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Handbook of Power Systems II

Springer

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To our families.

Preface of Volume II

Power systems are undeniably considered as one of the most important infrastructures of a country. Their importance arises from a multitude of reasons of technical, social and economical natures. Technical, as the commodity involved requires continuous balancing and cannot be stored in an efficient way. Social, because power has become an essential commodity to the life of every person in the greatest part of our planet. Economical, as every industry relates not only its operations but also its financial viability in most cases with the availability and the prices of the power.

The reasons mentioned above have made power systems a subject of great interest for the scientific community. Moreover, given the nature and the specificities of the subject, sciences such as mathematics, engineering, economics, law and social sciences have joined forces to propose solutions.

In addition to the specificities and inherent difficulties of the power systems problems, this industry has gone through significant changes. We could refer to these changes from an engineering and economical perspective. In the last 40 years, important advances have been made in the efficiency and emissions of power generation, and in the transmission systems of it along with a series of domains that assist in the operation of these systems. Nevertheless, the engineering perspective changes had a small effect comparing to these that were made in the field of economics where an entire industry shifted from a long-standing monopoly to a competitive deregulated market.

The study of such complex systems can be realized through appropriate modelling and application of advance optimization algorithms that consider simultaneously the technical, economical, financial, legal and social characteristics of the power system considered. The term technical refers to the specificities of each asset that shall be modelled in order for the latter to be adequately represented for the purpose of the problem. Economical characteristics reflect the structure and operation of the market along with the price of power and the sources, conventional or renewable, used to be generated. Economical characteristics are strongly related with the financial objectives of each entity operating a power system, and consist in the adequate description and fulfillment of the financial targets and risk profile. Legal specificities consist in the laws and regulations that are used for the operation of the power system. Social characteristics are described through a series of parameters that have to be considered in the operation of the power system and reflect the issues related to the population within this system.

The authors of this handbook are from a mathematical and engineering background with an in-depth understanding of economics and financial engineering to apply their knowledge in what is know as modelling and optimization. The focus of this handbook is to propose a selection of articles that outline the modelling and optimization techniques in the field of power systems when applied to solve the large spectrum of problems that arise in the power system industry. The above mentioned spectrum of problems is divided in the following chapters according to its nature: Operation Planning, Expansion Planning, Transmission and Distribution Modelling, Forecasting, Energy Auctions and Markets, and Risk Management.

Operation planning is the process of operating the generation assets under the technical, economical, financial, social and legal criteria that are imposed within a certain area. Operation is divided according to the technical characteristics required and the operation of the markets in real time, short term and medium term. Within these categories the main differences in modelling vary in technical details, time step and time horizon. Nevertheless, in all three categories the objective is the optimal operation, by either minimizing costs or maximizing net profits, while considering the criteria referred above.

Expansion planning is the process of optimizing the evolution and development of a power system within a certain area. The objective is to minimize the costs or maximize the net profit for the sum of building and operation of assets within a system. According to the focus on the problem, an emphasis might be given in the generation or the transmission assets while taking into consideration technical, economical, financial, social and legal criteria. The time-step used can vary between 1 month and 1 quarter, and the time horizon can be up to 25 years.

Transmission modelling is the process of describing adequately the network of a power system to apply certain optimization algorithms. The objective is to define the optimal operation under technical, economical, financial, social and legal criteria. In the last 10 years and because of the increasing importance of natural gas in power generation, electricity and gas networks are modelled jointly.

Forecasting in energy is applied for electricity and fuel price, renewable energy sources availability and weather. Although complex models and algorithms have been developed, forecasting also uses historical measured data, which require important infrastructure. Hence, the measurement of the value of information also enters into the equation where an optimal decision has to be made between the extent of the forecasting and its impact to the optimization result.

The creation of the markets and the competitive environment in power systems have created the energy auctions. The commodity can be power, transmission capacity, balancing services, secondary reserve and other components of the system. The participation of the auction might be cooperative or non-cooperative, where players focus on the maximization of their results. Therefore, the market participant focus on improving their bidding strategies, forecast the behavior of their competitors and measure their influence on the market.

