Panoramic Imaging
Sensor-Line Cameras and Laser Range-Finders

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Panoramic Imaging

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I find panoramic vision fascinating. Imagine, if we humans had panoramic vision, we would not have words for in front and back (only relative to something else). As a result, we wouldn’t use metaphors based on them, like “this is a backward society” or “this is the frontier of science”. Perhaps, then, we would be thinking differently. . . . Change eyes, change thinking. . . . (Yiannis Aloimonos)

This book is about accurate panoramic imaging in the context of close-range photogrammetry, computer vision, computer graphics, or multimedia imaging, for applications where (very) high-resolution data need to be recorded for documenting or visualizing three-dimensional (3D) scenes or objects. For example, large-screen virtual reality systems often use panoramic images for various types of virtual tours.

One of the deplorable consequences of computers being used for image processing is that many grossly distorted pictures are provided on the Internet, and some have even been published in books and journals. Many landscape photographs get ignorantly stretched horizontally, to produce fake panoramic views. This book addresses high-quality panoramic imaging rather than panoramic imaging as already popular (at various levels of quality) for non-professional digital cameras, typically using some “stitching software” for combining a set of images into a larger image.

Sensor-line cameras have been designed since the 1980s, originally for “push-broom” viewing from an aircraft or satellite. (The book briefly addresses recent applications of airborne sensor-line cameras in Chapter 10.) A sensor-line camera has basically just a line of CCD or CMOS pixel sensors, and records images line by line. Such a camera may appear “somehow” simpler with respect to its hardware design than a sensor-matrix camera, where images are captured by a rectangular array of pixel sensors. However, having only one image line at a time defines particular difficulties for creating a “consistent” two-dimensional (2D) image.

Panoramic images may be captured by a sensor-line camera through rotation. In the case of a free-hand rotation, subsequent geometric rectification of recorded image lines (e.g., on a cylindrical surface) would be really difficult. In this book we basically assume that rotation is accurately performed by using a computer-controlled turntable. During such a rotation, a panoramic image is captured on a cylindrical surface (in contrast to a planar image when using a sensor-matrix camera).

The geometry of this cylindrical surface comes with new challenges with respect to sensor calibration, stereo viewing, stereo analysis, and so forth. (Note that stereo analysis or stereo
visualization of panoramic images becomes more and more essential for various imaging applications.)

A line of pixel sensors consists of “many” (say, 10,000) sensor elements. During the rotation, we take, say, more than 100,000 individual shots, line by line. All these line images define a panoramic image of 1,000 megapixels (i.e., of at least several gigabytes) or more on a cylindrical surface. Additional efforts (when using a rotating sensor-line camera) thus pay already off with benefits regarding image resolution.

The architecture of the rotating sensor-line camera is also a perfect match with the architecture of a (rotating) laser range-finder, which becomes more and more popular for generating 3D object or scene models. A (rotating) laser range-finder produces a dense “cloud of points”, representing 3D locations of visible surface points in a given scene. These “clouds of points” need to be processed and triangulated. Color images taken by a rotating sensor-line camera can then be used for rendering of triangulated surfaces. This is of value in many applications such as (environmental) surveillance or for architectural photogrammetry.

This book illustrates that a multi-sensor approach (i.e., combining laser range-finder and rotating sensor-line camera) allows a digitization or 3D modeling of indoor or outdoor scenes, illustrates important progress, but also indicates areas of future research. Typically, cameras or laser range-finders have to be placed at various attitudes or poses, each defined by a location (say, in an assumed world coordinate system), an up-vector, and a direction of an initial viewing axis (so as to give a unique specification of a sensor’s attitude; obviously, viewing directions change during rotation). The book discusses the calibration of sensor attitudes, and the fusion of data recorded at multiple viewpoints.

The authors have used rotating sensor-line cameras for nearly ten years; experience with laser range-finders is a little more recent. Working with (rotating) sensor-line cameras has proved to be more difficult in general than using a sensor-matrix camera. But the two stated benefits (high resolution of recorded image data, and matching sensor architectures) are worth the extra effort.

This book reports on theoretical issues and also on practical experience with rotating sensor-line cameras, and laser range-finders, with regard to image correspondence, stereo sampling, image acquisition, camera design, and so forth. One common question is how to choose suitable camera parameters to obtain an optimized pair of stereo viewable panoramic images. We show that the decision may be based on simple distance estimates in a given 3D scene.

