

## Screw Compressors

A. Kovacevic N. Stosic I. Smith

---

# Screw Compressors

Three Dimensional Computational Fluid Dynamics  
and Solid Fluid Interaction

With 83 Figures

Ahmed Kovacevic  
Nikola Stosic  
Ian Smith

School of Engineering and Mathematical Sciences  
City University  
Northampton Square  
London  
EC1V 0HB  
United Kingdom

Library of Congress Control Number: 2006930403

ISBN 3-540-36302-5 Springer-Verlag Berlin Heidelberg New York

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in any other way, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer-Verlag. Violations are liable for prosecution under the German Copyright Law.

Springer-Verlag is a part of Springer Science+Business Media

© Springer-Verlag Berlin Heidelberg 2007  
springer.com

The use of general descriptive names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Cover-Design: Estudio Calamar, Viladasens  
Typesetting by the authors and SPi

Printed on acid-free paper      SPIN: 11741589      62/3100/SPi      5 4 3 2 1 0

---

## Preface

Screw compressors are rotary positive displacement machines, which are compact, have few moving parts and which operate at high efficiency over a wide range of speeds and pressure differences. Consequently a substantial proportion of all industrial compressors now produced are of this type.

There are few published books on the principles of their operation and how best to design them, especially in English. The authors made a first step to fill this void with an earlier work on mathematical modelling and performance calculation of these machines. This described analytical procedures which are generally adequate for most applications, especially when dealing with oil flooded machines, in which temperature changes are relatively small and thus the effect of changes of shape of the key components on performance may be neglected. This assumption permitted the use of analytical procedures based on flow through passages with dimensions that are invariant with temperature and pressure.

As manufacturing accuracy increases, clearances can be reduced and compressors thereby made smaller and more efficient. To obtain full advantage of this at the design stage, more accurate procedures are required to determine the internal fluid flow patterns, the pressure and temperature distribution and their effects on the working process. This is especially true for oil free machines, in which temperature changes are much larger and thus make thermal distortion effects more significant.

The present volume was prepared, as a sequel to the authors' earlier work and describes the most up to date results of methods, which are still being developed to meet this need. These are based on the simulation of three-dimensional fluid flow within a screw machine.

Such an approach requires the simultaneous solution of the governing equations of momentum, energy, mass, concentration and space conservation. In order to be solved, the equations are accompanied by constitutive relations for a Newtonian fluid in the form of Stokes, Fourier's and Fick's laws. Although this model is generally applicable, at least three features have been derived, without which the solution of the flow pattern within a screw machine would not be possible. Firstly, there is the application of the Euler-Lagrange method for the solution of multi-phase flow of both the injected fluid and the liquid phase of the main fluid. Its equations define mass, energy and concentration source terms for the main equations. The introduction of a 'boundary domain' in which the pressure is kept constant by injection or by subtraction of mass, is the next innovation in and allows simulation of the real operational mode of a screw compressor. Finally, a rather simple method is used here for calculation of the properties of real fluids. This

## VI Preface

permits a fast but still reasonably accurate procedure even for the calculation of processes like evaporation or condensation within the working chamber.

A computer program has been developed for Windows or UNIX oriented machines, based on the methodology, thus described. It automatically generates files which contain node, cell and region specifications as well as user subroutines for a general CFD solver of the finite volume type. The method and program have been tested on a commercial CFD solver and good simulations of positive displacement screw machines were obtained.

Four examples outline the scope of the applied mathematical model for three dimensional calculation of fluid flow and stresses in the solid parts of the screw machine. In the first example, the results for two oil-free air screw compressors with different rotor profiles are compared with each other and with results obtained from one-dimensional calculation. Advantages are found in the use of the three-dimensional simulation model, in which the suction and discharge dynamic losses are accounted for in the results.

The second example verifies the results of three-dimensional calculations with measurements obtained in an experimental test rig. The influence of turbulence upon the process in a screw compressor is also investigated. It is concluded that, despite the excessive dissipation of kinetic energy of turbulence, the overall parameters of a positive displacement machine do not significantly change if it is calculated either as turbulent or laminar flow. Investigation of the grid size influence on the accuracy of 3-D calculations is performed and it shows that coarser numerical meshes cannot capture all of the flow variations within the compressor accurately. However, the integral parameters are in all cases reasonably accurate.

