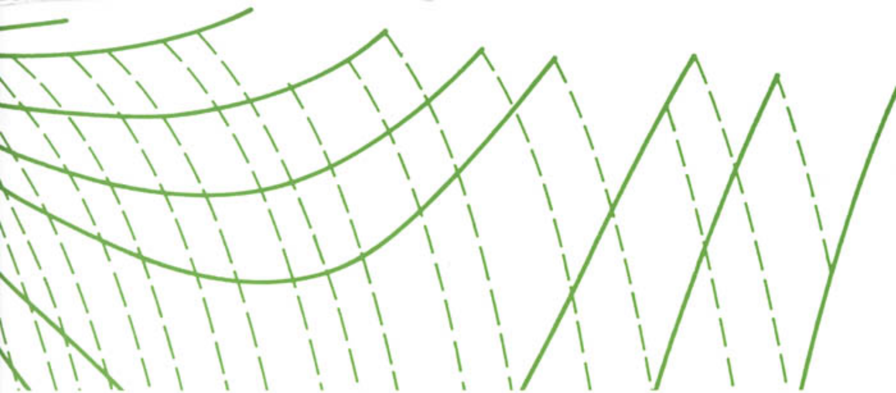

DETECTION, ESTIMATION, AND MODULATION THEORY

Part III

Radar-Sonar Signal Processing and Gaussian Signals in Noise

HARRY L. VAN TREES

Detection, Estimation, and Modulation Theory

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Detection, Estimation, and Modulation Theory

Radar-Sonar Processing and Gaussian Signals in Noise

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To Diane

and Stephen, Mark, Kathleen, Patricia,
Eileen, Harry, and Julia

and the next generation—
Brittany, Erin, Thomas, Elizabeth, Emily,
Dillon, Bryan, Julia, Robert, Margaret,
Peter, Emma, Sarah, Harry, Rebecca,
and Molly

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Preface for Paperback Edition

In 1968, Part I of *Detection, Estimation, and Modulation Theory* [VT68] was published. It turned out to be a reasonably successful book that has been widely used by several generations of engineers. There were thirty printings, but the last printing was in 1996. Volumes II and III ([VT71a], [VT71b]) were published in 1971 and focused on specific application areas such as analog modulation, Gaussian signals and noise, and the radar–sonar problem. Volume II had a short life span due to the shift from analog modulation to digital modulation. Volume III is still widely used as a reference and as a supplementary text. In a moment of youthful optimism, I indicated in the the Preface to Volume III and in Chapter III-14 that a short monograph on optimum array processing would be published in 1971. The bibliography lists it as a reference, *Optimum Array Processing,* Wiley, 1971, which has been subsequently cited by several authors. After a 30-year delay, *Optimum Array Processing,* Part IV of *Detection, Estimation, and Modulation Theory* will be published this year.

A few comments on my career may help explain the long delay. In 1972, MIT loaned me to the Defense Communication Agency in Washington, D.C. where I spent three years as the Chief Scientist and the Associate Director of Technology. At the end of the tour, I decided, for personal reasons, to stay in the Washington, D.C. area. I spent three years as an Assistant Vice-President at COMSAT where my group did the advanced planning for the INTELSAT satellites. In 1978, I became the Chief Scientist of the United States Air Force. In 1979, Dr. Gerald Dinneen, the former Director of Lincoln Laboratories, was serving as Assistant Secretary of Defense for C3I. He asked me to become his Principal Deputy and I spent two years in that position. In 1981, I joined M/A-COM Linkabit. Linkabit is the company that Irwin Jacobs and Andrew Viterbi had started in 1969 and sold to M/A-COM in 1979. I started an Eastern operation which grew to about 200 people in three years. After Irwin and Andy left M/A-COM and started Qualcomm, I was responsible for the government operations in San Diego as well as Washington, D.C. In 1988, M/A-COM sold the division. At that point I decided to return to the academic world.

I joined George Mason University in September of 1988. One of my priorities was to finish the book on optimum array processing. However, I found that I needed to build up a research center in order to attract young research-oriented faculty and doctoral students. The process took about six years. The Center for Excellence in Command, Control, Communications, and Intelligence has been very successful and has generated over $300 million in research funding during its existence. During this growth period, I spent some time on array processing but a concentrated effort was not possible. In 1995, I started a serious effort to write the Array Processing book.

Throughout the *Optimum Array Processing* text there are references to Parts I and III of *Detection, Estimation, and Modulation Theory*. The referenced material is available in several other books, but I am most familiar with my own work. Wiley agreed to publish Part I and III in paperback so the material will be readily available. In addition to providing background for Part IV, Part I is still useful as a text for a graduate course in Detection and Estimation Theory. Part III is suitable for a second level graduate course dealing with more specialized topics.