Risk management in the financial field has emerged in the power systems in the last two decades and plays actually an important role. In this field the entities that participate in the market while looking to maximize their net profits are heavily concerned with their exposure to financial risk. The latter is directly related to the operation of the assets and also with a variety of external factors. Hence, risk mangers model their portfolios and look to combine optimally the operation of their assets by using the financial instruments that are available in the market.

We take this opportunity to thank all contributors and the anonymous referees for their valuable comments and suggestions, and the publisher for helping to produce this volume.

February 2010

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Part I
Transmission and Distribution Modeling

Recent Developments in Optimal Power Flow Modeling Techniques

Rabih A. Jabr

Abstract This article discusses recent advances in mathematical modeling techniques of transmission networks and control devices within the scope of optimal power flow (OPF) implementations. Emphasis is on the newly proposed concept of representing meshed power networks using an extended conic quadratic (ECQ) model and its amenability to solution by using interior-point codes. Modeling of both classical power control devices and modern unified power flow controller (UPFC) technology is described in relation to the ECQ network format. Applications of OPF including economic dispatching, loss minimization, constrained power flow solutions, and transfer capability computation are presented. Numerical examples that can serve as testing benchmarks for future software developments are reported on a sample test network.

Keywords Economic dispatching · Interior-point methods · Load flow control · Loss minimization · Nonlinear programming · Optimization methods · Transfer capability

1 Introduction

The optimal power flow (OPF) is an optimization problem that seeks to minimize or maximize an objective function while satisfying physical and technical constraints on the power network. The versatility of the OPF has kept it amongst the active research problems since it was first discussed by Carpentier (1962). The OPF has numerous applications, which include the following (Wang et al. 2007; Wood and Wollenberg 1996):

1. Coordinating generation patterns and other control variables for achieving minimum cost operation

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2. Setting of generator voltages, transformer taps, and VAR sources for loss minimization
3. Implementing both preventive and corrective control strategies to satisfy system security constraints
4. Stress testing of a planned transmission network, for instance, computation of the operating security limit in the context of maximum transfer capability
5. Providing a core pricing mechanism for trading in electricity markets, for instance, the computation of the locational marginal cost at any bus in the network
6. Providing a technique for congestion management in restructured power networks
7. Controlling of flexible AC transmission systems (FACTS) for better utilization of existing power capacities

This article centers on recent developments that have occurred in OPF modeling approaches. In particular, it covers aspects related to the physical network representation, operational constraints, classical power flow control and FACTS devices, OPF objective functions and formulations, and optimization techniques. It also includes numerical examples, which could serve as benchmarks for future OPF software research and development.

A recent advancement that has appeared in the power systems literature is the extended conic quadratic (ECQ) format (Jabr 2007; 2008) for OPF modeling and solution via primal-dual interior-point methods. In retrospect, previous research on interior-point OPF programs reported the use of the voltage polar coordinates model (Granville 1994; Jabr 2003; Rider et al. 2004; Wang et al. 2007; Wu et al. 1994), the voltage rectangular model (Capitanescu et al. 2007; Torres and Quintana 1998; Wei et al. 1998), and the current mismatch formulation (Zhang et al. 2005). The advantages of the ECQ format include the simple and efficient computation of the Jacobian and Lagrangian Hessian matrices in interior-point methods and the use of linear programming scaling techniques for improving the numerical conditioning of the problem.

2 Physical Network Representation

Consider a power system operating in steady-state under normal conditions. The system is assumed to be balanced and is represented by a single-phase network formed of N buses. Denote the complex rectangular representation of an element in the $N \times N$ bus admittance matrix by $\hat{Y}_{in} = G_{in} + jB_{in}$. In OPF formulations, the network model is accounted for via the enforcement of the real and reactive power injection constraints:

$$P_i = P_{gi} - P_{di}, \tag{1}$$

$$Q_i = Q_{gi} - Q_{di} + Q_{ci}, \tag{2}$$

where

- P_{gi}/Q_{gi} is the real/reactive power generated at bus $i\,(i = 1, \ldots, N)$.
- P_{di}/Q_{di} is the real/reactive power demand of the load at bus $i\,(i = 1, \ldots, N)$.
- Q_{ci} is the reactive power injected by a capacitor at bus $i\,(i = 1, \ldots, N)$.