In the third example the procedures for real fluid properties enabled fast calculation of ammonia compressor parameters. The importance of the oil injection port position is outlined here through the oil distribution obtained by the three dimensional calculations. Such figures of oil distribution within the screw compressor have never been found in the open literature. This achievement is therefore a step forward in understanding screw compressor internal flows.

The fourth application covers the simultaneous calculation of the solid fluid interaction. The influence of the rotor deformation on the integral screw compressor parameters caused by the change in clearance is presented together with how rotor clearances are reduced due to the enlargement of the rotors caused by temperature dilatations. This results in an increase in both, the compressor flow and power input. The influence of pressure causes the rotors to bend. For a moderate compressor pressure, the clearances gap is enlarged only slightly and hence has only a negligible influence on the delivery and power consumption. In the case of high working pressures, the rotors deform more and the decrease in the delivery and rise in specific power becomes more pronounced.

*Ahmed Kovacevic  
Nikola Stosic  
Ian Smith*

London,  
February 2006

---

## Notation

$A$	- area of the cell surface
$\mathbf{a}_1, \mathbf{b}_1$	- radius vectors of boundary points
$A_1, A_2$	- constants in the saturation temperature equation
$b_i$	- constant
$B_1, B_2$	- constants in the compressibility factor equation
$c_i$	- concentration of species
$c_1-c_4$	- tension spline coefficients
$C_1, C_2$	- coefficients of the orthogonalisation procedure
$C_{drag}$	- drag coefficient
$C_p$	- specific heat capacity at constant pressure
$C, \sigma$	- constants in $k-\varepsilon$ model of turbulence
$d_i$	- distance in transformed coordinate system
$D_i$	- mass diffusivity of the dispersed phase
$d_o$	- Sauter mean droplet diameter
$D_1-D_4$	- constants in the vapour specific heat equation
$\mathbf{D}$	- rate of strain tensor
$e_1, e_2$	- cell edges maximal values in coordinate directions
$f(s)$	- adaptation variable
$\mathbf{f}_b$	- resultant body force
$f_e$	- expansion factor
$f_k$	- weight function
$F^i(s)$	- integrated adaptation variable
$h_1-h_8$	- blending functions of Hermite interpolation
$h$	- enthalpy
$h_L$	- enthalpy of evaporation
$k$	- turbulent kinetic energy
$K_1-K_4$	- coefficients for Hermite interpolation
$k_p$	- point counter
$K$	- number of points
$m$	- mass
$m_o$	- mass of species
$m_i$	- mass in the numerical cell
$\Delta m_L$	- mass of evaporated or condensed fluid
$n$	- rotor speed
Nu	- Nusselt number
$p$	- pressure
$P$	- production of turbulence energy

## VIII Notation

$Pr$	- Prandtl number
$\mathbf{q}_{ci}$	- diffusion flux of species
$\mathbf{q}_h$	- heat flux
$\mathbf{q}_k$	- diffusion flux in kinetic energy equation
$\mathbf{q}_\varepsilon$	- diffusion flux in dissipation equation
$\mathbf{q}_{\phi S}$	- flux source in the generic transport equation
$\mathbf{q}_{\phi V}$	- volume source in the generic transport equation
$\dot{Q}_{con}$	- convective heat flux
$\dot{Q}_{mass}$	- heat flux due to phase change
$\mathbf{r}$	- radius vector
$Re$	- Reynolds number
$R_i$	- grid point ratio
$S$	- cell surface
$s$	- transformed coordinate
$\mathbf{s}$	- area vector
$S_{ci}$	- source term of species
$S_h$	- heat source term
$\mathbf{S}$	- viscous part of stress tensor
$t$	- time
$T$	- temperature
$\mathbf{T}$	- stress tensor
$\mathbf{u}$	- displacement vector
$\mathbf{v}$	- fluid velocity
$\mathbf{v}_{ci}$	- velocity of the dispersed phase
$v_i$	- Cartesian component of velocity vector
$\mathbf{v}_s$	- surface velocity
$V$	- cell volume
$V_{CM}$	- volume of the control mass
$V_{CV}$	- control volume
$w$	- weight factor
$W$	- weight function
$X_\xi$	- grid spacing
$x, y, z$	- physical coordinates
$X, Y, Z$	- points on physical boundaries
$x_p, y_p$	- coordinates of the calculation point
$y_p', y_p''$	- first and second derivatives in vicinity of the point $P$
$y^+$	- dimensionless distance from the wall
$z$	- compressibility factor

