Dietmar Schulze

Powders and Bulk Solids

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Behavior, Characterization, Storage and Flow

With 352 Figures

Springer

Professor Dr.-Ing. Dietmar Schulze
Institute for Recycling
University of Applied Sciences Braunschweig/Wolfenbüttel
Robert-Koch-Platz 8a
D-38440 Wolfsburg
Germany
e-mail: D.Schulze@fh-wolfenbuettel.de

Co-Translator:
em. Professor Dr.-Ing. Jörg Schwedes
Schwedes + Schulze Schüttguttechnik GmbH
Petzvalstr. 56
38104 Braunschweig
Germany

Review English text:
John W. Carson, Ph.D.
President
Jenike & Johanson, Inc.
400 Business Park Dr.
Tyngsboro MA 01879
USA

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Preface

In the engineering community the handling of powders and bulk solids is called bulk solids technology being, at least in Germany, part of mechanical process engineering. Process engineering involves study of the change and transformation of material properties. If mechanical processes are used for this transformation, the engineering discipline is called mechanical process engineering. The best known unit operations of mechanical process engineering are grinding, agglomeration, mixing, and separation. Due to the mechanical treatment, the particles – either single particles or a collection of particles – increase in value. To profit from this value increase industry and academia perform research and development. When handling bulk solids, e.g. storing, dosing and conveying, no value increase can be achieved, because the material properties are not changed. What can be achieved is "at most" the elimination of problems which is less interesting research and development compared to the development of new products or processes with the chance to obtain a patent.

A silo for the intermediate storage of bulk solids often seems to be only a single, unimportant element for the realization of a larger process. Bulk solid is filled into the silo continuously or discontinuously and discharged later at predetermined points of time in desired quantities. That is not always unproblematic. The known problems of arching and ratholing can lead to irregular discharge or complete blockage. The hammer often found close to a silo is a known auxiliary means which often is subsequently resorted to due to incorrect silo design. Nearly forty years ago the author of this preface wrote a monograph on bulk solids storage, called this discipline as belonging to the "stepchildren" of technology, and summarised that it often happens that a problem is accepted as given due to its frequent appearance. Has that changed much up to today?

Today the characterization and handling of bulk solids is taught at many (in Germany, nearly all) universities and technical schools in faculties of mechanical and chemical engineering. For more than twenty years seminars and short courses have been offered which cover the basics and application potential of modern bulk solids technology. The most important theoretical fundamentals are known, from which reliable silo design is possible. The necessary information is available, but it is often not considered or understood. Avoidable failures, unnecessary overtime and production shortages or stoppages have to occur before those who have lived with the mentioned problems become willing to adopt the available design method in which the silo geometry is fitted to measurable bulk solid properties.

When designing silos for flow a procedure has to be followed that is similar to the one used when designing a heat exchanger. The material properties of the concerned fluids have to be known to design the heat exchanger. This procedure is state-of-the-art. The necessary data can be found in equivalent text books (e.g., “Heat Exchanger Design Handbook”, “VDI-Wärmeatlas”) or have to be measured following standardized test methods. The procedure to design a silo is similar. Unfortunately there are no equivalent text books (no “Silo Design Handbook” containing material parameters). So the experimental determination of relevant bulk solid properties is much more important. If these properties are known the silo geometry can be fitted to the measured values and trouble-free operation can be achieved. A prerequisite is that the bulk solid properties measured with laboratory equipment are representative. But this prerequisite also holds regarding a heat exchanger. If, for example, “fouling” takes place on the heat exchanger surfaces the material parameters have changed, and the heat exchanger can no longer fulfil the demanded requirements. In a similar way a silo might not provide trouble-free operation when bulk solids with different properties are stored.

A text book (“Silo Design Handbook”) containing reliable quantitative data concerning silo design for flow will never exist. A fluid for which the composition is known always has identical material properties. This only very seldom holds for bulk solids. Besides chemical composition, which is sufficient for characterizing a fluid, further parameters based on the disperse nature of bulk solids have an influence on their properties. Of influence are particle size, particle size distribution, particle shape, porosity, humidity and many more. If one would determine the influence of all these parameters on the relevant properties for silo design, many experiments would be necessary. Thus it is more practical to measure relevant bulk solid properties directly on representative samples. To obtain these properties shear testers are used. But only when those shear tests are performed correctly can data be obtained which enable reliable silo design for flow. Experience indicates that the prerequisite of correct performance of a shear test is often not realized, due to which it is often concluded – usually too quickly – that shear tests only have limited usefulness.

Today I would not judge bulk solid storage anymore as belonging to the “stepchildren” of technology. The most important theoretical fundamentals are known, and there are sufficiently many examples available which prove that silos fulfil the demanded requirements when they have been designed according to the present state-of-the-art on the basis of properties measured in shear testers. What still can be called shabby and therefore could be improved is the broad application, confidence in the method and the necessity to apply it. This is all the more remarkable when considering that the percentage of bulk solids used in process industry is enormous. A few years ago the president of the EFCE (European Federation of Chemical Engineering) judged that about 60% of all products produced in the chemical industries in Europe are bulk solids. An additional 20% of products use bulk solids in the processes.

