

# QUANTITATIVE REVIEW

## 2ND EDITION

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The only study guide with  
300 past GMAT® questions  
—and their answers—  
by the creators  
of the test.

~The~  
**OFFICIAL**  
Guide

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THE OFFICIAL GUIDE FOR  
GMAT® QUANTITATIVE REVIEW  
2ND EDITION

- Actual questions from past GMAT tests, including 75 questions new to this edition
  - 300 past Problem Solving and Data Sufficiency questions and answer explanations spanning Arithmetic, Algebra, Geometry, and Word Problems
  - Questions organized in order of difficulty to save study time
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From the Graduate Management Admission Council®

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## THE OFFICIAL GUIDE FOR GMAT® QUANTITATIVE REVIEW, 2ND EDITION

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## **1.0 What Is the GMAT®?**

## 1.0 What Is the GMAT®?

The Graduate Management Admission Test® (GMAT®) is a standardized, three-part test delivered in English. The test was designed to help admissions officers evaluate how suitable individual applicants are for their graduate business and management programs. It measures basic verbal, mathematical, and analytical writing skills that a test taker has developed over a long period of time through education and work.

The GMAT test does not measure a person's knowledge of specific fields of study. Graduate business and management programs enroll people from many different undergraduate and work backgrounds, so rather than test your mastery of any particular subject area, the GMAT test will assess your acquired skills. Your GMAT score will give admissions officers a statistically reliable measure of how well you are likely to perform academically in the core curriculum of a graduate business program.

Of course, there are many other qualifications that can help people succeed in business school and in their careers—for instance, job experience, leadership ability, motivation, and interpersonal skills. The GMAT test does not gauge these qualities. That is why your GMAT score is intended to be used as one standard admissions criterion among other, more subjective, criteria, such as admissions essays and interviews.

## 1.1 Why Take the GMAT® Test?

GMAT scores are used by admissions officers in roughly 1,800 graduate business and management programs worldwide. Schools that require prospective students to submit GMAT scores in the application process are generally interested in admitting the best-qualified applicants for their programs, which means that you may find a more beneficial learning environment at schools that require GMAT scores as part of your application.

Because the GMAT test gauges skills that are important to successful study of business and management at the graduate level, your scores will give you a good indication of how well prepared you are to succeed academically in a graduate management program; how well you do on the test may also help you choose the business schools to which you apply. Furthermore, the percentile table you receive with your scores will tell you how your performance on the test compares to the performance of other test takers, giving you one way to gauge your competition for admission to business school.

### *Myth* -vs- **FACT**

**M** – If I don't score in the 90th percentile, I won't get into any school I choose.

**F** – Very few people get very high scores.

Fewer than 50 of the more than 200,000 people taking the GMAT test each year get a perfect score of 800. Thus, while you may be exceptionally capable, the odds are against your achieving a perfect score. Also, the GMAT test is just one piece of your application packet. Admissions officers use GMAT scores in conjunction with undergraduate records, application essays, interviews, letters of recommendation, and other information when deciding whom to accept into their programs.

Schools consider many different aspects of an application before making an admissions decision, so even if you score well on the GMAT test, you should contact the schools that interest you to learn more about them and to ask about how they use GMAT scores and other admissions criteria (such as your undergraduate grades, essays, and letters of recommendation) to evaluate candidates for admission. School admissions offices, school Web sites, and materials published by the school are the best sources for you to tap when you are doing research about where you might want to go to business school.

For more information about how schools should use GMAT scores in admissions decisions, please read Appendix A of this book. For more information on the GMAT, registering to take the test, sending your scores to schools, and applying to business school, please visit our Web site at [www.mba.com](http://www.mba.com).

## 1.2 GMAT® Test Format

The GMAT test consists of four separately timed sections (see the table on the next page). You start the test with two 30-minute Analytical Writing Assessment (AWA) questions that require you to type your responses using the computer keyboard. The writing section is followed by two 75-minute, multiple-choice sections: the Quantitative and Verbal sections of the test.