*Greek symbols*

$\alpha_t$	- linear thermal expansion coefficient
$\alpha, \beta$	- tension spline coefficients
$\alpha_1, \beta_1$	- blending functions
$\delta$	- Kroneker delta function
$\delta t$	- time step
$\varepsilon$	- dissipation of turbulent kinetic energy
$\phi$	- transported property in generic transport equation
$\varphi$	- interlobe angle of male rotor
$\Gamma_\phi$	- Diffusive term in generic transport equation
$\eta, \lambda$	- Lamé coefficients
$\kappa$	- thermal conductivity
$\mu$	- viscosity
$\mu_t$	- turbulent viscosity
$\pi$	- the ratio of a circle's circumference to its diameter
$\omega$	- angular velocity
$\rho$	- density
$\sigma$	- tension spline parameter
$\sigma_o$	- oil surface tension
$\sigma_{cv}$	- normalised cell volume
$\xi, \eta$	- computational coordinates
$\hat{\xi}_o, \hat{\xi}_1$	- one-dimensional stretching functions
$\hat{\xi}$	- multi-dimensional stretching function

*Subscripts*

$1$	- male rotor
$2$	- female rotor
$add$	- injected / subtracted fluid
$const$	- constant prescribed value
$D$	- discharge bearings
$i$	- dispersed phase
$L$	- evaporated/condensed fluid
$l$	- liquid
$M$	- mixture
$max$	- maximum
$min$	- minimum
$o$	- oil
$ref$	- reference value
$s$	- grid values
$S$	- surface



X Notation

<i>sat</i>	- saturation
<i>t</i>	- turbulence
<i>v</i>	- vapour
<i>V</i>	- volume

*Superscripts*

'	- fluctuating components for time averaging
''	- fluctuating component for density-weighted averaging
<i>k</i>	- number of time steps

---

# Contents

<b>1 Introduction</b> .....	<b>1</b>
1.1 Screw Machines.....	1
1.2 Calculation of Screw Machine Processes.....	3
1.3 Fluid Flow Calculation.....	3
<b>2 Computational Fluid Dynamics in Screw Machines</b> .....	<b>7</b>
2.1 Introduction.....	7
2.2 Continuum Model applied to Processes in Screw Machines.....	8
2.2.1 Governing Equations.....	9
2.2.2 Constitutive Relations.....	12
2.2.3 Multiphase Flow.....	14
2.2.4 Equation of State of Real Fluids.....	19
2.2.5 Turbulent Flow.....	22
2.2.6 Pressure Calculation.....	23
2.2.7 Boundary Conditions.....	23
2.3 Finite Volume Discretisation.....	27
2.3.1 Introduction.....	27
2.3.2 Space Discretisation.....	29
2.3.3 Time Discretisation.....	29
2.3.4 Discretisation of Equations.....	30
2.4 Solution of a Coupled System of Nonlinear Equations.....	32
2.5 Calculation of Screw Compressor Integral Parameters.....	33
<b>3 Grid generation of Screw Machine Geometry</b> .....	<b>39</b>
3.1 Introduction.....	39
3.1.1 Types of Grid Systems.....	42
3.1.2 Properties of a Computational Grid.....	44
3.1.3 Grid Topology.....	47
3.2 Decomposition of a Screw Machine Working Domain.....	48
3.3 Generation and Adaptation of Domain Boundaries.....	52
3.3.1 Adaptation Function.....	53
3.3.2 Adaptation Variables.....	55
3.3.3 Adaptation Based on Two Variables.....	55
3.3.4 Mapping the Outer Boundary.....	58
3.4 Algebraic Grid Generation for Complex Boundaries.....	62
3.4.1 Standard Transfinite Interpolation.....	63