There are different representations of the power injections in terms of the elements of the bus admittance matrix and the bus voltages, namely the classical model with voltages in polar or rectangular coordinates and the more recent extended conic quadratic model.

2.1 Classical Model

Assume that bus voltages are expressed in polar form ($\tilde{U}_i = U_i \angle \theta_i$). The real and reactive injected power at an arbitrary bus i is given by (Grainger and Stevenson 1994)

$$P_i = U_i^2 G_{ii} + \sum_{\substack{n=1 \\ n \neq i}}^{N} [U_i U_n G_{in} \cos(\theta_i - \theta_n) + U_i U_n B_{in} \sin(\theta_i - \theta_n)], \tag{3}$$

$$Q_i = -U_i^2 B_{ii} - \sum_{\substack{n=1 \\ n \neq i}}^{N} [U_i U_n B_{in} \cos(\theta_i - \theta_n) - U_i U_n G_{in} \sin(\theta_i - \theta_n)]. \tag{4}$$

2.2 Extended Conic Quadratic Model

The ECQ network model is obtained by defining (Jabr 2007)

$$R_{in} = U_i U_n \cos(\theta_i - \theta_n) \text{ for every branch } i - n, \tag{5}$$

$$T_{in} = U_i U_n \sin(\theta_i - \theta_n) \text{ for every branch } i - n, \tag{6}$$

$$u_i = U_i^2/\sqrt{2} \text{ for every bus } i. \tag{7}$$

The substitution of (5)–(7) in the nonlinear power flow equations (3)–(4) yields the following linear equations:

$$P_i = \sqrt{2} G_{ii} u_i + \sum_{\substack{n=1 \\ n \neq i}}^{N} [G_{in} R_{in} + B_{in} T_{in}], \tag{8}$$

$$Q_i = -\sqrt{2} B_{ii} u_i - \sum_{\substack{n=1 \\ n \neq i}}^{N} [B_{in} R_{in} - G_{in} T_{in}]. \tag{9}$$

From the definitions (5)–(7), it follows that

$$R_{in}^2 + T_{in}^2 - 2u_i u_n = 0, \tag{10}$$

$$\theta_i - \theta_n - \tan^{-1}(T_{in}/R_{in}) = 0. \tag{11}$$

Equations (8)–(11) are referred to as the extended conic quadratic format of the load flow equations (Jabr 2008). In the ECQ format, it is understood that the angle at the slack bus is zero and both u_i and R_{in} take only non-negative values. The bus voltage magnitudes can be deduced from the ECQ variables by solving (7)

$$U_i = (\sqrt{2}u_i)^{1/2}. \tag{12}$$

3 Operational Constraints

Several operational restrictions must be taken into account in the OPF formulation, including generator capability constraints, voltage constraints, and branch flow limits.

3.1 Generator Capability Constraints

A generator must be operated such that it stays within the limits of its stability and power rating. The power rating usually depends on thermal restrictions. If the rating is exceeded for a long time, it may cause damage; unless it is exceeded by a large amount, the machine will continue to function. On the other hand, the stability limit if exceeded even for a short period of time may cause the machine to lose synchronism (Sterling 1978).

The generator capability constraints are most accurately accounted for using the capability chart, which shows the normal loading and operation limits of the generator (Sterling 1978). In OPF, it is possible to model the capability chart using a trapezoidal approximation (Chebbo and Irving 1995); however, a further simplification is obtained by using box constraints:

$$P_{gi}^{\min} \le P_{gi} \le P_{gi}^{\max}, \tag{13}$$

$$Q_{gi}^{\min} \le Q_{gi} \le Q_{gi}^{\max}, \tag{14}$$

where

- $P_{gi}^{\min}/Q_{gi}^{\min}$ is the minimum real/reactive power generated at bus i
- $P_{gi}^{\max}/Q_{gi}^{\max}$ is the maximum real/reactive power generated at bus i

3.2 Voltage Constraints

In OPF, the generator voltage refers to the voltage maintained at the high voltage side of the generator transformer. The voltage limits are usually a few points off the rated terminal voltage (Debs 1988):

$$U_i^{\min} \le U_i \le U_i^{\max}, \tag{15}$$

where $U_i^{\min}$ and $U_i^{\max}$ are the minimum and maximum allowable limits of the generator bus voltage magnitude. This enables the stator terminal voltage to be maintained at a constant value, normally the design figure, and allows the necessary reactive power contribution to be achieved by manually tapping the generator transformer against the constant stator volts (British Electricity International 1991).