### *Myth* -vs- **FACT**

**M** – Getting an easier question means I answered the last one wrong.

**F** – Getting an easier question does not necessarily mean you got the previous question wrong.

To ensure that everyone receives the same content, the test selects a specific number of questions of each type. The test may call for your next question to be a relatively hard problem-solving item involving arithmetic operations. But, if there are no more relatively difficult problem-solving items involving arithmetic, you might be given an easier item.

Most people are not skilled at estimating item difficulty, so don't worry when taking the test or waste valuable time trying to determine the difficulty of the questions you are answering.

The GMAT is a computer-adaptive test (CAT), which means that in the multiple-choice sections of the test, the computer constantly gauges how well you are doing on the test and presents you with questions that are appropriate to your ability level. These questions are drawn from a huge pool of possible test questions. So, although we talk about the GMAT as one test, the GMAT test you take may be completely different from the test of the person sitting next to you.

Here's how it works. At the start of each GMAT multiple-choice section (Verbal and Quantitative), you will be presented with a question of moderate difficulty. The computer uses your response to that first question to determine which question to present next. If you respond correctly, the test usually will give you questions of increasing difficulty. If you respond incorrectly, the next question you see usually will be easier than the one you answered incorrectly. As you continue to respond to the questions presented, the computer will narrow your score to the number that best characterizes your ability. When you complete each section, the computer will have an accurate assessment of your ability.

Because each question is presented on the basis of your answers to all previous questions, you must answer each question as it appears. You may not skip, return to, or change your responses to previous questions. Random guessing can significantly lower your scores. If you do not know the answer to a question, you should try to eliminate as many choices as possible, then select the answer you think is best. If you answer a question incorrectly by mistake—or correctly by lucky guess—your answers to subsequent questions will lead you back to questions that are at the appropriate level of difficulty for you.

Each multiple-choice question used in the GMAT test has been thoroughly reviewed by professional test developers. New multiple-choice questions are tested each time the test is administered. Answers to trial questions are not counted in the scoring of your test, but the trial questions are not identified and could appear anywhere in the test. Therefore, you should try to do your best on every question.

The test includes the types of questions found in this guide, but the format and presentation of the questions are different on the computer. When you take the test:

- Only one question at a time is presented on the computer screen.
- The answer choices for the multiple-choice questions will be preceded by circles, rather than by letters.
- Different question types appear in random order in the multiple-choice sections of the test.
- You must select your answer using the computer.
- You must choose an answer and confirm your choice before moving on to the next question.
- You may not go back to change answers to previous questions.

<b>Format of the GMAT®</b>		
	Questions	Timing
Analytical Writing		
Analysis of an Issue	1	30 min.
Analysis of an Argument	1	30 min.
Optional break		
Quantitative	37	75 min.
Problem Solving		
Data Sufficiency		
Optional break		
Verbal	41	75 min.
Reading Comprehension		
Critical Reasoning		
Sentence Correction		
Total Time:		210 min.

## 1.3 What Is the Content of the Test Like?

It is important to recognize that the GMAT test evaluates skills and abilities developed over a relatively long period of time. Although the sections contain questions that are basically verbal and mathematical, the complete test provides one method of measuring overall ability.

Keep in mind that although the questions in this guide are arranged by question type and ordered from easy to difficult, the test is organized differently. When you take the test, you may see different types of questions in any order.

## 1.4 Quantitative Section

The GMAT Quantitative section measures your ability to reason quantitatively, solve quantitative problems, and interpret graphic data.

Two types of multiple-choice questions are used in the Quantitative section:

- Problem solving
- Data sufficiency

Problem solving and data sufficiency questions are intermingled throughout the Quantitative section. Both types of questions require basic knowledge of:

- Arithmetic
- Elementary algebra
- Commonly known concepts of geometry

To review the basic mathematical concepts that will be tested in the GMAT Quantitative questions, see the math review in chapter 3. For test-taking tips specific to the question types in the Quantitative section of the GMAT test, sample questions, and answer explanations, see chapters 4 and 5.

## 1.5 Verbal Section

The GMAT Verbal section measures your ability to read and comprehend written material, to reason and evaluate arguments, and to correct written material to conform to standard written English. Because the Verbal section includes reading sections from several different content areas, you may be generally familiar with some of the material; however, neither the reading passages nor the questions assume detailed knowledge of the topics discussed.

