IUTAM Symposium on Vibration Control of Nonlinear Mechanisms and Structures

SOLID MECHANICS AND ITS APPLICATIONS
Volume 130

Aims and Scope of the Series

The fundamental questions arising in mechanics are: *Why?, How?,* and *How much?* The aim of this series is to provide lucid accounts written by authoritative researchers giving vision and insight in answering these questions on the subject of mechanics as it relates to solids.

The scope of the series covers the entire spectrum of solid mechanics. Thus it includes the foundation of mechanics; variational formulations; computational mechanics; statics, kinematics and dynamics of rigid and elastic bodies: vibrations of solids and structures; dynamical systems and chaos; the theories of elasticity, plasticity and viscoelasticity; composite materials; rods, beams, shells and membranes; structural control and stability; soils, rocks and geomechanics; fracture; tribology; experimental mechanics; biomechanics and machine design.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

For a list of related mechanics titles, see final pages.

IUTAM Symposium on

Vibration Control of Nonlinear Mechanisms and Structures

Proceedings of the IUTAM Symposium held in Munich, Germany, 18–22 July 2005

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DEDICATION

Contributions of the IUTAM Symposium on Vibration Control of Nonlinear Mechanisms and Structures are dedicated to Professor Friedrich Pfeiffer on the occasion of his seventieth birthday, which he celebrated on 22 February 2005.

Friedrich Pfeiffer originated from Wiesbaden, where he was awarded his university-entrance diploma from the Realgymnasium in 1955 as the best of his class. Afterwards, he studied mechanical engineering at the Technische Hochschule Darmstadt from 1955 to 1961, supported by a scholarship from the "Studienstiftung des deutschen Volkes". From 1961 to 1965, he was a research assistant at the TH Darmstadt Institute for Aeronautics, under the supervision of Professor Günther Bock. He received his Dr.-Ing. summa cum laude in 1965 based on his doctoral thesis "Abwindkorrekturen für Flügel beliebiger Pfeilung in offenen und geschlossenen kreisrunden Windkanälen mit Bodenplatte".

This was the foundation of his storybook career in industry. In the year 1966 he started as a development engineer at the department of aerospace of Bölkow GmbH in Ottobrunn, where he soon became project manager, then department manager, head of the department, and finally a member of the company management. There he was responsible for research and development, supervising about 1000 employees and with annual sales of approximately 100 million Euro.

During his 16 years at the company, Professor Pfeiffer was always at the top of the ongoing current research in the field of the dynamics of rockets and satellites. When he was appointed to a professorship at the newly founded "Bundeswehrhochschule Hamburg" in 1973 he declined the offer. But almost 10 years later he was eventually attracted to university life and became a full professor for mechanics at the "Technische Universität München – Lehrstuhl B für Mechanik", where he was the successor of Professor Magnus.

Professor Pfeiffer has educated many thousands of students in the basic principles of mechanics and in the field of multi-body dynamics of robots and walking robots. He was very popular among students, who respected him as an excellent teacher. And as an outstanding researcher, it was easy for him to inspire his research and teaching assistants, of which more than 80 did their Ph.D. under his supervision.

More than 200 papers in international journals and five books demonstrate Professor Pfeiffer's exceptional talents in research and teaching, especially his remarkable ability to put complex theory into engineering practice. He focused his scientific research mainly on dynamics and control of large mechanical systems with friction and clearance but also always kept an eye on the mechanical fundamentals and worked on different problems from industry. The name Pfeiffer is primarily linked to groundbreaking work about non-smooth dynamics of multi-body systems with unilateral constraints and biological oriented walking robots, for which he was awarded the "Körber Preis". Professor Pfeiffer is editor and co-editor of many internationally renowned journals in the area of non-linear dynamics and robotics. For many years he was active on different committees, among those the board of "Studienstiftung des deutschen Volkes", member of the senate of the DFG (German Research Foundation), dean of the Faculty of Mechanical Engineering, a member of the senate of the "Technische Universität München" and, since 2000, president of the Society of Applied Mathematics and Mechanics ("Gesellschaft für Angewandte Mathematik und Mechanik, GAMM"). He has received numerous honours of which only a few are listed here: honorary doctorate (Dr. h. c. and Dr.-Ing. E. h.) from the universities of Moscow and Dresden, "Bundesverdienstkreuz am Bande", and the appointment as an IEEE fellow and ASME fellow.

Students, colleagues, and fellow scientists wish him first of all health, happiness, and vitality. We hope that he continues being active and giving advice in the years to come.

Munich, July 2005 Heinz Ulbrich

PREFACE

During the last decades, the growth of micro-electronics has reduced the cost of computing power to a level acceptable to industry and has made possible sophisticated control strategies suitable for many applications. Vibration control is applied to all kinds of engineering systems to obtain the desired dynamic behavior, improved accuracy and increased reliability during operation. In this context, one can think of applications related to the control of structures' vibration isolation, control of vehicle dynamics, noise control, control of machines and mechanisms and control of fluid-structure-interaction. One could continue with this list for a long time.

Research in the field of vibration control is extremely comprehensive. Problems that are typical for vibration control of nonlinear mechanisms and structures arise in the fields of modeling systems in such a way that the model is suitable for control design, to choose appropriate actuator and sensor locations and to select the actuators and sensors.

The objective of the Symposium was to present and discuss methods that contribute to the solution of such problems and to demonstrate the state of the art in the field shown by typical examples. The intention was to evaluate the limits of performance that can be achieved by controlling the dynamics, and to point out gaps in present research and give links for areas of future research. Mainly, it brought together leading experts from quite different areas presenting their points of view.