This book by Dietmar Schulze certainly will help to improve the understanding of bulk solids behavior, especially for people who only occasionally deal with bulk solids. After having obtained his diploma at the Technical University of Braunschweig he has been continuously concerned with all aspects of the characterization of bulk solids, application of measured bulk solids properties for silo design, and theoretical fundamentals of bulk solids technology. He has written important contributions to nearly all aspects of bulk solids technology which are documented in many publications. His effort in the determination of relevant bulk solid properties has been very intensive. He developed a special ring shear tester that is today known worldwide as the “Schulze Ring Shear Tester”. It is often erroneously asserted when mentioning shear testers that they are too complicated or too time consuming for many applications. Neither statement is correct. Depending on the task and on the problem, only a few experiments with a ring shear tester which has been adapted to the problem yield results that are more reliable than those which follow from empirical tests or from the application of so-called “simple testers”.

This book was published in German in autumn 2006. One of the main merits of the book is the fact that all chapters have been written by the same person who is an expert in all of the mentioned topics. The wording, nomenclature and mode of presenting graphs and sketches are identical throughout the book. This helps in understanding and searching for special questions. This has been missing in the English literature. Indeed there are some good books on bulk solids technology in English that are worth reading. But the single chapters of these books were mostly written by different authors using different nomenclature, etc. Thus, Dietmar Schulze was pressed by many of us to prepare an English version of his book, which is now available.

It is my hope this book will help to improve the understanding of bulk solids and the problems caused by them. This book treats nearly all problems which can arise in the technical application of storage and flow of bulk solids. The broad understanding and know how are based on the author's own experience in the measurement of bulk solids properties, planning and performing investigations of large silos and laboratory equipment, consulting with companies in solving problems or planning new silo facilities, and on a continuous and broad literature study.

Jörg Schwedes Braunschweig, in July 2007

Foreword

This book provides an introduction into bulk solids technology starting from the flow properties of particulate solids to the flow of powders and bulk solids in hoppers, bins, and silos. Besides the basics some newer findings are also included. Furthermore, frequent questions that have been addressed to the author in the past are taken into account.

The level of the book should make it well-suited to a wide range of readers who are interested in an introduction into the topic. The major part of the contents is based on contributions to seminars and conferences, and on lectures at the University of Applied Sciences Braunschweig/Wolfenbüttel.

Regarding the topics "measurement of flow properties" and "silo design", the book follows the great work of Andrew W. Jenike. About 45 years ago Jenike developed a theory on the flow of bulk solids in silos which is still applied today, and he presented a shear tester which can be regarded as the basis for the quantitative measurement of flow properties. Starting from these results, many research projects on bulk solids technology have been performed at many places around the world for several decades. Thus, a lot of knowledge is available today, but, unfortunately, sometimes it can be observed that the corresponding publications, some of which are older than twenty years, are not known, not found, or just ignored. Therefore, it is a further goal of this book to spread and to preserve at least a part of the existing knowledge.

Here as well as in the title of the present book the words "powder" and "bulk solid" are intentionally mentioned side-by-side since in some industries, e.g., the pharmaceutical industry, the word powder is common, whereas materials like cement, sand, and coal are known as bulk solids. For reasons of simplification, in the present book the term bulk solid is used to represent all materials consisting of particles, i.e., fine, cohesive powders, granulates, and coarse bulk materials.

The first chapters deal with the flow behavior and flow properties of bulk solids. Knowledge of flow properties is necessary to design hoppers, silos and other equipment to avoid flow problems (flow obstructions, segregation, ...). However, even when handling small amounts of bulk solids, e.g. when filling the dies of tabletting machines, or when operating dosage equipment, sufficiently good flow behavior is important. Furthermore, to an increasing extent quantitative information on flowability is needed, e.g., for comparative tests, product development, and quality control.

In serious bulk solids technology shear testers are mostly used for the measurement of flow properties. Thus, shear testers can be regarded as the standard testers in bulk solids technology, especially if quantitatively applicable results with defined physical meaning are required. In this book their principle is explained in great detail, because their function and theoretical background are sometimes felt to be complicated by newcomers and non-experts.

In the second half of the book the flow of bulk solids in hoppers, bins, silos, etc. is considered. This includes, above all, the procedure to design bins in order to avoid problems such as flow obstructions and segregation. Additional topics like, for example, stresses in bulk solids, feeders, and flow promoting devices, are considered in order to support those who have to design or operate plants or equipment where bulk solids are stored or transported.

I would particularly like to thank all who helped me to gain experience in bulk solids technology, and, thus, to finally prepare this book. About twenty years ago I started as a research assistant at the Institute of Mechanical Process Engineering of the Technical University of Braunschweig, where I got my first insight into bulk solids technology with financial support of the German Research Foundation (Deutsche Forschungsgemeinschaft) and supervision of Professor Jörg Schwedes. After I obtained my Ph.D., the consultancy “Schwedes + Schulze Schüttgutechnik” was founded where I could make use of my experience from research work and also gain more experience. Also my work as a Professor at the University of Applied Sciences Braunschweig/Wolfenbüttel has allowed me to learn more about the behavior of bulk solids, supported, besides others, by the Deutsche Forschungsgemeinschaft, the Lower Saxony’s Federation of Innovative Projects (AGIP), and the university itself.

Special thanks are given to emeritus Prof. Dr. Jörg Schwedes for his kind assistance in the translation of this book into English, and to Dr. John Carson, President of Jenike & Johanson, Inc., for his highly appreciated assistance in the review of the English text and many helpful comments and discussions.

Finally, my sincere thank goes to Springer for the agreeable and uncomplicated cooperation.

Wolfenbüttel, in July 2007 Dietmar Schulze

Contents

1. Introduction

Powders and bulk solids have to be handled or stored in nearly all industries, from powder coating to food, from nano-scale powders and pharmaceutical substances to products like cement, coal, and ore, from dry materials like fly ash to moist bulk solids like filter cake and clay. All these materials have to be transported, conveyed, or handled otherwise. Thus the characterization of powders and bulk solids regarding their flow properties plays an important role, e.g., for product development and optimization, customer support and the response to customer complaints.

