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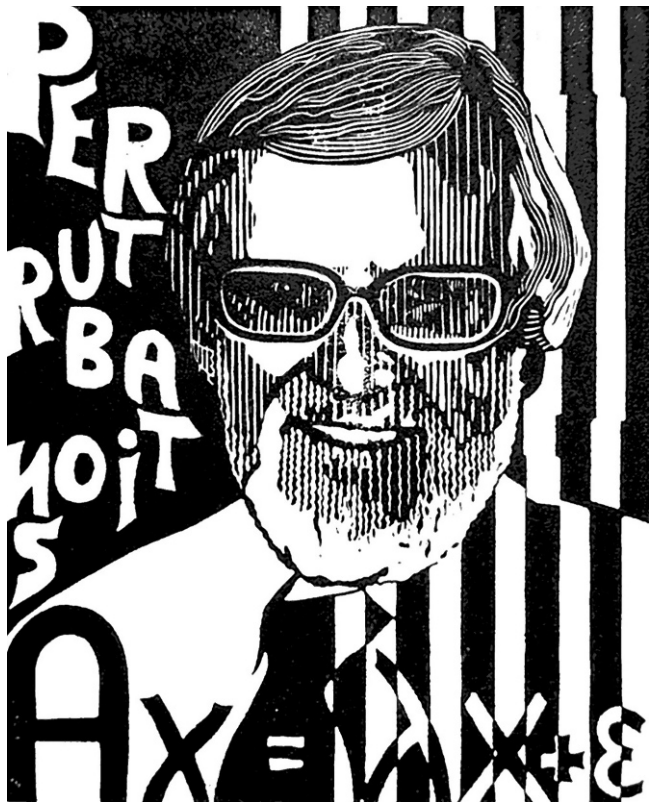
G.W. Stewart

Selected Works with Commentaries



Linocut by Henk van der Vorst

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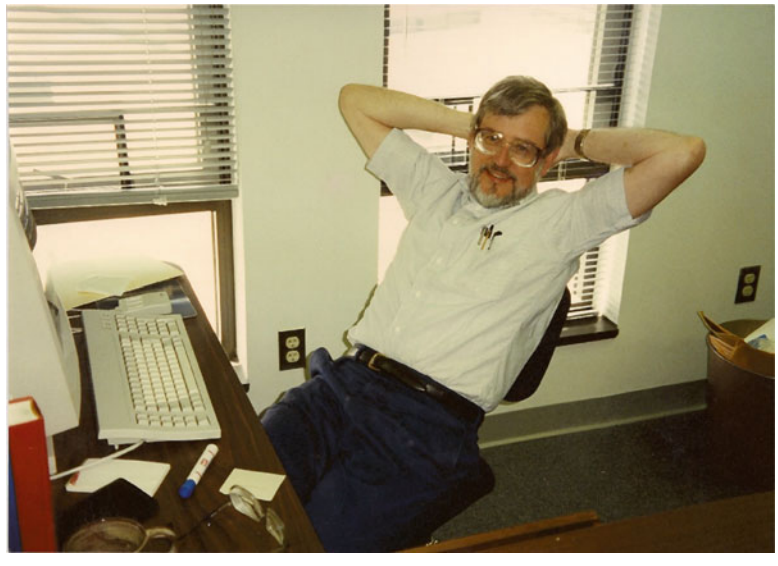
Pete circa 1944

Jack Dongarra, Cleve Moler, Pete Stewart, and Jim Bunch, with Cleve's car and license plate, late 1970s

Pete with Oak Ridge colleagues Michael Heath, Alston Householder (his Ph.D. advisor), and Robert Funderlic, circa 1970

Pete lecturing in the 1980s

Pete and his wife, Astrid Schmidt-Nielsen, circa 1987



Pete in his office at the University of Maryland, circa 1998

Contents

Foreword

G.W. (Pete) Stewart is a world-renowned expert in computational linear algebra. It is widely accepted that he is the successor to James Wilkinson, the first giant in the field, taking up the perturbation theory research that Wilkinson so ably began and complementing it with algorithmic innovation. Stewart's results on rounding error in numerical computations provide basic understanding of floating-point computation. His results on perturbation of eigensystems, generalized inverses, least squares problems, and matrix factorizations are fundamental to numerical practice today. His algorithms for the singular value decomposition, updating and downdating matrix factorizations, and the eigenproblem broke new ground and are still widely used in an increasing number of applications. His papers, widely cited, are characterized by elegance in theorems and algorithms and clear, concise, and beautiful exposition. His six popular textbooks are excellent sources of knowledge and history.