The book evolved from the International Symposium on Vibration Control of Nonlinear Mechanisms and Structures, held in Munich, Germany, from 18 to 22 July 2005. This Symposium was initiated by the International Union of Theoretical and Applied Mechanics. The main topics of the Symposium were:

- Control of machines and mechanisms
- Control of vehicles
- Active control of noise and vibration in structures
- Active vibration isolation of mechanisms
- Control of nonsmooth dynamics
- Actuators and sensors
- Vibration control of fluid-structure interaction

- Numerical methods in real-time-control

A Scientific Committee was appointed by the Bureau of IUTAM with the following members:

- Heinz Ulbrich, Munich, Germany (Chairman)
- Sunil K. Agrawal , Delaware, USA
- Steven R. Bishop, London, UK
- Felix L. Chernousko, Moscow, Russia
- Mikio Horie, Yokohama, Japan
- Ali H. Nayfeh, Blacksburg, USA
- Werner Schiehlen, Stuttgart, Germany
- Gábor Stépán, Budapest, Hungary

This committee selected the participants to be invited and the papers to be presented at the Symposium. As a result of this procedure, more than 60 active scientific participants from 14 different countries followed the invitation. There were three key lectures selected to give an overview of the different fields to be covered by the Symposium followed by 31 papers. All the papers presented at the Symposium are included in this book.

Since many of the presentations are related to more than one of these topics, the papers in this book are arranged in alphabetical order with respect to the family name of the first author, starting at the beginning with the three key papers. The papers cover a wide range of engineering applications of the vibration control of nonlinear mechanisms and structures. The presentations and discussions during the Symposium will certainly stimulate further theoretical and experimental investigations in the related research fields.

The editors wish to thank both the participants of this IUTAM Symposium and the authors of the papers for their valuable contributions to the important field of vibration control of nonlinear mechanisms and structures. Special thanks are given to the invited lecturers, all the lecturers and the sessions chairmen for making this Symposium a success.

The organizer gratefully acknowledges the financial support and/or effective help in the preparation of the Symposium

- Deutsche Forschungsgemeinschaft (German Research Foundation)
- International Union of Theoretical and Applied Mechanics (IUTAM)
- BMW Research Group
- Bayerisches Staatsministerium für Wissenschaft und Kunst (Bavarian Ministry of Science and Arts)
- Technical University of Munich (TUM)

The main contribution to the success of the Symposium was the great help and excellent work of the staff of the Institute of Applied Mechanics of the TUM and the Local Organizing Committee.

Special thanks are given to Dipl.-Ing. Sandor Riebe and to Dipl.-Ing. Wolfgang Günthner, they did an excellent job, as well as to Ms. Rita Schneider and Ms. Manuela Müller-Philipp for their great administrative help.

In addition, many thanks are due to Springer (formerly Kluwer Academic Publishers) and Karada Publishing Services for their efficient cooperation.

Munich, July 2005 Heinz Ulbrich

WELCOME ADDRESS

Magnifizenz Professor Schilling, Chairman Professor Ulbrich,
Dear Colleagues from all over the world,
Ladies and Gentlemen,

It is my honor and pleasure to welcome all of you on behalf of the International Union of Theoretical and Applied Mechanics, and its President Professor Ben Freund from Brown University. Let me use this Opening Ceremony for a short look on the past and present activities of IUTAM.

Organized meetings between scientists in the field of mechanics were initiated 83 years ago, namely in 1922, when Professor Theodore von Kármán and Professor Tullio Levi-Civita organized the world's first conference in hydro- and aero-mechanics. Two years later, in 1924, the First International Congress was held in Delft, the Netherlands, encompassing all fields of mechanics, that means analytical, solid and fluid mechanics, including their applications. From then on, with the exception of the year 1942, International Congresses in Mechanics have been held every four years.

In particular, when the mechanics community reassembled in Paris for the Sixth Congress in 1946, out of the Congress series an international union was formed, and as a result IUTAM was created and statutes were adopted. After one year, in 1947, the Union was admitted to ICSU, the International Council for Science. This council coordinates activities among various other scientific unions to form a tie between them and the United Nations Educational, Scientific and Cultural Organization, well known as UNESCO.

Today, IUTAM forms the international umbrella organization of about 50 national Adhering Organizations of mechanics from nations all over the world. Furthermore, a large number of international scientific organizations of general or more specialized branches of mechanics are connected with IUTAM as Affiliated Organizations. As a few examples, let me mention: the European Mechanics Society (EUROMECH), the International Association of Computational Mechanics (IACM), the International Association for Vehicle System Dynamics (IAVSD), and the International Commission of Acoustics (ICA).

Within IUTAM the only division used so far is related to solid and fluid mechanics as indicated by our two Symposia Panels. But more recently nine

Working Parties with up to five members each have been established by the General Assembly of IUTAM devoted to specific areas of mechanics. These areas are:

- Non-Newtonian Fluid Mechanics and Rheology,
- Dynamical Systems and Mechatronics,
- Mechanics of Materials,
- Materials Processing,
- Biomechanics,
- Nano- and Micro-Scale Phenomena in Mechanics,
- Geophysical and Environmental Mechanics,
- Education in Mechanics and Capacity Building.

The terms of reference of the Working Parties include to make recommendations to the General Assembly regarding timely subjects for IUTAM Symposia, to maintain contact with the relevant Affiliated Organizations and sister International Unions, to identify important growth areas of the field, and to assist the Bureau and the General Assembly in discussions on position statements. Professor Friedrich Pfeiffer whom I am greeting too, is Chairman of the Working Party on Dynamical Systems and Mechatronics.

