Fuzzy Systems Engineering
Toward Human-Centric Computing

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To Ewa, Thais, Adam, Tiago, Barbara, Flavia, Arthur, Ari, and Maria de Lourdes
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Preface

Over 40 years have already passed since the inception of fuzzy sets. During this period we have witnessed a truly impressive wealth of theoretical developments and conceptual pursuits, emergence of novel methodologies, algorithmic environments, and a variety of applications. Contemporary technologies in the areas such as information storage and retrieval, web search, image processing and understanding, control, pattern recognition, bioinformatics and computational biology, e-markets, autonomous navigation, and guidance have benefited considerably from the developments in fuzzy sets. What becomes equally important is that we have accumulated a body of knowledge, developed sound design practices, and gained a comprehensive insight into the role of the technology of fuzzy sets in system design and analysis.

With the existing affluence and apparent diversity of the landscape of intelligent systems, fuzzy sets exhibit an important and unique position by forming a unified framework supporting various facets of human-centric computing. Given the current trends in the information technology, it becomes apparent that the increasing level of intelligence, autonomy, and required flexibility comes hand in hand with the increased human centricity of resulting systems. This manifests at the end level when the delivered systems are expected to exhibit flexibility, significant communication abilities, user awareness, and a substantial level of adaptive behavior.

The human-centric facet of processing (or human centricity, briefly) supported by the use of fuzzy sets is concerned with (a) user-friendly nature of the resulting systems (manifesting though a high level of context awareness, realization of relevance feedback, etc.), (b) forming a sound trade-off between accuracy and transparency (interpretability), and (c) incorporation of designer-friendly mechanisms of system development facilitating an efficient aggregation of various sources of available information being present at several quite different levels of abstraction (say, a highly seamless integration of domain knowledge and numeric experimental data).

LEITMOTIV

The fundamental objective of this book is to offer a comprehensive, systematic, fully updated, and self-contained treatise of fuzzy sets that will be of vital interest to a broad audience of students, researchers, and practitioners. Our ultimate goal is to offer solid conceptual fundamentals, a carefully selected collection of design methodologies, a wealth of development guidelines, and pertinent, carefully selected illustrative material. The book constitutes a departure from the conventional approach to fuzzy systems engineering used to date. We explicitly cover concepts, design methodologies, and algorithms inherently coupled with interpretation,
analysis, and underlying engineering knowledge. This holistic view of the discipline is helpful in stressing the role of fuzzy sets as a fundamental component of computational intelligence (CI) and human-centric systems.

FOCAL POINTS

There are several focal points that make this book highly unique and relevant from the perspective of the key fundamentals and practice of fuzzy sets

- **Systematic exposure of the concepts, design methodology, and detailed algorithms:** Overall, we adhere to the top-down strategy starting with the concepts and motivating arguments and afterward proceeding with the detailed design that materializes in some specific algorithms.

- **A wealth of illustrative material:** All concepts covered are illustrated with a series of small, numeric examples to make the material more readable, motivating, and appealing.

- **Self-containment of the material:** No specific prerequisites are required (standard calculus, linear algebra, probability, and logic are deemed to be fully sufficient).

- **More advanced concepts explained in great detail and augmented by pertinent illustrative material:** Appendices offer a concise and focused coverage of the subjects of neural networks and biologically inspired optimization.

- **Down-to-earth exposure of the material:** Although we maintain a required level of formalism and necessary mathematical rigor, our ultimate goal is to present the material in the way it emphasizes its applied side so that the reader becomes fully aware of direct applicability and limitations of the presented concepts, algorithms, and modeling techniques.

- **Auxiliary editorial features:** Historical and bibliographical notes included in each chapter help the reader view the developments of fuzzy sets in a broader perspective. Each chapter comes with a suite of well-balanced exercises and problems.

CHAPTER SUMMARY

The following offers a concise summary of the topics covered in each chapter and underlines the essential aspects of each of them.

In Chapter 1—**Introduction**—we introduce the reader to the subject, highlight several motivating factors, elaborate on the origin of fuzzy sets, and cast them in a certain historical perspective. Similarly, it is shown what role fuzzy sets play vis-à-vis existing technologies. This discussion helps emphasize the enabling role of the technology of fuzzy sets as well as highlight its key role in human-centric systems when addressing the acute and widely spread problem of the semantic gap. Fuzzy sets are also linked to the ideas of granular computing treated as a generalized conceptual
and algorithmic environment. The fundamental areas of Artificial Intelligence and CI are concisely discussed and contrasted in the context of the development of intelligent systems. Some illustrative examples and case studies are covered.

