International Symposium on History of Machines and Mechanisms

Hong-Sen Yan · Marco Ceccarelli
Editors

International Symposium on History of Machines and Mechanisms

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Preface

The International Symposium on the History of Machines and Mechanisms is the main activity of the Permanent Commission (PC) for the History of Mechanism and Machine Science (HMM) of the International Federation for the Promotion of Mechanism and Machine Science (IFToMM). The first symposium, HMM2000, was initiated by Dr. Marco Ceccarelli and was held at the University of Cassino (Cassino, Italy) on May 11–13, 2000. The second symposium, HMM2004, was chaired by Dr. Marco Ceccarelli and held at the same venue on May 12–15, 2004. The third symposium, HMM2008, was chaired by Dr. Hong-Sen Yan and held at the National Cheng Kung University (Tainan, Taiwan) on November 11–14, 2008.

The mission of IFToMM is to promote research and development in the field of machines and mechanisms by theoretical and experimental methods, along with their practical applications. The aim of HMM2008 is to establish an international forum for presenting and discussing historical developments in the field of Mechanism and Machine Science (MMS). The subject area covers all aspects of the development of HMM, such as machine, mechanism, kinematics, design method, etc., that are related to people, events, objects, anything that assisted in the development of the HMM, and presented in the forms of reasoning and arguments, demonstration and identification, and description and evaluation.

The HMM2008 Proceedings contain 26 papers by authors from all over the world. The topics include historical development on mechanism and machine theory, historical figures and their works, history of mechanical engineering, ancient machines and mechanisms, reconstruction design of ancient devices, mechanism and machine design, and engineering education. This book is of interest to researchers, graduate students and engineers specializing or promoting the history of science and technology, in particular on mechanism and machine science. It is believed that the book would provide the readers with extensive background information on the origin and the history of the invention of fundamental machines and mechanisms, and will undoubtedly provide further understanding and motivation for their own research and/or consultancy work.

The figure on the cover was provided by Dr. Tsung-Yi Lin. It shows a pictorial view of the water-powered armillary sphere and celestial globe invented by Su Song (蘇頌) of the Northern Song Dynasty in ancient China around 1088 AD. This was a water-powered mechanical clock with an escapement regulator. On the top was a massive spherical astronomical instrument for observing the stars. Inside the tower was a celestial globe, whose movements were synchronized with those of the sphere above. At the front of the tower was a pagoda-like structure of five floors, each with a door through which wooden puppets appeared at regular intervals throughout the day and night.

The evolutionists see history as the steady progression through time; the space enthusiasts press on with the exploration of the universe. However, by looking back in time, there is much to learn from the vast treasure of ancient science and technology, from the accumulated knowledge of our ancestors. And, we truly hope the publication of this book will create more interest from people around the world toward the research and publication of the history of machines and mechanisms.

Finally, we would like to express our sincere gratitude to the members of the Organizing Committee of the Symposium: Prof. Hanfried Kerle (Chair of the PC for History of MMS), Prof. Alexander Golovin (Russia), Prof. Teun Koetsier (The Netherlands), Prof. Carlos López-Cajún (Mexico), Prof. Jammi S. Rao (India), Prof. Jae Kyung Shim (Korea), Prof. Junichi Takeno (Japan), and Prof. Lu Zhen (China). We are grateful to the hard work of all the contributing authors and of the reviewers. We would also like to thank the sponsors of the Symposium: IFToMM, Ancient Chinese Machinery Cultural Foundation, National Cheng Kung University (NCKU) and the University Museum, Southern Taiwan University, and National Pingtung University of Science and Technology. Special thanks are also due to the following friends and colleagues for which without their generous support and help, this book would not have been possible: Dr. Sanly Hsin-Hui Huang, Dr. Tsung-Yi Lin, Dr. Kuo-Hung Hsiao, Dr. Sin Sin Hsu, Ms. Gretle Yu-Lin Chu, and students in the Creative Machine Design Research and Education Lab in the Department of Mechanical Engineering at NCKU.

November 2008, Tainan, Taiwan.

Hong-Sen Yan
Chairman, HMM2008
International Symposium on History of Machines and Mechanisms

Marco Ceccarelli
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Contents

History of Dynamics of Machines and Mechanisms from Leonardo to Timoshenko

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Abstract In this paper we review the use of dynamic analysis in the evolution of machine and mechanism design. Our thesis is that the application of analytical methods in dynamics to machines and mechanisms lagged behind the application of these methods to non-machine areas of science and engineering such as planetary dynamics and structural dynamics. The early works of Mertzalov, Den Hartog and Timoshenko are reviewed.

Keywords Dynamics, Machines, Mechanisms, Leonardo da Vinci, Reuleaux, Timoshenko

Introduction

Historical reviews of dynamics usually treat the motions of particles and rigid bodies but rarely mention applications to constrained rigid bodies in mechanisms and machines (see e.g. [24,83]). In the same spirit, historical reviews of kinematics of mechanisms do not discuss dynamical problems in machines. (see e.g. [61,62,33,27]) The author has recently published a book on the history of kinematics in machines from the time of Leonardo to Reuleaux. [63]. Often dynamics histories describe the evolution of general principles of physics but do not consider the application to technology. In this survey we attempt to put together an outline the history of technical dynamics as applied to mechanisms and machines.

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The histories of kinematics of mechanisms and dynamics have evolved on different paths and time tracks. In some cases the applied dynamics of mechanisms such as clocks were understood before theoreticians had a deep understanding of theoretical dynamics. Galileo described the motions of the pendulum and Huygens invented a pendulum clock in 1658 before Newton published his *Principia* in 1686 [39]. Even then it was not until 1760 before Euler had written his treatise on dynamics of rigid bodies. In addition to the history of general principles there are many dynamical phenomenon connected with machines that have only now been understood mathematically such as nonholonomic dynamics of rolling and control of robotic systems. Even in the realm of general principles, the phenomenon of unpredictable dynamics known of chaos theory has only recently been understood and even less so in its application to machines (see e.g. [59]).