IUTAM carries out an exceptionally important task of scientific cooperation in mechanics on the international scene. Each national Adhering Organization of IUTAM, like the German Committee for Mechanics (DEKOMECH), is represented by a number of scientists in IUTAM's General Assembly. In particular, the German delegates with IUTAM are

Professor Ulrich Gabbert, Otto von Guericke University of Magdeburg;
Professor Christian Miehe, University of Stuttgart;
Professor Wolfgang Schröder, Rheinisch-Westfälische Technische Universität Aachen (RWTH Aachen);
Professor André Thess, Ilmenau University of Technology.

Mechanics is a very well developed science in Germany represented at most universities and some national laboratories. Since 1949 there has been held more than 280 IUTAM symposia worldwide. Out of them 31 symposia where organized in Germany with two symposia in Munich.

In 1977, just 28 years ago, the first IUTAM Symposium was held at the Technical University of Munich under the chairmanship of Professor Kurt Magnus. This first Munich Symposium was devoted to the Dynamics of Multibody Systems with the result that a new branch of mechanics called "Multibody Dynamics" was created.

Twenty-one years later in 1998 a second IUTAM Symposium took place here in Munich. Professor Friedrich Pfeiffer was chairman of the Symposium on Unilateral Multibody Contacts which was very successful again. Since that time contact problems are considered as an attractive research field in multibody dynamics, too.

As I mentioned before, IUTAM organizes not only symposia but also international congresses all over the world. Last year the 21st International Congress of Theoretical and Applied Mechanics was held in Warsaw, Poland. With 1515 participants the Warsaw Congress was a major event in mechanics also described as the Olympics of Mechanics. The Twenty-second International Congress of Theoretical and Applied Mechanics will be held in Adelaide, Australia, from 24th to 30st August 2008, which means in three years from now. Announcements of this forthcoming congress will be widely distributed and published in many scientific journals. The German members elected to the standing Congress Committee of IUTAM are Professor Edwin Kreuzer, Technical University Hamburg-Harburg, and Professor André Thess, Ilmenau University of Technology.

The present Symposium is exceptionally interesting because it deals with new developments in mechanics. The Symposium covers important approaches:

- Control of machines and mechanisms,
- Active control of vibration in structures,
- Control of multibody dynamics,
- Sensors and observers,
- Applications to cranes and robots.

IUTAM found that the proposal of Professor Ulbrich for such a symposium was not only very timely, but also well justified in the outstanding research carried out in this field at the Technical University of Munich. Thus, the proposal for the Symposium was readily accepted and granted by the General Assembly of IUTAM. There is no doubt that IUTAM considers vibration control of nonlinear systems as an important field of mechanics.

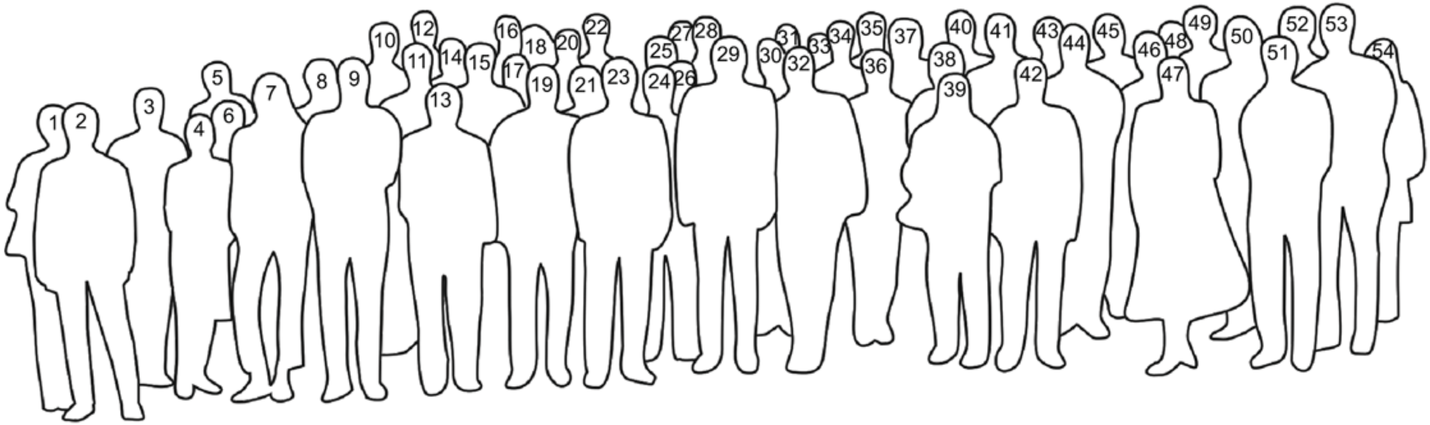
Finally, I would like to mention that to sponsor a scientific meeting is one thing, but to organize one is another. A heavy burden is placed on the shoulders of the Chairman and his associates who are in charge of the scientific program and the practical local arrangements. All who have tried this before know very well how much work has to be done in organizing a meeting like this one.

Thus, we should be thankful, not only to the International Scientific Committee, but also to the Chairman, Professor Heinz Ulbrich, and his associates who assisted him in carrying the heavy load and responsibility.

It is up to you now, Ladies and Gentlemen, to harvest the fruits of the Organizer's work. Contribute your share to make this IUTAM Symposium a meeting that will be long remembered as a very successful one!

On behalf of IUTAM, I greet you all and wish you great success!