Sensor calibration is an important subject in this book. A least-squares minimization-based approach is proposed for the rotating sensor-line camera, and two more options are briefly discussed, suggesting areas of future research.

The book may be used as a textbook for graduate courses in computer science, engineering, information technology, or multimedia imaging. Each chapter contains exercises, which are supported by an accompanying website (see also links on authors’ home pages).

Chapter 1 provides a historic review of panoramic imaging and used sensor technology, also briefly discussing those areas which are not covered in this book. Chapter 2 discusses used sensors (rotating line cameras and range-finders). Chapter 3 is basically about the mathematical fundamentals used throughout the book, with a focus on spatial positioning of sensors. Chapter 4 provides epipolar curve equations, which will prove to be useful not only in stereo applications, but also in sensor calibration, which is the subject of Chapter 5. The potential distribution of stereo samples is discussed in Chapter 6, and formulas for optimization of sample distributions are provided in Chapter 7. A more refined analysis of stereo geometry
and parameters in relation to given 3D scenes is contained in Chapter 8. Chapters 9 and 10 discuss how sampled depth values can be visualized by means of triangulated surfaces, also using captured panoramas for surface texturing. Chapter 10 also discusses some applications towards 3D rendering of aerial sceneries, such as city scenes captured from an airplane.

The authors acknowledge comments by, collaboration with or support from various students and colleagues at The University of Auckland and DLR (Berlin and Munich) over the past seven years, and we also thank friends and family for very important patience and help. We also thank Illustrated Architecture (Berlin) for their collaboration.

Fay Huang, Reinhard Klette, and Karsten Scheibe
Yi-lan, Auckland and Berlin
My first introduction to panoramic imaging was my junior high school (in the 1950’s) and senior high school class pictures. The class members, segregated by height, were lined up on four rows of bleacher seats that extend over 40 feet. We were asked to hold still for about 30 seconds as the “strange” camera did its “scan” of the bleacher bound class. Some students, of course, ran from one end of the formation to the other to get into the picture twice. The students in the center of the picture were in sharp focus and those at the ends looked a little out of focus and slightly distorted. None of this bothered me at the time for I knew just the rudimentary aspects of optics, cameras and film. I had no idea if the camera had a curve film plane to go with a rotating lens and slit or was just a large format camera with a wide-angle lens. As I became more interested in photography and purchased a good 35mm SLR, I would attempt to take panoramic images by using a good tripod, some form of level indicator and a variety of lenses. I, like so many other amateur photographers, would then try to piece the images together to form a useable print, never really satisfied with the results due to changes in exposure, color balance and translational (vertical) variations. All of this was before the “age of digital images” where one could scan negatives or prints into digital files and use some sort of software to align them properly, make the exposure and color balance uniform and then crop to get the best looking aspect ratio for the final print. While there were many specialized cameras developed to take panoramic images, very few of them were used by the causal picture taker. The Advantix™ cameras provided a “panoramic look” by using a wide-angle lens (28 mm) and a framing aperture that gave the look of a panoramic image. Digital still and video cameras, introduced in the mid-90’s and now clearly dominant in most fields of imaging, when combined with the “cheap” computer power and ample image oriented application software now makes it possible to create panoramic images at home. These historical antidotes provide a reference point for what is now possible with modern panoramic cameras.

While consumer imaging will always drive the digital camera markets, there are many scientific, industrial, cartographic, robotic, and military uses of panoramic imaging, often combined with range finding equipment to form three-dimensional representations of the original scene. These new panoramic cameras provide researchers and end-users new tools to study and evaluate real-world scenes to a degree not possible a decade ago. The sixth offering of the Wiley-IS&T Series in Imaging Science and Technology by Dr. Fay Huang, Dr. Reinhard Klette, and Dr. Karsten Scheibe, Panoramic Imaging: Rotating Line Cameras and Laser Range Finders, provides the imaging community with a detailed primer on how to understand the nature of panoramic imaging, a guide on how to build cameras using rotating, linear arrays,
and how to combine the imaging data with depth information from laser rangefinders to create accurate three-dimensional images. The reader will appreciate the clarity and detail of the complex mathematical formulations of the geometric projections required to reconstruct an accurate, three-dimensional panoramic image. Every aspect of understanding the nature of the scene and its representation by the panoramic camera is covered in great detail. This text is a must for any researcher in computer vision, robotic imaging, remote sensing, image understanding, image synthesis and related topics.

