



# Physics of Multiantenna Systems and Broadband Processing

**Tapan K. Sarkar**  
**Magdalena Salazar-Palma**  
**Eric L. Mokole**

*With Contributions from:*

Santana Burintramart  
Jeffrey T. Carlo  
Wonsuk Choi  
Arijit De  
Debalina Ghosh  
Seunghyeon Hwang  
Jinhwan Koh  
Raúl Fernández Recio  
Mary Taylor  
Nuri Yilmazer  
Yu Zhang



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# Preface

The objective of the book is to present a scientific methodology that can be used to analyze the physics of multiantenna systems. The multiantenna systems are becoming exceedingly popular because they promise a different dimension (spatial diversity) than what is currently available to the communication systems engineers. Simultaneously using multiple transmit and receive antennas provides a means to perform spatial diversity, at least from a theoretical standpoint. In this way, one can increase the capacities of existing systems that already exploit time and frequency diversity. The deployment of multiantenna systems is equivalent to using an overmoded waveguide, where information is simultaneously transmitted via not only the dominant mode but also through all the higher-order modes. We look into this interesting possibility and study why communication engineers advocate the use of such a system, whereas electromagnetic and microwave engineers have avoided such propagation mechanisms in their systems. Most importantly, we study the physical principles of multiantenna systems through Maxwell's equations and utilize them to perform various numerical simulations to observe how a typical system will behave in practice. The first five chapters of this book are devoted to this topic.

Specifically, Chapter 1 describes Maxwell's equations in the frequency and time domains and shows how to solve practical problems in both domains.

Chapter 2 presents the frequency domain properties of antennas, and specifically what is meant by near field and far field of antennas, which are relevant to our discussions as an antenna beam can only be defined in the far field. In particular, an antenna has no nulls in the near field, which is independent of distance, and is only a function of the azimuth and elevation angles. We also study how the presence of a ground plane, namely the earth, modifies our concepts and how it affects the electrical performance of a system.

Chapter 3 describes the properties of antennas in the time domain and illustrates how a broadband antenna should behave. Using the terminology broadband implies a finite width time domain pulse that can be either transmitted or received by an antenna without severe distortion. From this perspective, a spread spectrum system will not be considered broadband, since the instantaneous spectrum of its signals is still small. In dealing with wideband signals, one observes that the impulse response of the antenna in the transmit mode is the time derivative of the impulse response of the antenna in the receive

mode. We also look at the impulse response of some of the conventionally used wideband antennas, including a century bandwidth antenna.

Chapter 4 looks at the concept of channel capacity from a Maxwellian viewpoint. The concept of channel capacity is intimately connected with the concept of entropy – hence related to physics. We present two forms of the channel capacity, the usual Shannon capacity which is based on power; and the seldomly used definition of Hartley which uses values of the voltage. These two definitions of capacities are shown to yield numerically very similar values if one is dealing with conjugately matched antennas. However, from an engineering standpoint, the voltage-based form of the channel capacity is more useful as it is related to the sensitivity of the receiver to an incoming electromagnetic wave. Furthermore, we illustrate through numerical simulations how to apply the channel capacity formulas in an electromagnetically proper way. To perform the calculations correctly, first in the simulations, the input power fed to the antennas need to remain constant in a comparison. Second, the expression of power often used by most communication engineers in the channel capacity is related to the radiated power and not to the input power, which is not correct. In a fair comparison, one should deal with the gain of antenna systems and not their directivities, which is an alternate way of referring to the input power fed to the antennas rather than to the radiated power. The problem is, the radiated power essentially deals with the directivity of an antenna and theoretically one can get any value for the directivity of an aperture. Hence, the distinction needs to be made between gain and directivity in a proper way to compare systems. Finally, one needs to use the Poynting's theorem to calculate the power in the near field and not using exclusively either the voltage or the current. This applies to the power form of the Shannon channel capacity theorem. For the voltage form of the capacity due to Hartley is applicable to both near and far fields. Use of realistic antenna models in place of representing antennas by point sources further illustrates the above points, as the point sources by definition generates only far field.

Chapter 5 presents the concept of a multi-input-multi-output (MIMO) antenna system and illustrates the strengths and the weaknesses of this multiantenna deployments in both the transmitters and the receivers. Sample simulations show that only the classical phased array mode out of the various spatial modes that characterize spatial diversity is useful and the other spatial modes are not efficient radiators. Hence, it is more useful to use the concept of adaptive beam forming using a phased array mode.