Werner Schiehlen
Past President of IUTAM, University of Stuttgart, Germany



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ACTIVE CONTROL OF STRUCTURAL VIBRATION

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Abstract: In its most general form, active control can be used to arbitrarily modify the vibration response of a structure, although in engineering applications the objective is normally to attenuate the vibration. Many actuators and sensors are required to control the vibrations of a large structure, and conventionally these are connected using a single centralised controller, designed using a detailed model of the structure. Very selective control is possible using this approach, which can, for example, modify some modes with little effect on others. However such centralised controllers may not be stable if the response of the structure changes or individual transducers fail, and can become very complicated as the structure becomes large. An alternative approach is to use multiple local control loops in a decentralised arrangement. With careful choice of the actuators and sensors, these loops are simple to design and can be guaranteed stable even if the structural response of the system changes or some transducers fail. Each actuator, sensor and controller can be constructed as a self-contained module, and although the resulting vibration control is not selective such a decentralised system can perform almost as well as a centralised system in reducing the global response of the structure.

Decentralised control for vibration isolation with multiple active mounts will be discussed in this paper as an example of an engineering structure, and this will be contrasted with the way that decentralised control loops enhance the vibration of the inner ear in the active hearing mechanism.

Key words: active control isolation, decentralised control, active hearing.

1. INTRODUCTION

The control of sound and vibration in systems with many degrees of freedom requires multiple actuators and multiple sensors. Conventionally, all the actuators are driven by a single, centralised controller, which is also supplied with signals from all the sensors. Such centralised controllers are often designed using a model of the system under control, such as a modal model, and have the advantage that the controller can be designed so that individual

H. Ulbrich and W. Günthner (eds), IUTAM Symposium on Vibration Control of Nonlinear Mechanisms and Structures, 1–19.

modes can be influenced to different extents [1–3]. The type of actuators and sensors used and their positioning are chosen to best influence or observe these modes. There are two potential disadvantages of centralised controllers First, their performance, and sometimes their stability, can be threatened if the system changes, or if a transducer fails, so that the assumed model is no longer an accurate one. Second, a great deal of wiring is required on large systems to connect all the actuators and sensors to the single controller, and the complexity of the controller rises sharply as the size of the system increases.

In a fully decentralised system, however, a number of independent controllers are used to drive individual actuators from individual sensors. Care must be taken in the selection of the types of sensors and actuators used, however, to ensure their stability. Such a control strategy has the advantages of reduced complexity and wiring and, if all the controllers are the same, it also has the advantage of modularity, so that increasingly complex systems can be controlled with a larger number of identical simple units, and scalability, so the complexity of the system scales only linearly with the number of modes. Because such units only act locally, however, individual global properties of the system, such as mode amplitudes, cannot be selectively controlled.

In this paper decentralised control will be discussed in both the design of controllers for the reduction of vibration in engineering structures, and also in the active mechanism that appears to operate in the inner ear to enhance the vibration of the basilar membrane.

The centralised and decentralised control of vibration in engineering structures is discussed first, using collocated pairs of dual actuators and sensors. Ideal duality in the actuator and sensor pair can rarely be achieved in practical active vibration control systems. The concept of a passive controller, however, which relies on such an idealisation of dual actuator and sensor pairs, can be used, to motivate the combined selection of the control strategy and the details of the actuator and sensor design. The practical example described is the active isolation of a piece of delicate equipment from base vibration. Here conventional passive mounts provide good isolation at high frequencies, but can amplify the base vibration at low frequency resonances. By applying an active force in parallel with the active mount, controlled by the absolute velocity above the mount, "skyhook" damping can be realised to attenuate these low frequency resonances, without affecting the high frequency performance of the passive mounts. The reaction force acting on the base structure, which is generally flexible, spoils the duality of the force acting on the equipment and the sensed velocity. It is fortunate, however, that although this reaction force can be significant, it does not unduly affect the passivity of the system under control. Thus local control loops closed round individual mounts can be shown to be unconditionally stable, even in the event of failures.

This decentralised approach to vibration reduction is then contrasted with the active mechanism in the inner ear, which also appears to be achieved with local feedback loops. Although the details of these control loops are not well understood, they are clearly nonlinear in their action and this nonlinearity gives rise to a number of important auditory effects. Although the mechanics of the inner ear are distributed and nonlinear, and their details are rather complicated, the study of nonlinearities in the ear using simple engineering models appears to hold promise.

2. CENTRALISED AND DECENTRALISED CONTROL OF VIBRATION

The conventional approach to controlling the dynamics of large structures, with many degrees of freedom, is to work in terms of the structure's modes. The dynamic behaviour is approximated by the sum of a finite number of modal contributions, actuators and sensors are chosen so that they can observe and control these modes, and a feedback controller is designed to modify their response. Often the objective is not to actively change the shapes of the modes, but to independently modify the natural frequencies and damping ratios of a number of modes of interest, which has been termed independent modal-space control, IMSC, by Meirovitch [1].

In a large, well defined, structure it may not be difficult to numerically model the first 10 or so modes, using finite elements for example, or to experimentally identify a similar number of modes using modal analysis. It becomes increasingly difficult, however, to distinguish between the different modal contributions beyond this number, particularly in built-up structures with a high modal overlap. Under these conditions the modal approach to control becomes inappropriate. If we are considering the control of large structures, particularly at the higher frequencies for which sound radiation is important, then a reliable modal model of the structure is generally not available and alternative approaches to active control have to be found.