XII Contents

3.4.2 Ortho transfinite interpolation.....	65
3.4.3 Simple Unidirectional Interpolation .....	73
3.4.4 Grid Orthogonalisation .....	75
3.4.5 Grid Smoothing.....	78
3.4.6 Moving Grid.....	79
3.5 Computer Program.....	81
<b>4 Applications.....</b>	<b>83</b>
4.1 Introduction .....	83
4.2 Flow in a Dry Screw Compressor .....	84
4.2.1 Grid Generation for a Dry Screw Compressor .....	86
4.2.2 Mathematical Model for a Dry Screw Compressor .....	87
4.2.3 Comparison of the Two Different Rotor Profiles .....	87
4.3 Flow in an Oil Injected Screw Compressor .....	94
4.3.1 Grid Generation for an Oil-Flooded Compressor .....	97
4.3.2 Mathematical Model for an Oil-Flooded Compressor .....	97
4.3.3 Comparison of the Numerical and Experimental results for an Oil-Flooded Compressor .....	98
4.3.4 Influence of Turbulence on Screw Compressor Flow.....	106
4.3.5 The Influence of the Mesh Size on Calculation Accuracy .....	114
4.4 A Refrigeration Compressor.....	118
4.4.1 Grid Generation for a Refrigeration Compressor .....	118
4.4.2 Mathematical Model of a Refrigeration Compressor.....	119
4.4.3 Three Dimensional Calculations for a Refrigeration Compressor.....	119
4.5 Fluid-Solid Interaction .....	123
4.5.1 Grid Generation for Fluid-Solid Interaction.....	123
4.5.2 Numerical Solution of the Fluid-Solid Interaction .....	124
4.5.3 Presentation and Discussion of the Results of Fluid-Solid Interaction.....	125
<b>5 Conclusions.....</b>	<b>131</b>
<b>A Models of Turbulent Flow .....</b>	<b>133</b>
<b>B Wall Boundaries .....</b>	<b>139</b>
<b>C Finite Volume Discretisation .....</b>	<b>143</b>
<b>References .....</b>	<b>155</b>

---

## Introduction

### 1.1 Screw Machines

The operating principle of screw machines, as expanders or compressors, has been known for over 120 years. Despite this, serious efforts to produce them were not made until low cost manufacturing methods became available for accurate machining of the rotor profiles. Since then, great improvements have been made in performance prediction, rotor profile design and manufacturing techniques. Screw compressors are now highly efficient, compact, simple and reliable. Consequently, they have largely replaced reciprocating machines for the majority of industrial applications and in many refrigeration systems.

Screw compressors and expanders are positive displacement rotary machines. They consist essentially of a pair of meshing helical lobed rotors, which rotate within a fixed casing that totally encloses them, as shown in Figure 1-1.



Figure 1-1 Screw Compressor Components

Although screw machines can function as either expanders or compressors, their overwhelmingly common use is as compressors, of which there are two main types. These are oil flooded, commonly known as oil injected, and oil free compressors. An example of each, with similar rotor sizes, is shown in Figure 1-2.

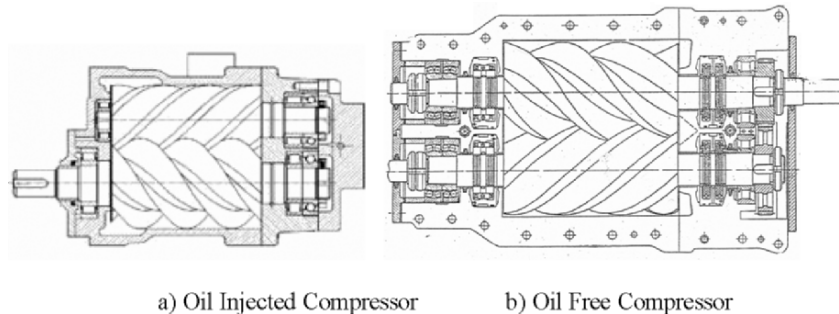


Figure 1-2 Types of Screw Compressor

In oil injected compressors, a relatively large mass, though a very small volume, of oil is admitted with the gas to be compressed. The oil acts as a lubricant between the contacting rotors, a sealant of any clearances between the rotors and between the rotors and the casing and as a coolant of the gas during the compression process. This cooling effect improves the compression efficiency and permits pressure ratios of up to approximately 15:1 in a single stage, without an excessive temperature rise, by maintaining an oil:gas mass ratio of 4:1 or even more. The effects of thermal expansion are then relatively small and now that screw compressor components can be manufactured with tolerances of the order of  $\pm 5\mu\text{m}$ , internal clearances can be as little as 30-60 $\mu\text{m}$ .