Stewart's 60th birthday was celebrated with a meeting at College Park, MD. His 70th birthday will be observed by a meeting in Austin, TX, and by a special issue of the journal *Linear Algebra and its Applications* dedicated to him. It is fitting that there be a collection of his selected works published on this occasion, and we were happy to undertake the task of editing this volume.

Pete chose the papers to include here, and we are grateful for the permission to reprint these papers. The publishers are ACM, AMS, Elsevier, ETNA, IEEE, Oxford University Press, SIAM, and Springer. We are especially grateful to SIAM for the permission to reprint 19 papers.

We thank Iain S. Duff for providing a lively biography of Pete, based on interviews. We are very grateful to our collaborators in writing the commentaries: Zhaojun Bai, James W. Demmel, Lars Eldén, Howard C. Elman, Ilse C.F. Ipsen, and Charles F. Van Loan. These leading experts in their fields produced commentary with depth and breadth.

Each chapter of the commentary was reviewed for accuracy and completeness, and we are grateful to Jesse Barlow, Åke Björck, James Demmel, Nick Higham,

Chris Paige, Yousef Saad, Michael Saunders, Nick Trefethen, and David Watkins for doing these reviews. We are also grateful to a number of individuals who provided us with remarks regarding the impact of Pete's scholarship on their own research: M.W. Berry, R.A. Brualdi, J.R. Bunch, Z. Jia, C.-R. Lee, K.J.R. Liu, C.C. Paige, B.N. Parlett, Y. Saad, and M.A. Saunders. Comments and quotations from these individuals are interspersed within the commentaries.

Finally, we thank Henk van der Vorst for providing the wonderful linocut image used on the title page.

We present this volume as a gift to Pete, gathering some of his most important contributions. He is a good friend and an inspiring colleague, and we are honored to be associated with him.

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Part I

G. W. Stewart

1

Biography of G. W. Stewart

Iain S. Duff

If one is asked to name the most influential people in numerical linear algebra, then Pete (G.W.) Stewart would come very high on the list. Pete has had a major influence on the field and, in several ways, on my own career. It is with great pleasure that I pen these words as a biography and tribute to him. I am grateful to Pete not only for spending the time to discuss his life with me but also for going carefully over a draft and finding many instances where accuracy had been sacrificed for the sake of the narrative. I should, however, stress that any rounding errors that remain are purely my responsibility but hopefully do not now contaminate the result.

Pete was born in Washington, DC in the fall of 1940 and spent his first 5 years in Arlington, Virginia, just across the Potomac river from Washington. His father was a journalist who spent the year when Pete turned six at Harvard as a Nieman Fellow. Thus Pete started first grade in Cambridge, coincidentally at the same time that David Young arrived at Harvard as a new graduate student. Pete's father then accepted a position in the State Department attached to the United Nations, and Pete attended elementary school for 3 years in Jamaica, NY, and 3 more years in Hempstead, Long Island.

Pete's long association with the sour mash state of Tennessee began in the fall of 1953, when his father accepted a position in Knoxville as assistant director of the Information Department of the Tennessee Valley Authority. Pete completed his high school education at Bearden High School. Although at this point it was not clear that a future star for our field was in the making, Pete discovered his talent for mathematics in high school. With the encouragement of his math teacher, Ms. Ival Aslinger, he completed the high school mathematics curriculum in 3 years and taught himself calculus. Indeed this self-teaching is characteristic of Pete; and it is fair to say that at most periods of his life, he has engaged in this form of learning, whether it be in mathematics, languages, or history. Perhaps one of his main interests outside of mathematics at this time (and to some extent even today)

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was in history. He still recalls reading Gibbon's *Decline and Fall* for the first time. He graduated from high school with good grades in 1957.