If local, fixed-gain, control loops are closed around dual actuator/sensor pairs [4], the control system is an example of direct output feedback control [1] and has some very attractive stability properties [5]. Since each local feedback loop can only absorb power from the structure, this is an example of a dissipative controller [6, 7], and no combinations of positive gains can destabilise the system. Each force actuator driven by a collocated velocity sensor in fact synthesises an entirely passive damper. In practice vibration velocity is often estimated by integrating the output of an accelerometer, and so the velocity is measured relative to the inertial reference frame. The integrator singularity at dc is avoided since rigid-body modes are not being controlled. The activelysynthesised dampers thus operate with respect to this reference frame and are called skyhook or inertial dampers [8].

Ideal local velocity control thus has very attractive robustness properties, since it is unconditionally stable, whatever the response of the structure to which it is attached, and in spite of any failures that may occur in other control loops. The design of such local loops to achieve a specific performance objective in controlling the structure is less obvious than it was for modal control, however. Clearly the actuator/sensor pairs should be positioned so that they efficiently couple into the modes to be controlled. The individual gains could be optimised to minimise a specific quadratic cost function, using state space methods for example. In general, however, decentralised control is not selective since it does not allow the independent specification of the modal responses that is the attractive feature of modal control. If, for example, a large number of force actuator/velocity sensor pairs with equal feedback gains are uniformly distributed over a structure then the damping of an equal number of the structural modes is increased to approximately the same extent [9].

Apart from the completely decentralised controller described above, there are a number of other strategies for reducing the complexity of fully-coupled controllers in systems with many actuators and sensors, for example based on clustering or hierarchical control [10–12].

3. ACTIVE ISOLATION OF VIBRATION

One important application of active vibration control is to the isolation of sensitive equipment from the vibrations of a base structure. Compliant passive mounts are often used for vibration isolation, and their performance can be quantified in terms of the transmissibility of the complete system, which is the ratio of the velocity of the equipment to be isolated to the velocity of the base structure, as a function of frequency. The transmissibility of a compliant passive mount, modelled as a spring and a damper, when isolating a simple equipment structure, modelled as a lumped mass, from base vibrations are shown in Figure 1. The transmissibility for this single degree of freedom system can be written as

$$\mathrm{T} = \frac{v_e}{v_b} = \frac{1 + 2j\zeta_{\text{pass}}\Omega}{1 - \Omega^2 + 2j\zeta_{\text{pass}}\Omega}, \tag{1}$$

where v_e and v_b are the complex velocities of the equipment and base at a given excitation frequency. $\Omega = \omega/\sqrt{k/m}$ is the normalised excitation frequency where k and m are the stiffness of the mount and the mass of the equipment and the passive damping ratio is $\zeta_{\text{pass}} = c/2\sqrt{km}$ where c is the passive damping in the mount.

Figure 1 shows the calculated transmissibility for two values of mount damping, corresponding to the damping ratios $\zeta_{\text{pass}} = 0.1$ and $\zeta_{\text{pass}} = 0.6$.

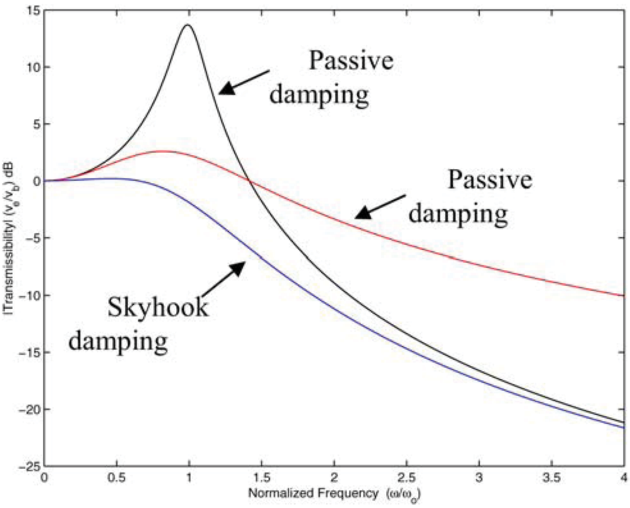

Figure 1. Transmissibility of a single degree of freedom system for the isolation of base vibration with passive damping, having damping ratio of $\zeta_{\text{pass}} = 0.1$, $\zeta_{\text{pass}} = 0.6$ and skyhook damping with damping ratio $\zeta_{\text{act}} = 0.6$.

If the mount is lightly damped the transmissibility is low at higher frequencies, and so the isolating performance is good, but the transmissibility is greater than 0 dB, i.e. the vibration is amplified, at excitation frequencies close to the natural frequency of the equipment mass on the isolator stiffness. If the passive damping is increased, the amplification at resonance is reduced, but significant degradation of high frequency performance results from the increased impedance of the damper at high frequencies. This illustrates a classic trade-off in the design of passive isolation systems [13].

One method of actively controlling the resonance without degrading high frequency performance is to use a feedback system to generate a force on the equipment that is proportional to the absolute velocity of the equipment as illustrated in Figure 2. This generates skyhook damping [8] and if the base structure has a low mobility so that the reaction force acting on the base structure in Figure 2 does not affect the equipment velocity, the action of the feedback loop is to synthesise the additional skyhook damper, as also shown in Figure 2. The transmissibility then becomes

$$\mathrm{T} = \frac{1 + 2j\zeta_{\text{pass}}\Omega}{1 - \Omega^2 + 2j(\zeta_{\text{pass}} + \zeta_{\text{act}})\Omega}, \tag{2}$$

where the active damping ratio is $\zeta_{\text{act}} = h/2\sqrt{km}$ where h is the gain in the feedback loop. The transmissibility with $\zeta_{\text{act}} = 0.6$ is also shown in Figure 1 and it can be seen that the performance is significantly better than the passive