It is also common to consider the minimum and maximum voltage limits (15) at load buses. The load voltage limits are chosen such that they do not cause damage to the electric system or customer facilities. In terms of the ECQ model variables, the voltage constraints reduce to

$$(U_i^{\min})^2 \Big/ \sqrt{2} \le u_i \le (U_i^{\max})^2 \Big/ \sqrt{2}. \tag{16}$$

3.3 Branch Flow Constraints

Thermal limits establish the maximum amount of current that a transmission facility can conduct for a specified period of time without sustaining permanent damage or violating public safety (North American Electric Reliability Council 1995). By using the ECQ format, it is possible to limit the (squared) magnitude of the line current in line $i - n$ and leaving bus i using a linear equation (Ruíz Muñoz and Gómez Expósito 1992):

$$I_{in}^2 = \sqrt{2}A_{in}u_i + \sqrt{2}B_{in}u_n - 2C_{in}R_{in} + 2D_{in}T_{in} \le (I_{in}^{\max})^2, \tag{17}$$

where

$$\begin{aligned} A_{in} &= g_{in}^2 + (b_{in} + b_{sh}/2)^2, \\ B_{in} &= g_{in}^2 + b_{in}^2, \\ C_{in} &= g_{in}^2 + b_{in}(b_{in} + b_{sh}/2), \\ D_{in} &= g_{in}b_{sh}/2. \end{aligned}$$

In the above equations, g_{in} and b_{in} are the series conductance and susceptance in the π equivalent model, $b_{sh}/2$ is the $1/2$ charging susceptance, and $I_{in}^{\max}$ is the line current rating. It is a common practice to limit the line current magnitude at both the sending and receiving ends of each branch.

For short lines, the thermal limit dictates the maximum power transfer. However, practical stability considerations set the transfer limit for longer lines (in excess of 150 miles) (Saadat 1999). The transient stability limits can be roughly approximated by constraints on active power flow:

$$P_{in} = \sqrt{2} g_{in} u_i - g_{in} R_{in} - b_{in} T_{in} \leq P_{in}^{\max}, \tag{18}$$

where $P_{in}^{\max}$ is the stability limit expressed through the real power in line $i - n$ and leaving bus i. This limit is obtained from stability studies.

4 Tap-Changing and Regulating Transformers

Almost all transformers are equipped with taps on windings to adjust the ratio of transformation. Regulating transformers are also used to provide small adjustments of voltage magnitudes or phase angles. Tap-changers are mainly employed to control bus voltage magnitudes, whereas phase-shifters are limited to the control of active power flows. Some transformers regulate both the magnitude and phase angle (Grainger and Stevenson 1994). Previous researchers studied methods for accommodating tap-changers and phase-shifters in Newton's method. These techniques are well documented in Acha et al. (2004).

The tap-changing/voltage regulating and phase-shifting transformers can be accounted for using the regulating transformer model in Fig. 1, where the admittance $\hat{y}_{t(ij)}$ is in series with an ideal transformer representing the complex tap ratio $1 : \hat{a}_{(ij)}$. The subscript (ij) is dropped to simplify the notation. Because the complex power on either side of the ideal transformer is the same, the equivalent power injection model of the regulating transformer can be represented as in Fig. 2 in which the quantities at the fictitious bus x are constrained as follows (Jabr 2003):

$$a^{\min} \leq \frac{U_j}{U_x} \leq a^{\max}, \tag{19}$$

$$\varphi^{\min} \leq \theta_j - \theta_x \leq \varphi^{\max}. \tag{20}$$

In the above equations, $[a^{\min}, a^{\max}]$ and $[\phi^{\min}, \phi^{\max}]$ are the intervals for the magnitude and angle of the complex tap ratio $\hat{a} = a\angle\phi$. The tap-changing (or voltage

