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# OPTIMIZATION OF POWER SYSTEM OPERATION

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Principal Engineer, AREVA T&D Inc. Redmond, WA, USA  
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Mohamed E. El-Hawary, *Series Editor*



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To My Wife and Son



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# PREFACE

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I have been undertaking the research and practical applications of power system optimization since the early 1980s. In the early stage of my career, I worked in universities such as Chongqing University (China), Brunel University (UK), National University of Singapore, and Howard University (USA). Since 2000 I have been working for AREVA T&D Inc (USA). When I was a full-time professor at Chongqing University, I wrote a tutorial on power system optimal operation, which I used to teach my senior undergraduate students and postgraduate students in power engineering until 1996. The topics of the tutorial included advanced mathematical and operations research methods and their practical applications in power engineering problems. Some of these were refined to become part of this book.

This book comprehensively applies all kinds of optimization methods to solve power system operation problems. Some contents are analyzed and discussed for the first time in detail in one book, although they have appeared in international journals and conferences. These can be found in Chapter 9 “Steady-State Security Regions”, Chapter 11 “Optimal Load Shedding”, Chapter 12 “Optimal Reconfiguration of Electric Distribution Network”, and Chapter 13 “Uncertainty Analysis in Power Systems.”

This book covers not only traditional methods and implementation in power system operation such as Lagrange multipliers, equal incremental principle, linear programming, network flow programming, quadratic programming, nonlinear programming, and dynamic programming to solve the economic dispatch, unit commitment, reactive power optimization, load shedding, steady-state security region, and optimal power flow problems, but also new technologies and their implementation in power system operation in the last decade. The new technologies include improved interior point method, analytic hierarchical process, neural network, fuzzy set theory, genetic algorithm, evolutionary programming, and particle swarm optimization. Some new topics (wheeling model, multiarea wheeling, the total transfer capability computation in multiareas, reactive power pricing calculation, congestion management) addressed in recent years in power system operation are also dealt with and put in appropriate chapters.

In addition to having the rich analysis and implementation of all kinds of approaches, this book contains much hand-on experience for solving power system operation problems. I personally wrote my own code and tested the presented algorithms and power system applications. Many materials presented in the book are derived from my research accomplishments and publications when I worked at Chongqing University, Brunel University, National University of Singapore, and Howard University, as well as currently with AREVA T&D Inc. I appreciate these organizations for providing me such good working environments. Some IEEE papers have been used as primary sources and are cited wherever appropriate. The related publications for each topic are also listed as references, so that those interested may easily obtain overall information.

I wish to express my gratitude to IEEE book series editor Professor Mohammed El-Hawary of Dalhousie University, Canada, Acquisitions Editor Steve Welch, Project Editor Jeanne Audino, and the reviewers of the book for their keen interest in the development of this book, especially Professor Kit Po Wong of the Hong Kong Polytechnic University, Professor Loi Lei Lai of City University, UK, Professor Ruben Romero of Universidad Estadual Paulista, Brazil, and Dr. Ali Chowdhury of California Independent System Operator, who offered valuable comments and suggestions for the book during the preparation stage.

Finally, I wish to thank Professor Guoyu Xu, who was my PhD advisor twenty years ago at Chongqing University, for his high standards and strict requirements for me ever since I was his graduate student. Thanks to everyone, including my family, who has shown support during the time-consuming process of writing this book.

JIZHONG ZHU

## INTRODUCTION

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The electric power industry is being relentlessly pressured by governments, politicians, large industries, and investors to privatize, restructure, and deregulate. Before deregulation, most elements of the power industry, such as power generation, bulk power sales, capital expenditures, and investment decisions, were heavily regulated. Some of these regulations were at the state level, and some at the national level. Thus new deregulation in the power industry meant new challenges and huge changes. However, despite changes in different structures, market rules, and uncertainties, the underlying requirements for power system operations to be secure, economical, and reliable remain the same.

This book attempts to cover all areas of power systems operation. It also introduces some new topics and new applications of the latest new technologies that have appeared in recent years. This includes the analysis and discussion of new techniques for solving the old problems and the new problems that are arising from deregulation.