Pete matriculated at the University of Tennessee, where at one time or other, he declared several majors: engineering physics, psychology, pre-medicine, in addition to mathematics and physics. Perhaps, the first inkling of what was to happen came when at the end of his sophomore year in 1959 he became a summer student employee at the Gaseous Diffusion Plant in Oak Ridge. This brought him in contact with computational mathematics and with issues in numerical analysis, many of which he was to develop, refine, and define in his later works. He worked with the supercomputer of the day, an IBM 704 with 16K of 36-bit word memory. His work at the plant led to a fairly well-paid consulting contract that enabled him to enjoy life (another characteristic that endures to this day!) while completing his undergraduate studies. He graduated *summa cum laude* in 1962 with a major in mathematics and a minor in physics.

In his undergraduate work, Pete also took courses in pure mathematics, including topology. He was awarded a Woodrow Wilson/NSF fellowship to Princeton, where he intended to study algebraic topology. But he soon switched to logic and the foundations of mathematics, taking a class from Alonzo Church. Much to the later benefit of our field, however, he only stayed a year at Princeton, before returning to Oak Ridge on a full-time basis in 1963. At that time he married one of his co-workers, Lynn Tharp, and they moved to Phoenix, Arizona, where he worked in the General Electric Computer Division (1964–1965). The job was not particularly demanding, and Pete had plenty of time to hone his personal skills, especially when the company sent him to Fairbanks, Alaska for a summer of working at the Nimbus weather satellite tracking station in Fairbanks. During this time he devoted himself to mastering the classics by Feller on probability theory and Sheffé on the analysis of variance. Perhaps more importantly, Bob Funderlic, a friend and colleague from Oak Ridge, passed on some bootleg notes by Jim Wilkinson from the Michigan Engineering Summer Conference in Ann Arbor. This conference on Numerical Analysis, organized by Bob Bartels, consisted of 2 weeks of lectures by distinguished speakers to a class of about 40 students. After the lectures, the speakers adjourned for rounds of liquid conviviality at Ann Arbor's Old German Restaurant. In his spare time in Phoenix, Pete coded algorithms from Wilkinson's notes, including the beautiful, double implicit shift QR algorithm of Francis. Following a lead from W. Ross Burrus, who had been using plane rotations to solve least squares problems at Oak Ridge National Laboratory, Pete devised a least squares algorithm that used Householder transformations and wrote it up in what he acknowledges to be a thoroughly amateurish paper. Fortunately, Gene Golub saved Pete from embarrassing himself by publishing essentially the same algorithm (and much more) in his classic *Numerische Mathematik* paper.

Pete returned to Oak Ridge in 1965 and enrolled as a graduate student in mathematics at the University of Tennessee, where among other required courses he took one from Alston Householder on the theory of matrices in numerical analysis.

It was at this time that his first journal paper, on a derivative-free version of Davidon's method, appeared in the *Journal of the ACM*. As we might expect, this included a rigorous error analysis of the effect of rounding errors on the difference approximations of derivatives.

The defining moment in Pete's professional life was when he and another Oak Ridge friend, Bert Rust, attended the Michigan Summer Conference. The lecturers in this summer school included Alston Householder, Jim Wilkinson, Dick Varga, John Todd, and John Rice. Pete was entranced by the talks, and at that moment realized his destiny and embraced the topic of numerical linear algebra that was to define and guide his life.

In the beginning, Pete did not have too much interaction with Alston Householder, who was at that time the Director of the Mathematics and Computer Division of ORNL. He was also Ford Professor at the University of Tennessee, where he taught a class that met on Wednesday afternoons and Saturday mornings. Pete took two of these classes – one on numerical linear algebra, as mentioned above, and one on the solution of nonlinear equations. Although, in Pete's words, Householder's style of lecturing left something to be desired, his subject matter more than made up for it. Among the influential texts that Pete can recall reading as a graduate student were Wilkinson's *Rounding Errors in Algebraic Processes*, and his *Algebraic Eigenvalue Problem.* In the spring of 1967, Pete passed his comprehensives (qualifying exam) that comprised both written and oral exams in algebra, analysis, topology, and the foundations of mathematics, and Householder agreed to take him on as his student. His interaction with Householder was only at the level of once a month; but Pete was already quite advanced in his research, largely because of his good background in computation and computational analysis, and he finished his thesis in record time by the end of 1967. Pete was supported during his thesis year by an Oak Ridge Associates Fellowship that also entailed his moving from the Gaseous Diffusion Plant to Oak Ridge National Laboratory – from K25 to X10 in the local coordinate system.