In oil free compressors, only gas is admitted into the working chamber. External timing gear is therefore needed, in order to prevent rotor contact, and internal shaft seals have to be located between the bearings at each end of the rotor shaft and the main body of the rotor. The shaft seals are needed to prevent oil, which is supplied to the bearings through an external lubrication system, from entering the working chamber and thereby contaminating the gas being compressed. Because there is no injected oil to cool the gas in this type of machine, the temperature rise of the compressed gas is much higher than in oil flooded compressors and pressure ratios are therefore limited to approximately 3:1, depending on the type of gas being compressed. Above this value the temperature rise associated with compression creates problems related to rotor and casing distortions. Clearances therefore have to be much larger in order to avoid contact between the rotors or between the rotors and their casing caused by thermal distortion. It is believed that the adiabatic efficiency of oil free compressors could be increased by as much as 10%, if minimum safe clearances could be predicted accurately.

## 1.2 Calculation of Screw Machine Processes

Up till approximately 1980, screw compressors were designed assuming an ideal gas in a leak proof working chamber going through a compression process which could reasonably be approximated in terms of pressure-volume changes by the choice of a suitable value of exponent “n” in the relationship  $pV^n = \text{Constant}$ .

To improve on this procedure it was first necessary to obtain an algorithm which could be used to estimate the trapped volume between the rotors and the casing and the areas of all leakage passages, at any rotational angle. The latter are formed by clearances between the rotors and between the rotors and the casing. In addition, the area of the inlet or exit passage exposed to bulk flow of fluid into or out of the working chamber had to be obtained where applicable.

The assumption of dimensionless non-steady bulk fluid flow and steady one dimensional leakage flow through the working chamber, together with suitable flow coefficients through the passages and an equation of state for the working fluid, made it possible to develop a set of non-linear differential equations which describe the instantaneous flow of fluid, work and heat transfer through the system. These could be solved numerically to estimate pressure-volume changes through the suction, compression and delivery stages and hence determine the net torque, power input, fluid delivery and isentropic and volumetric efficiencies of a compressor. More details of this are given in the authors’ earlier volume *Stosic, Smith and Kovacevic*, (2005).

## 1.3 Fluid Flow Calculation

In recent years there has been a steady growth in the use of Computational Fluid Dynamics (CFD) as a means of calculating 3-D external and internal flow fields. It is widely used today for estimating flow in rotating machinery and specialised codes have been developed for this to allow faster calculations.

Many books on fluid dynamics such as *Bird et al* (1962), *Fox and McDonald* (1982) and *White* (1986) contain a detailed derivation of general conservation laws. Three main groups of methods have been developed through the years as described by *Ferziger and Perić* (1995). These are the finite difference, finite element and finite volume methods. It is believed that the finite difference was first described by *Euler* in the 18<sup>th</sup> century but, more recently, *Smith* (1985) gave a comprehensive account of all its aspects.

The finite element method was initially developed for structural analysis, but later has also been used for the study of fluid flow. It has been described extensively by many authors, such as *Oden* (1972), *Fletcher* (1991) and *Zienkiewicz and Taylor* (1991).

A summary of the finite volume method is given by *Versteeg and Malalasekera* (1995). Since the finite volume method has already been used to solve problems involving unsteady flow with moving boundaries and strong pressure-velocity-density coupling, it is of particular interest for this book. The ‘space conservation