According to the different characteristics and types of the problems as well as their complexity, power systems operation is divided into the following aspects that are addressed in the book:

- Power flow analysis (Chapter 2)
- Sensitivity analysis (Chapter 3)
- Classical economic dispatch (Chapter 4)
- Security-constrained economic dispatch (Chapter 5)
- Multiarea systems economic dispatch (Chapter 6)

- Unit commitment (Chapter 7)
- Optimal power flow (Chapter 8)
- Steady-state security regions (Chapter 9)
- Reactive power optimization (Chapter 10)
- Optimal load shedding (Chapter 11)
- Optimal reconfiguration of electric distribution network (Chapter 12)
- Uncertainty analysis in power system (Chapter 13)

From the view of optimization, the various techniques including traditional and modern optimization methods, which have been developed to solve these power system operation problems, are classified into three groups [1–13]:

- (1) Conventional optimization methods including
  - Unconstrained optimization approaches
  - Nonlinear programming (NLP)
  - Linear programming (LP)
  - Quadratic programming (QP)
  - Generalized reduced gradient method
  - Newton method
  - Network flow programming (NFP)
  - Mixed-integer programming (MIP)
  - Interior point (IP) methods
- (2) Intelligence search methods such as
  - Neural network (NN)
  - Evolutionary algorithms (EAs)
  - Tabu search (TS)
  - Particle swarm optimization (PSO)
- (3) Nonquantity approaches to address uncertainties in objectives and constraints
  - Probabilistic optimization
  - Fuzzy set applications
  - Analytic hierarchical process (AHP)

## 1.1 CONVENTIONAL METHODS

### 1.1.1 Unconstrained Optimization Approaches

Unconstrained optimization approaches are the basis of the constrained optimization algorithms. In particular, most of the constrained optimization problems in power system operation can be converted into unconstrained

optimization problems. The major unconstrained optimization approaches that are used in power system operation are gradient method, line search, Lagrange multiplier method, Newton–Raphson optimization, trust-region optimization, quasi–Newton method, double dogleg optimization, and conjugate gradient optimization, etc. Some of these approaches are used in Chapter 2, Chapter 3, Chapter 4, Chapter 7, and Chapter 9.

### 1.1.2 Linear Programming

The linear programming (LP)-based technique is used to linearize the nonlinear power system optimization problem, so that objective function and constraints of power system optimization have linear forms. The simplex method is known to be quite effective for solving LP problems. The LP approach has several advantages. First, it is reliable, especially regarding convergence properties. Second, it can quickly identify infeasibility. Third, it accommodates a large variety of power system operating limits, including the very important contingency constraints. The disadvantages of LP-based techniques are inaccurate evaluation of system losses and insufficient ability to find an exact solution compared with an accurate nonlinear power system model. However, a great deal of practical applications show that LP-based solutions generally meet the requirements of engineering precision. Thus LP is widely used to solve power system operation problems such as security-constrained economic dispatch, optimal power flow, steady-state security regions, reactive power optimization, etc.

### 1.1.3 Nonlinear Programming

Power system operation problems are nonlinear. Thus nonlinear programming (NLP) based techniques can easily handle power system operation problems such as the OPF problems with nonlinear objective and constraint functions. To solve a nonlinear programming problem, the first step in this method is to choose a search direction in the iterative procedure, which is determined by the first partial derivatives of the equations (the reduced gradient). Therefore, these methods are referred to as first-order methods, such as the generalized reduced gradient (GRG) method. NLP-based methods have higher accuracy than LP-based approaches, and also have global convergence, which means that the convergence can be guaranteed independent of the starting point, but a slow convergent rate may occur because of zigzagging in the search direction. NLP methods are used in this book from Chapter 5 to Chapter 10.

### 1.1.4 Quadratic Programming

Quadratic programming (QP) is a special form of nonlinear programming. The objective function of QP optimization model is quadratic, and the constraints are in linear form. Quadratic programming has higher accuracy than LP-based

approaches. Especially, the most often-used objective function in power system optimization is the generator cost function, which generally is a quadratic. Thus there is no simplification for such objective function for a power system optimization problem solved by QP. QP is used in Chapters 5 and 8.

### 1.1.5 Newton's Method

Newton's method requires the computation of the second-order partial derivatives of the power flow equations and other constraints (the Hessian) and is therefore called a second-order method. The necessary conditions of optimality commonly are the Kuhn–Tucker conditions. Newton's method is favored for its quadratic convergence properties, and is used in Chapters 2, 4, and 8.

### 1.1.6 Interior Point Methods

The interior point (IP) method is originally used to solve linear programming. It is faster and perhaps better than the conventional simplex algorithm in linear programming. IP methods were first applied to solve OPF problems in the 1990s, and recently, the IP method has been extended and improved to solve OPF with QP and NLP forms. The analysis and implement of IP methods are discussed in Chapters 8 and 10.