Dr. Klette, who has a Ph.D. in Mathematics and a Dr. Sc. degree in Computer Science, has been involved in imaging research for over 35 years with over 280 refereed publications and numerous presentations. His current research interests focus on Image and Video Technology, Digital Geometry and Topology, Computer Vision, Panoramic Imaging, and Biomedical Image Analysis. He was the head of an interdisciplinary research project ‘Intelligent Microscopy’ at Jena University and Carl-Zeiss Company (1979–1984), the head of an AI research unit at the Academy of Sciences, Berlin (1984–1990), Professor for Computer Vision at the Technical University of Berlin (1990–1996), and currently a Professor for Information Technology at The University of Auckland (since 1996). He also served as the Director of the Center for Image Technology and Robotics (1996–2005), and Deputy Head of Department (1996–2002, 2005). Dr. Klette is an active participant and editor in many professional scientific and engineering societies. He has mentored 17 Ph.D. students and 99 M.S. students over his distinguished career.

Dr. Fay Huang received her Ph.D. in Computer Science from The University of Auckland, New Zealand in 2002 and is currently an Assistant Professor at the Institute of Computer Science and Information Engineering, National Ilan University in Taiwan, Republic of China. Her research interests are in Computer Vision, Computer Graphics, Virtual Reality and Computer Art. Dr. Huang has over 40 publications (journal and conference papers, book chapters, and technical reports, for example at IMA Minneapolis) in her relatively short, but active career.

Dr. Karsten Scheibe received his Ph.D. in Computer Science from Georg-August-Universität Göttingen in 2006. He is currently working at the German Aerospace Center (DLR), Optical Information Systems, Information Processing for Optical Systems. Dr. Scheibe research focuses on image and signal processing and modeling and simulation of imaging systems. Dr. Scheibe has published 30 papers in the area of computer graphics and robotic vision. He contributes at DLR to major satellite imaging, 3D visualization and aerial 3D imaging projects.

Michael A. Kriss
Formerly of the Eastman Kodak Research Laboratories
and the University of Rochester
Website and Exercises


Some of the exercises are labeled as being possible lab projects (say, assignments in a course). There is example source code available on the site. (Just download the zip file numbered as the corresponding exercise.) Example source code needs typically some visualization or window management, and therefore it uses some common platform libraries (e.g., Microsoft Foundation Class, DirectX, OpenGL, Windows GDI, etc.). Those have to be adapted to the used platform. In principle, the source code for most of the exercises is edited and compiled with Microsoft Visual C++ .NET, without using a special platform library.

Solutions to exercises, which are simple console programs, should be compilable with your own favorite developer environment. Readers who are using Microsoft Visual C++ .NET just load the project file with the extension *.vcproj. If you are using a different developer environment make sure that you load all *.cpp and *.h files as sources into your project. Additionally, link external libraries (e.g., glut.lib) as documented in your developer environment. Solutions to exercises, which need those external libraries, also contain a separate readme file where these external libraries are listed. Make sure that you are downloading the compatible library for your platform (e.g., glut.lib needs to be available for the majority of platforms).

Sometimes, source code requires more detailed documentation, and this is not discussed in this book. In such cases, the zip also contains a pdf file detailing some aspects of the source code (e.g., drawings for the geometric understanding of the actual implementation).

All the sources provided are for educational purposes only, and the authors and publisher cannot be made responsible for any outcome using those sources.
List of Symbols

**Image notations**

- $E_P$: panoramic image (two-dimensional image)
- $E_{P_i}$: the $i$th panoramic image, for integer $i$, when multiple panoramic images are considered at the same time
- $E_{P_L}, E_{P_R}$: the left or the right panoramic image in the stereo image case
- $M$: planar matrix image width in pixels
- $W$: panoramic image width in pixels
- $H$: image height in pixels
- $(x_o, y_o)$: image center (measurements, in real numbers)
- $(i_c, j_c)$: image center (pixel position, in integers)

