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Contents

Contents ... 5
Preface ... 7
Curriculum Vitae of László Lovász ... 11
Publications of László Lovász ... 15
I. Bárány: On the Power of Linear Dependencies ... 31
J. Beck: Surplus of Graphs and the Lovász Local Lemma ... 47
A. A. Benczúr and M. X. Goemans: Deformable Polygon Representation and Near-Mincuts ... 103
K. Bérczi and A. Frank: Variations for Lovász' Submodular Ideas 137
A. Björner: Random Walks, Arrangements, Cell Complexes, Greedoids, and Self-organizing Libraries ... 165
A. Blokhuis and F. Mazzocca: The Finite Field Kakeya Problem . 205
B. Bollobás and V. Nikiforov: An Abstract Szemerédi Regularity Lemma ... 219
S. C. Brubaker and S. S. Vempala: Isotropic PCA and Affine-Invariant Clustering ... 241
U. Feige: Small Linear Dependencies for Binary Vectors of Low Weight ... 283
K. Gyarmati, M. Matolcsi and I. Z. Ruzsa: Plünnecke's Inequality for Different Summands ... 309
R. Kannan: Decoupling and Partial Independence ... 321
B. Korte and J. Vygen: Combinatorial Problems in Chip Design .. 333
J. Nešetřil and P. Ossona de Mendez: Structural Properties of Sparse Graphs ... 369
M. D. Plummer: Recent Progress in Matching Extension ... 427
H. E. Scarf: The Structure of the Complex of Maximal Lattice Free Bodies for a Matrix of Size $(n+1) \times n$ ... 455

A. Schrijver: Graph Invariants in the Edge Model 487
J. Solymosi: Incidences and the Spectra of Graphs 499
J. Spencer: The Maturation of the Probabilistic Method 515
V. Vu: A Structural Approach to Subset-Sum Problems 525

Preface

László Lovász, briefly called Laci by his friends, turned sixty on March 9, 2008. To celebrate this special birthday two conferences have been held in Hungary, one in Budapest (August 5–9, 2008) with invited speakers only and one in Keszthely (August 11–15, 2008). Several top mathematicians and computer scientists have not only lectured at these meetings but also dedicated (together with some coauthors) research papers to this occasion. This volume is the collection of their articles. The contributions to the conferences and this book honor a person who has not only made an almost uncountable number of fundamental contributions to mathematics and computer science, but who also broke down many borders between mathematical disciplines and built sustainable bridges between mathematics and computer science.

Laci has been a role model for many young researchers, he inspired lots of colleagues, and guided quite a few scientific careers. In addition, he is an extremely nice person and very pleasant colleague, and that is why so many researchers have come to the "Lovász meetings" in Hungary to present their best recent work and celebrate with him.

In the Fazekas Mihály Gimnázium in Budapest, a breeding place of world class mathematicians, Laci's outstanding talent became visible at very young age. He did not only win various mathematics competitions in Hungary, Lovász also won three gold medals and one silver medal in the International Mathematical Olympiad. The solution of an open problem in lattice theory gained him his first international visibility and soon after, in 1972, his proof of the perfect graph theorem earned him lasting fame in graph theory. An unparalleled sequence of scientific achievements followed and is continuing till today. It is impossible to mention even a small fraction of Laci's results here. The list of his publications (up to summer 2008) is contained in this volume to indicate the breadth and depth of his contributions.

Being a combinatorialist at heart, like so many Hungarian mathematicians, it has been natural for Laci to employ combinatorial techniques in other areas of mathematics; but he also brought topology, algebra, analysis, stochastics and other mathematical fields to combinatorics, often in quite unusual ways. In this way he opened up quite a number of new flourishing fields of research. Algorithmic issues such as polynomial time solvability and general complexity theory opened his eyes for computer science where he particularly contributed to the interface between computer science and discrete mathematics.

After a distinguished academic career with employments and visiting positions in Szeged, Budapest, Waterloo, Bonn, Cornell, Princeton, Yale (and guest professorships in many other places) Lovász had worked for Microsoft Research in Redmond from 1998 to 2006. He returned thereafter to Budapest to become director of the Mathematical Institute of Eötvös Loránd University.