In the 30-year period, there has been a dramatic change in the signal processing area. Advances in computational capability have allowed the implementation of complex algorithms that were only of theoretical interest in the past. In many applications, algorithms can be implemented that reach the theoretical bounds.

The advances in computational capability have also changed how the material is taught. In Parts I and III, there is an emphasis on compact analytical solutions to problems. In Part IV, there is a much greater emphasis on efficient iterative solutions and simulations. All of the material in parts I and III is still relevant. The books use continuous time processes but the transition to discrete time processes is straightforward. Integrals that were difficult to do analytically can be done easily in Matlab®. The various detection and estimation algorithms can be simulated and their performance compared to the theoretical bounds. We still use most of the problems in the text but supplement them with problems that require Matlab® solutions.

We hope that a new generation of students and readers find these reprinted editions to be useful.

HARRY L. VAN TREES

Fairfax, Virginia
June 2001

Preface

In this book I continue the study of detection, estimation, and modulation theory begun in Part I [1]. I assume that the reader is familiar with the background of the overall project that was discussed in the preface of Part I. In the preface to Part II [2] I outlined the revised organization of the material. As I pointed out there, Part III can be read directly after Part I. Thus, some persons will be reading this volume without having seen Part II. Many of the comments in the preface to Part II are also appropriate here, so I shall repeat the pertinent ones.

At the time Part I was published, in January 1968, I had completed the "final" draft for Part II. During the spring term of 1968, I used this draft as a text for an advanced graduate course at M.I.T. and in the summer of 1968, I started to revise the manuscript to incorporate student comments and include some new research results. In September 1968, I became involved in a television project in the Center for Advanced Engineering Study at M.I.T. During this project, I made fifty hours of videotaped lectures on applied probability and random processes for distribution to industry and universities as part of a self-study package. The net result of this involvement was that the revision of the manuscript was not resumed until April 1969. In the intervening period, my students and I had obtained more research results that I felt should be included. As I began the final revision, two observations were apparent. The first observation was that the manuscript has become so large that it was economically impractical to publish it as a single volume. The second observation was that since I was treating four major topics in detail, it was unlikely that many readers would actually use all of the book. Because several of the topics can be studied independently, with only Part I as background, I decided to divide the material into three sections: Part II, Part III, and a short monograph on *Optimum Array Processing* [3]. This division involved some further editing, but I felt it was warranted in view of increased flexibility it gives both readers and instructors.

In Part II, I treated nonlinear modulation theory. In this part, I treat the random signal problem and radar/sonar. Finally, in the monograph, I discuss optimum array processing. The interdependence of the various parts is shown graphically in the following table. It can be seen that Part II is completely separate from Part III and *Optimum Array Processing*. The first half of *Optimum Array Processing* can be studied directly after Part I, but the second half requires some background from Part III. Although the division of the material has several advantages, it has one major disadvantage. One of my primary objectives is to present a unified treatment that enables the reader to solve problems from widely diverse physical situations. Unless the reader sees the widespread applicability of the basic ideas he may fail to appreciate their importance. Thus, I strongly encourage all serious students to read at least the more basic results in all three parts.

	Prerequisites
Part II	Chaps. I-5, I-6
Part III	
Chaps. III-1 to III-5	Chaps. I-4, I-6
Chaps. III-6 to III-7	Chaps. I-4
Chaps. III-8-end	Chaps. I-4, I-6, III-1 to III-7
Array Processing	
Chaps. IV-1, IV-2	Chaps. I-4
Chaps. IV-3-end	Chaps. III-1 to III-5, AP-1 to AP-2

The character of this book is appreciably different that that of Part I. It can perhaps be best described as a mixture of a research monograph and a graduate level text. It has the characteristics of a research monograph in that it studies particular questions in detail and develops a number of new research results in the course of this study. In many cases it explores topics which are still subjects of active research and is forced to leave some questions unanswered. It has the characteristics of a graduate level text in that it presents the material in an orderly fashion and develops almost all of the necessary results internally.

The book should appeal to three classes of readers. The first class consists of graduate students. The random signal problem, discussed in Chapters 2 to 7, is a logical extension of our earlier work with deterministic signals and completes the hierarchy of problems we set out to solve. The last half of the book studies the radar/sonar problem and some facets of the digital communication problem in detail. It is a thorough study of how one applies statistical theory to an important problem area. I feel that it provides a useful educational experience, even for students who have no ultimate interest in radar, sonar, or communications, because it demonstrates system design techniques which will be useful in other fields.

The second class consists of researchers in this field. Within the areas studied, the results are close to the current research frontiers. In many places, specific research problems are suggested that are suitable for thesis or industrial research.