**Parameters of rotating sensor-line camera**

- $\tau$: CCD size or pixel size (in micrometers, assuming square CCD cells)
- $f$: camera’s focal length (in millimeters)
- $\nu$: camera’s vertical field of view
- $R$: off-axis distance (in meters)
- $\omega$: principal angle (i.e., the camera’s viewing angle)
- $\gamma$: angular unit (i.e., angular distance between adjacent projection centers)

**Stereo analysis**

- $d$: image disparity in pixels
- $\theta$: angular image disparity

**A scene’s range of interest (RoI)**

- $D_1$: radius of the RoI’s inner cylindrical frontier (in meters)
- $D_2$: radius of the RoI’s outer cylindrical frontier (in meters)
- $V_1$: height of the RoI’s inner cylindrical frontier (in meters)
- $H_1$: distance between the camera’s optical center and the RoI’s inner cylindrical frontier with respect to the camera’s viewing direction (in meters)
### Coordinate systems

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<td>W</td>
<td>origin of world coordinate system</td>
</tr>
<tr>
<td>O</td>
<td>origin of sensor coordinate system</td>
</tr>
<tr>
<td>C</td>
<td>origin of camera coordinate system</td>
</tr>
<tr>
<td>(X, Y, Z)</td>
<td>coordinates in a 3D Euclidean coordinate system</td>
</tr>
<tr>
<td>(x, y)</td>
<td>coordinates in a 2D Euclidean image coordinate system</td>
</tr>
<tr>
<td>(θ, φ)</td>
<td>coordinates in a 2D non-Euclidean spherical image coordinate system</td>
</tr>
<tr>
<td>(φ, L)</td>
<td>coordinates in a 2D non-Euclidean cylindrical image coordinate system</td>
</tr>
<tr>
<td>(i, j)</td>
<td>coordinates of an image pixel</td>
</tr>
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</table>

### Geometric transforms

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>rotation matrix</td>
</tr>
<tr>
<td>t</td>
<td>translation vector</td>
</tr>
</tbody>
</table>

### Geometric objects

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ℋ</td>
<td>a circle defined by possible positions of a camera’s optical center in 3D space</td>
</tr>
<tr>
<td>ℋ</td>
<td>projection ray or line in 3D</td>
</tr>
<tr>
<td>ℋ</td>
<td>plane in 3D</td>
</tr>
<tr>
<td>v</td>
<td>image vector</td>
</tr>
</tbody>
</table>

### Screen parameter

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>H_s</td>
<td>the height of the displaying screen in pixels</td>
</tr>
</tbody>
</table>

### General mathematical symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, B, C</td>
<td>real numbers</td>
</tr>
<tr>
<td>M, N</td>
<td>integers</td>
</tr>
<tr>
<td>α, β</td>
<td>angles</td>
</tr>
<tr>
<td>i, j, k, m, n</td>
<td>indices</td>
</tr>
<tr>
<td>p, q</td>
<td>2D points or vectors</td>
</tr>
<tr>
<td>P, Q</td>
<td>3D points</td>
</tr>
<tr>
<td>M, R</td>
<td>matrices</td>
</tr>
<tr>
<td>f, g</td>
<td>functions</td>
</tr>
<tr>
<td>f'</td>
<td>derivative of a unary function</td>
</tr>
<tr>
<td>A, B, S</td>
<td>sets</td>
</tr>
</tbody>
</table>
This chapter provides a general introduction to panoramic imaging, mostly at an informal level. Panoramas have an interesting history in arts and multimedia imaging. Developments and possible applications of panoramic imaging are briefly sketched in a historic context. The chapter also discusses the question of accuracy, and introduces rotating sensor-line cameras and laser range-finders.

1.1 Panoramas

A panorama is defined by a wide field of view. Obviously, a single panoramic image thus contains more information or features than a “normal” image. This has potential for understanding the geometry of three-dimensional (3D) scenes, or for estimating the locations of panoramic sensors within a 3D scene.

1.1.1 Accurate Panoramic Imaging

Panoramic images are already part of our daily lives. They may be generated with relatively inexpensive tools, and basically by anyone with a digital camera after spending a few minutes reading the manual.