Dynamics of mechanisms developed as a formal sub-area of the theory of machines around the beginning of the 20th century. For example in the textbook *Mechanisms* written by S. Dunkerley [25], who was Director of the Whitworth Laboratory at the University of Manchester, three areas of machine design are outlined: kinematics, machine element design and dynamics of machines. Some aspects of dynamics of machines can be found in the work of Rankine [67] and Redtenbacher [69] in which mention is made of the dynamic principal of virtual velocities. However in the major books of Franz Reuleaux who is often called the "father of kinematics" there is no discussion of dynamical principles in either *Kinematics of Machinery* [74–76] or *Der Constucktor* [73,77].

Dynamics vis a vis Kinematics of Mechanisms

Before we proceed too far it is necessary to define our terminology because in classical mechanisms the terms kinematics and dynamics of connected rigid links are sometimes used synonymously. We define a *dynamics-based* problem in machines and mechanisms when either accelerations are required to determine forces (*indirect* dynamics problem) or if differential equations of motion, based on the Newton-Euler principles of mechanics, must be solved to determine the motion of all the parts (*direct* dynamics problem). Included in the direct problems are regulated and controlled machines, including modern robotics that may require control theory in addition to Newton-Euler theory. On the other hand classical engine balancing is considered an *indirect* problem in dynamics of machines.

Dynamics of Machines in Antiquity

To begin our review we survey what is known about dynamics of machines in the historical records of the ancient Greeks and Romans and European civilizations.

Aristotle (384–322 BCE)
Of particular interest to us is his *MHXANIKA*, or *Mechanical Problems,* published in modern editions under the *Minor Works* of Aristotle. *Mechanical Problems* is more a mathematics text than an engineering manual. The focus of the Greek mathematics was on the force equilibrium nature of the simple machines and less on the motion or dynamics character of the devise. However the chariot, potters wheel and catapult were by nature dynamic machines. Aristotle's simple machines were focused on forces and not motions. The idea of the inherent kinematic nature of many mechanisms did not aappear until the time of the French school at the Polytechnique in Paris under Gaspard Monge at the end of the 19th century.

Archimedes (287–212 BCE)
His work as a designer of machines however is usually ascribed through other ancient writers. For example, Plutarch in writing about the Roman general Marcellus, tells how Archimedes designed machines for war against the Romans in 212 BCE. These included dynamic machines to hurl missiles and large stones at the enemy as well as an underwater mechanism of levers and pulleys that could destabilize and overturn a ship entering a harbor.

Ctesibius (2nd C. BCE)
Ctesibius' is another engineer schooled in Alexandria and who is often credited with inventions but of whom we have no extant works to document his contributions. Nevertheless, he has been identified with clock mechanisms and geared devices. In the Roman work by Vitruvius Pollio (circa 50 BCE), Ctesibius's water clock is described as the first to have a regulator that would maintain a constant head of water in the effluent part of the clock in order to improve the accuracy. Ctesibius is also recorded as inventing various automata or moving mechanical animals driven by his clock.

Hero of Alexandria (2nd C. BCE)
Hero is one of the few Greek engineer-mathematicians whose written works have come down to us. Among his dynamic machines are catapults and balisti. These devices could launch both stones and arrows. He also published a book on automata. An Italian translation of 1589 by Bernardino Baldi contains a drawing attempting to reconstruct one of these devices for a fountain offering wine and milk and having a rotating figure on top. The automata is driven by a hidden falling weight that creates a torque on a rotating cylinder.

Vitruvius Pollio (c. 37 BCE)
In *de Architectura libri decem (c. 37 BCE)* he proceeds in Book X to describe the existing machines of Roman times as well as their methods of construction. There is some discussion of pumps in Book VIII, as well as water clocks in Book IX. However in Book X we can find an encyclopedia of descriptions of many applications of machines. Here he described numerous dynamic machines of war including scorpions, catapultae and ballistae. In Chapter 11 he gave detailed instructions for a catapult capable of hurling stones. It is likely that he must have had experience in the

dynamics of such machines because his dimensions are very detailed and specific. In Chapter 13 he described a battering ram that is essentially a pendulum comprised of a large beam supported by ropes swung back and forth to develop sufficient kinetic energy to transfer into impact to knock down the enemies defenses.

Dynamics of Machines in the Manuscripts of Leonardo da Vinci

The Renaissance in the 15th century produced important machine engineers in both Siena and Florence (see Moon [63], for a review of history of machines in the Renaissance). Before the work of Leonardo, there was a school of machine inventors in Florence's rival city Siena; the principal engineers were Taccola and Francesco di Giorgio Martini. The latter's work was reproduced in several books and included dynamic machines such as catapults and trebuchets. One of these books was in the library of Leonardo da Vinci.

There has also much been written about Leonardo's writings on mechanics and dynamics. Criticism has been made of Leonardo's observations on the laws of dynamics, statics and fluid mechanics due to often contradictory writings (see e.g. [86]); in one place noting prescient observations of laws of physics that were discovered later, while in other places espousing outdated concepts from classical Aristotelian physics. However our focus here is on his observations of dynamics as it applied to constrained mechanical systems such as machines and mechanisms of which Leonardo made a few contributions.

For example, one of his most direct discussions of a dynamic nature is on flywheels, and particularly about masses on chains attached to a spinning axis (Fig. 1). Here is Reti's translation from *Codex Madrid I*;

"Why do the weights which hang perpendicularly at the beginning and at the end [of the motion] take up, together with their chains a horizontal line while they are in motion?"

[Observations about the use of flywheels can also be found in the machine book of Francesco di Giorgio, c. 1460.]

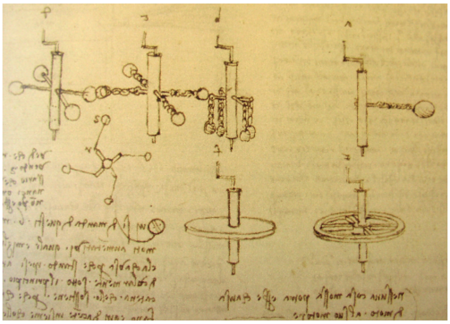
In remarks relevant to another drawing of a flywheel turned by a rope wrapped around a shaft and under a weight under gravity, he writes;

"The question here is: how many times would the wheel turn by itself, once the cord of the counterweight is completely unwound—?" This drawing is somewhat analogous to the sketch in 1589 by Baldi of Hero's automata fountain.