Three types of multiple-choice questions are used in the Verbal section:

- Reading comprehension
- Critical reasoning
- Sentence correction

These question types are intermingled throughout the Verbal section.

For test-taking tips specific to each question type in the Verbal section, sample questions, and answer explanations, see *The Official Guide for GMAT Review*, 12th Edition, or *The Official Guide for GMAT Verbal Review*, 2nd Edition; both are available for purchase at [www.mba.com](http://www.mba.com).

## 1.6 What Computer Skills Will I Need?

You only need minimal computer skills to take the GMAT Computer-Adaptive Test (CAT). You will be required to type your essays on the computer keyboard using standard word-processing keystrokes. In the multiple-choice sections, you will select your responses using either your mouse or the keyboard.

To learn more about the specific skills required to take the GMAT CAT, download the free test-preparation software available at [www.mba.com](http://www.mba.com).

## 1.7 What Are the Test Centers Like?

The GMAT test is administered at a test center providing the quiet and privacy of individual computer workstations. You will have the opportunity to take two optional breaks—one after completing the essays and another between the Quantitative and Verbal sections. An erasable notepad will be provided for your use during the test.

## 1.8 How Are Scores Calculated?

Your GMAT scores are determined by:

- The number of questions you answer
- Whether you answer correctly or incorrectly
- The level of difficulty and other statistical characteristics of each question

Your Verbal, Quantitative, and Total GMAT scores are determined by a complex mathematical procedure that takes into account the difficulty of the questions that were presented to you and how you answered them. When you answer the easier questions correctly, you get a chance to answer harder questions—making it possible to earn a higher score. After you have completed all the questions on the test—or when your time is up—the computer will calculate your scores. Your scores on the Verbal and Quantitative sections are combined to produce your Total score. If you have not responded to all the questions in a section (37 Quantitative questions or 41 Verbal questions), your score is adjusted, using the proportion of questions answered.

Appendix A contains the 2007 percentile ranking tables that explain how your GMAT scores compare with scores of other 2007 GMAT test takers.

## 1.9 Analytical Writing Assessment Scores

The Analytical Writing Assessment consists of two writing tasks: Analysis of an Issue and Analysis of an Argument. The responses to each of these tasks are scored on a 6-point scale, with 6 being the highest score and 1, the lowest. A score of zero (0) is given to responses that are off-topic, are in a foreign language, merely attempt to copy the topic, consist only of keystroke characters, or are blank.

The readers who evaluate the responses are college and university faculty members from various subject matter areas, including management education. These readers read holistically—that is, they respond to the overall quality of your critical thinking and writing. (For details on how readers are qualified, visit [www.mba.com](http://www.mba.com).) In addition, responses may be scored by an automated scoring program designed to reflect the judgment of expert readers.

Each response is given two independent ratings. If the ratings differ by more than a point, a third reader adjudicates. (Because of ongoing training and monitoring, discrepant ratings are rare.)

Your final score is the average (rounded to the nearest half point) of the four scores independently assigned to your responses—two scores for the Analysis of an Issue and two for the Analysis of an Argument. For example, if you earned scores of 6 and 5 on the Analysis of an Issue and 4 and 4 on the Analysis of an Argument, your final score would be 5:  $(6 + 5 + 4 + 4) \div 4 = 4.75$ , which rounds up to 5.

Your Analytical Writing Assessment scores are computed and reported separately from the multiple-choice sections of the test and have no effect on your Verbal, Quantitative, or Total scores. The schools that you have designated to receive your scores may receive your responses to the Analytical Writing Assessment with your score report. Your own copy of your score report will not include copies of your responses.

## 1.10 Test Development Process

The GMAT test is developed by experts who use standardized procedures to ensure high-quality, widely appropriate test material. All questions are subjected to independent reviews and are revised or discarded as necessary. Multiple-choice questions are tested during GMAT test administrations. Analytical Writing Assessment tasks are tried out on first-year business school students and then assessed for their fairness and reliability. For more information on test development, see [www.mba.com](http://www.mba.com).