Especially the discharge of powders and bulk solids from silos, hoppers, transport containers etc. may result in severe problems, e.g., due to flow obstructions, segregation, shocks and vibrations, or unsteady flow. To avoid such complications, solutions have to be found considering the flow properties of the bulk solid.

1.1 Common problems with bulk solids

When a plant is designed, silos, hoppers, and bins are usually not the main objective of the design work, because they do not contribute much to the processing and refinement of bulk materials like, for example, a mill or a mixer. On the other hand, poorly designed silos can reduce throughput and product quality. Nevertheless many silos and bins are built without considering the flow behavior of the bulk solid to be stored. This results in well-known problems, e.g., the occurrence of flow obstructions. Typical indications for the latter are hammering traces on hopper walls (Fig. 1.1).

Even if bulk solids are handled on a smaller scale (e.g., dosage), the problems are similar to those in silos, because the basic processes are the same and independent of the scale.

Figure 1.2 shows several problems which can emerge during the operation of a silo. If the outlet opening is too small, a stable arch can form above the outlet and the flow stops (Fig. 1.2.a). If particles are large with respect to the opening, the reason for arch formation is the interlocking of particles. Materials of very fine particle size can form cohesive arches as a result of the compressive strength caused by consolidation and interparticle adhesive forces.

Another possible source of problems is funnel flow (Fig. 1.2.b). One reason for funnel flow is a hopper wall which is too shallow or too rough. In this case in a filled silo the bulk solid cannot slide downwards along the hopper walls. Thus, so-called stagnant zones build up and the material flow is limited to a flow zone above the outlet opening. In a silo used as a buffer and never discharged completely, bulk material can remain in the stagnant zones over long periods of time and change its properties (e.g., decomposition of food products). Furthermore, the bulk solid in the stagnant zones can consolidate with time to such an extent that it will not be able to flow out after the flow zone has emptied out. The latter results in a “pipe” or “rathole” reaching from the outlet opening to the top of the filling (Fig. 1.2.c).

Fig. 1.1. Traces of hammering at a hopper wall indicating flow problems

The residence time of the bulk solid in the flow zone of a funnel flow silo can be extremely short (Fig. 1.2.d), so that the material, which has just been fed into the silo, is immediately discharged. Within this short time an easily fluidized bulk solid (e.g., flour, fine chalk) cannot sufficiently deaerate. Hence, it will flood out of the outlet opening like a fluid, resulting in increased dust generation and flooding of the feeder.

Funnel flow can also result in reduced product quality due to segregation (Fig. 1.2.e). When filling a silo, one has to take into account that the product can segregate across the cross-section of the silo. When the silo is filled centrically, one often will observe an increased amount of fines close to the silo axis, and more coarse particles close to the silo wall. If funnel flow takes place at discharge, at first the material from the center (the fines) flows out, followed by the coarser material from the silo periphery. The time-dependent composition of the discharged bulk solid can be, for example, a quality problem during the filling of small packages, or when steady-state downstream processes have to be charged.

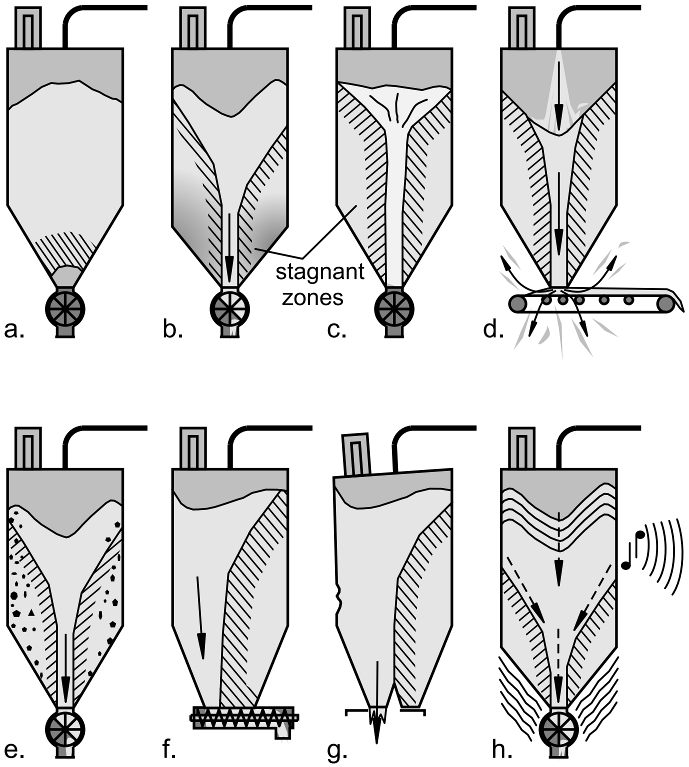

Fig. 1.2. Possible problems during the operation of silos; a. arching; b. funnel flow with wide residence time distribution and deterioration in product quality; c. ratholing; d. flooding; e. segregation; f. non-uniform discharge with screw feeder; g. buckling caused by eccentric flow; h. vibrations (silo quaking and silo noise)

Less well-designed feeders can result in a non-uniform withdrawal of the bulk solid and, thus, in funnel flow. The screw feeder shown in Fig. 1.2.f discharges the bulk solid at its rear end and then conveys it horizontally to its outlet, i.e., the screw is filled already at the rear end of the feeder so that along the rest of the outlet length no further bulk solid can enter the screw. Another reason for eccentric flow is, for example, the discharge from only one of multiple outlets (Fig. 1.2.g). Eccentric flow results in unfavorable non-symmetrical loads on the silo structure thus increasing the danger of buckling of thin-walled metal silos and cracking of reinforced concrete silos that have only a single layer of reinforcing steel.