Part of Pete's thesis concerned a rigorous analysis of Lehmer's method for solving polynomial equations, including a scaling algorithm to avoid overflow. He also translated the work of Bauer on *Treppeniteration* (a.k.a. subspace iteration), and was inspired to analyze a variant that used Rayleigh–Ritz approximations to speed up the calculations. Unknown to him at the time, Rutishauser had come up with the same algorithm, and the perturbation theory that Pete devised for the analysis was about to be published in a more general form by Chandler Davis and W. (Velvel) Kahan. It was around that time that Pete first met Gene Golub, who visited the department in the fall of 1967.

With his PhD assured, Pete went job hunting and was invited for interviews at Florida State University, the University of Virginia, and the University of Texas, accepting the offer of the latter to become an Assistant Professor in the fall of 1968. This was no mean achievement given the state of the academic market, which had just gone into a nosedive. A year later he accepted a half-time research appointment

in David Young's Institute for Numerical Analysis. Tenure and promotion to Associate Professor followed rapidly in the following year, with Pete's abilities at that time being recognized by the fact that this took less than half the expected period for translating tenure track to tenured position. A glance at Pete's publication record at that time shows that he was becoming something of a paper-generating machine, with four or more journal papers a year being published in the late sixties and early seventies. It should be noted that these were substantial contributions in high quality journals. In addition to his research, Pete was for the first time heavily into teaching within the Math and Computer Science Departments, teaching not only mathematics but the theory of programming languages. Pete got to know Cleve Moler when the latter visited Texas, and in an afternoon session during his visit they sketched the outlines of the QZ algorithm for the generalized eigenvalue problem. Cleve extended his stay so that they could flesh out their ideas. While in Texas, Pete's two children were born: his son Michael (who has to some extent followed in his father's footsteps) in 1969 and his daughter Laura in 1970.

Pete then spent 2 years at Carnegie–Mellon University (1972–1974) with a joint appointment in the Department of Mathematics and the Department of Computer Science, the latter headed by Joe Traub. At the end of his stay in Pittsburgh, his *Introduction to Matrix Computations* was published, arguably the first of the great modern texts and the book of choice for supporting courses in numerical linear algebra until the publication of Golub and Van Loan's book in 1983. Another significant event during the Carnegie–Mellon period was Pete's divorce from Lynn in 1974.

Werner Rheinboldt, who with Jim Ortega had written an influential book on nonlinear equations and optimization, was the technical go-between for Pete and his publisher. In 1974, he arranged an appointment for Pete in the Computer Science Department at the University of Maryland with a half-time research appointment in the Institute for Fluid Dynamics and Applied Mathematics (later to become the Institute for Physical Sciences and Technology). Pete was promoted to Full Professor in 1976. At this point, it is impressive to think that Pete was still only 35 – a meteoric career indeed. It is fair to say that although Pete enjoyed the teaching aspects of being a professor, his dedication to research meant that his love of teaching was somewhat tempered and, in his words, he loved teaching in moderation.

Although Pete's parents were still living in Knoxville, he had good family connections in the Washington area through his uncle Bill and aunt Carleen. (Bill was a ground crew soldier in the Air Force during WWII who came to Washington on rotation and met and fell in love with Carleen, Pete's babysitter.) Shortly after Pete arrived at Maryland, he met Astrid Schmidt-Nielsen at a dinner party, and 3 years later they were married (with Alston Householder in attendance). Astrid has been a constant, influential, and inspiring partner to Pete from that day until today.

In the summer of 1975, Pete visited the Division of Mathematical Sciences headed by Jim Poole at Argonne National Laboratory. The Division had sponsored

the development of EISPACK, a collection of routines to solve eigenvalue problems. Jim asked Pete to chair a public meeting on the possibility of producing software for linear systems – what eventually became LINPACK. It was decided to confine the effort to dense matrices with Pete, Cleve Moler, and Jim Bunch designing and coding the algorithms (Jack Dongarra came on board later). At that time granting agencies were not funding program development, arguing that it was not research. Pete returned to Argonne in the fall, and wrote a proposal that billed the effort as *research* in how to develop software. As we know, the proposal was successful. The efforts of the group were assisted by the visits of Jim Wilkinson to Argonne during the summer. Work on LINPACK was perhaps Pete's most visible contribution for the next 3 years, with annual visits to Argonne culminating in the release of LINPACK in the summer of 1978. Pete played a leading role in the coding of the routines, which was greatly helped by the decision to use the BLAS. These were of course just the Level 1 BLAS, and thus the decision was more important for clarity and modularity than for efficiency. An important aspect of the package was its *Users' Guide,* which was intended to explain the workings of the algorithms, not just how to use them. The introduction, which was largely written by Pete, resonated, I am sure, with the NSF and certainly contributed greatly into making the manual a SIAM best seller.