His international reputation is stellar. One proof of this claim is a remarkable series of prestigious honors; among them are the Grünwald, Pólya, Fulkerson, Wolf, Knuth, Gödel, von Neumann, Bolyai, and Széchenyi Prizes and various other distinctions such as honorary degrees and professorships. In 2006 László Lovász has been elected president of the International Mathematical Union for the years 2007 to 2010. A few more details can be found in his (very brief) Curriculum Vitae on pages 11 to 13 in this volume.

Laci has always been a family man, a loving husband, father, and meanwhile also grandfather. With Kati Vesztergombi, his wife for almost 40 years, he has not only shared family and friends; Kati and Laci are also closely linked by their common love for mathematics. Their relationship has not only resulted in three wonderful daughters and a son but also in two joint books and quite a number of papers. Kati has been a mainstay in Laci's life since their common time in high school. That is why we have chosen to display a husband and wife portrait of the couple on page 9 of this volume.

Budapest
August 2008

Martin Grötschel
Gyula Katona

Kati Vesztergombi and László Lovász

BOLYAI SOCIETY
MATHEMATICAL STUDIES, 19

Building Bridges
pp. 11–13.

Curriculum Vitae of László Lovász

Born: March 9, 1948 in Budapest, Hungary

Family: Married, 4 children

Citizenship: Hungary, United States

Degrees:
Dr. rer. nat., Eötvös Loránd University, Budapest, Hungary, 1971;
Candidate of mathematical sciences, Hungarian Academy of Sciences, Budapest, Hungary, 1970;
Dr. of mathematical sciences, Hungarian Academy of Sciences, Budapest, Hungary, 1977.

Academies:
Hungarian Academy of Sciences, corresp. member, 1979; full member, 1985;
European Academy of Arts, Sciences and Humanities, 1981;
Academia Europaea, 1991;
Nordrhein-Westfälische Akademie der Wissenschaften, corresp. member, 1993;
Deutsche Akademie der Naturforscher Leopoldina, 2002;
Russian Academy of Sciences, 2006;
Royal Netherlands Academy of Arts and Sciences, 2006;
Royal Swedish Academy of Sciences, 2007.

Positions in scientific societies:
Executive Committee of the International Mathematical Union, 1987–1994;
Presidium of the Hungarian Academy of Sciences, 1990–1993 and since 2008;
Abel Prize Committee, 2004–2006;
President of the International Mathematical Union, 2007–2010;

Positions held:
Research Associate, Eötvös Loránd University, Budapest, 1971–75;
Docent, József Attila University, Szeged, 1975–78;
Professor, Chair of Geometry, József Attila University, Szeged, 1978–82;

Professor, Chair of Computer Science, Eötvös Loránd University, Budapest, 1983–93;
Professor, Dept. of Computer Science, Yale University, 1993–2000;
Senior Researcher, Microsoft Research, 1999–2006;
Director, Mathematical Institute of the Eötvös Loránd University, Budapest, 2006–.

Visiting positions:
Vanderbilt University, 1972/73;
University of Waterloo, 1978/79;
Universität Bonn, 1984/85;
University of Chicago, Spring 1985;
Cornell University, Fall 1985;
Mathematical Sciences Research Institute, Berkeley, Spring 1986;
Princeton University, Fall 1987, Spring 1989, 1990/91, 1992/93.

Honorary degrees and positions:
Adjunct Professor, University of Waterloo, Waterloo, Ontario, Canada, 1980–1990;
A. D. White Professor-at-Large, Cornell University, Ithaca, NY, 1982–1987;
Honorary Professor, Universität Bonn, 1984; Academia Sinica, 1988;
John von Neumann Professor, Universität Bonn, 1985;
Doctor Honoris Causa: University of Waterloo, Ontario, Canada, 1992; University of Szeged, Hungary, 1999; Budapest University of Technology, 2002; University of Calgary, 2006.

Awards:
Grünwald Géza Prize, Bolyai Society, 1970;
George Pólya Prize, Soc. Ind. Appl. Math., 1979;
Best Information Theory Paper Award, IEEE, 1981;
Ray D. Fulkerson Prize, Amer. Math. Soc. – Math. Prog. Soc., 1982;
State Prize, Hungary, 1985;
Tibor Szele Medal, Bolyai Society, 1992;
Brouwer Medal, Dutch Matematical Society – Royal Netherl. Acad. Sci., 1993;
National Order of Merit of Hungary, 1998;
Bolzano Medal, Czech Mathematical Society, 1998;
Wolf Prize, Israel, 1999;
Knuth Prize, ACM–SIGACT, 1999;
Corvin Chain, Hungary, 2001;
Goedel Prize, ACM–EATCS, 2001;

John von Neumann Medal, IEEE, 2005;
John von Neumann Theory Prize, INFORMS, 2006;
Bolyai Prize, Hungary, 2007;
Széchenyi Grand Prize, Hungary, 2008.