For different reasons the flow of a bulk solid in a silo can result in vibrations and shocks (Fig. 1.2.h). Depending on the frequency, the effect is called silo noise (audible) or silo quaking (low frequency, individual shocks).

The problems described above result on the one hand from the given equipment (e.g., design of the feeder), on the other hand from the properties of the bulk solid (e.g., strength, friction). When designing silos, hoppers, bins etc., at first the properties of the bulk solid have to be determined. This data in combination with proven design procedures yields the appropriate geometry (e.g., hopper slope, outlet size). Therefore in the present book the flow properties of bulk solids and their measurement as well as design procedures (e.g., stresses, avoidance of flow problems) and design considerations (e.g., hopper shape, feeder) are described.

1.2 Milestones of bulk solids technology

As much as Bernoulli's equation can be regarded as a milestone in fluid dynamics, the work of two persons, variously referred to in the present book, need to be mentioned with respect to their impact on bulk solids technology: H.A. Janssen and A.W. Jenike.

Janssen was an engineer living in Bremen, Germany, at the end of the 19th century, when increasing amounts of corn were imported from overseas and, thus, needed to be stored. In the United States corn was stored in silos (corn being the first bulk solid stored in large silos; cement and flour followed around 1900 [1.1]). Engineering books at the time did not list design procedures for silos, but from the literature [1.2,1.3] Janssen knew that stress at the bottom of a silo does not increase linearly with filling height as in the case of fluids, but becomes constant from a certain filling height.

In order to investigate the dependence of the stress on the filling height, Janssen used the experimental set-up shown in Fig. 1.3 [1.4]. Model silos with square cross-section of different size made from wood were placed above a balance. At different filling levels the force acting on the balance, and thus the vertical stress, was measured. This way Janssen could confirm that the vertical stress is not proportional to the filling level (Fig. 1.4). The bulk solid did not show the linear increase of stress with filling height typical for fluids.

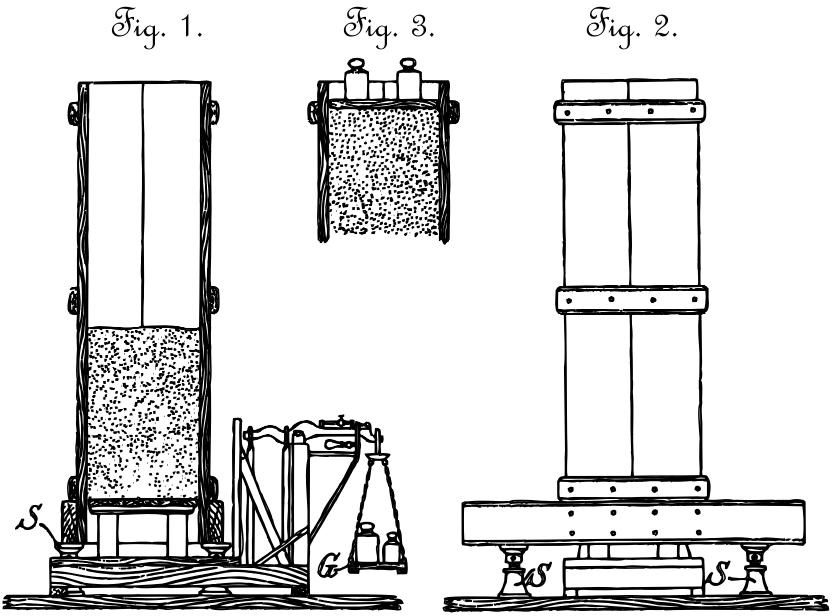

Fig. 1.3. Janssen's test set-up for the measurement of vertical stress (figure copied from [1.4])

Janssen recognized that the bulk solid in the silo is supported by friction at the silo wall. For the measurement of the coefficient of friction between the wall material and the bulk solid he employed the set-up shown in Fig. 1.5, where a platen made from the wall material (wood) was loaded vertically with weights and moved horizontally across the surface of the bulk solid whereby the force F was measured. Furthermore, Janssen derived an equation from the equilibrium of forces on a differential slice of bulk solid in the silo. This equation, which allows the calculation of stresses in the vertical section of a silo, is still applied today and known as the "Janssen equation". It is part of the silo design codes in several countries and can be found, for example, in the old German code DIN 1055 part 6 [1.5] and the new Eurocode EN 1991-4:2006-12 [1.6].