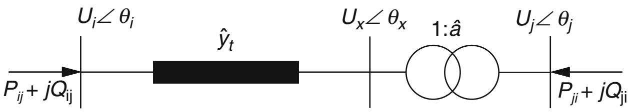

Fig. 1 Regulating transformer equivalent circuit

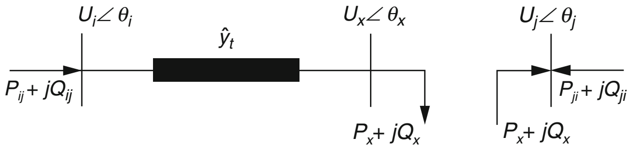

Fig. 2 Regulating transformer power injection model

regulating) transformer model can be obtained by setting

$$\varphi^{\min} = \varphi^{\max} = 0. \tag{21}$$

Similarly,

$$a^{\min} = a^{\max} = 1 \tag{22}$$

results in the phase-shifter model.

Equation (20) can be used with the ECQ format. Equation (19) can be easily placed in a form compatible with this format by using the substitutions in Sect. 2.2 for U_j and U_x, so that (19) becomes

$$(a^{\min})^2 u_x \leq u_j \leq (a^{\max})^2 u_x. \tag{23}$$

Figure 2 shows that the lossless ideal transformer model requires that the active/reactive power extracted from bus x is injected into bus j. It is possible to account for this constraint without introducing the additional variables P_x and Q_x by adding the active/reactive injection equation at bus x to the active/reactive injection equation at bus j. The result is equivalent to combining buses x and j into one super-node and writing the active/reactive injection (8)/(9) at this node. The super-node equation sets the summation of power flows leaving buses x and j to zero.

5 FACTS Devices

FACTS provide additional degrees of freedom in the power network operation by allowing control of bus voltages and power flows. The unified power flow controller (UPFC) is one of the most comprehensive FACTS devices. When installed at one end of a line, it can provide full controllability of the bus voltage magnitude, the active power line flow, and the reactive power line flow (Acha et al. 2004; Zhang et al. 2006).

The principle of operation of the UPFC has been previously reported in the power systems literature (Gyugyi 1992). Figure 3 shows its equivalent circuit under steady-state operating conditions (Acha et al. 2004; Zhang et al. 2006). The equivalent circuit includes two voltage sources operating at the fundamental frequency and

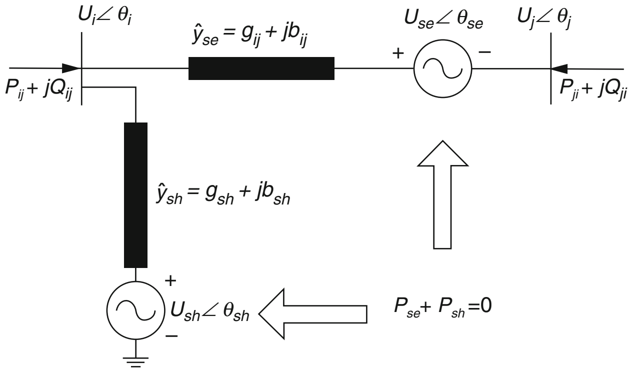

Fig. 3 UPFC equivalent circuit

two impedances. The voltage sources represent the fundamental Fourier series component of the AC converter output voltage waveforms and the impedances model the resistances and leakage inductances of the coupling transformers. To simplify the presentation, the resistances of the coupling transformers are assumed to be negligible and the losses in the converter valves are neglected.