With respect to dynamics of machines, Leonardo only addresses dynamics issues in the context of specific machines and not in terms of general principles.

One of the more striking drawings and directed discussions of a dynamic mechanism is the perpetual motion wheel. (*Codex Madrid I* 147 verso and 148 recto). Leonardo seemed to describe a sequence of motions and impacts that might allow the wheel to continue moving. However in characteristic Leonardo style, on the second page, he emphatically states the impossibility of such a device.

"therefore, as it has been demonstrated, such a wheel is sophistical".



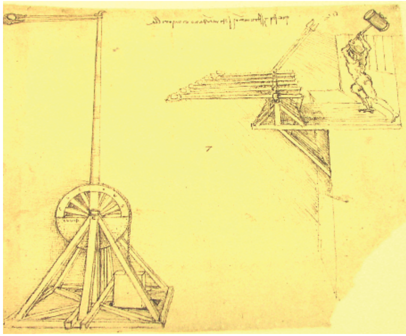


Fig. 1 Drawings of flywheels showing centrifugal effects from Leonardo da Vinci's Codex Madrid, Folio 114 recto (*left*)

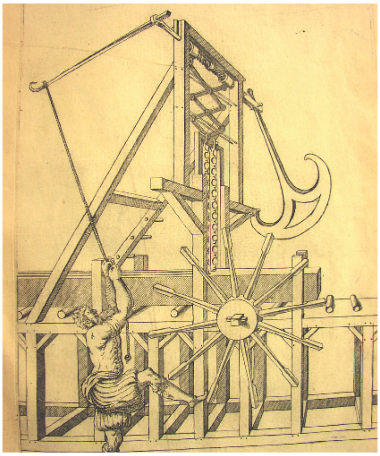
Fig. 2 Leonardo da Vinci drawing of a double pendulum trebuchet war machine from the *Codex Atlanticus* (*right*)

This perpetual motion wheel appeared in other sources in the time of the Renaissance besides Leonardo and the concept of conservation of energy would not be formulated until the mid 19th century. His conclusion on the implausibility of this device was not made from general principles but as stated above, from his attempt to analyze the specific sequence of dynamic events were the wheel to work. It is interesting to note that although Leonardo da Vinci drew one of these perpetual motion wheels, he declared that they could not work. In later books, such as *Theatrum Machinarum Novum* by Böckler [14] and Jacob Leupold [49] [*Theatrum Machinarum Generale*], one can find machine designs that would supposedly run continuously without power input.

Among the most dynamic mechanisms is Leonardo da Vinci's work is his drawings of trebuchets in the Codex Atlanticus that are essentially nonlinear double pendulum devices (Fig. 2). Certainly Leonardo da Vinci's designs for human flying machines were dynamics dependent.

Theatre of Machine Books: Besson (1569) and Böckler (1661)

In the 16th, 17th and 18th centuries there were published books illustrating and describing a wide range of machines and applications. Some of these portray machines that use dynamic principles for their operation. For example in the 16th century Book of Besson called the "theatre of machines" he illustrated the use of pendulum resonance to actuate pumps using a mangle mechanism (Fig. 3). The worker supposedly applies a periodic torque through a crank on the pendulum at the resonance frequency of the pendulum thereby setting it into large oscillations that are used to drive the pump. A similar scheme can be found in later "theatre of machines" books. Another example is the use of large verge and foliot escapement to control a pump (Böckler, 17th C.) shown in Fig. 4.



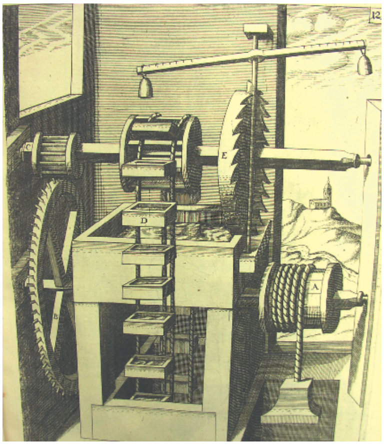


Fig. 3 Woodcut of a saw mill machine using the resonance of a pendulum [7] (*left*)
Fig. 4 Engraving of pump escapement [13] (*right*)

Analytical Dynamics in the Industrial Age

Although the greatest advances in analytical dynamics of particles and rigid bodies took place in the pre-industrial and industrial age, little of this theory was utilized in the design of machines by either the machine practitioners nor their theorists in the universities. Isaac Newton's *Principia*, published in 1686, was a treatise on dynamics of particles and their behavior in gravitational force fields. The Swiss mathematician Leonard Euler was the principal theorist on rigid body dynamics whose formalism we still use today. His major work in this area was *Theoria motus corporum solidorum seu rigidorum*, published in 1760, 40 years after Newcommen's steam engine was deployed in the Cornwall mines in England.

Joseph-Louis de Lagrange (1736–1813) in his *Mecanique Analytique* [44], developed powerful mathematical tools to study the motions of constrained systems such as connected rigid bodies in a machine or mechanism. This work was published during Watt and Boulton's monopoly on steam engine manufacture that saw the installation of hundreds of steam engines. Yet machine dynamics was not studied with these new techniques in the analytical dynamics community.

In the new technical universities such as Ecole Polytechnique in Paris, in the late 18th century, Gaspar Monge introduced descriptive geometry methods that became a mathematical underpinning of kinematics of machines. Ecole Polytechnique was established in 1894 and the mathematician Gaspard Monge was one of its founders. Shortly thereafter, Lagrange received an appointment to teach analysis there. He had a reputation however of being a poor lecturer as he had an Italian accent, having been born in Turin. Later books on mechanisms by Hachette, Lanz and Betancourt [30] and Borgnis [15], were inspired by the work of Monge in descriptive geometry, but did not include any material of a dynamics favor from Lagrange.