Chapter 2—*Notions and Concepts of Fuzzy Sets*—serves as a coherent and systematic introduction of the fundamental concept of a fuzzy set with focus on the ideas of partial membership conveyed by membership functions, underlying rationale, examples, and most commonly encountered categories (classes) of membership functions. The ideas of fuzzy quantities and numbers are also introduced.

In Chapter 3—*Characterization of Fuzzy Sets*—major properties of membership functions are studied along with their interpretation. Discussed are geometric features of fuzzy sets to help underline the differences between sets and fuzzy sets in terms of operations defined therein, and in the sequel some global characterization of fuzzy sets (expressed through energy, granularity, etc.) is offered. The properties of families of fuzzy sets defined in the same space are presented. We emphasize here the semantics of information granules represented as fuzzy sets.

Chapter 4—*The Design of Fuzzy Sets*—elaborates on the development of fuzzy sets (membership functions) by emphasizing their syntax and semantics and linking those concepts with the user- and data-driven mechanisms of elicitation of membership functions. Some typical mechanisms supporting the construction of fuzzy sets are presented and contrasted. A great deal of attention is paid to fuzzy clustering that is regarded as one of the dominant technologies of information granulation.

In Chapter 5—*Operations and Aggregations of Fuzzy Sets*—we are concerned with operations (union, intersection, complement) on fuzzy sets. The presentation addresses the issues of formal requirements, interpretations, and realizations and possible parametric adjustments. Covered are triangular norms (t-norms and t-conorms) along with their conceptual and computing refinements such as, for example, ordinal sums, uninorms, and nullnorms.Outlined are the aspects of semantics conveyed by such logic operators and their possible parametric refinements invoked by available experimental data.

The predominant concept presented in Chapter 6—*Fuzzy Relations*—is relations. Fuzzy relations are fundamental concepts expanding the idea of fuzzy sets to a multivariable case. The very concept is introduced and illustrated. The closely linked ideas of Cartesian products, projections, and cylindric extensions of fuzzy relations are discussed.

In Chapter 7—*Transformations of Fuzzy Sets*—we introduce an idea of mappings of fuzzy sets between spaces and elaborate on its realization in the case of functions (extension principle) and relations (relational calculus). In particular, discussed are the principles of fuzzy arithmetic.

In Chapter 8—*Generalizations and Extensions of Fuzzy Sets*—various concepts and ideas that augment fuzzy sets and discussed. In this chapter, fuzzy sets are discussed in the framework of granular computing involving various formalisms of information granulation including interval analysis and rough sets. Through an extended contrastive analysis, we are able to emphasize the role played by fuzzy sets. The generalizations in the form of type-2 and order-2 fuzzy sets are covered. The concept of shadowed sets is presented as a vehicle of a more qualitative (three-valued) interpretation of fuzzy sets.
The orthogonality of fuzzy sets and probability is underlined and illustrated. Furthermore, hybrid constructs resulting through a joint treatment of fuzzy sets and probability, such as fuzzy probabilities, are discussed.

Chapter 9—Interoperability Aspects of Fuzzy Sets—is concerned with various dependencies between fuzzy sets and other environments of granular computing and numeric settings. In the latter case, the ideas of encoding and decoding (referred to as a fuzzification and defuzzification mechanism, respectively) are introduced and studied in detail. Both scalar and vector cases are investigated. The linkages between fuzzy sets and sets are revealed and articulated in the language of $\alpha$-cuts.

Chapter 10—Fuzzy Modeling: Principles and Methodology—offers an in-depth discussion on the principles and underlying methodology of fuzzy modeling, their design objectives (accuracy, interpretability, etc.), an overall design process, and pertinent verification and validation procedures.

In Chapter 11—Rule-based Fuzzy Models—we concentrate on a class of models that play a dominant and highly noticeable role in fuzzy modeling. We introduce the main concepts and underlying terminology, classes of architectures, and discuss a variety of design processes. The mechanisms of structural and parametric learning with examples of the ensuing optimization vehicles are discussed as well.

The focal point of Chapter 12—From Logic Expressions to Fuzzy Logic Networks—is a category of fuzzy systems exhibiting logic-driven semantics and significant parametric flexibility. Different classes of logic neurons are introduced and afterward exploited as generic building components in the formation of highly heterogeneous logic networks. The underlying interpretability issues are raised and investigated in great detail.