### 1.1.7 Mixed-Integer Programming

The power system problem can also be formulated as a mixed-integer programming (MIP) optimization problem with integer variables such as transformer tap ratio, phase shifter angle, and unit on or off status. MIP is extremely demanding of computer resources, and the number of discrete variables is an important indicator of how difficult an MIP will be to solve. MIP methods that are used to solve OPF problems are the recursive mixed-integer programming technique using an approximation method and the branch and bound (B&B) method, which is a typical method for integer programming. A decomposition technique is generally adopted to decompose the MIP problem into a continuous problem and an integer problem. Decomposition methods such as Benders' decomposition method (BDM) can greatly improve efficiency in solving a large-scale network by reducing the dimensions of the individual subproblems. The results show a significant reduction of the number of iterations, required computation time, and memory space. Also, decomposition allows the application of a separate method for the solution of each subproblem, which makes the approach very attractive. Mixed-integer programming can be used to solve the unit commitment, OPF, as well as the optimal reconfiguration of electric distribution network.

### 1.1.8 Network Flow Programming

Network flow programming (NFP) is special linear programming. NFP was first applied to solve optimization problems in power systems in 1980s. The early applications of NFP were mainly on a linear model. Recently, nonlinear convex network flow programming has been used in power systems' optimization problems. NFP-based algorithms have the features of fast speed and simple calculation. These methods are efficient for solving simplified OPF problems such as security-constrained economic dispatch, multiarea systems economic dispatch, and optimal reconfiguration of an electric distribution network.

## 1.2 INTELLIGENT SEARCH METHODS

### 1.2.1 Optimization Neural Network

Optimization neural network (ONN) was first used to solve linear programming problems in 1986. Recently, ONN was extended to solve nonlinear programming problems. ONN is completely different from traditional optimization methods. It changes the solution of an optimization problem into an equilibrium point (or equilibrium state) of nonlinear dynamic system, and changes the optimal criterion into energy functions for dynamic systems. Because of its parallel computational structure and the evolution of dynamics, the ONN approach appears superior to traditional optimization methods. The ONN approach is applied to solve the classic economic dispatch, multiarea systems economic dispatch, and reactive power optimization in this book.

### 1.2.2 Evolutionary Algorithms

Natural evolution is a population-based optimization process. The evolutionary algorithms (EAs) are different from the conventional optimization methods, and they do not need to differentiate cost function and constraints. Theoretically, like simulated annealing, EAs converge to the global optimum solution. EAs, including evolutionary programming (EP), evolutionary strategy (ES), and GA are artificial intelligence methods for optimization based on the mechanics of natural selection, such as mutation, recombination, reproduction, crossover, selection, etc. Since EAs require all information to be included in the fitness function, it is very difficult to consider all OPF constraints. Thus EAs are generally used to solve a simplified OPF problem such as the classic economic dispatch, security-constrained economic power dispatch, and reactive optimization problem, as well as optimal reconfiguration of an electric distribution network.

### 1.2.3 Tabu Search

The tabu search (TS) algorithm is mainly used for solving combinatorial optimization problems. It is an iterative search algorithm, characterized by the use of a flexible memory. It is able to eliminate local minima and to search areas beyond a local minimum. The TS method is also mainly used to solve simplified OPF problems such as unit commitment and reactive optimization problems.

### 1.2.4 Particle Swarm Optimization

Particle swarm optimization (PSO) is a swarm intelligence algorithm, inspired by social dynamics and an emergent behavior that arises in socially organized colonies. The PSO algorithm exploits a population of individuals to probe promising regions of search space. In this context, the population is called a swarm and the individuals are called particles or agents. In recent years, various PSO algorithms have been successfully applied in many power engineering problems including OPF. These are analyzed in Chapters 7, 8 and 10.

## 1.3 APPLICATION OF FUZZY SET THEORY

The data and parameters used in power system operation are usually derived from many sources, with a wide variance in their accuracy. For example, although the average load is typically applied in power system operation problems, the actual load should follow some uncertain variations. In addition, generator fuel cost, VAR compensators, and peak power savings may be subject to uncertainty to some degree. Therefore, uncertainties due to insufficient information may generate an uncertain region of decisions. Consequently, the validity of the results from average values cannot represent the uncertainty level. To account for the uncertainties in information and goals related to multiple and usually conflicting objectives in power system optimization, the use of probability theory, fuzzy set theory, and analytic hierarchical process may play a significant role in decision-making.