It is also interesting that Lagrange's analytical dynamics did not make it into the German texts in machine design though he spent 20 years at the Academy of Sciences in Berlin from 1767 to 1787. He also wrote his famous *Mecanique analytique* there in 1782 that was later published in 1788 in Paris. Lagrange had been recommended for the Berlin position by Euler who himself had been in Berlin and had moved to the court at St Petersburg. Euler, Lagrange, and D'Alembert all exchanged letters and ideas during this time, but somehow in the critical period of 1798–1810, these dynamical theories were not absorbed into the culture of machine theorists.

During the 19th century, theoretical work of Hamilton, Jacobi, Caughy, Navier, and Poincare in dynamics developed but again these methods were not incorporated into machine design teaching or practice until the mid 20th century.

Dynamics of Machines in the Industrial Age 1750–1900

James Watt (c.1780): Regulators

Historians of kinematics are quick to point out that James Watt's proudest invention was the approximate straight-line mechanism [87]. A portrait in the National Gallery of London of Watt shows him contemplating a drawing of this linkage applied to the steam engine. However, for those who write of the history of control engineering, it was his use of the rotating ball speed regulator that was his most important contribution and the first large scale use of feedback control (some might argue that the directional control of windmills was the earliest automatic controller. See Bennett [5]), Boulton and Watt did not seek patent protection for the ball governor but thought it might go unnoticed. However it was soon copied and improved as can be seen in the version of Poncelet from the mid 19th C. The ball mechanism was only a form of sensor-actuator and also required linkages and control valves. In some engine controllers, instead of maintaining constant speed there was the dynamic phenomenon of hunting. This sparked an effort by some to develop an analytical model to derive stable operating condition for these controllers [see discussion below on servomechanisms and Maxwell].

J.-A. Borgnis (1818)

The Italian J.-A, Borgnis in 1818 published a treatise in French in which he categorized machines into six orders as "recepteurs, communicateurs, modificateurs, supports, regulateurs and operateurs" [15]. It is within the class of regulators that he included dynamic mechanisms such as escapements. He wrote about their use in clocks and their connection with pendula. Borgnis discussed in detail the contributions of several figures in the history of clock design including Hugyens, Gramham, Berthoud, Tompion, Le Roy, Thomas, Mudge, Bre'guet and others. In particular he discussed clock regulator subcomponents such as the fusee, remontoir, and different types of escapements. He also discussed the temperature compensation of pendulums for clocks. This work is completely descriptive with 36 plates of about 8–10 figures each and no mathematical equations.

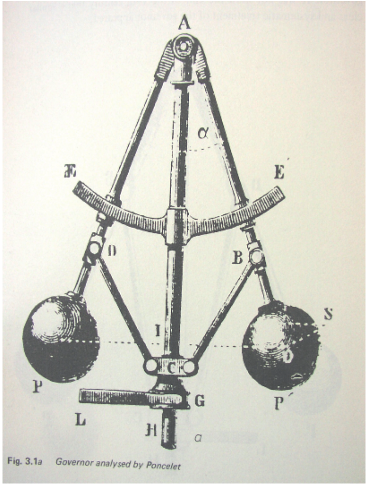


Fig. 5 Rotating ball governor of Poncelet (from [4])

Haton de la Goupillie're (1864)
This is a mid 19th century French book on the theory of mechanisms: *Traite des Me'canismes* [37]. Haton cites the work of Lanz and Betancourt [45], Willis [89] and Laboulaye [43]. The major part of this work discusses more than 200 mechanisms with over 250 drawings. However the second part of this book is titled "Etude Dynamique des Me'canismes". The major part of this section treats the laws of friction in machines. There is one section however that derives the differential equation of oscillation for a cylinder rolling inside of another cylinder in a style that anticipates vibration texts 80 years later such as Den Hartog [22]. Haton also discussed clock escapements including the anchor and cylinder escapements but did not give any analytical treatment of them. In a section on regulators he analyzed Watt's rotating ball regulator and similar devices and wrote equations of motion balancing the centripetal force moment with the gravity force moment. This is the extent of Haton's treatment of dynamics of mechanisms. It basically contains no general principles and a few ad hoc dynamics related problems.

Julius Weisbach (1848)
Julius Weisbach (1806–1871), like Redtenbacher and Rankine wrote general books that helped define the teaching of what would become mechanical engineering [88]. Weisbach's books were translated into English and thus his work was familiar to engineers in both Great Britain and North America. For example, his *Principles of Machinery and Engineering*, first published in America in 1848 was a two volume work that was expanded in its 1870 edition into three volumes. This work presented some general material on dynamics of rigid bodies, dynamic stability and the theory of oscillations.

Franz Reuleaux (1859, 1876, 1893)
The Reuleaux ''School'' of kinematics, which included Kennedy [41] in England and Burmester [18], Hartmann [36], and Grübler [29] in Germany, influenced the kinematics of machines to this day [72,75–77]. Although Reuleaux's theory of machines were important contributions, his theories were based largely on geometric ideas, or Phoronomy, and not on dynamic principles, that were later incorporated into the theory of machines (see, e.g. [34]). Nor did Reuleaux treat the problem of nonholnomic constraints. In general, however, ideas about the importance of elastic vibrations, resonance, or structure-borne noise in machines did not make it into Reuleaux's work.

There are examples in his oeuvre where he exhibited interest in dynamics and control. One was a 1859 paper in the new journal of the German society of engineers or VDI. He titled this paper [72] "Regulatorfrage" or regulator questions. Reuleaux also published a paper in 1876 titled, "Das Zentrifugalmoment: Ein Beitrag zur Dynamik". [The centrifugal moment: A contribution to dynamics]. Reuleaux found a general equation for the centrifugal force of extended rigid bodies rotating about an axis. He used integration methods to relate the force moment to the principal moments of inertia of the rotating body and designed an experimental apparatus to measure this moment (Fig. 6). However, this result in dynamics did not appear in his major books.