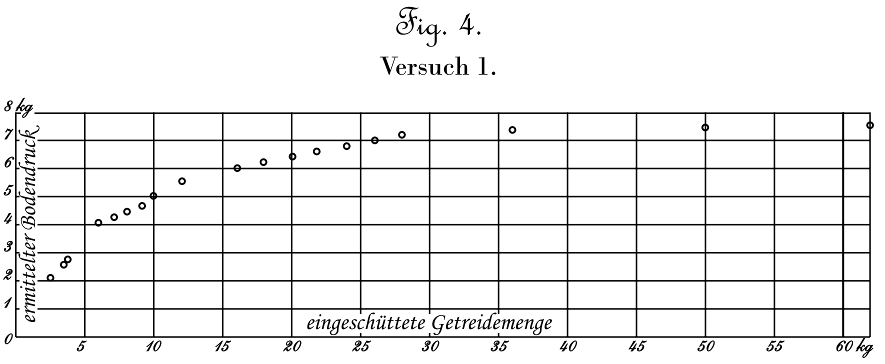

Fig. 1.4. Janssen's test results: Measured vertical stress ("ermittelter Bodendruck") in dependence on the amount of corn ("eingeschüttete Getreidemenge") filled into the silo (figure copied from [1.4])

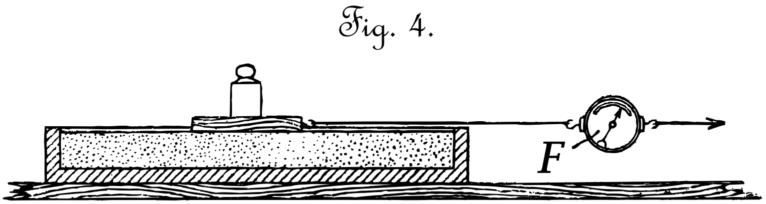

Fig. 1.5. Set-up for the measurement of the friction coefficient between bulk solid and wall material (figure copied from [1.4])

Dr. Andrew W. Jenike graduated in Mechanical Engineering from Warsaw Polytechnic Institute in 1939. After World War II he obtained his Ph.D. in Structural Engineering in England, immigrated to Canada and later to the United States. In the Fifties, when he was approaching the age of 40, he looked for a field in which he could make a unique and significant engineering and scientific contribution. Within one year he studied about 40 different fields of technology. Finally Jenike chose bulk solids technology, because in this field he identified a very low level of technology. With financial support from different sources, Jenike set up the "Bulk Solids Flow Laboratory" at the University of Utah [1.7].

On the basis of principles known from soil mechanics, Jenike described the behavior of bulk solids by introducing the yield locus. Furthermore, he derived a theory describing the stresses in silos, especially in the hopper section [1.8,1.9]. Jenike defined the terms *mass flow* and *funnel flow* [1.10], which are the most important criteria for the assessment of the flow regime in a silo. In case of funnel flow, only a portion of the bulk solid in the silo moves downwards during discharge whereby the rest of the bulk solid is at rest in the stagnant zones (Fig. 1.6.a). In case of mass flow the whole silo contents, i.e., every particle, move during discharge (Fig. 1.6.b).

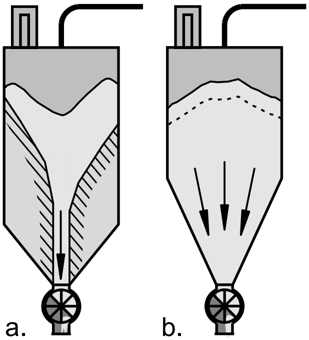

Fig. 1.6. a. Funnel flow; b. mass flow

On the basis of his theoretical considerations, Jenike derived a procedure for designing silos for unobstructed flow. From this point on it was possible to calculate the slope of the hopper walls to ensure mass flow (Fig. 1.6.b), and to predict the minimum outlet dimensions required to avoid stable arches (Fig. 1.2.a) or ratholes (Fig. 1.2.c). Furthermore, Jenike worked on the practical design of silos, e.g., regarding the influence of the feeder on the flow in the silo (e.g. Fig. 1.2.f). For the determination of the flow properties, which have to be known for the application of Jenike's design procedure, he developed a tester, which is widely known as the Jenike shear tester. Still today the Jenike design procedure is the most widely applied method, although in the meantime other (similar) procedures have been published. Jenike's work, especially his most important publication for practical application, Bulletin 123 "Storage and flow of solids" [1.9], is still cited as fundamental literature for bulk solid and silo technology.

With the introduction of the yield locus for the representation of flow properties and the presentation of a suitable tester, Jenike laid the foundation for the quantitative measurement of flow properties (e.g., flowability), which goes far beyond archaic, relatively inaccurate methods like, for example, the measurement of the angle of repose.

2 Fundamentals

2.1 Particles or continuum?

A bulk solid consists of individual particles. In principle it is possible to describe the behavior of a bulk solid by regarding the particle-particle interactions. But, as is easily understandable, this would be a difficult procedure, because the number of particles in a powder-handling system is usually very large (e.g., 10^9 particles of a diameter of 10 μm are contained in one cm^3), each particle has a different shape, and the adhesive forces between particles can hardly be accurately calculated. Although in the last few years more and more calculations based on particle-particle interactions (DEM - discrete element method) have been presented, the output with respect to the number of particles and the complexity of particle shapes is limited by the available processing power.

Another approach is to regard the bulk solid as a continuum. Therefore, instead of forces between individual particles, one regards forces or stresses, respectively, on boundary areas of volume elements and the resulting deformations, similar to procedures in fluid mechanics and strength theory. The volume elements have to be sufficiently large with respect to the particle size so that local interactions between individual particles do not need to be considered.

The last mentioned procedure, i.e., the application of the methods of continuum mechanics, is the classic way of proceeding in bulk solids technology and will be outlined in the present book. This is true for the measurement of flow properties, where usually stresses and deformations on a defined volume of bulk solid are investigated, as well as for calculation procedures.

2.2 Forces and stresses

The state of load on a bulk solid is described using the methods of continuum mechanics: One does not consider the forces at the individual particles of the bulk solid, but the forces on the boundary areas of individual volume elements. Resolution of force *F* acting on an area *A* (Fig. 2.1) in an arbitrary direction leads to:

- the normal force, F_N: force acting perpendicular (“normal”) to area *A*.
- the shear force, F_S: force acting parallel to area *A*.

