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Practical Goal Programming

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Dedicated to our parents

Rahmat Tamiz and Mahtalat Helmi
and
James Jones and Brenda Jones

Preface

The setting and attainment of goals is a fundamental aspect of human decision making, which is manifest in the modern discipline of operational research by the technique of goal programming. Influences from the fields of mathematical programming and multiple criteria decision making (MCDM) can be found in goal programming, and it is our view that in order to use goal programming most effectively, users should be aware of the basic aspects and concepts of both fields.

We are aware that this will not be the first book on goal programming and have ourselves learnt about the topic from the previous works of Charnes and Cooper (1961), Lee (1970), Ignizio (1976, 1982, 1985, 1994), and Romero (1991). We have noted, however, that most of these excellent books are now out of print, and practitioners and post-graduate students in the field are reporting increasing difficulty in obtaining a work that will allow them to grasp the fundamentals of goal programming and hence utilise the full power and flexibility of the technique. It is also our desire to see goal programming continue to develop and be applied in a practical and correct manner. We would also like users of the technique to have access to and the ability to choose from the full range of variants and extensions of goal programming and its analysis tools in order to build models that best reflect the preferences and desires of their decision maker(s).

The purpose of this book is therefore to empower academics and practitioners to be able to build effective goal programming models, as well as to detail the current state of the art in the topic and lay the foundation for its future development and continued application to new and varied fields of application as they arise.

The notation and terminology used in this book for investigating GP and its variants have been designed and refined in collaboration with the leading experts in the field. We believe they give the best description of the subject and would want them to become the standards.

This book is divided into nine chapters. Chapter 1 gives a brief history of goal programming and details the fundamental definitions arising from the fields of mathematical programming and multiple criteria decision making that are used throughout the text. A section on the underlying philosophies of goal programming is also included. Chapter 2 details the goal programming variants and defines them algebraically.

Chapter 3 is particularly important to readers unfamiliar with goal programming who wish to learn how to build effective goal programming models that avoid common modelling pitfalls and formulation errors. It details the step-by-step formulation of a basic goal programming model in the form of each of the three main variants, as well as discussing basic modelling techniques. Chapter 4 details more advanced modelling issues and highlights some recently proposed extensions as well as giving a new and pragmatic weight sensitivity algorithm.

Chapter 5 details the solution methodologies of goal programming, concentrating on computerised solution by the Excel Solver and LINGO packages for each of the three main variants. A discussion of the viability and use of specialised goal programming packages is also included. Chapter 6 discusses the linkages between Pareto efficiency and goal programming. The state of the art in detecting Pareto inefficiency and restoring Pareto efficiency for each of the major variants is given.

Chapters 3–6 are supported by a set of 10 exercises drawn from our two decades of experience of applying goal programming to practical decision-making situations. An Excel spreadsheet giving the basic solution of each example can be found on the accompanying website (www.mopgp.com).

Chapter 7 details the current state of the art in terms of the integration of goal programming with other techniques from operational research and artificial intelligence. This is a key area which we believe will be an important topic of future research.

The text concludes with two case studies which are chosen to demonstrate the application of goal programming in practice and to illustrate the principles developed in Chapters 1–7. Chapter 8 details an application in health care and Chapter 9 describes applications in portfolio selection.

We are indebted to the many researchers in the field whose works on goal programming, conversations, and presentations on the topic have helped shape this text. In particular we would like to acknowledge the seminal work of James Ignizio and Carlos Romero. We would also like to thank the many academic members, international visitors, and doctoral students of the Management Mathematics Group at the University of Portsmouth, who have contributed to our research on the theory and application of goal programming in the past two decades. These include Simon Mardle, Rishma Hasham, Keith Fargher, Bijan Hesni, Keyvan Mir-Razavi, Sita Patel, Zul Mohd-Nopiah, Richard Treloar, John-Paul Oddoye, Jon Large, Patrick Beullens, Reza Khorramshahgol, XiaoDong Li, Kevin Willis, Rania Azmi, Blanca Perez, Amelia Bilbao, Ali Foroughi, Mohammad Ali Yaghoobi, Mohammad Afzalinejad, and Ersilia Liguigli. We are also grateful to Dr Paul Schmidt of the Queen Alexandra Hospital in Portsmouth for his help in developing the case study described in Chapter 8. We would also like to thank the British Royal Society for providing Dr Jones with an international visiting fellowship to the University of Malaga, Spain, where writing of this book was started. Dr Jones would also like to thank Rafael Caballero and the members of the Department of Applied Economics at the University of Malaga; Maria-Victoria Rodriguez and the members of the Department of Quantitative Economics at the University of Oviedo; and Sydney Chu and the members of the Department of Mathematics, University of Hong Kong,

for their hospitality and useful discussions about goal programming during his stay at their respective institutions.