To register for the GMAT test go to [www.mba.com](http://www.mba.com)

## 2.0 How to Prepare

## 2.0 How to Prepare

### 2.1 How Can I Best Prepare to Take the Test?

We at the Graduate Management Admission Council® (GMAC®) firmly believe that the test-taking skills you can develop by using this guide—and *The Official Guide for GMAT® Review*, 12th Edition, and *The Official Guide for GMAT® Verbal Review*, 2nd Edition, if you want additional practice—are all you need to perform your best when you take the GMAT® test. By answering questions that have appeared on the GMAT test before, you will gain experience with the types of questions you may see on the test when you take it. As you practice with this guide, you will develop confidence in your ability to reason through the test questions. No additional techniques or strategies are needed to do well on the standardized test if you develop a practical familiarity with the abilities it requires. Simply by practicing and understanding the concepts that are assessed on the test, you will learn what you need to know to answer the questions correctly.

### 2.2 What About Practice Tests?

Because a computer-adaptive test cannot be presented in paper form, we have created GMATPrep® software to help you prepare for the test. The software is available for download at no charge for those who have created a user profile on [www.mba.com](http://www.mba.com). It is also provided on a disk, by request, to anyone who has registered for the GMAT test. The software includes two practice GMAT tests plus additional practice questions, information about the test, and tutorials to help you become familiar with how the GMAT test will appear on the computer screen at the test center.

We recommend that you download the software as you start to prepare for the test. Take one practice test to familiarize yourself with the test and to get an idea of how you might score. After you have studied using this book, and as your test date approaches, take the second practice test to determine whether you need to shift your focus to other areas you need to strengthen.

#### *Myth* -vs- **FACT**

**M** – You may need very advanced math skills to get a high GMAT score.

**F** – The math skills test on the GMAT test are quite basic.

The GMAT test only requires basic quantitative analytic skills. You should review the math skills (algebra, geometry, basic arithmetic) presented both in this book (chapter 3) and in *The Official Guide for GMAT® Review*, 12th Edition, but the required skill level is low. The difficulty of GMAT Quantitative questions stems from the logic and analysis used to solve the problems and not the underlying math skills.

## 2.3 Where Can I Get Additional Practice?

If you complete all the questions in this guide and think you would like additional practice, you may purchase *The Official Guide for GMAT® Review*, 12th Edition, or *The Official Guide for GMAT® Verbal Review*, 2nd Edition, at [www.mba.com](http://www.mba.com).

**Note:** There may be some overlap between this book and the review sections of the GMATPrep® software.

## 2.4 General Test-Taking Suggestions

Specific test-taking strategies for individual question types are presented later in this book. The following are general suggestions to help you perform your best on the test.

### 1. Use your time wisely.

Although the GMAT test stresses accuracy more than speed, it is important to use your time wisely. On average, you will have about 1¾ minutes for each verbal question and about 2 minutes for each quantitative question. Once you start the test, an onscreen clock will continuously count the time you have left. You can hide this display if you want, but it is a good idea to check the clock periodically to monitor your progress. The clock will automatically alert you when 5 minutes remain in the allotted time for the section you are working on.

### 2. Answer practice questions ahead of time.

After you become generally familiar with all question types, use the sample questions in this book to prepare for the actual test. It may be useful to time yourself as you answer the practice questions to get an idea of how long you will have for each question during the actual GMAT test as well as to determine whether you are answering quickly enough to complete the test in the time allotted.

### 3. Read all test directions carefully.

The directions explain exactly what is required to answer each question type. If you read hastily, you may miss important instructions and lower your scores. To review directions during the test, click on the Help icon. But be aware that the time you spend reviewing directions will count against the time allotted for that section of the test.

### 4. Read each question carefully and thoroughly.

Before you answer a multiple-choice question, determine exactly what is being asked, then eliminate the wrong answers and select the best choice. Never skim a question or the possible answers; skimming may cause you to miss important information or nuances.

### 5. Do not spend too much time on any one question.

If you do not know the correct answer, or if the question is too time-consuming, try to eliminate choices you know are wrong, select the best of the remaining answer choices, and move on to the next question. Try not to worry about the impact on your score—guessing may lower your score, but not finishing the section will lower your score more.