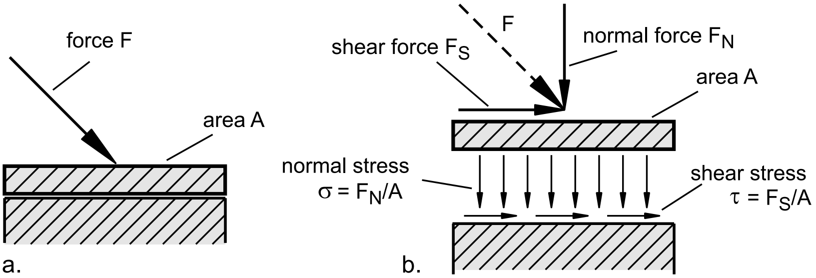

Fig. 2.1. a. applied force, *F*; b. resolution of force, *F*; stresses

To be able to describe the load on the bulk solid independently of the dimensions of the area considered above, stresses are calculated from the forces. A stress is defined through the relationship of force to area. If one divides the above-described forces, F_S and F_N, by area *A*, one obtains:

- the normal stress $\sigma = F_N/A$: stress acting perpendicularly (“normally”) on area *A;*
- the shear stress $\tau = F_S/A$: stress parallel to area *A*.

If a force of any direction is acting on the plane, a resolution of this force in a component perpendicular to the plane and another component parallel to the plane yields the normal and shear component of this force and thus normal and shear stresses acting on the plane can be calculated as described above.

In bulk solids technology, shear stresses always emerge due to frictional effects: If, for example, a bulk solid is located on a horizontal plane (Fig. 2.2.a) and is subjected only to gravity acting perpendicularly to the plane, (in the mean) no shear stress is acting between plane and bulk solid. The bulk solid remains at rest.

If the plane becomes inclined to the horizontal by a sufficiently large angle α, the bulk solid will slide downwards (Fig. 2.2.c). In contrast to this, the bulk solid will remain at rest at a smaller inclination (Fig. 2.2.b). In both cases a shear stress, τ, is acting between bulk solid and plane. The directions of the stresses acting on the bulk solid are shown by the arrows in Fig. 2.2. If the force transferred through the shear stress is not less than the force that pulls the bulk solid downwards, the bulk solid will remain at rest. If the transferable shear stress is too small, the bulk solid will slip downwards.

The magnitude of the transferable shear stress is dependent on the friction between the bulk solid and the plane surface: A rough surface will be able to transfer larger shear stresses than a smooth surface, i.e., on an inclined plane with a rough surface the bulk solid will slip downwards only at a larger angle α than on a plane with a smooth surface. If the surface were frictionless (ideally smooth), the bulk solid would slip downwards at any inclination $\alpha > 0°$, because there would be no shear stress for the plane to transfer.

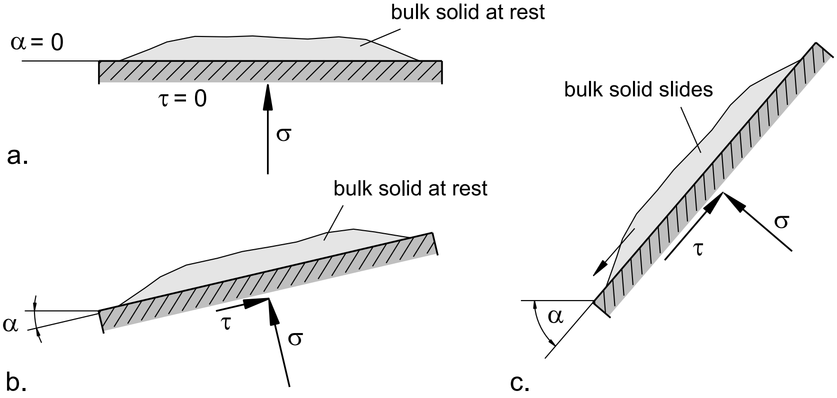

Fig. 2.2. Shear stresses (example)

In bulk solids technology, e.g., storage of a bulk solid in a container, normal stresses are mainly compressive stresses. This means that compressive normal stresses are acting on arbitrary volume elements within the bulk solid (similar to Fig. 2.1). Tensile forces are found rarely in bulk solids technology. In contrast to classic engineering mechanics, but similar to the pressure in fluid dynamics, compressive forces are defined as positive forces, and compressive stresses as positive normal stresses. Tensile forces and tensile stresses are characterized by negative values ($\sigma < 0$). There are also rules for the direction of positive and negative shear stresses, but these are not important for the purpose of this book (for more about this see [2.1,2.2,2.3]).

The unit used for stress is Pa (Pascal) according to the International System of Units (SI). 1 Pa is equal to 1 Newton per square meter (1 N/m^2). 1 bar is equal to 100 000 Pa, 1 psi is 6 894.76 Pa.

2.3 Stresses in bulk solids

Figure 2.3 shows a bulk solid element in a container (assumptions: infinite filling height, frictionless internal walls). In the vertical direction, positive normal stress ($\sigma_v > 0$, compressive stress) is exerted on the bulk solid.

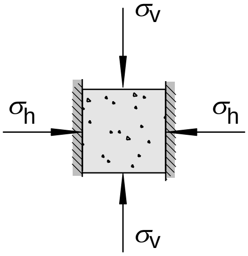

Fig. 2.3. Bulk solid element

If the bulk solid were to behave like a Newtonian fluid, the stresses in the horizontal and vertical direction (and in all other directions) would be of equal magnitude. In reality the behavior of a bulk solid is quite different from that of a fluid, so the assumption of analogies is often misleading.