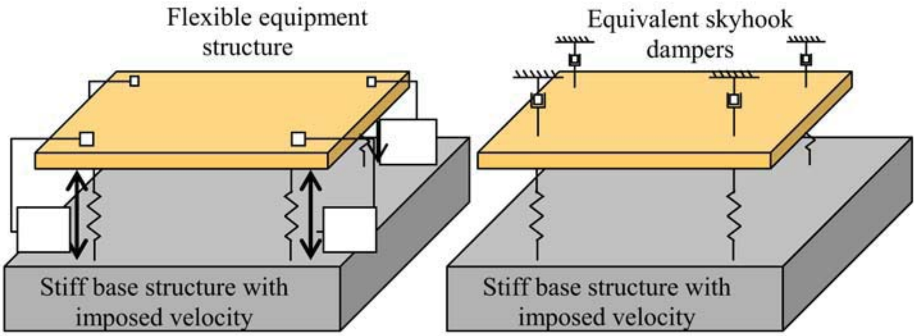

Figure 2. A multichannel active sky damper in which the velocity at each mount is fed back only to a force actuator in parallel with the passive mount (left), the equivalent mechanical system (centre) and the block diagram (right).

system with the same damping ratio at all frequencies, and is as good as the lightly damped passive system at high frequencies.

Apart from the clear performance advantages, the control loop shown in Figure 2 is also very robust [14]. This can be illustrated by considering a single active feedback loop in which $G(j\omega)$ is the plant response from secondary force input to velocity sensor output. If the mobility of the base structure is low then $G(j\omega)$ is equal to the input, or point, mobility of the mounted equipment structure, so that $v = G(j\omega)f$. The mechanical power supplied to equipment structure by this force is

$$W = \tfrac{1}{2}\mathrm{Re}[f^* v] = \tfrac{1}{2}|f|^2\mathrm{Re}[G(j\omega)] \tag{3}$$

and since W must be positive then so must $\mathrm{Re}[G(j\omega)]$, and the plant is said to be passive. The stability properties of the feedback loop can be described in terms of the system's Nyquist plot. Since $H(j\omega) = h > 0$ and the real part of $G(j\omega)$ is also positive, the Nyquist plot for the single degree of freedom active isolation feedback loop is entirely on the right hand side of the imaginary axis. However complicated the dynamics of the equipment are, its input mobility must be passive and so the Nyquist plot will always stay on the right hand side of the imaginary axis and is thus well away from the instability point $(-1, 0)$ so that the control strategy is inherently robust. Serrand [15] and Elliott et al. [14, 16] have shown that even if the mobility of the base structure does become significant, then the stability of the feedback loop is not generally threatened. In practice it is the low frequency phase shifts in the transducer conditioning amplifiers that limit the maximum feedback gains that can be used before instability, although with careful design, reductions in transmissibility of 30 dB at resonance can readily be achieved.

We now consider the stability and performance of a multichannel vibration isolation system, in which each active mount is modelled as above and is controlled by an independent feedback control loop.

One has to be careful in analysing the stability of this multichannel feedback system. The feedback loops can no longer be regarded as independent since they are coupled via the overall dynamics of the equipment structure, which is potentially flexible. The stability of a multichannel feedback control system can again be determined using a generalisation of the Nyquist plot, in which the locus of the real and the imaginary parts of eigenvalues of the matrix $\mathbf{G}(j\omega)\mathbf{H}(j\omega)$ are plotted as ω varies from $-\infty$ to ∞ [17]. In this case the controller is decentralised and each loop is assumed to have a constant gain, so that $\mathbf{H}(j\omega) = h\mathbf{I}$. Also the vector of velocities at the mounting points is related to the vector of actuator forces by $\mathbf{v} = \mathbf{G}(j\omega)\mathbf{f}$, where $\mathbf{G}(j\omega)$ is the fully populated plant response. If the actuators and sensors are collocated and the base structure is solid, $\mathbf{G}(j\omega)$ is the input mobility matrix for the mounted equipment structure, which is symmetric due to reciprocity. The total mechanical power supplied by all the actuator forces can then be written as

$$W = \tfrac{1}{2}\mathrm{Re}[\mathbf{v}^H\mathbf{f}] = \tfrac{1}{2}\mathrm{Re}[\mathbf{f}^H\mathbf{G}(j\omega)\mathbf{f}] \tag{4}$$

and since W must be positive, then $\mathbf{G}(j\omega)$ must be passive. In this case passivity implies that both

$$\mathrm{Eig}[\mathrm{Re}\,\mathbf{G}(j\omega)] \geq 0 \quad \text{and} \quad \mathrm{Re}[\mathrm{Eig}\,\mathbf{G}(j\omega)] \geq 0. \tag{5}$$

The second condition is less well known than the first and can be demonstrated by writing the eigendecomposition of the normal, symmetric but complex matrix $\mathbf{G}$ as $\mathbf{Q}^H\mathbf{\Lambda}\mathbf{Q}$ where $\mathbf{Q}$ and $\mathbf{\Lambda}$ are generally complex. The power supplied to the system is thus proportional to

$$\mathrm{Re}[\mathbf{f}^H\mathbf{G}\mathbf{f}] = \mathrm{Re}[\mathbf{q}^H\mathbf{\Lambda}\mathbf{q}] = \mathrm{Re}\left[\sum_i \lambda_i |q_i|^2\right], \tag{6}$$

where $\mathbf{q} = \mathbf{Q}\mathbf{f}$, and since the power must be positive for all inputs, so must the real parts of the eigenvalues of $\mathbf{G}$. This condition ensures that each of the loops in the generalised Nyquist diagram behave in a similar way to those of the single channel system and are strictly on the right hand side of the imaginary axis, ensuring that the control system is very robust. In particular, the stability is ensured for any degree of flexibility in the equipment structure, no matter how strongly the actuators and sensors are coupled. Exactly the same argument can be used to show that a control system with only three independently-acting active mounts is robustly stable, and so the stability of the system is not compromised by failure of individual mounts.