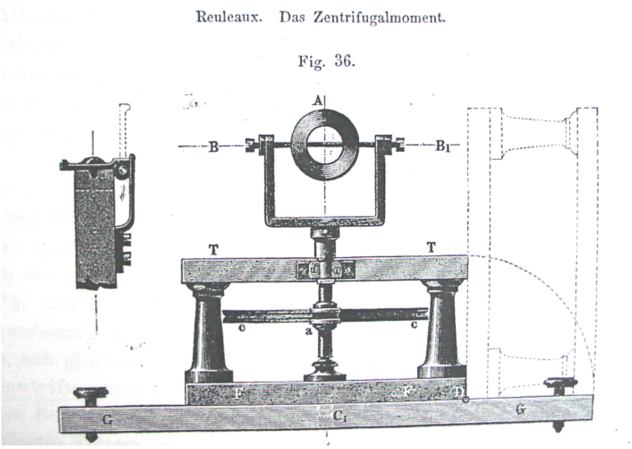


Fig. 6 Drawing of Reuleaux's design for an experimental apparatus to measure the moment of inertia of balls for a Watt type governor

Reuleaux was also interested in the regulation of steam and gas engine motions which he discussed in the 4th Edition of his The Constructor (1893). In these engines, slide and rotary valves were opened and closed during each machine cycle to admit steam or air-fuel mixtures or to exhaust steam or gas from the engine cylinder. In an early 20th century book by Bevan [9], he refers to "The Reuleaux Diagram", a phase diagram to describe the timing between the valves and crank motions of a steam engine. There is evidence that Reuleaux understood the concept of feedback control in a figure from The Constructor, Figs. 1012, 1213, p. 231, in which he tried to explain the workings of two coupled regulators of a commercial machine. This figure has the features of a block diagram, an idea that did not appear in control theory until several decades into the 20th century.

M. J. Callon (1875)

As an example of a French text on design of machines we examine the work of M.J. Callon whose course on machines at the L'Ecole des Mines de Paris largely dealt with the steam engine of mid 19th century Europe [19]. In Chapter 18 titled "Details on the parts of the steam engine" Callon discussed Watt's rotating ball speed regulator. He used the balance of the centrifugal force moment on the rotating balls and the gravity force moment to derive design equations (Figs. 255–265). This method converted a dynamics problem into a static equilibrium problem. However, it was Maxwell who about this time, examined the stability of governors in steam engines from a more modern point of view.

Alexander Kennedy (1886): Acceleration diagrams

Kennedy translated Reuleaux's major work in kinematics of machines in 1876, one year after the German publication. Both Kennedy [41] and Burmester [18] introduced the acceleration diagram that became a staple part of the kinematics pedagogy for the next half century.

One example in Kennedy's book describes the acceleration history of a piston in a pumping engine called the "Bull Engine" [Page 336, §45; see KMODDL website for digital copy; http://kmoddl.library.cornell.edu] Kennedy notes the importance of dynamics in this machine: "Kinematically the combination is nothing but a sliding pair of elements, there is no crank or rotating parts of any kind. Dynamically the machine is of much more interest, and its action much more complex. For although the form of the piston and cylinder prevent any relative motions—they do not in any way affect or control the velocity of motion and the length of the stroke of the engine is entirely dependent on the acceleration forces in action,—".

Ludwig Burmester (1888)

Burmester was an ardent admirer of Reuleaux's kinematics of machinery. In his *Lehrbuch der Kinematik,* he advanced and embellished Reuleaux's geometric theories especially in the area of centrode theory of rolling [18]. Unlike Reuleaux, Burmester included a long section on accelerations in mechanisms and presented graphical methods to calculate these terms especially four-bar and slider crank mechanisms [Chapter 11: "Die Lehre von der Beschleunigung und irhe Anwendung"]. It is odd that in his introduction Burmester mentions the work of Newton, D'Alembert, Euler, Carnot and Kant. He even cited Euler's 1765 theory on the motion of rigid bodies that was translated into German in 1853. However after an extremely detailed treatment of acceleration diagrams, Burmester did not present a general treatment of dynamics or vibrations in machines and mechanisms.

Clock Dynamics

The first pendulum clock is attributed to Huygens in 1657, although even here there is some posthumous claim to the invention by Galileo and his son. The Huygens clock is a combination of the verge and the pendulum. Huygens also recognized that the period of the pendulum increased with the amplitude and he designed a cycloidal clamp for the pendulum which decreased the effective length of the swinging bob to produce a constant period, independent of amplitude. This is one of the first solutions in nonlinear vibrations.

The next major improvement was the invention of the anchor escapement that replaced the verge with a two-arm device. Other contributors at this time were Pierre Le Roy and Ferdinand Berthoud of France as well as Arnold and Earnshaw in England. Many other escapements were invented such as the detent, cylinder, duplex, pin wheel, and gravity escapement, over a period of four centuries of clock invention, design and development [16] (see Fig. 7). Despite this progress, the historical record is replete with evidence and discussion of the irregularities, inaccuracies and unpredictability in the mechanical clock.

Early works on the dynamics of clocks include George Biddell Airy [1], James Mackenzie Bloxam [11], and Edmond Beckett Denison (Lord Grimthorpe) [23]. Other important mathematical analyses in the 20th century were those of the Russians Andronov, Chaiken and Witt (circa 1940–1960) [2]. Recently there have appeared a series of papers on the mathematical analysis of escapement dynamics such as, Kauderer [40], Kesteven [42], Lepschy et al. [48], Bernstein [6] and Roup and Bernstein et al. [78] as well as a work by the Author [64].