In Chapter 13—Fuzzy Systems and Computational Intelligence—fuzzy systems are discussed vis-à-vis the research agenda and main concerns of CI. The synergistic linkages between fuzzy sets and other leading technologies of CI such as neural networks and evolutionary methods are discussed. Several representative examples are studied including recurrent neurofuzzy systems.

Human centricity of fuzzy systems is studied in Chapter 14—Granular Models and Human-Centric Computing—This chapter serves as a carefully organized compendium of human-centric architectures in the areas of data analysis, clustering, and granular modeling. It involves a general methodological discussion and formulates a series of guidelines. We highlight an important and active role of fuzzy sets in learning processes.

Chapter 15—Emerging Trends in Fuzzy Systems—ventures into several emerging and already promising areas of further developments of fuzzy sets with emphasis placed on their applied side. In particular, this concerns examples of relational ontology, information retrieval, and multiagent systems.

To make the material highly self-contained, we have included three appendices. The first one is a concise summary of the most useful and commonly encountered ideas and concepts of linear algebra and unconstrained and constrained optimization. The two others offer a brief view of the essence of neurocomputing and biologically inspired optimization that plays a vital role in the development of fuzzy systems and various constructs of CI, in general.
READERSHIP

Given the content of the book and an arrangement of the material, it will appeal to at least three large communities of readers:

Senior undergraduate students: The key objective is to present fuzzy sets as a coherent enabling technology that offers a unique and highly functional environment for building human-centric systems in numerous subject areas to which the students have been already exposed during the earlier years of their undergraduate programs. The pedagogy of the resulting course may succinctly highlight the capabilities fuzzy sets can offer as a coherent analysis platform augmenting, formalizing, and expanding the existing detailed subject knowledge. The fundamental design practices supported by fuzzy sets build upon the existing body of design knowledge being now substantially enriched by fuzzy sets. The book can be also used to deliver a standalone one-term course on fuzzy sets. Depending upon the objectives of the instructor, the material could be structured to emphasize the fundamentals of fuzzy sets or concentrate on their applied facet including modeling, classification, and data analysis.

Graduate students: The book supports the need of a broad audience of graduate students in engineering and science. Given this audience, we anticipate that the detailed presentation of the fundamentals of fuzzy sets (along with their necessary mathematical details) and the comprehensive design principles would be equally appealing to them. Again, we envision that this might involve students working in the realm of fuzzy sets or pursuing advanced research in other disciplines.

Researchers and practitioners: The organization and coverage of the material will appeal to all those who are already familiar with fuzzy sets and are interested in exploring further advancements in the area. The readers can benefit from a thorough, in-depth, and critical assessment of the current state of the art of the area. Along with the presentation of novel pursuits within the realm of the well-established domains of fuzzy sets, the book embarks on a number of emerging areas of fuzzy sets. For those who are looking for a brief yet highly informative introduction to fuzzy sets, the core of the book brings solid exposure to the area. The holistic view of the discipline embracing the fundamentals with the practice of fuzzy sets could greatly appeal to those interested in pursuing the applied side of fuzzy sets.

Throughout the book we emphasize the role of fuzzy sets as an enabling technology whose impact, contributions, and methodology stretch far beyond any specific community and research area. Taking this into account, a substantial interest arises from a vast array of disciplines like engineering, computer science, business, medicine, bioinformatics, computational biology, and so on.

THE ROADMAP

The book is intended to serve the needs of a broad audience by covering a wealth of territory of the discipline of fuzzy sets. Depending on the needs, several possible routes can be projected:
A one-semester undergraduate course could cover Chapters 1–4 (possibly excluding Sections 4.5, 4.8, and 4.10), Chapter 5–11.

A one-term graduate course could be composed of Chapters 1–12 with some selective choice of content of Chapters 13–15.

For some specialized, short-term courses one could consider covering Chapters 1–11.

INSTRUCTOR RESOURCES

Instructors will be provided with the following classroom-ready electronic resources:

- Viewgraphs to be used in class. They aim to be customized when used in more specialized presentations or short courses.
- Solution manual with graphics presenting answers to selected problems.
- Sample assignments and examinations.

Although we strived for the delivery of a flawless material, we are aware that some typos may be inevitable. Some concepts could have been presented differently. Some algorithms could have been outlined in a more readable manner. Some interesting generalizations could have been included. We greatly appreciate your comments; please drop us a line (pedrycz@ee.ualberta.ca or gomide@dca.fee.unicamp.br).