However, the accuracy which is required makes a difference: if panoramic images or 3D models, derived from panoramas, have to satisfy high-quality demands (as in close-range photogrammetry, wide-screen visualization, or in many computer vision applications, such as in industrial inspection or accurate object modeling), then the geometry of the panoramic sensor needs to be understood, sensors have to be calibrated, and image capturing has to follow strict rules. This is the starting point for this book, which is about accurate panoramic imaging, where accuracy requirements might be described in terms of fractions of a millimeter, even for large objects such as an important building.
The authors predict that accurate panoramic imaging will become available for a wider community in the near future, for example by means of tools for correcting perspective or radial distortions, or perhaps automated incremental generation of 3D models, and mapping of images onto those models for further refinement.

1.1.2 Importance of Panoramas

Potentially, panoramas provide improved support for the visualization or inference of 3D world features; but the tradeoff is that a larger amount of data is acquired (in comparison to conventional approaches with more restricted fields of view). However, as faster computers and larger bandwidth became available, applications and studies using panoramas became widely accessible. We mention a few aspects which highlight the importance of panoramas:

*Immersion.* The importance of a panoramic image rests, obviously, in its large field of view. The impression of immersion can be fully achieved by a display method (e.g., a facility known as “The Cave”) which allows an equally wide field of view, possibly enhanced by 3D stereo viewing.

Figure 1.1 shows an important panoramic image, documenting the impact of the first weapon of mass destruction (a small nuclear weapon, unleashed from the US B-29 bomber *Enola Gay*). The bomb detonated 600 m above the center of Hiroshima, a city of about 400,000 people, on 6 August 1945 at 8.15 a.m. local time. An actual panorama (not shown here), formed from those 16 photographs, is on display at the Hiroshima Peace Memorial Museum. Images like these provide shocking evidence of the disasters at Hiroshima and Nagasaki. In order to understand the importance of a worldwide ban on weapons of mass destruction we must not be allowed to forget events like these, and wide-angle views may help to initiate wide-angle thinking.

*Realism.* A photorealistic quality of a “virtual world” can be achieved by a “dense” (with respect to locations of viewpoints) set of panoramas within a scene of interest. As with other virtual reality systems, stereo visualization can also be implemented with free navigation in a virtual panoramic world. A virtual reality world of synthetic scenes, generated by computer graphics approaches only, can hardly achieve a comparable photorealistic quality for complex 3D scenes.

*Simplification.* For example, stereo reconstruction of 3D scenes can be based on 360° stereo panoramic images; this bypasses a complicated (and possibly erroneous) process of merging multiple depth-maps in traditional multi-pinhole-image approaches.

*Localization.* A robot or aircraft equipped with panoramic vision can recognize a location by comparing an incoming panorama with memorized maps or images, and this strategy improves robustness because of more information, provided by a larger visual field.

*Compression.* A panorama composed from an image sequence of a video camera may be seen as a compact version of the original video data, without redundancy.

Before embarking on a more technical discussion of panoramas, we provide a brief but informative historic overview, starting with panoramic paintings, continuing with panoramic photographs, and adding a few comments on panoramic digital imaging in general.
1.2 Panoramic Paintings

The word “panorama” was introduced by the Irish painter Robert Barker (1739–1806) when describing his 1787 paintings of Edinburgh. However, panoramic paintings date back to earlier times. Panoramic presentations in Asia, such as Chinese scrolls, may be cited as historic origins.
1.2.1 Chinese Scrolls

The name “panorama” is typically not used in art history publications on Chinese landscape paintings. A typical format of a Chinese landscape painting is either a handscroll or a hanging scroll.

The handscroll – so called because it is unrolled by hand horizontally – is a unique way to view the landscape one section at a time (from one end to the other), as if touring through the picture. Vertical hanging scrolls are a dominant form of wall decoration in Chinese domestic architecture. They are easily rolled up and stored, and are changed frequently or viewed only on special occasions.

Landscape paintings are a popular Chinese tradition, showing poetry in nature. For over a thousand years, since the T’ang dynasty (618–907), Chinese landscape paintings have been understood as both an intimate expression of nature and a way of conveying profound emotions. They play an important role in expressing Chinese spirituality and philosophical values. Literally translated, the Chinese characters for landscape mean “mountain and water”. Almost all Chinese landscape paintings depict mountains or water in some abstract style. The landscape is a composite of many elements of nature that invites the viewer to a spiritual journey of the imagination.