Figure 3. Experimental arrangement of 4 active mounts isolating the equipment structure from the vibration of the buss structure.

Figure 3 shows an experimental arrangement in which a simple equipment structure is supported on a vibrating plate by four active mounts [18]. The active force at each mount is provided by a shaker that acts through the mount, as illustrated in Figure 4, and is driven from the integrated output of an accelerometer mounted on the equipment close to the mount. Each active mount is thus of exactly the same design and contains actuator, sensor and independently-acting controller. A number of such identical modules could thus be used to isolate more complicated pieces of equipment with no increase in the complexity of the system.

Figure 5 shows the magnitude of the individual frequency responses of the elements of the 4×4 plant matrix $\mathbf{G}(j\omega)$, all on the same scales, as measured on the experimental system. The diagonal terms correspond to input mobilities and have alternating poles and zeros [3], whereas the off-diagonal terms correspond to transfer mobilities. The measured matrix is reasonably symmetric at each frequency as one would expect from reciprocity. Figure 6 shows the generalised Nyquist plot for this control system, as calculated from the measured plant frequency responses. It should be noted that the calculation of a set of eigenvalues that vary smoothly with frequency from a matrix of experimental data is not a trivial matter, as discussed by Huang [19]. Even though the plant matrix in Figure 5 has strong cross coupling, its eigenvalues still have almost entirely positive real parts, as one would expect from equation (5). The

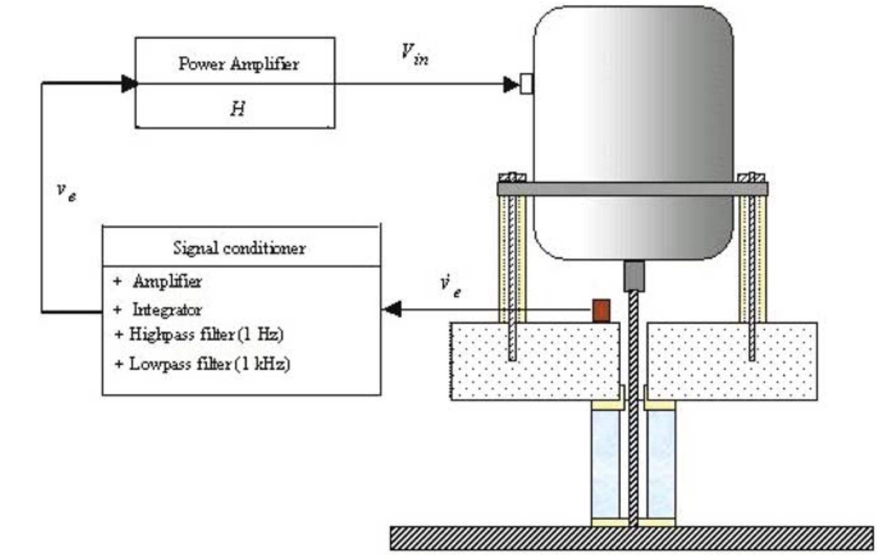

Figure 4. Details of a single control loop, in which the integrated output of the accelerometer is fed back to the shaker to generate skyhook damping.

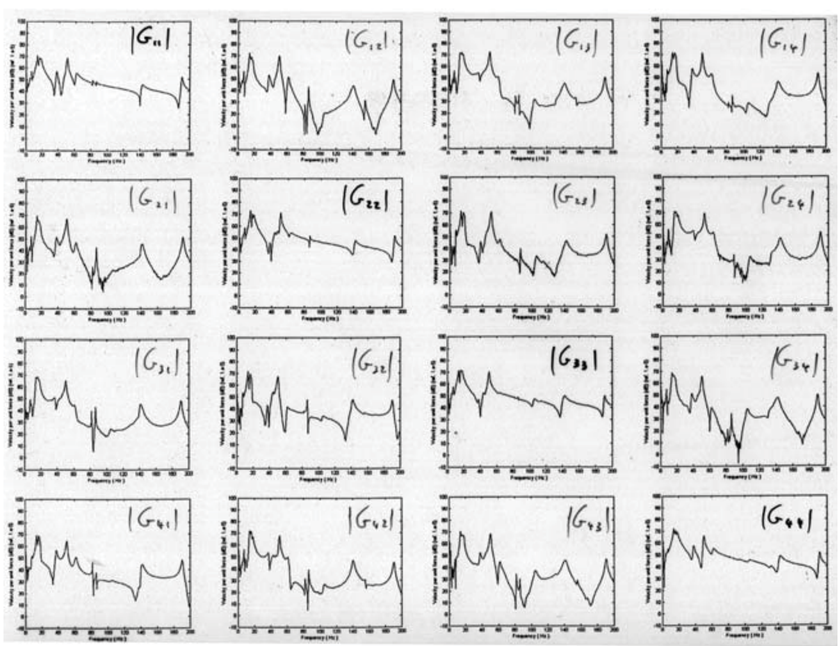

Figure 5. The magnitudes of the elements of the 4×4 plant matrix from shaker inputs to accelerometer outputs measured on the experimental active isolation system.