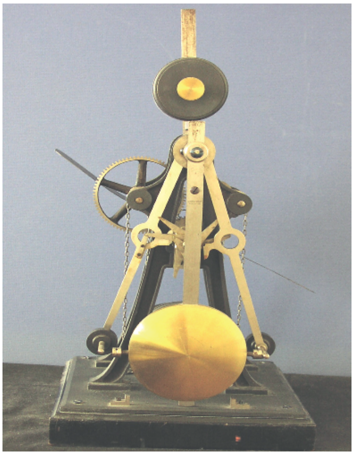


Fig. 7 Reuleaux model of a three pendula gravity escapement [from the Cornell Reuleaux kinematic models collection]

Governors, Servomechanisms and Control Theory

The two principal historical texts in this area are Otto Mayr's *The Origins of Feedback Control* [54] and S. Bennett's *A History of Control Engineering 1800–1930,* written in 1979 [4]. Mayr's work covers the period from antiquity to around 1800. There have been many other reviews of control engineering since. However for us the focus is how this subject of control and its associated questions of dynamics and stability entered the teaching of machine design. Many writers credit James Watt with the invention of the rotating ball regulator in 1788 to achieve speed control of the steam engine. However Mayr cites earlier use of the rotating ball governor for control in windmills by Mead around 1787.

In Maxwell's paper [53], which was published in the Proceedings of the Royal Society, there are no sketches or pictures of any governor mechanisms, although there are several in Bennett's book. There is reference to several governors of Watt, Jenkins, Siemens etc. but Maxwell's analysis uses general abstract terms to describe forces or torques in these machines and no specifics. He obtained a third-order dynamic system coupling the governor dynamics to the machine or "plant" motions and thus found a stability criterion for the controller to avoid instabilities. Although his mathematical models have some generality, it is not clear if they apply to actual devices since, none of the parameters are estimated by Maxwell. There is also a hint in Maxwell's paper that there was anecdotal evidence for engine instabilities with governors as Maxwell calls it, "oscillating and jerking motion, increasing in violence until it reaches the limit of action of the governor".

Maxwell and others such as E.J. Routh [79] and the Russian work of J. Wischnegradski (1876–1879) in St Petersburg and A.M. Lyapunov [50] laid out the ideas of stability of motion in mechanical and electrical systems by the end of the 19th century. It is interesting to note that Lyapunov was a student of Chebyshev at St Petersburg. The latter had spent many years analyzing the kinematic geometry of linkages and mechanisms. There is the question of whether the teaching of dynamics of machines and kinematics of mechanisms was more unified in Russia than that in Europe and North America.

As outlined in Bennett [4], the use of speed controllers in the 19th century, evolved into the field of servomechanisms (Fig. 8). Initially both feedback and control actuation were accomplished with mechanical linkages but were gradually replaced with electromechanical sensors and actuation in the early 20th century. Still, the teaching of control theory in the late 20th century was often devoid of specific machine knowledge. Two exceptions were in gyro design and aircraft control, in which detailed knowledge of the plant was part of the control culture.

Gyroscopes

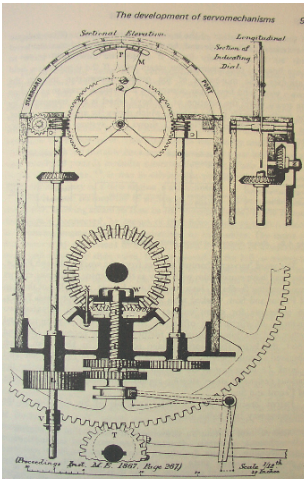
Spinning tops have always had a fascination with people and can be traced back to ancient civilizations in China and the Middle East and Mediterranean cultures. In 1760, The dynamic theory of rotating rigid bodies was first formulated by Euler.

A century later, J.B. Leon Foucault, using his famous pendulum in the Paris Pantheon determined that the spinning of the Earth could be detected in the precession of the plane of rotation of a pendulum. Around the same time he invented a spinning top that could also detect the Earth's rotation. He was the first to give such a device the name "gyroscope".

Despite Foucault experiments with the use of a spinning body as an angular reference, the first practical use of the gyro compass for navigation was developed in Germany by Hermann Anschütz around 1908. Later in 1911, Elmer Sperry in the US developed a gyrocompass for ship navigation that was easier to manufacture (see e.g. [91]).

A theoretical treatise on these principles appeared in 1897 by F. Klein and A. Sommerfeld; *Über die Theorie des Kreisels,* which was published later in several editions. Despite the span of theoretical and mathematical discussion of the dynamics of rotating bodies from Euler [26] to Klein, there remained technical problems to overcome before this technology became practical machinery. To quote the book by the MIT Draper lab engineers, Wrigley et al. [91]: "The history of precision gyro technology is, as is the case for most practical devices, a story of methods, materials, engineering skill, and perseverance rather than of scientific breakthrough or new physical principles".

However it still does not explain why the design of the gyro mechanism was absent from most kinematics of machines and machine design books of the 20th century.



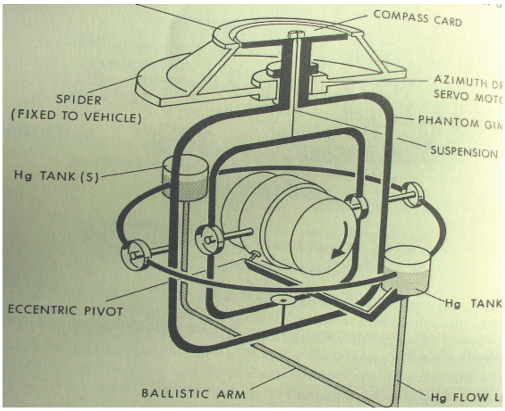


Fig. 8 Servomechanism for ship steerage of McFarlane Grey (from Bennett) (*left*)

Fig. 9 Sketch of design for gyrocompass of Sperry (from Wrigley et al.) (*right*)

Early 20th Century Dynamics of Machines

The lack of common ground between practitioners of analytical dynamics and technical mechanics of machines is illustrated in the career of Arnold Sommerfeld (1868–1951). Born and educated in Köingsberg, Prussia, he taught at Göttingen, Clausthal, Aachen and Munich. In Clausthal (1897–1899) he was professor of mathematics, while at Aachen (1899–1905) he held the chair of Technical Mechanics at the Technische Hochschule. Although known for his contributions in applied mathematics applied to quantum mechanics, Sommerfeld gave lectures in classical physics and mechanics. For example, his lectures in mechanics at Munich were published in English after World War II [81]. Scattered throughout this text are a number of examples relating to the dynamics of engines, including the slider-crank mechanism, balancing of a four piston marine engine and the equations of motion of an automobile differential, juxtaposed amidst mathematical exposition on Hamilton–Jacobi theory and the quantum treatment of Kepler's orbital mechanics. Although the first German edition of these lectures was published in 1942, it is likely that Sommerfeld's interest in dynamics of machines was born during his tenure at the Technical University at Aachen at the turn of the century. As in the case of Lagrange at the Ecole Polytechnique a century earlier, the mathematical theory that Sommerfeld brought to mechanics of machines was not incorporated into engineering textbooks and practice in a systematic way until the late 1920s and early 1930s. For example in Timoshenko's 1928 book *Vibration Problems in Engineering* [84], he references an experiment of Sommerfeld in 1904 on the resonant vibrations of an unbalanced rotor.