Landscape painting reached a (first) period of prosperity during the Five Dynasties (906–960) and the Song (960–1279) era. Chinese paintings, especially in the horizontal handscroll format, are read like a traditional written text. They begin on the right and come to a conclusion at the left-hand end. Like a conventional narrative, these landscapes have a beginning, middle, and end. Painted around the year 1000, Fan Kuan’s Travelers among Streams and Mountains (2,064 × 1,035 mm) is one of the most famous hanging scroll landscapes in Chinese art history; see Plate 1. It shows tiny figures on a road which passes through a landscape dominated by a huge, looming mountain peak.

Later landscape paintings during the Ming (1368–1644) and Ch’ing (1644–1911) dynasties begin to introduce more people into the scene. In comparison to the “mountains and water” style, there is also a return to greater realism. Typically, the landscapes become intricately detailed visions, almost as if they are painted from the air, from a bird’s-eye view of the world. In the Ch’ing dynasty, a landscape painting may also include a painting of a city. One of the best-known, long handscroll format paintings of the Ch’ing dynasty is Along the River during the Ch’ing-ming Festival (356 × 11,528 mm); see Plate 2.

This work is based on an original painting by Chang Tse-tuan (painted in early 12th century). It beautifully illustrates life and customs during the Sung dynasty at the capital of Pien (K’ai-feng). This theme, popular in the Northern Sung dynasty (960–1126), has been copied often throughout the years. The one shown in Plate 2 is one of the most famous, made by court painters of the imperial painting academy in the time of the Ch’ien-lung Emperor, who reigned 1736–1795. Street entertainment, commerce, folk customs of the Ming and Ch’ing dynasties, daily household life, and architecture as shown reflect very much the life and appearance of the period when it was painted. The lively activities include a theatrical performance, monkey show, acrobatics, and a martial arts ring to lend a festive air to the scenery.

These Chinese paintings can certainly be classified as panoramas, meaning wide-angle views of real-world scenes. Today, “panorama” is considered to be an English word, and there are different ways to translate it into Chinese. The format of a handscroll painting is most relevant to “cylindrical panoramas” as discussed in this book, although they are very often...
presented in a “linear” (i.e., translational) multi-perspective approach, while the ones to be discussed in this book typically have projection centers along a circular path.

1.2.2 European Panoramic Paintings

Figure 1.2 shows (in gray levels only) one of the early panoramic paintings by Robert Barker and his son, Henry Aston Barker (1774–1856). The first exhibition of a panoramic painting by Robert Barker was in 1787, with the city of Edinburgh painted around the (inner) wall of a rotunda. It had to be viewed from the center of the room to give an illusion of reality.

![Figure 1.2](image)

_Figure 1.2_ Six plates forming a 360° panorama, *London from the Roof of the Albion Mills*, by Robert Barker and his son, Henry Aston Barker, London, 1791. The figure shows gray-level copies of the originally 1792 color aquatints by Frederick Birnie, a panoramic print series; the size of each of these six prints is 425 mm × 540 mm, with an overall length of 3,250 mm. Courtesy of the University of Exeter Library, Bill Douglas Centre.
Barker obtained a patent for these panoramic paintings. His exhibitions of panoramas were very influential: panoramas or dioramas (a related way of displaying wide-angle scenery, often also by decorating the foreground with various, sometimes moving, objects, often also with audio) became an early form of mass entertainment in the larger cities of Europe or North America, before the advent of the cinema.

However, panoramic paintings were popular in Europe much earlier than the late 18th century. The Bayeux Tapestry (which is actually embroidery, not tapestry; see Plate 3) might be one of the earliest examples. It tells a story, similar to Chinese scrolls, and was probably commissioned in the 1070s by Bishop Odo of Bayeux, half-brother of William the Conqueror, and (again, probably) embroidered in Kent, England.

At an exhibition in London (around 1870) a panoramic view of the entire Earth could be seen from a platform at the centre of a large hollow sphere, which was painted on the inside to represent the Earth’s outer surface.

Panoramic pictures or dioramas developed into “multimedia shows” (which are still shown today in Gettysburg, Uljanowsk, Bad Frankenhausen, and other historic sites). Wide-angle paintings of military battles, historic scenes, or landscapes became popular in a few countries such as United States, Russia, France, UK, and Germany.