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Chapter 1

Introduction

We live in the world of digital technology that surrounds us and without which we can barely function. There are myriads of examples (which we take for granted) in which computers bring a wealth of services. Computers constitute an omnipresent fabric of the society (Vasilakos and Pedrycz, 2006). As once succinctly captured by Weiser (1991), “the most profound technologies are those that disappear. They weave themselves into the fabric of everyday life until they are indistinguishable from it.”

There is an ongoing challenge of building intelligent systems whose functionality could make them predominantly human centric. Human centricity is one of the driving forces of ubiquitous and pervasive computing. Although there are interesting developments along this line, there is a still a long way to go. Some important milestones have been achieved, yet a lot of challenges lie ahead.

In this chapter, we investigate some fundamental features of human centricity of intelligent systems and in this context raise a need for comprehensive studies in information granulation and fuzzy sets, in particular.

1.1 DIGITAL COMMUNITIES AND A FUNDAMENTAL QUEST FOR HUMAN-CENTRIC SYSTEMS

Problem solving, design, and creative thinking—these are all endeavors in which we are inherently faced with conflicting requirements, incomplete information, numerous constraints, and finally collections of alternative solutions. All of these lead us to situations in which we have to effectively manage enormous amounts of heterogeneous data, deal with conflicting or missing evidence, and arrive at meaningful conclusions being aware of the confidence associated with our findings.

In spite of ever growing complexity of the problems, we somewhat manage to develop solutions. Both in analysis and in design (synthesis), we follow the key principles of abstraction and decomposition that help us handle a phenomenon of complexity and arrive at meaningful solutions. In essence, the effective use of abstraction means that instead of being buried in a flood of details and mountains
of data, we establish certain, perhaps most suitable conceptual perspective and set up a framework in which the problems could be tackled. Granularity of the problem representation is a fundamental manifestation of the principle of abstraction. The decomposition is a meaningful and commonly used strategy in which on the basis of some prudently established granularity we solve the problem by isolating its loosely connected subproblems and handling them on an individual basis.

Computing systems that are around us in so visible abundance operate on completely different principles of binary (Boolean logic), numeric information and solutions, and predefined models of the world of two-valued logic and human information processing. It becomes apparent that we are concerned with two conceptually distinct worlds. To make them work together and take full advantage of the computing faculties, we need a well-developed interface through which both worlds could talk to each other. This is the key rationale behind the emergence of human-centric systems and human-centric computing (HC$^2$). The primary objective of the HC$^2$ is to make computers adjust to people by being more natural and intuitive to use and seamlessly integrated within the existing environment. Various pursuits along the line of e-society include intelligent housing, ambient intelligence (Vasilakos and Pedrycz, 2006) and ubiquitous computing, semantic web, e-health, e-commerce and manufacturing, sensor networks, intelligent data analysis, and wearable hardware. All of these are concrete examples of the general tendency existing in the development of HC$^2$ systems. Referring to the general architectural framework as portrayed in Figure 1.1, we easily note that in such endeavors a middleware of the semantic layer plays a crucial role in securing all necessary efficient interaction and communication between various sources of data and groups of users coming with their diversified needs and objectives. In the development of HC$^2$ systems, we are ultimately faced with an omnipresent challenge known as a semantic gap. To alleviate its consequences, we have to focus on how to reconcile and interpret detailed numeric information with the qualitative, descriptive, and usually linguistic input coming from the user. For instance, in the design of a typical HC$^2$ system, such

![Figure 1.1](image_url)  
**Figure 1.1** An overall architecture of human-centric systems; note a critical role of the semantic layer linking the layers of computing and humans together.
as a personalized digital photo album, we encounter a lot of detailed numeric data (pixels of images) and have to accommodate a significant and highly descriptive user’s input that comes in the form of some relevance feedback. The context awareness and personalization invoke numerous collaborative aspects of processing involving various sources of data and information (including those available directly from the users). The user-based processing capability is an important aspect of HC² systems that has to be taken into account in any design considerations.

The crux of the semantic layer lies in the formation and usage of entities that are easily perceived and processed by humans. The difficulty is that the world of numeric processing has to interact with humans who are quite resistant to the explicit use of numbers and uncomfortable to process them. We operate at the higher level of abstraction, and this essential design perspective has to be embraced by human-centric systems through their underlying functionality.