Dynamics of Engines: Inertia Forces and Balancing

Although there were a few serious mathematical studies of a dynamical nature in technology at the beginning of the 20th century, these tended to be carried out by mathematical scientists and not engineers or as in the case of Van der Pol for the new field of electric circuits. Practitioners in the field of kinematic design of machines generally took only small steps toward full dynamic analysis. One example of such books was by Ham and Crane [32] of the University of Illinois (Fig. 10). The text reviews all the standard elements of kinematic analysis, pairs, chains, inversions, point paths etc. Near the end of the text however there is an analysis of acceleration and "inertial forces". The problem of engine balancing became important with the expansion of the use of the automobile and became more prominent in machine engineering books such as Den Hartog [22]. *Dynamics of Machines: Den Hartog and Timoshenko (1928–1948).*

The reduction of dynamic problems in machines to equilibrium of static forces and "inertia forces" was soon replaced by full dynamic analysis with the recognition that machine elements are elastic components. This often involved the use of vibration theory, a mathematical method that appeared in the 19th century in works such as Rayleigh [68] in his theory of sound.

One of the important English books on the dynamics of machines was *Mechanical Vibrations* by J.P. Den Hartog of Harvard University written in 1934 [22]. Den Hartog had worked for the Westinghouse Corporation when he arrived from Europe in the 1920s as did Steven Timoshenko. Den Hartog's book acknowledges his indebtedness to his former colleagues at Westinghouse as well as to Professor Timoshenko who first went to the University of Michigan and then to Stanford University. Many of the problems in Den Hartog's book arose out of machine related problems including gear systems, vibration absorbers in machines, ship stabilization problems, automobile shock systems, multi-cylinder engine dynamics, balancing or rotors, hunting of steam engine governors, wheel shimmy and self excitation of fluid valves (Fig. 11). At the same time Den Hartog introduced advanced analytical methods such as coupled linear systems, subharmonic resonance, nonlinear vibrations, relaxation oscillations. Both Den Hartog and Timoshenko brought advanced analytical techniques to America from Europe and Russia and combined them with experience with practical problems in industry.

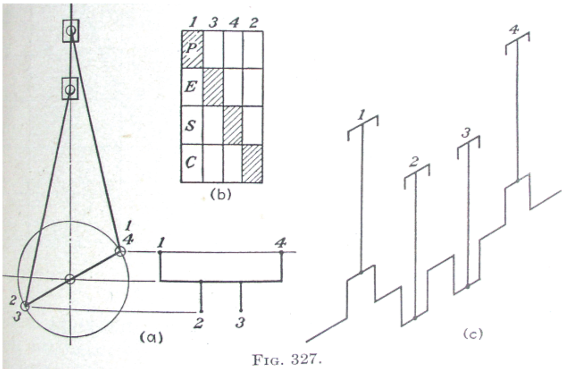


Fig. 10 Balancing of a 4 cylinder engine (Ham and Crane)

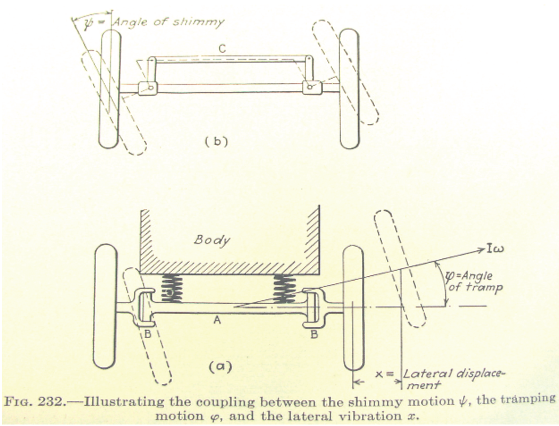


Fig. 11 Wheel Shimmy model from Den Hartog [22]

Steven Timoshenko (1878–1972) was born in the Ukraine and educated in St Petersburg. Ten years younger than Sommerfeld, he worked in railroad engineering. He came to the United States in 1922. His first dynamics book was published in 1928 under the title *Vibration Problems in Engineering*, when he was at the University of Michigan [84]. He also acknowledged his indebtedness to the Westinghouse Corporation as well as to Den Hartog with whom he had worked on several dynamics problems on electric generating systems for Westinghouse. His work not only dealt with linear systems, but also "non-harmonic vibrations", using the dynamics of locomotives as an example. He also dealt with vibrations of turbine blades and described several vibration measuring instruments including seismic vibrographs that had a substantial mechanism component to them. A large part of the book treats the vibration of continuous beams and plates without much reference to specific machine applications however.

The 1948 book of S. Timoshenko and D.H. Young *Advanced Dynamics,* both at Stanford University at the time, truly combined advanced analytical methodology with many problems of direct machine dynamics [85]. This includes balancing of reciprocating engines, dynamics of rotating ball governors, dynamics of constrained systems including linkages, flywheel governors, gyroscopic motions, the gyrocompass, ship stabilizers, and vibration absorbers. Analytical techniques included Lagrange's equations, linear systems, Mathieu's equation, Rayleigh's method, and nonlinear vibrations. The Authors acknowledged their access to several European books such as E.I. Nikolai's *Theoretical Mechanics* [65], Routh's *Elementary Rigid [Body] Dynamics* [80], and Biezeno and Grammel's *Technische Dynamik* [10].