Last, but not least, we initiated the international series of bi-annual conferences in multi-objective and goal programming (MOPGP). The seventh conference was held in Portsmouth in September 2008. We would particularly like to thank Belaid Aouni and Ralph Steuer for their on-going commitment and help in ensuring the success of this conference series over the years. We also wish to thank all the delegates of these conferences for their participation, useful discussions, and indirect contributions to the completion of this book.

Finally, we would like to thank our families: Sherry, Yasaman, and Sam Rahmat Tamiz; and Katia, Thais, Isabella, and Owen Jones for their support and encouragement during the sometimes time-consuming process of writing this book.

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September 2009

Contents

Chapter 1
History and Philosophy of Goal Programming

> *Man is a goal seeking animal. His life only has meaning if he is reaching out and striving for his goals.*
>
> Aristotle 384–322 BC

The above quote demonstrates that goal-based behaviour and decision making has a long history. This goal-based philosophy has been formalised in the modern field of operational research and management science by the technique of goal programming. The earliest goal programming formulation was introduced by Charnes et al. (1955) in the context of executive compensation. At that point the term ‘goal programming’ was not used and the model was seen as an adaptation of linear programming. A more formal theory of goal programming is given by Charnes and Cooper (1961). Further development took place by Ijiri (1965), and seminal textbooks by Lee (1972) and Ignizio (1976) brought the technique into common usage as an operational research tool. This led to large number of applications being reported in the literature from the mid-1970 onwards. The relatively straightforward ease of which a goal programme could be formulated and the familiarity of practitioners and academics with linear programming methodology ensured that goal programming quickly rose to become the most popular technique within the field of multi-criteria decision making (MCDM). This also, however, brought problems. Although goal programming can correctly be viewed as a generalisation of linear programming, it is also a bona fide multi-criteria decision-making technique. Therefore, users of goal programming should be aware of the theories, methodologies, and pitfalls of multiple criteria decision making if they are to build ‘effective’ models. Thus goal programming came under criticism in the 1980s because of some basic errors caused, in our opinion, by lack of awareness of good MCDM practice. These included the generation of Pareto-inefficient solutions, the use of excessive numbers of priority levels leading to redundancy, lack of weight sensitivity analysis, direct comparison of incommensurable goals, and ineffectual elicitation and representation of decision maker preferences. This debate culminated in the publication of a key textbook by Romero (1991) in which good goal programming practice is detailed and the problems shown to be due more to poor modelling practice rather than any fundamental deficiency in goal programming.

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During the 1990s there were further theoretical developments and more complex and varied applications of goal programming. Greater awareness of the multiple criteria aspects of goal programming was in evidence in portions of the literature, with more care being taken to avoid the modelling pitfalls outlined by Romero (1991). There were advances in the user-friendliness of computer software, with the introduction of the GPSYS system which had automated checking for many common modelling errors as well as suggesting ways of overcoming them (Jones et al., 1997). In addition, modern mathematical programming modelling and solution systems made it easier to model and solve all variants of goal programming. Goal programming was combined with other techniques from MCDM and the wider field of operational research with symbiotic advantages. This has been termed the 'trend of integration and combination' and will be further explored in Chapter 7. In addition, the MOPGP: Multiple Objective and Goal Programming – Theory and Application International Conference Series (Tamiz, 2009) started in 1994 and has been running on bi-annual basis since then. This series has provided a dedicated forum for academics and practitioners to discuss and advance the state of multiple objective and goal programming. Figure 1.1 shows an increasing use of goal programming in the early to mid-2000s, with a wide variety of modern fields of application utilising the simplicity and power of the technique.