Let us offer a sample of examples in which human centricity plays a pivotal role (Table 1.1) (Frias-Martinez et al., 2005; Perkowitz and Etzioni, 2000; Spott and Nauck, 2006). Most of them heavily rely on the idea of an effective relevance feedback that needs to be implemented in an efficient manner.

### Table 1.1 Selected Examples of Human-Centric Systems and their Underlying Objectives.

<table>
<thead>
<tr>
<th>Area</th>
<th>Key objectives, existing trends, and solutions</th>
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<tbody>
<tr>
<td>Intelligent data analysis</td>
<td>Effective explanatory analysis, delivery of findings at the level of information granules, and effective mechanisms of summarization.</td>
</tr>
<tr>
<td>System modeling</td>
<td>Building transparent models that could be easily interpreted and whose outcomes are readily understood. Models should help the user justify decisions being taken.</td>
</tr>
<tr>
<td>Adaptive hypermedia</td>
<td>Personalization of hypermedia to meet needs of individual users, development of specialized web services, building collaborative filtering, recommendation, content-based filtering, personalization of web engines, and so on.</td>
</tr>
<tr>
<td>e-commerce</td>
<td>Expressing preferences of customers formulated at different levels of specificity (granularity).</td>
</tr>
<tr>
<td>Intelligent interfaces</td>
<td>Face expression, emotion recognition and tracking, formation and use of face-related features.</td>
</tr>
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</table>

From the brief investigations covered above, it becomes apparent that in the realization of the quest for humancentricity of systems, the leitmotiv of many investigations is in building effective mechanisms of communication including various schemes of relevance feedback. Given that human processing is carried out at some level of
abstraction, a concept of information granules and information granulation plays a visible role. The question of dichotomy offered by some formal frameworks of information granules has to be revisited as well.

The concept of dichotomy becomes profoundly imprinted into our education, philosophy, and many branches of science, management, and engineering. Although the formalism and vocabulary of Boolean concepts being effective in handling various discrimination processes involving binary quantification (yes–no, true–false) has been with us from the very beginning of our education, it becomes evident that this limited, two-valued view at world is painfully simplified and in many circumstances lacks rapport with the reality. In real world, there is nothing like black–white, good–bad, and so on. All of us recognize that the notion of dichotomy is quite simple and does not look realistic. Concepts do not possess sharp boundaries. Definitions are not binary unless they tackle very simple concepts (say odd–even numbers). Let us allude here to the observation made by Russell (1923)

"... the law of excluded middle is true when precise symbols are employed, but it is not true when symbols are vague, as, in fact, all symbols are."

In reality, we use terms whose complexities are far higher and which depart from the principle of dichotomy. Consider the notions used in everyday life such as warm weather, low inflation, long delay, and so on. How could you define them if you were to draw a single line? Is 25°C warm? Is 24.9°C warm? Or is 24.95°C warm as well? Likewise in any image: Could you draw a single line to discriminate between objects such as sky, land, trees, and lake. Evidently, as illustrated in Figure 1.2, identifying boundaries delineating the objects in this way is a fairly futile task and in many cases produces pretty much meaningless results. Objects in images do not exhibit clear and unique boundaries (the location of the horizon line is not obvious at all) (Fig. 1.2(a)). Experimental data do not come in well-formed and distinct clusters; there are always some points in-between (Fig. 1.2(b)).

One might argue that these are concepts that are used in everyday language and, therefore, they need not possess any substantial level of formalism. Yet, one has to admit that the concepts that do not adhere to the principle of dichotomy are also

![Figure 1.2](image.png) **Figure 1.2** Objects, as we perceive and describe them, do not exhibit sharp boundaries. Such boundaries implementing a principle of dichotomy exhibit limitations. Practically, they may not exist at all: (a) images and (b) experimental data.
visible in science, mathematics, and engineering. For instance, we often carry out a linear approximation of nonlinear function and make a quantifying statement that such linearization is valid in some small neighborhood of the linearization point. Under these circumstances, the principle of dichotomy does not offer too much.