Bear in mind that if you do not finish a section in the allotted time, you will still receive a score.

### 6. Confirm your answers ONLY when you are ready to move on.

Once you have selected your answer to a multiple-choice question, you will be asked to confirm it. Once you confirm your response, you cannot go back and change it. You may not skip questions, because the computer selects each question on the basis of your responses to preceding questions.

### 7. Plan your essay answers before you begin to write.

The best way to approach the two writing tasks that comprise the Analytical Writing Assessment is to read the directions carefully, take a few minutes to think about the question, and plan a response before you begin writing. Take care to organize your ideas and develop them fully, but leave time to reread your response and make any revisions that you think would improve it.

#### Myth -vs- FACT

**M** – It is more important to respond correctly to the test questions than it is to finish the test.

**F** – There is a severe penalty for not completing the GMAT test.

If you are stumped by a question, give it your best guess and move on. If you guess incorrectly, the computer program will likely give you an easier question, which you are likely to answer correctly, and the computer will rapidly return to giving you questions matched to your ability. If you don't finish the test, your score will be reduced greatly. Failing to answer five verbal questions, for example, could reduce your score from the 91st percentile to the 77th percentile. Pacing is important.

#### Myth -vs- FACT

**M** – The first 10 questions are critical and you should invest the most time on those.

**F** – All questions count.

It is true that the computer-adaptive testing algorithm uses the first 10 questions to obtain an initial estimate of your ability; however, that is only an *initial* estimate. As you continue to answer questions, the algorithm self-corrects by computing an updated estimate on the basis of all the questions you have answered, and then administers items that are closely matched to this new estimate of your ability. Your final score is based on all your responses and considers the difficulty of all the questions you answered. Taking additional time on the first 10 questions will not game the system and can hurt your ability to finish the test.

## 3.0 Math Review

## 3.0 Math Review

Although this chapter provides a review of some of the mathematical concepts of arithmetic, algebra, and geometry, it is not intended to be a textbook. You should use this chapter to familiarize yourself with the kinds of topics that are tested in the GMAT® test. You may wish to consult an arithmetic, algebra, or geometry book for a more detailed discussion of some of the topics.

Section 3.1, “Arithmetic,” includes the following topics:

- |                           |                                |
|---------------------------|--------------------------------|
| 1. Properties of Integers | 7. Powers and Roots of Numbers |
| 2. Fractions              | 8. Descriptive Statistics      |
| 3. Decimals               | 9. Sets                        |
| 4. Real Numbers           | 10. Counting Methods           |
| 5. Ratio and Proportion   | 11. Discrete Probability       |
| 6. Percents               |                                |

Section 3.2, “Algebra,” does not extend beyond what is usually covered in a first-year high school algebra course. The topics included are as follows:

- |   |                   |
|---|-------------------|
| 1. Simplifying Algebraic Expressions              | 7. Exponents      |
| 2. Equations                                      | 8. Inequalities   |
| 3. Solving Linear Equations with One Unknown      | 9. Absolute Value |
| 4. Solving Two Linear Equations with Two Unknowns | 10. Functions     |
| 5. Solving Equations by Factoring                 |                   |
| 6. Solving Quadratic Equations                    |                   |

Section 3.3, “Geometry,” is limited primarily to measurement and intuitive geometry or spatial visualization. Extensive knowledge of theorems and the ability to construct proofs, skills that are usually developed in a formal geometry course, are not tested. The topics included in this section are the following:

- |                                  |                                     |
|----------------------------------|-------------------------------------|
| 1. Lines                         | 6. Triangles                        |
| 2. Intersecting Lines and Angles | 7. Quadrilaterals                   |
| 3. Perpendicular Lines           | 8. Circles                          |
| 4. Parallel Lines                | 9. Rectangular Solids and Cylinders |
| 5. Polygons (Convex)             | 10. Coordinate Geometry             |

Section 3.4, “Word Problems,” presents examples of and solutions to the following types of word problems:

- |                      |                         |
|----------------------|-------------------------|
| 1. Rate Problems     | 6. Profit               |
| 2. Work Problems     | 7. Sets                 |
| 3. Mixture Problems  | 8. Geometry Problems    |
| 4. Interest Problems | 9. Measurement Problems |
| 5. Discount          | 10. Data Interpretation |

## 3.1 Arithmetic

### 1. Properties of Integers

An *integer* is any number in the set  $\{ \dots -3, -2, -1, 0, 1, 2, 3, \dots \}$ . If  $x$  and  $y$  are integers and  $x \neq 0$ , then  $x$  is a *divisor* (*factor*) of  $y$  provided that  $y = xn$  for some integer  $n$ . In this case,  $y$  is also said to be *divisible* by  $x$  or to be a *multiple* of  $x$ . For example, 7 is a divisor or factor of 28 since  $28 = (7)(4)$ , but 8 is not a divisor of 28 since there is no integer  $n$  such that  $28 = 8n$ .

If  $x$  and  $y$  are positive integers, there exist unique integers  $q$  and  $r$ , called the *quotient* and *remainder*, respectively, such that  $y = xq + r$  and  $0 \leq r < x$ . For example, when 28 is divided by 8, the quotient is 3 and the remainder is 4 since  $28 = (8)(3) + 4$ . Note that  $y$  is divisible by  $x$  if and only if the remainder  $r$  is 0; for example, 32 has a remainder of 0 when divided by 8 because 32 is divisible by 8. Also, note that when a smaller integer is divided by a larger integer, the quotient is 0 and the remainder is the smaller integer. For example, 5 divided by 7 has the quotient 0 and the remainder 5 since  $5 = (7)(0) + 5$ .

Any integer that is divisible by 2 is an *even integer*; the set of even integers is  $\{ \dots -4, -2, 0, 2, 4, 6, 8, \dots \}$ . Integers that are not divisible by 2 are *odd integers*;  $\{ \dots -3, -1, 1, 3, 5, \dots \}$  is the set of odd integers.

If at least one factor of a product of integers is even, then the product is even; otherwise the product is odd. If two integers are both even or both odd, then their sum and their difference are even. Otherwise, their sum and their difference are odd.

A *prime* number is a positive integer that has exactly two different positive divisors, 1 and itself. For example, 2, 3, 5, 7, 11, and 13 are prime numbers, but 15 is not, since 15 has four different positive divisors, 1, 3, 5, and 15. The number 1 is not a prime number since it has only one positive divisor. Every integer greater than 1 either is prime or can be uniquely expressed as a product of prime factors. For example,  $14 = (2)(7)$ ,  $81 = (3)(3)(3)(3)$ , and  $484 = (2)(2)(11)(11)$ .

The numbers  $-2, -1, 0, 1, 2, 3, 4, 5$  are *consecutive integers*. Consecutive integers can be represented by  $n, n + 1, n + 2, n + 3, \dots$ , where  $n$  is an integer. The numbers 0, 2, 4, 6, 8 are *consecutive even integers*, and 1, 3, 5, 7, 9 are *consecutive odd integers*. Consecutive even integers can be represented by  $2n, 2n + 2, 2n + 4, \dots$ , and consecutive odd integers can be represented by  $2n + 1, 2n + 3, 2n + 5, \dots$ , where  $n$  is an integer.

*Properties of the integer 1.* If  $n$  is any number, then  $1 \cdot n = n$ , and for any number  $n \neq 0$ ,  $n \cdot \frac{1}{n} = 1$ .

The number 1 can be expressed in many ways; for example,  $\frac{n}{n} = 1$  for any number  $n \neq 0$ .

Multiplying or dividing an expression by 1, in any form, does not change the value of that expression.

*Properties of the integer 0.* The integer 0 is neither positive nor negative. If  $n$  is any number, then  $n + 0 = n$  and  $n \cdot 0 = 0$ . Division by 0 is not defined.

## 2. Fractions

In a fraction  $\frac{n}{d}$ ,  $n$  is the *numerator* and  $d$  is the *denominator*. The denominator of a fraction can never be 0, because division by 0 is not defined.