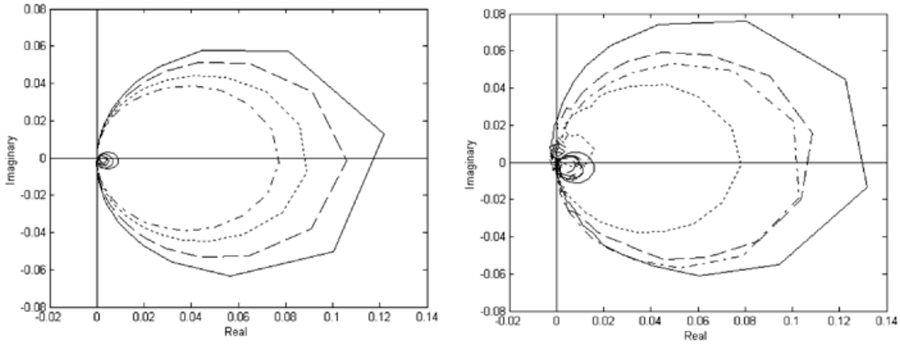

Figure 6. Generalised Nyquist plots of the 4 × 4 plant matrix for the isolation system; from simulation (left) and from experiments (right).

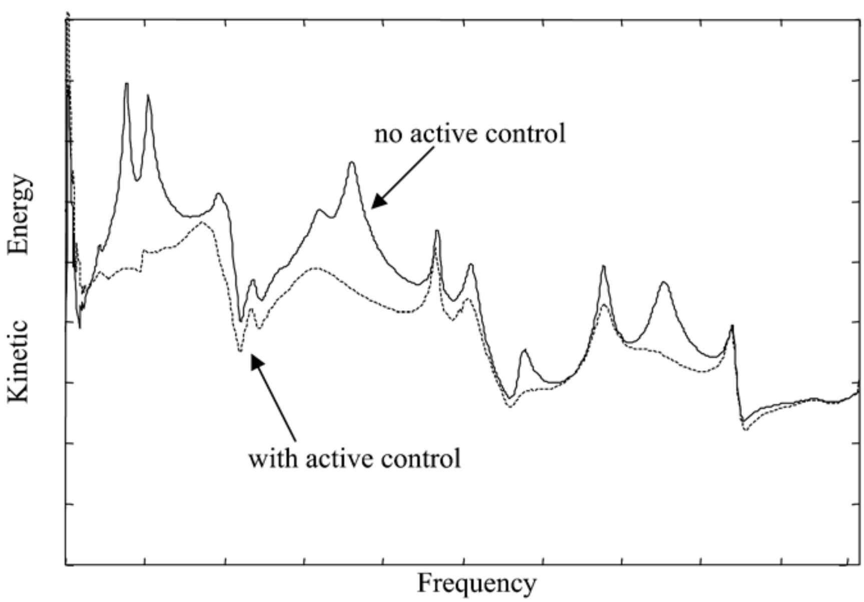

Figure 7. Total kinetic energy of the equipment structure, calculated from the four velocity signals measured at the mounting points of the experimental active isolation system before control (solid) and after 4 channels of local feedback control are implemented (dashed).

slight incursion of the plots onto the left of the imaginary axis is due to phase shifts in the transducer conditioning amplifiers, as mentioned above.

The measured velocities at the four corners of the equipment structure can be used to calculate the amplitudes of its rigid body modes and hence its total kinetic energy. Figure 7 shows the kinetic energy in the experimental system when driven by the randomly-excited base structure alone (no active control), and with all of the four feedback control systems connected (with active control). The peaks at about 20 Hz are due to the heave and pitch resonances of the equipment structure on its mounts, and these have been attenuated by more than 20 dB by the active mounts. The peaks in the kinetic energy above 20 Hz are due to resonances of the plate that forms the base structure in this case. Even though the feedback system was not originally designed to control these resonances, they make a strong contribution to the plant responses in Figure 5 and so the loop gain is large enough to achieve reductions of up to 15 dB in the equipment's kinetic energy at these frequencies. If the kinetic energy is measured over a wider frequency range, it is also seen that the action of the feedback control system does not significantly increase the vibration of the equipment at higher frequencies, as predicted by the simple models at the start of this section. The local feedback loops can thus be designed to control the low frequency transmissibility, while lightly-damped passive mounts can be used to provide high frequency isolation.

4. LOCAL CONTROL IN THE INNER EAR

The high sensitivity and exquisite frequency selectivity of our hearing has long suggested that there is an active amplification process taking place in the inner ear. These active processes are not well understood, but it is interesting to compare some of the current models for these active processes with the types of active vibration control system discussed above, in order to compare these two examples of distributed control. The snail-shaped cochlea in the inner ear contains two fluid chambers, separated by the basilar membrane, which are excited at the end by the bones of the middle ear [20]. The basilar membrane is much stiffer near the entrance to the cochlea than it is at the other end, which results in a distribution of natural or characteristic frequencies along its length. Figure 8 shows a diagrammatic representation of the ear, in which the spiral structure of the inner ear has been straightened out and the mass and stiffness of the individual parts of the limp basilar membrane are made explicit. The lower part of this figure shows the approximate distribution of characteristic frequencies, along the length of the cochlea in the human ear.

The coupling between the inertia of the fluid and the dynamics of the basilar membrane can, with various simplifying assumptions, be analysed to give a wave equation for the propagation of disturbances within the cochlea, see, for example, de Boer [21];

$$\frac{\partial^2 p(x,\omega)}{\partial x^2} - \frac{2j\omega\rho/h}{Z(x)} p(x,\omega) = -j\omega\rho q(x,\omega), \tag{7}$$

where $p(x,\omega)$ is the complex pressure difference between the two chambers, which are filled with fluid of density ρ, x is the position along the cochlea, h is the height of the chamber and $q(x,\omega)$ is a complex source term. $Z(x,\omega)$