The next seven chapters address a new phased array methodology for accurate and efficient adaptive processing. In Chapter 6, three classes of optimum filters are presented to illustrate in what sense they are optimal. Of the three classes, one has the promise of performing estimation rather than the usual detection process carried out in conventional adaptive processing. We illustrate that it is possible to perform adaptive processing using a single snapshot of the data, which may be more useful for a highly dynamic environment or in the presence of blinking jammers. A single snapshot based adaptive procedure

generates a least squares solution and does not require any statistical description of the signals. In fact, it has been illustrated in the literature and summarized in this book that processing a single snapshot of the data has essentially the same number of degrees of freedom for coherent interferers as a classical multiple-snapshot processing that is based on conventional sample matrix inversion techniques. In addition, this new method is at least an order of magnitude faster in computational speed than the sample matrix inversion techniques when using the same number of degrees of freedom. This new methodology is then extended to space-time adaptive processing, where a single snapshot is applied to a range cell and requires neither secondary data nor a statistical description of clutter. Recently, this methodology was applied to real airborne data and demonstrated to provide a better solution than conventional statistical methods.

In Chapter 7, we show that the minimum of the sum of the absolute value of the weights can be used for further or equivalently secondary processing for improving the estimation of the direction of arrival of the signal of interest in an adaptive processing methodology. In this way, one can further improve the estimates for both the direction of arrival and the Doppler frequency for the signal of interest in a space-time adaptive algorithm. In particular, the minimum value for the norm of the adaptive weights is obtained at the true value for the direction of arrival for adaptive processing or at the true value for direction of arrival and Doppler frequency in space-time adaptive processing (STAP).

Chapter 8 illustrates that the direct-data-domain least-squares (D3LS) adaptive methodology is quite flexible and it can easily be modified to deal with real values of the adaptive weights for both adaptive and space-time adaptive processing. How this adaptive processing approach can be achieved and implemented for phase-only weights is illustrated in Chapter 9. In Chapter 10, the D3LS method is used for simultaneously forming more than one main beam, which makes it possible to track multiple targets in the same adaptive process.

In Chapter 11, a performance comparison is made between four versions of the statistical-based STAP and D3LS STAP algorithms, when the number of training data is varied. The four statistical-based methods are: the full-rank statistical method; the relative importance of the eigenbeam (RIE) method; the principle component generalized sidelobe canceller (GSC) method; and the cross-spectral GSC method. In contrast to the D3LS approach utilizes only a single snapshot of data (space and time corresponding to one range cell only), one needs to know the rank of the interference covariance matrix for multiple-snapshots to make the statistically-based methods work. The D3LS performs better when the number of training data available for the statistical-based methods is less than the rank of the interference covariance matrix. The channel mismatch is also introduced to all methods to evaluate their performance.

Chapter 12 shows the effects of mutual coupling among the antenna elements in the array and illustrates how a nonplanar array with nonuniformly spaced elements can be used for adaptive processing. One method that can be used to compensate for the mutual coupling is using the embedded in-situ

element patterns. This simple widely used method, however, breaks down when the intensity of the interferer increases. In those situations, implementing a more accurate compensation technique through the transformation matrix approach is necessary. When the strengths of the interferers are comparable to the signal of interest, using dummy antenna elements at the edges of an array can minimize the effects of mutual coupling.

Chapter 13 illustrates how reciprocity can be used in directing a signal to a preselected receiver when there is a two way communication between a transmitter and the receiver. This embarrassingly simple method is much simpler in computational complexity than a traditional MIMO and can even exploit the polarization properties for effectively decorrelating multiple receivers in a multi-input-single-output (MISO) system.

The next three chapters treat the estimation of the direction of arrival (DOA). Chapter 14 describes the Matrix Pencil method for DOA estimation, as knowledge of the DOA for the signal of interest is often necessary in many problems. A unitary transform is applied to illustrate how this method can be implemented in a real system using real arithmetic. The Matrix Pencil method is a direct data domain approach as opposed to ESPRIT, which uses a correlation matrix of the data. For situations, where few available snapshots of the data are available, we show that the Matrix Pencil method provides a more accurate estimate of the DOA than the ESPRIT method. In Chapter 15, DOA estimation is carried out using electrically small antennas and presents the associated Cramer-Rao bound to illustrate the accuracy of this estimation procedure. It is shown that conjugately matched electrically small antennas can be as effective, if not more effective, than their resonant versions. Chapter 16 presents a nonconventional least squares methodology for DOA estimation using arbitrary shaped nonplanar conformal arrays.