Editorial boards:
Combinatorica (Editor-in-Chief),
Advances in Mathematics,
J. Combinatorial Theory (B),
Discrete Math.,
Discrete Applied Math.,
J. Graph Theory,
Europ. J. Combinatorics, Discrete and Computational Geometry,
Random Structures and Algorithms,
Acta Mathematica Hungarica,
Acta Cybernetica,
Electronic Journal of Combinatorics,
Geometric and Functional Analysis.

Fields of research: Combinatorial optimization, graph theory, theoretical computer science.

Publications: 9 books, 250 research papers.

BOLYAI SOCIETY
MATHEMATICAL STUDIES, 19

Building Bridges
pp. 15–29.

Publications of László Lovász

Books:

[1] Lovász L., Pelikán J., Vesztergombi K.: *Kombinatorika,* Tankönyvkiadó, Budapest, 1977 (German translation: Teubner, 1977; Japanese translation: 1985).

[2] Gács P., Lovász L.: *Algoritmusok,* Műszaki Könyvkiadó, Budapest, 1978; Tankönyvkiadó, Budapest, 1987.

[3] L. Lovász: *Combinatorial Problems and Exercises,* Akadémiai Kiadó – North Holland, Budapest, 1979 (Japanese translation: Tokai Univ.Press, 1988; Hungarian translation: Typotech, 1999; Second edition: North-Holland Publishing Co., Amsterdam, 1993.)

[4] L. Lovász, M.D. Plummer: *Matching Theory,* Akadémiai Kiadó – North Holland, Budapest, 1986 (Russian translation: Mir, 1998).

[5] L. Lovász: *An Algorithmic Theory of Numbers, Graphs, and Convexity,* CBMS-NSF Regional Conference Series in Applied Mathematics **50,** SIAM, Philadelphia, Pennsylvania 1986.

[6] M. Grötschel, L. Lovász, A. Schrijver: *Geometric Algorithms and Combinatorial Optimization,* Springer, 1988; Chinese edition: World Publishing Corp., Beijing, 1990.

[7] B. Korte, L. Lovász, R. Schrader: *Greedoids,* Springer, 1991.

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Research papers:

[1] Lovász L.: Független köröket nem tartalmazó gráfokról (On graphs containing no independent circuits), *Mat. Lapok,* **16** (1965), 289–299.

[2] L. Lovász: On decomposition of graphs, *Studia Math. Hung.,* **1** (1966), 237–238.

[3] L. Lovász: On connected sets of points, *Annales Univ. R. Eötvös,* **10** (1967), 203–204.

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MATHEMATICAL STUDIES, 19

Building Bridges
pp. 31–45.

On the Power of Linear Dependencies

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Simple as they may be, linear dependencies have proved very useful in many ways. In this survey several geometric applications of linear dependencies are discussed, focusing on rearrangements of sums and on sums with ± 1 signs.

1. Introduction

Linear algebra is a basic and powerful tool in many areas of mathematics. In combinatorics, for instance, there are several cases when the size of a set can be bounded by a number n because the elements of the set are associated with vectors in R^n, and these vectors turn out to be linearly independent. The excellent book [1] by Babai and Frankl (which is unfortunately, still unpublished) contains thousands of beautiful applications of the so-called linear algebra method.

This article describes another kind of use of linear algebra, this time in geometry. The method uses linear dependencies and is often referred to as the *method of floating variables*. The same method is used at other places as well, for instance in discrepancy theory, in the Beck–Fiala theorem [7] or [8], and in probability theory, [9]. Here we focus on rearrangement of sums and on sums with ± 1 signs.

In what follows the setting is the d-dimensional Euclidean space $\mathbb{R}^d$, together with a (Minkowski) norm, $\|\cdot\|$ whose unit ball is denoted by B or B^d. We write $\mathbb{N}$ for the set of natural numbers and $[n]$ for the set $\{1, 2, \dots, n\}$ where $n \in \mathbb{N}$. We assume that $V \subset B$ is a finite set.