Paintings of very large dimensions are popular in modern art. Typically, today they no longer aim to provide photo-realistic presentations of extensive scenes.

1.3 Panoramic or Wide-Angle Photographs

The first photograph was taken by Nicéphore Niepce in 1826 near Chalon-sur Saône, France (this photograph still survives, in recognizable form). Soon after, photographers began to capture wide-angle photos of landscapes (e.g., Edouard Baldus in France, and Maxime Du Camp during his travels in Egypt). General-purpose panoramic photography developed around 1900 into wide-angle architectural photography or aerial photography, contributing to the development of photogrammetry.

1.3.1 Historic Panoramic Cameras

Panoramic cameras have been built since the second half of the 19th century. For example, in England in 1860 Thomas Sutton designed a wide-angle (120°) camera whose “lens” was a hollow glass sphere filled with water; this camera was then built by Paul Eduard Liesegang in Germany. Albrecht Meydenbauer (1834–1921), a German architect, designed a camera in 1867 which used the first wide-angle (105°) optical lens.

As an alternative to a single wide-angle shot, cameras could also take a continuous shot during a full 360° rotation. For example, the sophisticated “Cyclographe” panoramic cameras of Jules Damoizeau, built between 1890 and 1894, rotated by means of a spring mechanism as the film was fed past the shutter at the same speed, but in the opposite direction. A camera with a pivoting lens, called a périphote, was built in 1901 by Lumière in Lyon. During exposure, the lens rotated 360° (see Figure 1.3); no film transport was needed for this model.

Of course, cameras could also be used for multiple shots in different directions, during a 360° rotation of the camera on a tripod. However, these separate images could not be used for creating a seamless panorama, until the occurrence of digital stitching techniques (see Section 1.4.2).
1.3.2 Photogrammetry

Engineers of many disciplines were also quick to realize the potential use of photography as a tool for both civilian and military applications. This was true for areas such as architecture, and more significantly in cartography. ¹ In particular, some pioneering photographers attempted to apply their art to the science of measuring physical spaces at an accuracy corresponding to their latest tools. Of course, photography is in general more concerned with creating images that meet aesthetic and commercial demands than with capturing accurate images required for scientific measurements.

Close-range photogrammetry, as a method for recording and monitoring architecture, originated in the work of Albrecht Meydenbauer (see previous subsection). He was the first to use the term “photogrammetry”, and he pioneered the photogrammetric recording of Islamic architecture in the Middle East in the 1870s.

Aerial photography or wide-range photogrammetry was pioneered by Aimé Laussedat, using balloon photography or aerial photography supported by a string of kites (see Figure 1.4).

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¹ Plane surveying (with the dioptra, an instrument used for measuring angles since about 300 BC) was developed in Hellenistic culture, and then revived in the Netherlands in the 16th century. The distance between one pair of points is measured, and then angle measurements are used to calculate a triangulation network. In the 1930s, the British Colonial Survey made a geodetic survey of Nigeria (which is very flat) by running a tape measure over that colony. Only a minor role was played by angle measurements (mostly for altitude estimation), and a network of triangles was calculated by trilateration (not by triangulation). Consequently, by 1939 Nigeria was the most accurately mapped country on Earth. (Computer vision techniques, which use projected light patterns, calculate 3D points by triangulation.)
Photogrammetry became the field of engineering concerned with the task of documenting and measuring scenes based on photographs. Photogrammetry is basically a measurement technology in which the 3D coordinates of points of an object are determined by measurements made in two or more photographic images taken from different positions or with different viewing directions.

The development of photogrammetry can be subdivided into four periods. Each period is characterized by technological or methodological innovations which made photogrammetry incrementally more flexible and more effective. Graphic or plane table photogrammetry (c.1850–1900) combined terrestrial photographs and topographic maps; early experiments with aerial photographs demonstrated the potential of this new discipline. Analog photogrammetry (c.1900–1960) emerged with the advent of stereoscopy (i.e., the use of corresponding points in binocular images for identifying 3D positions by triangulation) and of airplanes. The first cameras specially designed for aerial imaging were built around 1920.

Analytical photogrammetry (since around 1960, and still active) is characterized by a “rigorously correct least squares solution, the simultaneous solution of any number of photographs, and a complete study of error propagation” (Doyle, 1964), which was possible with computer