The probabilistic methods and their application in power systems operation with uncertainty are discussed in Chapter 13. The fuzzy sets may be assigned not only to objective functions, but also to constraints, especially the nonprobabilistic uncertainty associated with the reactive power demand in constraints. Generally speaking, the satisfaction parameters (fuzzy sets) for objectives and constraints represent the degree of closeness to the optimum and the degree of enforcement of constraints, respectively. With the maximization of these satisfaction parameters, the goal of optimization is achieved and simultaneously the uncertainties are considered. The application of fuzzy set theory to the OPF problem is also presented in Chapter 13. The analytic hierarchical process (AHP) is a simple and convenient method to analyze a complicated

problem (or complex problem). It is especially suitable for problems that are very difficult to analyze wholly quantitatively, such as OPF with competitive objectives, or uncertain factors. The details of the AHP algorithm are given in Chapter 7. AHP is employed to solve unit commitment, multiarea economic dispatch, OPF, VAR optimization, optimal load shedding, and uncertainty analysis in the power system.

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## POWER FLOW ANALYSIS

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This chapter deals with the power flow problem. The power flow algorithms include the Newton–Raphson method in both polar and rectangle forms, the Gauss–Seidel method, the DC power flow method, and all kinds of decoupled power flow methods such as fast decoupled power flow, simplified BX and XB methods, as well as decoupled power flow without major approximation.

### 2.1 MATHEMATICAL MODEL OF POWER FLOW

Power flow is well known as “load flow.” This is the name given to a network solution that shows currents, voltages, and real and reactive power flows at every bus in the system. Since the parameters of the elements such as lines and transformers are constant, the power system network is a linear network. However, in the power flow problem, the relationship between voltage and current at each bus is nonlinear, and the same holds for the relationship between the real and reactive power consumption at a bus or the generated real power and scheduled voltage magnitude at a generator bus. Thus power flow calculation involves the solution of nonlinear equations. It gives us the electrical response of the transmission system to a particular set of loads and generator power outputs. Power flows are an important part of power system operation and planning.

Generally, for a network with  $n$  independent buses, we can write the following  $n$  equations.

$$\left. \begin{aligned} Y_{11}\dot{V}_1 + Y_{12}\dot{V}_2 + \dots + Y_{1n}\dot{V}_n &= \dot{I}_1 \\ Y_{21}\dot{V}_1 + Y_{22}\dot{V}_2 + \dots + Y_{2n}\dot{V}_n &= \dot{I}_2 \\ &\dots\dots\dots \\ Y_{n1}\dot{V}_1 + Y_{n2}\dot{V}_2 + \dots + Y_{nn}\dot{V}_n &= \dot{I}_n \end{aligned} \right\} \quad (2.1)$$

The matrix form is

$$\begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \dots & Y_{nn} \end{bmatrix} \begin{bmatrix} \dot{V}_1 \\ \dot{V}_2 \\ \vdots \\ \dot{V}_n \end{bmatrix} = \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \vdots \\ \dot{I}_n \end{bmatrix} \quad (2.2)$$

or

$$[Y][V] = I \quad (2.3)$$

where  $I$  is the bus current injection vector,  $V$  is the bus voltage vector, and  $Y$  is called the bus admittance matrix. Its diagonal element  $Y_{ii}$  is called the self-admittance of bus  $i$ , which equals the sum of all branch admittances connecting to bus  $i$ . The off-diagonal element of the bus admittance matrix  $Y_{ij}$  is the negative of branch admittance between buses  $i$  and  $j$ . If there is no line between buses  $i$  and  $j$ , this term is zero. Obviously, the bus admittance matrix is a sparse matrix.

In addition, the bus current can be represented by bus voltage and power, that is,

$$\dot{I}_i = \frac{\hat{S}_i}{\hat{V}_i} = \frac{\hat{S}_{Gi} - \hat{S}_{Di}}{\hat{V}_i} = \frac{(P_{Gi} - P_{Di}) - j(Q_{Gi} - Q_{Di})}{\hat{V}_i} \quad (2.4)$$

where

$S$ : The complex power injection vector

$P_{Gi}$ : The real power output of the generator connecting to bus  $i$

$Q_{Gi}$ : The reactive power output of the generator connecting to bus  $i$

$P_{Di}$ : The real power load connecting to bus  $i$

$Q_{Di}$ : The reactive power load connecting to bus  $i$

Substituting equation (2.4) into equation (2.1), we have

$$\frac{(P_{Gi} - P_{Di}) - j(Q_{Gi} - Q_{Di})}{\hat{V}_i} = Y_{i1}\dot{V}_1 + Y_{i2}\dot{V}_2 + \dots + Y_{in}\dot{V}_n, \quad i = 1, 2, \dots, n \quad (2.5)$$