Within the bulk solid (Fig. 2.3) the horizontal stress, σ_h, is a result of the vertical stress, σ_v, where the resulting horizontal stress is less than the vertical stress exerted on the bulk solid from the top. The ratio of horizontal stress, σ_h, to vertical stress, σ_v, is the lateral stress ratio, K (alternative designation: λ):

$$K = \lambda = \frac{\sigma_h}{\sigma_v} \tag{2.1}$$

Typical values of K are between 0.3 and 0.6 [2.4].

2.3.1 Introduction of the Mohr stress circle

It follows from Fig. 2.3, that – in analogy to solids – in a bulk solid different stresses can be found in different cutting planes – even if the bulk solid is at rest. Stresses in cutting planes other than vertical and horizontal can be analyzed using a simple equilibrium of forces.

No shear stresses are exerted on the top or bottom surface of the bulk solid element in Fig. 2.3, i.e., the shear stresses, τ, in these planes are equal to zero. No shear stresses are acting at the lateral walls, since the lateral walls were assumed to be frictionless. Thus only the normal stresses shown are acting on the bulk solid from outside. Using a simple equilibrium of forces on a volume element with triangular cross-section cut from the bulk solid element shown in Fig. 2.3 (Fig. 2.4), the normal stress, σ_α, and the shear stress, τ_α, acting on a plane inclined by an arbitrary angle α, can be calculated. After some mathematical transformations (e.g. [2.1]), which are considered in the following section, it follows that:

$$\sigma_\alpha = \frac{\sigma_v + \sigma_h}{2} + \frac{\sigma_v - \sigma_h}{2}\cos(2\alpha) \tag{2.2}$$

$$\tau_\alpha = \frac{\sigma_v - \sigma_h}{2}\sin(2\alpha) \tag{2.3}$$

The pairs of values (σ_α, τ_α), which are to be calculated according to Eqs. (2.2) and (2.3) for all possible angles α, can be plotted in a σ,τ diagram (normal stress - shear stress diagram); see Fig. 2.4. If one joins all plotted pairs of values, a circle emerges, i.e., all calculated pairs of values form a circle in the σ,τ diagram.

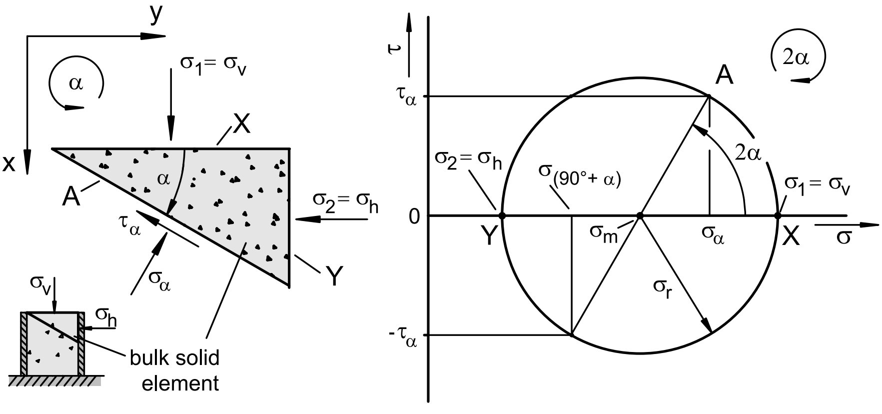

Fig. 2.4. Equilibrium of forces on a bulk solid element; Mohr stress circle

This circle is called the “Mohr stress circle”. Its center is located at $\sigma = \sigma_m = (\sigma_v + \sigma_h)/2$ and $\tau = 0$. The radius of the circle is $\sigma_r = (\sigma_v - \sigma_h)/2$. The Mohr stress circle represents the stresses in all cutting planes at arbitrary inclination angles α, i.e., in all possible cutting planes within a bulk solid element. Since the center of the Mohr stress circle is always located on the σ axis, each Mohr stress circle has two points of intersection with the σ axis. The normal stresses defined through these points of intersection are called the principal stresses, whereby the larger principal stress – the major principal stress – is designated as σ_1 and the smaller principal stress – the minor principal stress – is designated as σ_2 (Please note that, in contrast to bulk solids technology, in continuum mechanics the minor principal stress is usually designated as σ_3). If both principal stresses are given, the Mohr stress circle is defined.

In the example of Fig. 2.4 both horizontal and vertical planes are free from shear stresses ($\tau = 0$) and are thus principal stress planes. In this case the vertical stress, σ_v, which is greater than the horizontal stress, σ_h, is the major principal stress, σ_1, and the horizontal stress, σ_h, is the minor principal stress, σ_2.

The angle α also can be found with the Mohr stress circle. The stresses acting in a cutting plane that is rotated at an angle α to the first cutting plane are found at a point on the Mohr circle rotated by an angle 2α relative to the point representing the stresses in the first cutting plane, where the angle 2α on the Mohr stress circle is measured in the opposite direction from the angle α between the cutting planes in the bulk solid. For example, the horizontal and vertical cutting planes in Fig. 2.4 are perpendicular to each other, i.e., the angle inclined by these planes is $\alpha = 90°$. In the stress circle one finds the points that represent the stresses in the cutting planes at two positions inclined by $\alpha = 180°$.