In 1977–1978, Pete and Astrid spent time in Minnesota, Astrid as a Post Doc and Pete holding joint appointments in the Computer Science Department, headed by Ben Rosen, and the Applied Statistics Department, headed by Steve Fienberg. This was the beginning of Pete's interest in statistical computing.

Around this time Pete first met Dianne O'Leary, a student of Gene Golub's. Sometime after her arrival as an Assistant Professor at Maryland in 1978, they worked and published together on parallel computing – a goodly feat given that it would be several more years before practical, commercial parallel computers were readily available. They became involved in many aspects of parallel computing, including developing numerical algorithms and designing an operating system for a home grown parallel computer at Maryland. Pete says that his friendship and collegial relationship with Dianne has been one of the most satisfying aspects of his professional career.

At that time, much of his research was performed at the National Bureau of Standards (later to become NIST) where he consulted once a week and where he spent a half-year sabbatical in 1988. Some of Pete's more exciting and significant work at that time was his collaboration with Sun on matrix perturbation problems, culminating in their book *Matrix Perturbation Theory* published in 1990. The two-body problem (Astrid had a job with the Naval Research Laboratory in Washington) has kept Pete more at home than many researchers, although they both spent 6 months back in Minneapolis in 1992.

Pete has had a long and distinguished association with the Gatlinburg conferences, later to be renamed the Householder Symposia in honor of Pete's mentor and friend, Alston. It is amazing to think that Alston's 65th birthday was celebrated

in 1969 at the last of the conferences to be held in Gatlinburg (and the first attended by Pete). Pete was elected to the Householder Committee at the Asilomar conference in 1977 and remained an active and involved member until he stood down at the first meeting after his 60th birthday, almost 10 years ago. Pete well knew the meaning of the word symposium (Greek for drinking party), and I have many happy memories of the Gatlinburg/Householder meetings where high kudos was given to active attendance at the first talk of the morning after earnest early morning discussions fuelled by the waters of Scotland or Tennessee. This tradition was a staple of the community in all its meetings. In a sense, it is an embodiment of a singular aspect of the field of numerical linear algebra, where camaraderie, collaboration, and encouragement are the norm, rather than the cut-throat competitiveness of some other scientific and mathematical disciplines. This atmosphere was much appreciated by Pete and indeed is a major reason that he has spent his life working primarily in this area.

Pete was of course of an era when there were many fewer prizes and awards than are available today – no Fox, Householder, and Wilkinson prizes for example. Nevertheless, he has had recognition of his pioneering work in the award of the F.L. Bauer Prize by the Technical University of Munich in 1988 and his election to the National Academy of Engineering in 2004. He also holds the title of Distinguished University Professor, a signal honor at a university as prestigious as Maryland. He was an inaugural fellow of SIAM in 2009 with the citation "For contributions to numerical linear algebra."

So what can we say about the man who so influenced our field and our lives. He was certainly his own man, almost to the point of being a lone researcher (he has only 43 joint journal publications out of the 137 listed in his current curriculum vitae). He did not "father" many graduate students, with only six PhD students in his long career. Like Ramanujan, he ascribes some of his most creative work to an almost mystical crystallization of an incoherent muddle of ideas – the ideas bubbling around for some time before the eureka moment, perhaps as emerging from a dream. Unlike many of us, Pete actually enjoyed the duties of refereeing and found this at times to be another source of stimulation and inspiration. One thing that Pete shares with Don Knuth is that his magnum opus (*Matrix Algorithms* published by SIAM) was originally planned to be in four volumes but has stopped at two with no plans to continue the series. Pete is always happy to share his thoughts and was one of the first researchers to make his works available to all by anonymous ftp in the early 1990s. I recollect accessing his "Afternotes" on the web as early as 1993.