law' was introduced by *Trulio and Trigger* (1961) and used in conjunction with a finite difference method. The importance of the space conservation law was discussed by *Demirdžić and Perić* (1988) and introduced to the finite volume method for prediction of fluid flow in complex domains with moving boundaries by the same authors (1990) and also by *Demirdžić, Issa and Lilek* (1990). They followed the attempts of many other authors to apply it to solve some special cases. Typically, *Gosman and Watkins* (1977), *Gosman and Johns* (1978) and *Gosman* (1984) reported the calculations of flows in a cylinder with moving boundaries. *Stošić* (1982) applied the method to internal unsteady flows of a compressible fluid. *Thomas and Lombard* (1979) presented solutions of steady and unsteady supersonic flows while *Gosman* (1984) and *Durst et al* (1985) reported that simple transformation of the conservation equations enables easy discretisation when only one of the domain boundaries moves in one direction. *Perić* (1985) introduced a finite volume methodology for prediction of three-dimensional flows in complex ducts where, among others, he gave an evaluation of various pressure-derivation algorithms for orthogonal and non-orthogonal grids. Additional analysis on pressure-velocity coupling is given by *Perić* (1990) and later discussed and improved by *Demirdžić et al* (1992) and *Demirdžić and Muzaferija* (1995) where they applied the method simultaneously to fluid flow and solid body stress analysis. Turbulence modeling is discussed by many authors, among whom *Hanjalić* (1970) gave an essential introduction to its wider use. *Bradshaw* (1994) and *Hanjalić* (1994) gave good summaries on the subject. *Lumley* (1999) outlined important subjects on turbulence in internal flows of positive displacement machines.

Despite a large number of publications on CFD, little has been written on its use for the analysis of flow through screw machines. This is mainly due to the complexity of both, the machine configuration and the flow paths through them. Some existing commercial CFD codes have facilities that can cope with the complex geometry of screw machines. Unfortunately, these codes need to be improved in order to give useful results. In addition, a pre-processor needs to be developed to generate a numerical mesh that describes their shape with sufficient accuracy and allows for the complex stretching and sliding motion associated with the flow.

With the advance of computing, it is now possible to predict internal flow in screw compressors by 3-D methods so that the internal pressure and temperature distribution can be estimated throughout the machine. Further, this can be used as a basis for determining the distribution of injected oil in oil flooded machines, as a means to estimate thermal distortion within an oil free compressor and to design inlet and exit port passages with minimum flow losses.

Such a procedure makes it possible to reduce the size of screw compressors by bringing internal leakage to a minimum. This would improve the adiabatic efficiency of such machines by virtue of the reduced internal losses and greatly reduce the cost of developing new products by cutting the time and cost of experimental testing and development.

Despite a significant increase in the number of papers published recently in the area of computational fluid dynamics, only a few deal with the application of computational fluid dynamics to screw compressors. All of them are recent papers by *Kovacevic, Stosic and Smith* published between 1999 and 2005. These papers

introduced 3-D numerical analysis to the screw compressor world. Their latter papers are related to both, grid generation in screw compressors and 3D numerical performance estimation, *Kovacevic et al* (2003) and (2005). These include fluid solid interaction in screw machines, *Kovacevic et al* (2004).



---

## Computational Fluid Dynamics in Screw Machines

### 2.1 Introduction

As computer technology and its associated computational methods advance, the use of 3-D Computational Fluid Dynamics (CFD) to design and analyse positive displacement machinery working processes is gradually becoming more practicable. In general, the CFD modelling process can be split into four phases.

The first phase is concerned with defining the problem that has to be solved. Both the ease of solution and implementation of results into the design process are heavily dependent on this critical starting step. Two different approaches are available for screw machines. The first is to select one interlobe on the main rotor and the corresponding interlobe on the gate rotor in order to make a computational domain. This is probably the easiest to implement but takes no account of important phenomena such as interlobe leakage, blow-hole losses, oil injection and oil distribution. Another approach assumes that the whole domain of a screw machine is analysed. This includes the suction chamber and its port, the compression or expansion chamber with its moving rotor boundaries and the discharge system of the machine. By this means, the leakage paths and any additional inlet or outlet ports are included in the domain to be analysed. Realism in representing the machine working process gives a large advantage to this approach. The design procedure and the CFD numerical analysis can then be easily connected and interchanged and the calculation of the operational parameters of such machines is thereby facilitated. Unfortunately, such a complex geometry cannot be represented by a small number of computational points.

In the second phase, a mathematical model that is capable of describing the problem has to be selected. There are again two types of situation. The first is where an adequate mathematical description exists and can be used, e.g. heat conduction, elastic stress analysis and laminar fluid flow. The second is where such a description either does not exist or is impracticable to use, e.g. non-linear stress analysis and turbulent fluid flow. In the case of positive displacement machines, it is unlikely that any analytical solutions exist. This is because highly compressible flow appears inside both domains with turbulent flow regimes and domains with low Reynolds numbers. There is additional non-linearity introduced by two-phase flow, particle flow, moving and stretching domains and sliding boundaries. Due to