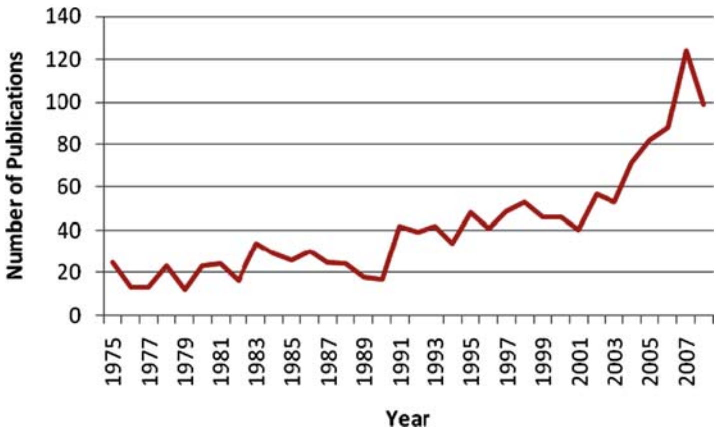

Fig. 1.1 Goal programming publications in the period 1975–2008. Source: ISI Web of Knowledge, published by Thomson Reuters

1.1 Terminology

The plethora of terminology used in the field of MCDM can be confusing, with Ehrgott (2005) listing eight different variants of the most fundamental definitions in the field. This section therefore details the basic definitions and concepts from the fields of MCDM and mathematical programming pertaining to goal programming that will be used throughout this book. These concepts will be developed further and brought together algebraically in Chapter 2.

Definition 1.1 – Decision Maker(s): The decision maker(s) in this text refer to the person(s), organisation(s), or stakeholder(s) to whom the decision problem under consideration belongs. For a description of group and organisational decision-making processes in the case of multiple decision makers the reader is referred to French et al. (2009).

Definition 1.2 – Decision Variable: A decision variable is defined as a factor over which the decision maker has control. An example is a manufacturing company which has to decide how many of a certain product to make in the next month. The set of decision variables fully describe the problem and form the decision to be made. The purpose of the goal programming model can be viewed as a search of all the possible combinations of decision variable values (known as decision space) in order to determine the point which best satisfies the decision maker's goals and constraints. Figure 3.1 in Chapter 3 is an example of a graph drawn in decision space.

Definition 1.3 – Criterion: A criterion is a single measure by which the goodness of any solution to a decision problem can be measured. There are many possible criteria arising from different fields of application but some of the most commonly arising relate at the highest level to

- Cost
- Profit
- Time
- Distance
- Performance of a system
- Company or organisational strategy
- Personal preferences of the decision maker(s)
- Safety considerations

A decision problem which has more than one criterion is therefore referred to as a multi-criteria decision making (MCDM) or multi-criteria decision aid (MCDA) problem. The space formed by the set of criteria is known as criteria space.

Definition 1.4 – 'Objective': An objective in this book will be referred to as a criterion with the additional information of the direction (maximise or minimise) in which the decision maker(s) prefer on the criterion scale, for example minimise cost or maximise the performance of a system. A decision problem with a set of objectives to be maximised or minimised is referred to as a **multi-objective optimisation problem**. In practice, these objectives will be conflicting, that is they cannot reach their optimal values simultaneously. If they could, then the model can be solved as a single-objective problem for any of the objectives. The space formed by the values of the set of objectives is known as objective space. Figure 1.2 shows an example of objective space for a bi-objective decision problem.