The next two chapters discuss broadband processing of signals operating at different frequencies or those having a finite bandwidth. Chapter 17 presents a broadband DOA estimation algorithm that uses the Matrix Pencil method, with the main objective of finding not only the azimuth and the elevation angles of arrival for the signals of interest but also their operating frequencies. Simulations illustrate how one can use realistic antennas to perform broadband DOA estimation. In Chapter 18, D3LS STAP of Chapter 6 is applied to show how broadband adaptive processing can be performed.

Finally, Chapter 19 analyzes how random position errors in the location of the antenna elements in an array can affect its STAP performance.

To recapitulate, the primary goal of this book is to develop a basic understanding of the physics of multiantenna and the concept of channel capacity by using Maxwell's theory. Since an antenna is a temporal filter as well as a spatial filter, any analysis dealing with antennas needs to merge both their spatial and temporal properties to obtain a physically meaningful solution. These two diverse properties are reflected in Maxwell's equations and throughly understanding these four century old equations, first articulated by Heinrich

Hertz in the scalar form and then by Oliver Heaviside in the vector form that we use nowadays, can address most of the problems dealing with space-time properties of antennas. Because, the classical phased array mode is dominant in multiantenna systems, we show how to do adaptive processing in a least squares fashion in an accurate and efficient way without requiring any statistical information as an a priori description of the signals. Demonstrating that this type of methodology is also amenable to broad band processing is a secondary goal of this book.

Every attempt has been made to guarantee the accuracy of the materials in the book. We would however appreciate readers bringing to our attention any errors that may have appeared in the final version. Errors and /or any comments may be emailed to any of the authors.



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*Tapan K. Sarkar (tksarkar@syr.edu)*  
*Magdalena Salazar-Palma (salazar@tsc.uc3m.es)*  
*Eric L. Mokole (eric.mokole@nrl.navy.mil)*  
*Syracuse, New York*  
*June 2008*

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# 1

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## WHAT IS AN ANTENNA AND HOW DOES IT WORK?

### 1.0 SUMMARY

An antenna is a structure that is made of material bodies that can be composed of either conducting or dielectric materials or may be a combination of both. Such a structure should be matched to the source of the electro-magnetic energy so that it can radiate or receive the electromagnetic fields in an efficient manner. The interesting phenomenon is that an antenna displays selectivity properties not only in frequency but also in space. In the frequency domain an antenna is capable of displaying a resonance phenomenon where at a particular frequency the current density induced on it can be sufficiently significant to cause radiation of electromagnetic fields from that structure. An antenna also possesses an impulse response that is a function of both the azimuth and elevation angles. Thus, an antenna displays spatial selectivity as it generates a radiation pattern that can selectively transmit or receive electromagnetic energy along certain spatial directions. As a receiver of electromagnetic fields, an antenna also acts as a spatial sampler of the electromagnetic fields propagating through space. The voltage induced in the antenna is related to the polarization and the strength of the incident electromagnetic fields. The objective of this chapter is to illustrate how the impulse response of an antenna can be determined. Another goal is to demonstrate that the impulse response of an antenna when it is transmitting is different from its response when the same structure operates in the receive mode. This is in direct contrast to antenna properties in the frequency domain as the transmit radiation pattern is the same as the receive antenna pattern. An antenna provides the matching necessary between the various electrical components associated with the transmitter and receiver and the free space where the electromagnetic wave is propagating. From a functional perspective an antenna is thus related to a loudspeaker, which matches the acoustic generation/receiving devices to the open space. However, in acoustics, loudspeakers and microphones are bandlimited devices and so their impulse responses are well behaved. On the other hand, an antenna is a high pass device and therefore the transmit and the receive impulse responses are not the same; in fact, the former is the time

derivative of the latter. An antenna is like our lips, whose instantaneous change of shapes provides the necessary match between the vocal cord and the outside environment as the frequency of the voice changes. By proper shaping of the antenna structure one can focus the radiated energy along certain specific directions in space. This spatial directivity occurs only at certain specific frequencies, providing selectivity in frequency. The interesting point is that it is difficult to separate these two spatial and temporal properties of the antenna, even though in the literature they are treated separately. The tools that deal with the dual-coupled space-time analysis are *Maxwell's equations*. We first present the background of Maxwell's equations and illustrate how to solve for them analytically. Then we utilize them in the subsequent sections and chapters to illustrate how to obtain the impulse responses of antennas both as transmitting and receiving elements and illustrate their relevance in the saga of smart antennas.