The third class consists of practicing engineers. In the course of the development, a number of problems of system design and analysis are carried out. The techniques used and results obtained are directly applicable to many current problems. The material is in a form that is suitable for presentation in a short course or industrial course for practicing engineers. I have used preliminary versions in such courses for several years.

The problems deserve some mention. As in Part I, there are a large number of problems because I feel that problem solving is an essential part of the learning process. The problems cover a wide range of difficulty and are designed to both augment and extend the discussion in the text. Some of the problems require outside reading, or require the use of engineering judgement to make approximations or ask for discussion of some issues. These problems are sometimes frustrating to the student but I feel that they serve a useful purpose. In a few of the problems I had to use numerical calculations to get the answer. I strongly urge instructors to work a particular problem before assigning it. Solutions to the problems will be available in the near future.

As in Part I, I have tried to make the notation mnemonic. All of the notation is summarized in the glossary at the end of the book. I have tried to make my list of references as complete as possible and acknowledge any ideas due to other people.

Several people have contributed to the development of this book. Professors Arthur Baggeroer, Estil Hoversten, and Donald Snyder of the M.I.T. faculty, and Lewis Collins of Lincoln Laboratory, carefully read and criticized the entire book. Their suggestions were invaluable. R. R. Kurth read several chapters and offered useful suggestions. A number of graduate students offered comments which improved the text. My secretary, Miss Camille Tortorici, typed the entire manuscript several times.

My research at M.I.T. was partly supported by the Joint Services and by the National Aeronautics and Space Administration under the auspices of the Research Laboratory of Electronics. I did the final editing

while on Sabbatical Leave at Trinity College, Dublin. Professor Brendan Scaife of the Engineering School provided me office facilities during this period, and M.I.T. provided financial assistance. I am thankful for all of the above support.

Harry L. Van Trees

Dublin, Ireland,

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Contents

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1

Introduction

This book is the third in a set of four volumes. The purpose of these four volumes is to present a unified approach to the solution of detection, estimation, and modulation theory problems. In this volume we study two major problem areas. The first area is the detection of random signals in noise and the estimation of random process parameters. The second area is signal processing in radar and sonar systems. As we pointed out in the Preface, Part III does not use the material in Part II and can be read directly after Part I.

In this chapter we discuss three topics briefly. In Section 1.1, we review Parts I and II so that we can see where the material in Part III fits into the over-all development. In Section 1.2, we introduce the first problem area and outline the organization of Chapters 2 through 7. In Section 1.3, we introduce the radar–sonar problem and outline the organization of Chapters 8 through 14.

1.1 REVIEW OF PARTS I AND II

In the introduction to Part I [1], we outlined a hierarchy of problems in the areas of detection, estimation, and modulation theory and discussed a number of physical situations in which these problems are encountered.

We began our technical discussion in Part I with a detailed study of classical detection and estimation theory. In the classical problem the observation space is finite-dimensional, whereas in most problems of interest to us the observation is a waveform and must be represented in an infinite-dimensional space. All of the basic ideas of detection and parameter estimation were developed in the classical context.

In Chapter I-3, we discussed the representation of waveforms in terms of series expansions. This representation enabled us to bridge the gap between the classical problem and the waveform problem in a straightforward manner. With these two chapters as background, we began our study of the hierarchy of problems that we had outlined in Chapter I-1.

In the first part of Chapter I-4, we studied the detection of known signals in Gaussian noise. A typical problem was the binary detection problem in which the received waveforms on the two hypotheses were

$$r(t) = s_1(t) + n(t), \qquad T_i \le t \le T_f : H_1, \tag{1}$$

$$r(t) = s_0(t) + n(t), \qquad T_i \le t \le T_f : H_0, \tag{2}$$

where $s_1(t)$ and $s_0(t)$ were known functions. The noise $n(t)$ was a sample function of a Gaussian random process.

We then studied the parameter-estimation problem. Here, the received waveform was

$$r(t) = s(t, \mathbf{A}) + n(t), \qquad T_i \le t \le T_f. \tag{3}$$

The signal $s(t, \mathbf{A})$ was a known function of t and $\mathbf{A}$. The parameter $\mathbf{A}$ was a vector, either random or nonrandom, that we wanted to estimate.

We referred to all of these problems as known signal-in-noise problems, and they were in the first level in the hierarchy of problems that we outlined in Chapter I-1. The common characteristic of first-level problems is the presence of a *deterministic signal* at the receiver. In the binary detection problem, the receiver decides which of the two deterministic waveforms is present in the received waveform. In the estimation problem, the receiver estimates the value of a parameter contained in the signal. In all cases it is the additive noise that limits the performance of the receiver.