I am notoriously bad at remembering my first meeting with anybody, even someone as influential as Pete. However, for sure I met him at the Gatlinburg meeting at Asilomar in 1977, and we co-edited the SIAM Sparse Matrix Proceedings in 1978. In the latter exercise, I overcame one of the latent prejudices of the UK toward the Americans with respect to language, when it was quite apparent that Pete's knowledge of the mother tongue was equal to that of anybody from my side of the Atlantic. Indeed he introduced me to Strunk and White, which is very much

a worthy competitor to Fowler's *Modern English Usage,* with which we in the UK were more familiar. Another early memory of Pete and Astrid was a visit to their house in January 1980 accompanied by our new baby, Catriona. We found that, as Pete apparently keeps UK time by his early start to the day, it was not the place to recover from any effects of jet lag. Pete and Astrid later attended more than one early birthday of Catriona, bringing with them on one trip a Fisher–Price boat named the SS Schmidt-Nielsen. That was in the days when Fisher–Price toys were little known in Europe, and so it was a rather special present that, having been later used by other village children, is now in our attic awaiting future grandchildren.

Pete has now added Emeritus to his title of Distinguished University Professor, but, like most of you, I do not believe that such an active and able mind will cease his research any time in the near future. I anticipate and hope that he will emulate Alston, who I recollect was still attending his eponymous meetings over the age of 90. So I thank the orchestrators of this volume, Misha Kilmer and Dianne O'Leary, for giving me the opportunity of wishing Pete (and Astrid) all the best for their "retirement" shared between their houses in Washington and Maine. You never know, he might even be persuaded to continue with the Matrix Algorithms series, and, given that I learned not only mathematics but some US Civil War history from the first volume, we could all gain much if this were to happen.

As a small postscript to my discussions with Pete while preparing this short biography, I can reveal a possible solution to the often discussed puzzle of why "G.W." became "Pete." Like his father and grandfather before him, he was christened Gilbert Wright, and like them he was also called Pete. This sobriquet was acquired by his preacher grandfather from cowboys in South Dakota just after he graduated from seminary. The story goes that when he reached town, he gave the saloon keeper an offer he couldn't refuse: if the keeper would close the bar for an hour on Sunday so his customers could attend church, he (Pete's grandfather) would not preach against liquor. Whether this inspired the cowboys to call him Pete and why they chose "Pete" in the first place is not clear. (Some family stories say that it was a college nickname). But apocryphal or not, his grandfather's compromise is one of which I am sure Pete (G.W.) the Third would approve.

Happy Birthday Pete (G.W.)

Iain Duff
Oxfordshire
11/11/2009

2

Publications, Honors, and Students

2.1. Publications of G. W. Stewart

2.1.1. Thesis

[T1] Dissertation: G. W. Stewart III, "Some Topics in Numerical Analysis," University of Tennessee. Published as Technical Report ORNL-4303, Oak Ridge National Laboratory, September 1968.
http://www.ornl.gov/info/reports/1968/3445605155079.pdf

2.1.2. Books

[B1] *Introduction to Matrix Computations*, Academic Press, New York (1973).
[B2] (with J. J. Dongarra, J. R. Bunch, and C. B. Moler), LINPACK *Users' Guide*, SIAM, Philadelphia (1979).
[B3] (with J.-G. Sun) *Matrix Perturbation Theory*, Academic Press, New York (1990).
[B4] Translation of Karl Friedrich Gauss, *Theoria Combinationis Observationum Erroribus Minimis Obnoxiae*, (Theory of the Combination of Observations Least Subject to Errors, Part One, Part Two, Supplement) SIAM, 1995.
[B5] *Afternotes on Numerical Analysis*, SIAM, 1996.
[B6] *Afternotes Goes to Graduate School*, SIAM, 1998.
[B7] *Matrix Algorithms Volume I: Basic Decompositions*, SIAM, 1998.
[B8] *Matrix Algorithms Volume II: Eigensystems*, SIAM, 2001.

2.1.3. Journal Publications

[J1] "A Modification of Davidon's Minimization Method to Accept Difference Approximations of Derivatives," *Journal of the ACM* **14** (1967) 72–83.