## 1.1 HISTORICAL OVERVIEW OF MAXWELL'S EQUATIONS

In the year 1864, James Clerk Maxwell (1831–1879) read his “Dynamical Theory of the Electromagnetic Field” [1] at the Royal Society (London). He observed theoretically that electromagnetic disturbance travels in free space with the velocity of light [1–7]. He then conjectured that light is a transverse electromagnetic wave by using dimensional analysis [7]. In his original theory Maxwell introduced 20 equations involving 20 variables. These equations together expressed mathematically virtually all that was known about electricity and magnetism. Through these equations Maxwell essentially summarized the work of Hans C. Oersted (1777–1851), Karl F. Gauss (1777–1855), André M. Ampère (1775–1836), Michael Faraday (1791–1867), and others, and added his own radical concept of displacement *current* to complete the theory.

Maxwell assigned strong physical significance to the magnetic vector and electric scalar potentials  $\mathbf{A}$  and  $\psi$ , respectively (bold variables denote vectors; italic denotes that they are function of both time and space, whereas roman variables are a function of space only), both of which played dominant roles in his formulation. He did not put any emphasis on the sources of these electromagnetic potentials, namely the currents and the charges. He also assumed a hypothetical mechanical medium called *ether* to justify the existence of displacement currents in free space. This assumption produced a strong opposition to Maxwell's theory from many scientists of his time. It is well known that Maxwell's equations, as we know them now, do not contain any potential variables; neither does his electromagnetic theory require any assumption of an artificial medium to sustain his displacement current in free space. The original interpretation given to the displacement current by Maxwell is no longer used; however, we retain the term in honor of Maxwell. Although modern Maxwell's equations appear in modified form, the equations introduced by Maxwell in 1864 formed the foundation of electromagnetic theory, which together is popularly referred to as *Maxwell's electromagnetic theory* [1–7].

Maxwell's original equations were modified and later expressed in the form we now know as Maxwell's equations independently by Heinrich Hertz (1857–1894) and Oliver Heaviside (1850–1925). Their work discarded the requirement of a medium for the existence of displacement current in free space, and they also eliminated the vector and scalar potentials from the fundamental equations. Their derivations were based on the impressed sources, namely the current and the charge. Thus, Hertz and Heaviside, independently, expressed Maxwell's equations involving only the four field vectors  $E$ ,  $H$ ,  $B$ , and  $D$ : the electric field intensity, the magnetic field intensity, the magnetic flux density, and the electric flux density or displacement, respectively. Although priority is given to Heaviside for the vector form of Maxwell's equations, it is important to note that Hertz's 1884 paper [2] provided the Cartesian form of Maxwell's equations, which also appeared in his later paper of 1890 [3]. Thus, the coordinate forms of the four equations that we use nowadays were first obtained by Hertz [2,7] in scalar form and then by Heaviside in 1888 in vector form [4,7].

It is appropriate to mention here that the importance of Hertz's theoretical work [2] and its significance appear not to have been fully recognized [5]. In this 1884 paper [2] Hertz started from the older action-at-a-distance theories of electromagnetism and proceeded to obtain Maxwell's equations in an alternative way that avoided the mechanical models that Maxwell used originally and formed the basis for all his future contributions to electromagnetism, both theoretical and experimental. In contrast to the 1884 paper, in his 1890 paper [3] Hertz postulated Maxwell's equations rather than deriving them alternatively. The equations, written in component forms rather than in vector form as done by Heaviside [4], brought unparalleled clarity to Maxwell's theory. The four equations in vector notation containing the four electromagnetic field vectors are now commonly known as Maxwell's equations. However, Einstein referred to them as *Maxwell–Heaviside–Hertz equations* [6,7].

Although the idea of electromagnetic waves was hidden in the set of 20 equations proposed by Maxwell, he had in fact said virtually nothing about electromagnetic waves other than light, nor did he propose any idea to generate such waves electromagnetically. It has been stated [6, Ch. 2, p. 24]: “*There is even some reason to think that he [Maxwell] regarded the electrical production of such waves as impossibility.*” There is no indication left behind by him that he believed such was even possible. Maxwell did not live to see his prediction confirmed experimentally and his electromagnetic theory fully accepted. The former was confirmed by Hertz's brilliant experiments, his theory received universal acceptance, and his original equations in a modified form became the language of electromagnetic waves and electromagnetics, due mainly to the efforts of Hertz and Heaviside [7].

Hertz discovered electromagnetic waves around the year 1888 [8]; the results of his epoch-making experiments and his related theoretical work (based on the sources of the electromagnetic waves rather than on the potentials) confirmed Maxwell's prediction and helped the general acceptance of Maxwell's electromagnetic theory. However, it is not commonly appreciated that