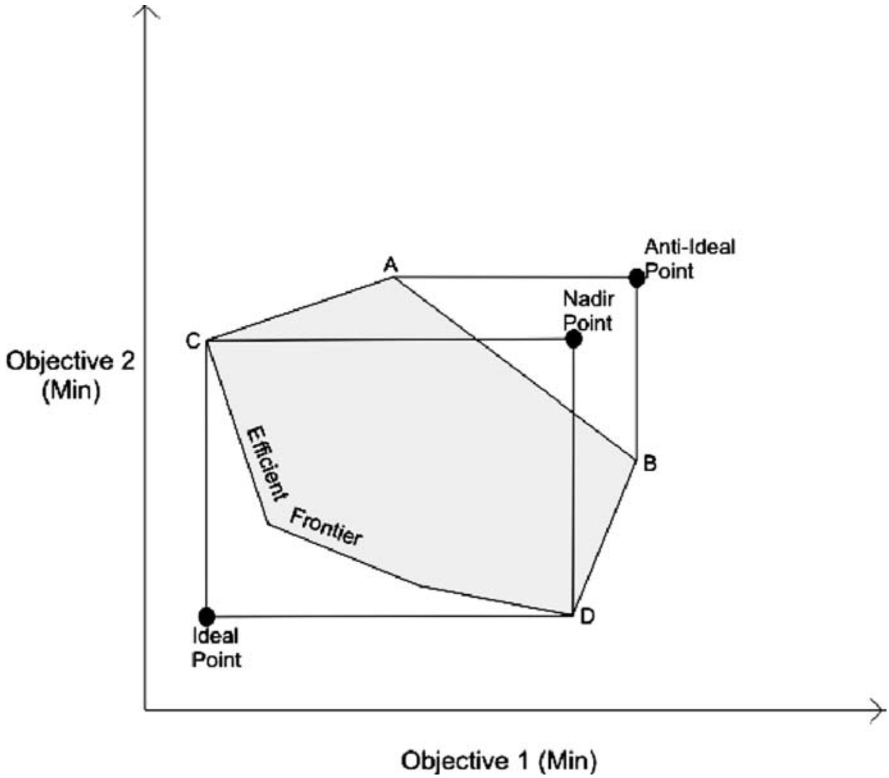

Fig. 1.2 Graphical representation of objective space for bi-objective model

Table 1.1 Three principal types of goals

Goal type	Significance	Example
1	Achieve at most the target level	Keep costs within a budget of $1 m
2	Achieve at least the target level	Aim to produce at least 20 items
3	Achieve the target level exactly	Aim to employ exactly 20 workers

Definition 1.5 – 'Goal': A goal in this book refers to a criterion and a numerical level, known as a **target level**, which the decision maker(s) desire to achieve on that criterion. There are three principal types of goals that can occur in a goal programming model, as listed in Table 1.1.

The relationship between these goals and the penalisation of deviations from the target level is given in Table 2.1.

Definition 1.7 – 'Deviational Variable': A deviational variable measures the difference between the target level on a criterion and the value that is able to be achieved in a given solution. If the achieved value is above the target level then the difference is given by the value of the **positive deviational variable**. If the achieved value is below the target level then the difference is given by the value of the **negative deviational variable**.

The essence of goal programming is the minimisation of unwanted deviational variables. For goal type 1 or 'less is better', the positive deviational variable is said to be the unwanted deviational variable For goal type 2 or 'more is better', the negative deviational variable is said to be the unwanted deviational variable. For goal type 3, both positive and negative deviational variables are said to be unwanted deviational variables. Algebraic examples of deviational variables and their minimisation are given in Section 2.1.

Definition 1.8 – 'Constraint': A constraint is a restriction upon the decision variables that must be satisfied in order for the solution to be implementable in practice. This is distinct from the concept of a goal whose non-achievement does not automatically make the solution non-implementable. A constraint is normally a function of several decision variables and can be an equality or an inequality.

Definition 1.9 – 'Sign Restriction': A sign restriction limits a single decision or deviational variable to only take certain values within its range. The most common sign restriction is for the variable to be non-negative and continuous.

Definition 1.10 – 'Feasible Region': The set of solutions in decision space that satisfy all constraints and sign restrictions in a goal programming form the feasible region. Any solution that falls within the feasible region is deemed to be implementable in practice.

Definition 1.11 – 'Pareto-Efficient Solution' (also known as Pareto optimal or in objective space as non-dominated): A solution to a multi-objective problem is **Pareto efficient** if no other feasible solution exists that is at least as good with respect to all objectives and strictly better with respect to at least one objective.

Definition 1.12 – 'Pareto-Inefficient Solution' (also known as Pareto sub-optimal or in objective space as dominated): A solution to a multi-objective problem is **Pareto inefficient** if another feasible solution exists that is at least as good with respect to all objectives and strictly better with respect to at least one objective.

A fundamental law of decision making states that no rational decision maker will knowingly choose a Pareto-inefficient solution, if they have knowledge of a Pareto-efficient solution that dominates it.

Definition 1.13 – 'Ideal Point': The point in objective space at which each objective in a multi-objective optimisation problem takes its optimal value when optimised individually, within the feasible region, is known as the ideal point. If the objectives are conflicting then this point will be outside the feasible region in objective space and hence an infeasible point. Nevertheless, it provides a useful point of reference to measure the goodness of any solution against. In goal programming, the ideal point is used to calculate the normalisation constants when using zero–one