From the explanation above it follows that the state of stress in a bulk solid cannot be completely described by only a single numerical value. Depending on the actual load acting on a bulk solid element, the corresponding Mohr stress circle can have a smaller or a larger radius, a center at a lesser or greater normal stress, and hence also different principal stresses σ_1 and σ_2. Generally, at a given major principal stress, σ_1, there are an infinite number of stress circles with different values for the lowest principal stress, σ_2. Therefore, a stress circle is defined clearly only if at least two numerical values are given, i.e., σ_1 and σ_2.

The relationships are actually more complicated because in reality a three-dimensional state of stress exists, which cannot be represented by only one stress circle in one plane as shown in Fig. 2.4. The Mohr stress circles in the two planes which are perpendicular to this plane and perpendicular to each other also are required for representation of the three-dimensional stress state [2.1,2.3,2.5,2.6]. Thus, altogether there are three principal stresses, which are perpendicular to each other (Fig. 2.5).

The volume element in Fig. 2.5 is cut along the principal stress planes, i.e., the surfaces of the volume element are parallel to the principal stress planes. For each principal stress plane the Mohr stress circle is plotted in the σ,τ diagram. The largest Mohr stress circle represents the stresses in the cutting plane perpendicular to the mean principal stress, σ_3, because in this plane the smallest minor and largest major principal stresses, σ_2 and σ_1, are acting. For many applications in bulk solids technology it is sufficient to consider only one plane. In those cases one refers to a "two-dimensional" state of stress. As a rule the plane considered is the one in which the smallest minor and largest major principal stresses, and, thus, the greatest shear stress, are acting [2.1,2.5,2.6]. Shear stress is especially important because it is responsible for the movement of particles relative to each other, i.e., the flow of a bulk solid.

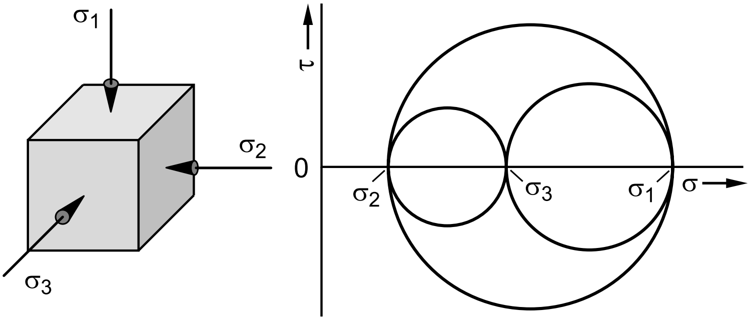

Fig. 2.5. Volume element with the three principal stresses being perpendicular to each other; stress circles for the three principal stress planes

For simple practical applications (e.g., flowability tests) it is often too complicated to characterize a two-dimensional stress state by two principal stresses (i.e., the minor and the major principal stresses), so that usually only the major principal stress, σ_1, is mentioned. Accordingly, one would mark the state of stress presented in Fig. 2.4 through the major principal stress, σ_1, which in this case is equal to the vertical stress, σ_v, exerted on the bulk solid. Therewith one assumes that the greatest of all normal stresses acting in different directions within a bulk solid element is responsible for the actual condition of the bulk solid.

This simplification or approximation is used, e.g., when loading a bulk solid sample with a certain vertical stress, for example, through weights of mass m (Fig. 2.6). Thereby, as a rule, one characterizes the stress state by the major principal stress, σ_1, which in this case is equal to the force of gravity, $m \cdot g$, of the weights divided by the area, A, and denotes this stress as the consolidation stress, σ_1.

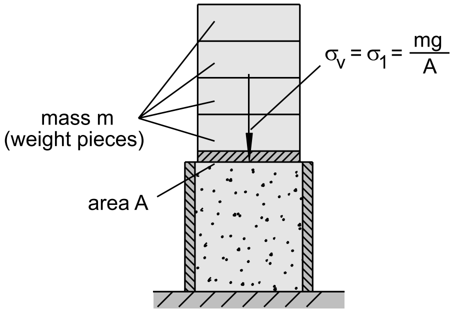

Fig. 2.6. Consolidation stress, σ_1, at uniaxial consolidation of a bulk solid in a hollow cylinder with frictionless walls

2.3.2 Derivation of the Mohr stress circle

In the following the mathematical derivation of the Mohr stress circle and Eqs.(2.2) and (2.3) is outlined starting from the equilibrium of forces on the volume element. Furthermore, the existence of two principal stress planes which are perpendicular to each other is shown.

The equilibrium of forces on the triangular bulk solid element in Fig. 2.4 in *x* and *y* direction yields (dimension of the bulk solid element perpendicular to the *x*,*y* plane is equal to 1):

$$\sum F_x = \sigma_v l \cos\alpha - \sigma_\alpha l \cos\alpha - \tau_\alpha l \sin\alpha = 0 \tag{2.4}$$

$$\sum F_y = \sigma_\alpha l \sin\alpha - \tau_\alpha l \cos\alpha - \sigma_h l \sin\alpha = 0 \tag{2.5}$$

It follows:

$$\tau_\alpha = \frac{\sigma_v \cos\alpha - \sigma_\alpha \cos\alpha}{\sin\alpha} \tag{2.6}$$

$$\sigma_\alpha = \frac{\tau_\alpha \cos\alpha + \sigma_h \sin\alpha}{\sin\alpha} \tag{2.7}$$

Substitution of τ_α in Eq.(2.7) by Eq.(2.6):

$$\sigma_\alpha = \frac{\sigma_v (\cos\alpha)^2 - \sigma_\alpha (\cos\alpha)^2}{(\sin\alpha)^2} + \sigma_h \,.$$