There were other works of this nature in the Russian and European literature during the quarter century, 1925–1950, that contained similar materials. The Author has come across a citation to a Russian work by N.I Mertsalov, *Dynamics of Mechanism* [55] which was a published set of lecture notes at the Imperial Technical School, in Russian. In a recent visit to Bauman Moscow State Technical University, the Author had an opportunity to examine a copy of Mertsalov's lecture notes. There is certainly some aspects that relate to the dynamics of machines, but this work is not as comprehensive as the later texts of Timoshenko and Den Hartog. (This reference is from Prof. Alexander Golovin of the Bauman Moscow State Technical Univ.)

In the English speaking engineering community, the works of Timoshenko and Den Hartog had considerable influence on the study and design of machine dynamics. One example is the book by James B. Hartman of Lehigh Universty, *Dynamics of Machinery* [35].

The presentation of analytical methods in dynamics in the mechanisms textbooks, were much abbreviated during this period. However in defense of the kinematicians, although the technical dynamics books of the time used examples from machine design, these examples were often used as a way to illustrate the mathematical methods of vibrations. Engine balancing seems to be the exception. In general, there was no attempt to address a general theory of dynamic design in machines and mechanisms nor to use dynamics in the context of design optimization and synthesis.

Summary and Concluding Observations

In the 18th, 19th and early 20th centuries, advanced ideas of analytical dynamics and concomitant mathematical tools of dynamical systems were developed decades, sometimes a century, before their adaptation into the theory of mechanisms and machines. What can be the reason for this delay? It cannot be for lack of communication channels as this period coincided with major advances in telecommunication as well as land and sea travel. One possible thesis was the existence of different scientific, mathematical and technical communities between dynamicists and machine designers.

One can advance the theory that it takes a community of scientists, mathematicians and engineers to develop a new idea or theory or to create new machines and technologies. Throughout most of the 19th century, machines were developed largely in workshops and factories although academic engineers such as Willis, Rankine, Weisbach, Redtenbacher and Reuleaux were creating an engineering-science of machine design. However this mathematics and science-based design methodology took at least half a century to mature and to be accepted in industry as well as in the academic engineering communities that provided the talent for these machine industries. Westinghouse Corporation for example hired both Steven Timoshenko from Russia as well as Den Hartog from Holland in the early 1920s to work on dynamic turbine failures in electric generators. Both of these engineers went on to write famous books illustrating the application of analytical mechanics, dynamics and mathematics to the design of machine elements and structures. The ASME Journal of Applied Mechanics began to publish some machine dynamics research at this time.

On the other hand, the analytical dynamics community that emerged in the 18th century and early 19th century, seemed more interested in so-called “natural” dynamics problems in astronomy, fluid mechanics of air and water, acoustics and electromagnetic fields as they still do today. The members of this community seldom had any overlap with the community of machine theorists or builders. Maxwell seems to have been one of the exceptions to this because of his interest in machine regulator stability.

One reason for the increasing interest by machine theorists in dynamics in the 20th century may have been the increasing speed of prime movers and the attempt at increasing power to weight ratios. Higher speeds in machines placed a greater emphasis on dynamic forces as well as on dynamic instabilities and machine component vibrations.

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On the Historical Overview of Geometric Algebra for Kinematics of Mechanisms

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Abstract In this article, a historical survey of geometric algebra also called Clifford algebra is first undertaken in chronological order. This new algebra is ascribed to Grassmann and Clifford. The quaternion algebra originated from Hamilton can be considered as its special version. Next, in terms of geometric algebra notation, we further deal with the representation of the classical problems about the single finite rotation, first derived by Euler, and the composition formula of two successive finite rotations, originally proposed by Rodriques. Finally, the rigid body motion in the four dimensional geometric algebra $\mathcal{G}_4$ is introduced for the basis of possible future applications using geometric algebra and a general rigid body motion related to the 4×4 homogeneous transformation matrix in Euclidean space is then elucidated.

Keywords Historical survey, Geometric algebra, Clifford algebra, Quaternion algebra, Rigid body motion

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Historic Survey on Geometric Algebra

Geometry stemmed from two Greek words meaning “earth measurement”. It was motivated by the need to make measurements of distances and areas on the Earth. Euclidean geometry has a broader meaning and the chief subject matter of the monumental 13-volume work called “The Elements”, written about 300 B.C. by Greek mathematician Euclid (365–265 B.C., Fig. 1) [1]. Geometry, as developed in Euclid, was a systematic body of mathematical knowledge, built by deductive reasoning upon a foundation of the definitions, axioms, and postulates.

Algebra was associated with geometry from its beginning, but the French philosopher René Descartes (1596–1650, Fig. 2) was the first to develop it systematically into a geometrical language in 1637. He gave the Greek notion of magnitude a symbolic form and made significant improvements in algebraic notations, putting algebra in a form close to the one we use today. Descartes united algebra and geometry by treating the arithmetic of scalars as a kind of arithmetic of line segments. His application of algebra to geometry in the book of *La Géométrie* [2] leads to Cartesian geometry.

Fig. 1 Euclid of Alexandria (365–265 B.C.)

Fig. 2 René Descartes (1596–1650)

The introduction of complex numbers, generally attributed to Jean Robert Argand (1768–1822) in his work of 1806 but in fact anticipated by Norwegian Caspar Wessel (1745–1818) in 1799 [3], provided a valuable way of describing a rotation in algebraic terms, namely modulus and phase. These contributions promoted many attempts to develop an algebra of *n*-dimensional space by analogy with the representation of the plane using complex number in the future.

In 1840, French mathematician Olinde Rodrigues (1795–1851, Fig. 3) proposed the well-known half-angle relations for calculating the compound effect of two finite rotations [4]. Three years later attention was deflected from Rodriques’s contribution by Hamilton’s celebrated paper of 1843 describing the quaternions. The quaternions, introduced by Irish mathematician Sir William Rowan Hamilton (1805–1865, Fig. 4) form the oldest and best known non-commutative algebra, and can be regarded as a special case of geometric or Clifford algebra by express-