Acha et al. (2004), Ambriz-Pérez et al. (1998), Zhang and Handschin (2001), and Zhang et al. (2006) present Newton and interior-point methods for including a detailed model of the UPFC in OPF studies, that is, the UPFC control parameters (voltage magnitude and angle in the series and shunt converters) are treated as independent variables in the optimization process. A downside of this comprehensive modeling is that the success of the iterative solution becomes sensitive to the choice of the initial UPFC control parameters. Another representation is the power injection model (PIM) proposed in Handschin and Lehmköster (1999). Because the PIM is a strict linear representation of the UPFC, it does not contribute to the nonconvexity of the power flow equations (Lehmköster 2002). Moreover, it does not suffer from problems related to initial point selection.

For a UPFC connected between buses i and j, let the series and shunt voltage sources be represented as phasors in polar form: $\tilde{U}_{se(ij)} = U_{se(ij)}\angle\theta_{se(ij)}$ and $\tilde{U}_{sh(ij)} = U_{sh(ij)}\angle\theta_{sh(ij)}$. To avoid clutter, the subscript (ij) is dropped below. Based on the equivalent circuit in Fig. 3, the active power injection at bus i is

$$P_i = -U_iU_jb_{ij}\sin(\theta_i - \theta_j) - U_iU_{se}b_{ij}\sin(\theta_i - \theta_{se}) - U_iU_{sh}b_{sh}\sin(\theta_i - \theta_{sh}). \quad (24)$$

The first term in (24) is identical to the conventional load flow equation of a transmission device with series susceptance b_{ij} and shunt susceptance b_{sh}. The last two terms can be used to define $-P_i^{FD}$, an active power injection at bus i attributed to the FACTS device's series and shunt voltage sources. Equation (24) can be written as

$$P_i - P_i^{FD} = -U_iU_jb_{ij}\sin(\theta_i - \theta_j), \quad (25)$$

where

$$P_i^{FD} = -U_i U_{se} b_{ij} \sin(\theta_i - \theta_{se}) - U_i U_{sh} b_{sh} \sin(\theta_i - \theta_{sh}). \tag{26}$$

Similarly, the FACTS device's active injection at bus j and reactive injections at buses i and j are

$$P_j^{FD} = U_j U_{se} b_{ij} \sin(\theta_j - \theta_{se}), \tag{27}$$

$$Q_i^{FD} = U_i U_{se} b_{ij} \cos(\theta_i - \theta_{se}) + U_i U_{sh} b_{sh} \cos(\theta_i - \theta_{sh}), \tag{28}$$

$$Q_j^{FD} = -U_j U_{se} b_{ij} \cos(\theta_j - \theta_{se}). \tag{29}$$

By assuming lossless converter valves, the active power exchange among converters via the DC link is zero (Acha et al. 2004; Handschin and Lehmköster 1999; Lehmköster 2002; Zhang et al. 2006), that is,

$$P_i + P_j = P_i^{FD} + P_j^{FD} = 0. \tag{30}$$

Therefore, the UPFC power injection model can be represented as in Fig. 4, where the UPFC active power injection into bus j, $-P_j^{FD}$, is equal to P_i^{FD}.

The UPFC PIM can be directly integrated into the ECQ OPF program by the following:

1. Including the series and shunt coupling transformers into the bus admittance matrix computation.
2. Treating the UPFC injection quantities as additional variables.

Equations (26)–(29) can be used to define upper and lower bounds on each of the UPFC injections. Moreover, the voltage magnitude and angle of the series and shunt voltage sources in Fig. 3 can be deduced from the UPFC PIM by solving (26)–(29) to yield the following closed-form solution (Jabr 2008):

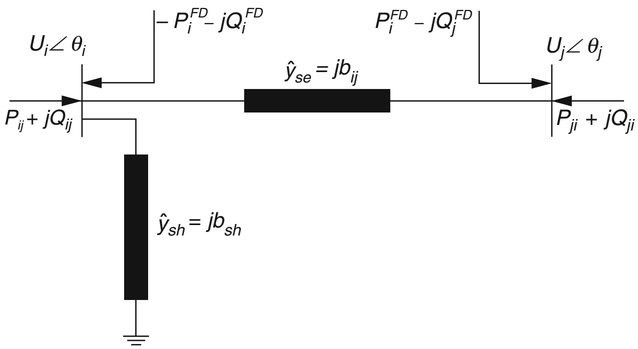

Fig. 4 UPFC power injection model