2. The Steinitz lemma

Assume $V \subset B$ is finite and $\sum_{v\in V} v = 0$. The question, due to Riemann and Lévy, is whether there is an ordering, $v_1, v_2, \dots, v_n$, of the elements of V such that all partial sums along this order are bounded by a number that only depends on B. The answer is yes. An incomplete proof came from Lévy [17] in 1905. The first complete proof, from 1913, is due to Steinitz [20], and that's why it is usually called the Steinitz Lemma.

Theorem 2.1. *Given a finite set $V \subset B$ with $\sum_{v\in V} v = 0$, where B is the unit ball of a norm in $\mathbb{R}^d$, there is an ordering $v_1, v_2, \dots, v_n$ of the elements of V such that for all $k \in [n]$*

$$\sum_{1}^{k} v_i \in dB.$$

So all partial sums are contained in a blown-up copy of the unit ball, with blow-up factor d. We will return to the value of the blow-up factor later. Let's see first the proof of Theorem 2.1, which is our first application of linear dependencies.

Proof. The key step is the construction of sets $V_{d+1} \subset \dots \subset V_{n-1} \subset V_n = V$ where $|V_k| = k$, together with functions $\alpha_k \colon V_k \to [0,1]$ satisfying

$$\sum_{v\in V_k} \alpha_k(v)v = 0$$

$$\sum_{v\in V_k} \alpha_k(v) = k - d.$$

So the functions $\alpha_k(.)$ are *linear dependencies* on V_k, with coefficients in $[0,1]$ that sum to $k-d$.

The construction goes by backward induction. The starting case $k = n$ is easy: $V_n = V$ and $\alpha_n = (n-d)/n$ satisfy the requirements. Assume now that V_k and α_k have been constructed, and consider the auxiliary system

$$\sum_{v\in V_k} \beta(v)v = 0,$$

$$\sum_{v \in V_k} \beta(v) = k - 1 - d,$$

$$0 \le \beta(v) \le 1 \quad \text{for all} \quad v \in V_k.$$

Write P for the set of functions $\beta\colon V_k \to [0,1]$ satisfying this auxiliary system. The elements of P can and will be regarded as vectors in $\mathbb{R}^k$ whose components are indexed by the elements of V_k.

Note that P is non-empty since $\beta(v) = \frac{k-1-d}{k-d}\alpha_k(v)$ belongs to P. Thus P is a convex polytope, lying in the unit cube of $\mathbb{R}^k$. Let $\beta^*(.)$ be an extreme point of P.

We claim now that $\beta^*(v) = 0$ for some $v \in V_k$. Indeed, assume $\beta^*(v) > 0$ for all $v \in V_k$. The auxiliary system has $d+1$ equations and k variables, so at least $k - (d+1)$ of the inequalities $\beta^*(v) \le 1$ are satisfied as equalities (the inequalities $\beta^*(v) \ge 0$ are all strict). Then $\sum_{v \in V_k} \beta^*(v) > k - d - 1$ (we use $k > d+1$ here), which contradicts one of the conditions defining P.

Let $v^* \in V_k$ be an element with $\beta^*(v^*) = 0$, and define $V_{k-1} = V_k \setminus \{v^*\}$ and $\alpha_{k-1}(v) = \beta^*(v)$ for all $v \in V_{k-1}$. All conditions are satisfied for V_{k-1} and α_{k-1}. The construction is finished.

Now we ready to order the elements of V. For $k = n, n-1, \ldots, d+2$ we set, quite naturally,

$$v_k = V_k \setminus V_{k-1}.$$

The remaining $d+1$ vectors are ordered arbitrarily.

We check, finally, that all partial sums are contained in dB. This is trivial for the first d partial sums. Assume now that $k \ge d+1$.

$$\sum_1^k v_i = \sum_{v \in V_k} v = \sum_{v \in V_k} v - \sum_{v \in V_k} \alpha_k(v) v = \sum_{v \in V_k} \big(1 - \alpha_k(v)\big) v.$$

Taking norms and using that $1 - \alpha_k(v) \ge 0$ and $\|v\| \le 1$ gives

$$\Big\| \sum_1^k v_i \Big\| \le \sum_{v \in V_k} \big(1 - \alpha_k(v)\big) = k - (k-d) = d. \qquad \blacksquare$$

This splendid proof, due to Grinberg and Sevastyanov [13], is a streamlined version of an earlier one by Sevastyanov [21]. Steinitz's original proof