The principle of dichotomy, or as we say an Aristotelian perspective at the description of the world, has been subject to a continuous challenge predominantly from the standpoint of philosophy and logic. Let us recall some of the most notable developments that have led to the revolutionary paradigm shift. Indisputably, the concept of a three-valued and multivalued logic put forward by Jan Lukasiewicz and then pursued by others, including Emil Post, is one of the earliest and the most prominent logical attempts made toward the direction of abandoning the supremacy of the principle of dichotomy. As noted by Lukasiewicz (1920, 1930,) the question of the suitability or relevance of two-valued logic in evaluating the truth of propositions was posed in the context of those statements that allude to the future. “Tomorrow will rain.” Is this statement true? If we can answer this question, this means that we have already predetermined the future. We start to sense that this two-valued model, no matter how convincing it could be, is conceptually limited if not wrong. The non-Aristotelian view of the world was vividly promoted by Korzybski (1933). Although the concept of the three-valued logic was revolutionary in 1920s, we somewhat quietly endorsed it over the passage of time. For instance, in database engineering, a certain entry may be two-valued (yes–no), but the third option of “unknown” is equally possible—here we simply indicate that no value of this entry has been provided.

1.3 GRANULAR COMPUTING

Information granules permeate human endeavors (Zadeh, 1973, 1979, 1996, 1997, 2005; Pedrycz, 2001; Bargiela and Pedrycz, 2003). No matter what problem is taken into consideration, we usually cast it into a certain conceptual framework of basic entities, which we regard to be of relevance to the problem formulation and problem solving. This becomes a framework in which we formulate generic concepts adhering to some level of abstraction, carry out processing, and communicate the results to the external environment. Consider, for instance, image processing. In spite of the continuous progress in the area, a human being assumes a dominant and very much uncontested position when it comes to understanding and interpreting images. Surely, we do not focus our attention on individual pixels and process them as such but group them together into semantically meaningful constructs—familiar objects we deal with in everyday life. Such objects involve regions that consist of pixels or categories of pixels drawn together because of their proximity in the image, similar texture, color, and so on. This remarkable and unchallenged ability of humans dwells on our effortless ability to construct information granules, manipulate them, and arrive at sound conclusions. As another example, consider a collection of time series. From our perspective, we can describe them in a semiqualitative manner by pointing at specific regions of such signals. Specialists can effortlessly interpret
electrocardiograms (ECG signals). They distinguish some segments of such signals and interpret their combinations. Experts can interpret temporal readings of sensors and assess the status of the monitored system. Again, in all these situations, the individual samples of the signals are not the focal point of the analysis and the ensuing signal interpretation. We always granulate all phenomena (no matter if they are originally discrete or analog in their nature). Time is another important variable that is subjected to granulation. We use seconds, minutes, days, months, and years. Depending on which specific problem we have in mind and who the user is, the size of information granules (time intervals) could vary quite dramatically. To the high-level management, time intervals of quarters of year or a few years could be meaningful temporal information granules on the basis of which one develops any predictive model. For those in charge of everyday operation of a dispatching power plant, minutes and hours could form a viable scale of time granulation. For the designer of high-speed integrated circuits and digital systems, the temporal information granules concern nanoseconds, microseconds, and perhaps seconds. Even such commonly encountered and simple examples are convincing enough to lead us to ascertain that (a) information granules are the key components of knowledge representation and processing, (b) the level of granularity of information granules (their size, to be more descriptive) becomes crucial to the problem description and an overall strategy of problem solving, and (c) there is no universal level of granularity of information; the size of granules is problem oriented and user dependent.

What has been said so far touched a qualitative aspect of the problem. The challenge is to develop a computing framework within which all these representation and processing endeavors could be formally realized. The common platform emerging within this context comes under the name of granular computing. In essence, it is an emerging paradigm of information processing. Although we have already noticed a number of important conceptual and computational constructs built in the domain of system modeling, machine learning, image processing, pattern recognition, and data compression in which various abstractions (and ensuing information granules) came into existence, granular computing becomes innovative and intellectually proactive in several fundamental ways:

- It identifies the essential commonalities between the surprisingly diversified problems and technologies used there, which could be cast into a unified framework we usually refer to as a granular world. This is a fully operational processing entity that interacts with the external world (that could be another granular or numeric world) by collecting necessary granular information and returning the outcomes of the granular computing.

- With the emergence of the unified framework of granular processing, we get a better grasp as to the role of interaction between various formalisms and visualize a way in which they communicate.

- It brings together the existing formalisms of set theory (interval analysis), fuzzy sets, rough sets, and so on under the same roof by clearly visualizing that in spite of their visibly distinct underpinnings (and ensuing processing), they exhibit some fundamental commonalities. In this sense, granular computing