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[J2] "A Generalization of a Theorem of Fan on Gershgorin Disks," *Numerische Mathematik* **10** (1967) 162.

[J3] (with D. W. Lick) "Numerical Solution of a Thin Plate Heat Transfer Problem," *Communications of the ACM* **11** (1968) 639–640.

[J4] "On the Continuity of the Generalized Inverse," *SIAM Journal on Applied Mathematics* **17** (1969) 33–45.

[J5] "Accelerating the Orthogonal Iteration for the Eigenvectors of a Hermitian Matrix," *Numerische Mathematik* **13** (1969) 362–376.

[J6] "Some Iterations for Factoring a Polynomial," *Numerische Mathematik* **13** (1969) 458–470.

[J7] "On Lehmer's Method for Finding the Zeros of a Polynomial," *Mathematics of Computation* **23** (1969) 829–836, s24–s30. Corrigendum: **25** (1971) 203.

[J8] "On Samelson's Iteration for Factoring Polynomials," *Numerische Mathematik* **15** (1970) 306–314.

[J9] "Incorporating Origin Shifts into the QR Algorithm for Symmetric Tridiagonal Matrices," *Communications of the ACM* **13** (1970) 365–367.

[J10] "Algorithm 384: Eigenvalues and Eigenvectors of a Real Symmetric Matrix," *Communications of the ACM* **13** (1970) 369–371. Remark: *Communications of the ACM* **13** (1970) 750.

[J11] "On the Convergence of Sebastião E Silva's Method for Finding a Zero of a Polynomial," *SIAM Review* **12** (1970) 458–460.

[J12] (with A. S. Householder) "The Numerical Factorization of a Polynomial," *SIAM Review* **13** (1971) 38–46.

[J13] "Error Analysis of the Algorithm for Shifting the Zeros of a Polynomial by Synthetic Division," *Mathematics of Computation* **25** (1971) 135–139.

[J14] "On a Companion Operator for Analytic Functions," *Numerische Mathematik* **18** (1971) 26–43.

[J15] "Error Bounds for Approximate Invariant Subspaces of Closed Linear Operators," *SIAM Journal on Numerical Analysis* **8** (1971) 796–808.

[J16] "On the Sensitivity of the Eigenvalue Problem $Ax = \lambda Bx$," *SIAM Journal on Numerical Analysis* **9** (1972) 669–686.

[J17] (with R. H. Bartels) "Algorithm 432: Solution of the Matrix Equation $AX + XB = C$," *Communications of the ACM* **15** (1972) 820–826.

[J18] (with C. B. Moler) "An Algorithm for Generalized Matrix Eigenvalue Problems," *SIAM Journal on Numerical Analysis* **10** (1973) 241–256.

[J19] "Error and Perturbation Bounds for Subspaces Associated with Certain Eigenvalue Problems," *SIAM Review* **15** (1973) 727–764.

[J20] "Conjugate Direction Methods for Solving Systems of Linear Equations," *Numerische Mathematik* **21** (1973) 285–297.

[J21] "Some Iterations for Factoring a Polynomial. II A Generalization of the Secant Method," *Numerische Mathematik* **22** (1974) 33–36.

[J22] "The Convergence of Multipoint Iterations to Multiple Zeros," *SIAM Journal on Numerical Analysis* **11** (1974) 1105–1120.

[J23] (with T. Söderström) "On the Numerical Properties of an Iterative Method for Computing the Moore-Penrose Generalized Inverse," *SIAM Journal on Numerical Analysis* **11** (1974) 61–74.

[J24] (with M. M. Blevins) "Calculating the Eigenvectors of Diagonally Dominant Matrices," *Journal of the ACM* **21** (1974) 261–271.

[J25] "Modifying Pivot Elements in Gaussian Elimination," *Mathematics of Computation* **28** (1974) 537–542.

[J26] "The Convergence of the Method of Conjugate Gradients at Isolated Extreme Points of the Spectrum," *Numerische Mathematik* **24** (1975) 85–93.

[J27] "Gershgorin Theory for the Generalized Eigenvalue Problem $Ax = \lambda Bx$," *Mathematics of Computation* **29** (1975) 600–606.

[J28] "An Inverse Perturbation Theorem for the Linear Least Squares Problem," *(ACM) SIGNUM Newsletter* **10**(2–3) (1975) 39–40.

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