, d_6$, while we consider two demand regions $D_1$ and $D_2$ in parts (c) and (d). All elements of $\mathcal{D}$ in parts (a) and (c) are assumed to have the Euclidean distance associated with them. In
part (b), $\gamma_{d_1}, \gamma_{d_2},$ and $\gamma_{d_3}$ equal the rectangular distance, while the remaining elements $d \in D_{total}$ again have $\gamma_d$ as Euclidean distance. In part (d), we assume $\gamma_{D_1}$ as rectangular distance and $\gamma_{D_2}$ as Euclidean. In parts (a) and (b) the cover consists of the filled small rectangles, in parts (c) and (d) the cover is given by the dashed area.

\begin{itemize}
\item (a)
\item (b)
\item (c)
\item (d)
\end{itemize}

**Fig. 2.2.** The cover for demand points (see (a) and (b)) and for demand regions (in (c) and (d)), both for the Euclidean distance (see (a) and (c)) and for mixed rectangular and Euclidean distances (in (b) and (d)).

We further need the following notation. Consider $d \in D_{total}$ with associated distance function $\gamma_d$. Let $B_d = \{x \in \mathbb{R}^2 : \gamma_d(x) \leq 1\}$ be the unit ball associated with $\gamma_d$, see Appendix C. Using the denotation

$$B^r_d = d + rB_d,$$

we get

$$\gamma_d(d, x) \leq r \text{ if and only if } x \in B^r_d.$$

Hence, we obtain:

**Lemma 2.6.** Let $d \in D_{total}$ and $S \subseteq T$. Then $d$ is covered by $S$ if and only if $S \cap B^r_d \neq \emptyset$.

We refer to Figure 2.3 for an illustration.