Two fractions are said to be *equivalent* if they represent the same number. For example,  $\frac{8}{36}$  and  $\frac{14}{63}$  are equivalent since they both represent the number  $\frac{2}{9}$ . In each case, the fraction is reduced to lowest terms by dividing both numerator and denominator by their *greatest common divisor* (gcd).

The gcd of 8 and 36 is 4 and the gcd of 14 and 63 is 7.

### Addition and subtraction of fractions.

Two fractions with the same denominator can be added or subtracted by performing the required operation with the numerators, leaving the denominators the same. For example,  $\frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \frac{7}{5}$  and  $\frac{5}{7} - \frac{2}{7} = \frac{5-2}{7} = \frac{3}{7}$ . If two fractions do not have the same denominator, express them as equivalent fractions with the same denominator. For example, to add  $\frac{3}{5}$  and  $\frac{4}{7}$ , multiply the numerator and denominator of the first fraction by 7 and the numerator and denominator of the second fraction by 5, obtaining  $\frac{21}{35}$  and  $\frac{20}{35}$ , respectively;  $\frac{21}{35} + \frac{20}{35} = \frac{41}{35}$ .

For the new denominator, choosing the *least common multiple* (lcm) of the denominators usually lessens the work. For  $\frac{2}{3} + \frac{1}{6}$ , the lcm of 3 and 6 is 6 (not  $3 \times 6 = 18$ ), so

$$\frac{2}{3} + \frac{1}{6} = \frac{2}{3} \times \frac{2}{2} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{5}{6}.$$

### Multiplication and division of fractions.

To multiply two fractions, simply multiply the two numerators and multiply the two denominators.

For example,  $\frac{2}{3} \times \frac{4}{7} = \frac{2 \times 4}{3 \times 7} = \frac{8}{21}$ .

To divide by a fraction, invert the divisor (that is, find its *reciprocal*) and multiply. For example,  $\frac{2}{3} \div \frac{4}{7} = \frac{2}{3} \times \frac{7}{4} = \frac{14}{12} = \frac{7}{6}$ .

In the problem above, the reciprocal of  $\frac{4}{7}$  is  $\frac{7}{4}$ . In general, the reciprocal of a fraction  $\frac{n}{d}$  is  $\frac{d}{n}$ , where  $n$  and  $d$  are not zero.

**Mixed numbers.**

A number that consists of a whole number and a fraction, for example,  $7\frac{2}{3}$ , is a mixed number:

$$7\frac{2}{3} \text{ means } 7 + \frac{2}{3}.$$

To change a mixed number into a fraction, multiply the whole number by the denominator of the fraction and add this number to the numerator of the fraction; then put the result over the

denominator of the fraction. For example,  $7\frac{2}{3} = \frac{(3 \times 7) + 2}{3} = \frac{23}{3}$ .

**3. Decimals**

In the decimal system, the position of the period or *decimal point* determines the place value of the digits. For example, the digits in the number 7,654.321 have the following place values:

Thousands		Hundreds		Tens		Ones or units		Tenths		Hundredths		Thousandths
7	,	6	5	4	.	3	2	1				

Some examples of decimals follow.

$$0.321 = \frac{3}{10} + \frac{2}{100} + \frac{1}{1,000} = \frac{321}{1,000}$$

$$0.0321 = \frac{0}{10} + \frac{3}{100} + \frac{2}{1,000} + \frac{1}{10,000} = \frac{321}{10,000}$$

$$1.56 = 1 + \frac{5}{10} + \frac{6}{100} = \frac{156}{100}$$

Sometimes decimals are expressed as the product of a number with only one digit to the left of the decimal point and a power of 10. This is called *scientific notation*. For example, 231 can be written as  $2.31 \times 10^2$  and 0.0231 can be written as  $2.31 \times 10^{-2}$ . When a number is expressed in scientific notation, the exponent of the 10 indicates the number of places that the decimal point is to be moved in the number that is to be multiplied by a power of 10 in order to obtain the product. The decimal point is moved to the right if the exponent is positive and to the left if the exponent is negative. For example,  $2.013 \times 10^4$  is equal to 20,130 and  $1.91 \times 10^{-4}$  is equal to 0.000191.