We then generalized the model by allowing the signal component to depend on a finite set of unknown parameters (either random or nonrandom). In this case, the received waveforms in the binary detection problem were

$$\begin{aligned} r(t) &= s_1(t, \boldsymbol{\theta}) + n(t), \qquad T_i \le t \le T_f : H_1, \\ r(t) &= s_0(t, \boldsymbol{\theta}) + n(t), \qquad T_i \le t \le T_f : H_0. \end{aligned} \tag{4}$$

In the estimation problem the received waveform was

$$r(t) = s(t, \mathbf{A}, \boldsymbol{\theta}) + n(t), \qquad T_i \le t \le T_f. \tag{5}$$

The vector $\boldsymbol{\theta}$ denoted a set of unknown and unwanted parameters whose presence introduced a new uncertainty into the problem. These problems were in the second level of the hierarchy. The additional degree of freedom in the second-level model allowed us to study several important physical channels such as the random-phase channel, the Rayleigh channel, and the Rician channel.

In Chapter I-5, we began our discussion of modulation theory and continuous waveform estimation. After formulating a model for the problem, we derived a set of integral equations that specify the optimum demodulator.

In Chapter I-6, we studied the linear estimation problem in detail. Our analysis led to an integral equation,

$$K_{dr}(t, u) = \int_{T_i}^{T_f} h_o(t, \tau) K_r(\tau, u)\, d\tau, \qquad T_i < t,\ u < T_f, \tag{6}$$

that specified the optimum receiver. We first studied the case in which the observation interval was infinite and the processes were stationary. Here, the spectrum-factorization techniques of Wiener enabled us to solve the problem completely. For finite observation intervals and nonstationary processes, the state-variable formulation of Kalman and Bucy led to a complete solution. We shall find that the integral equation (6) arises frequently in our development in this book. Thus, many of the results in Chapter I-6 will play an important role in our current discussion.

In Part II, we studied nonlinear modulation theory [2]. Because the subject matter in Part II is essentially disjoint from that in Part III, we shall not review the contents in detail. The material in Chapters I-4 through Part II is a detailed study of the first and second levels of our hierarchy of detection, estimation, and modulation theory problems.

There are a large number of physical situations in which the models in the first and second level do not adequately describe the problem. In the next section we discuss several of these physical situations and indicate a more appropriate model.

1.2 RANDOM SIGNALS IN NOISE

We begin our discussion by considering several physical situations in which our previous models are not adequate. Consider the problem of detecting the presence of a submarine using a *passive* sonar system. The engines, propellers, and other elements in the submarine generate acoustic signals that travel through the ocean to the hydrophones in the detection system. This signal can best be characterized as a sample function from a random process. In addition, a hydrophone generates self-noise and picks up sea noise. Thus a suitable model for the detection problem might be

$$r(t) = s(t) + n(t), \qquad T_i \le t \le T_f : H_1, \tag{7}$$

$$r(t) = n(t), \qquad T_i \le t \le T_f : H_0. \tag{8}$$

Now $s(t)$ is a sample function from a random process. The new feature in this problem is that the mapping from the hypothesis (or source output) to the signal $s(t)$ is no longer deterministic. The detection problem is to decide whether $r(t)$ is a sample function from a signal plus noise process or from the noise process alone.

A second area in which we decide which of two processes is present is the digital communications area. A large number of digital systems operate over channels in which randomness is inherent in the transmission characteristics. For example, tropospheric scatter links, orbiting dipole links, chaff systems, atmospheric channels for optical systems, and underwater acoustic channels all exhibit random behavior. We discuss channel models in detail in Chapters 9–13. We shall find that a typical method of communicating digital data over channels of this type is to transmit one of two signals that are separated in frequency. (We denote these two frequencies as ω_1 and ω_0). The resulting received signal is

$$\begin{aligned} r(t) &= s_1(t) + n(t), \qquad T_i \leq t \leq T_f : H_1, \\ r(t) &= s_0(t) + n(t), \qquad T_i \leq t \leq T_f : H_0. \end{aligned} \tag{9}$$

Now $s_1(t)$ is a sample function from a random process whose spectrum is centered at ω_1, and $s_0(t)$ is a sample function from a random process whose spectrum is centered at ω_0. We want to build a receiver that will decide between H_1 and H_0.

Problems in which we want to estimate the parameters of random processes are plentiful. Usually when we model a physical phenomenon using a stationary random process we assume that the power spectrum is known. In practice, we frequently have a sample function available and must determine the spectrum by observing it. One procedure is to parameterize the spectrum and estimate the parameters. For example, we assume

$$S(\omega, A) = \frac{A_1}{\omega^2 + A_2{}^2}, \qquad -\infty < \omega < \infty, \tag{10}$$

and try to estimate A_1 and A_2 by observing a sample function of $s(t)$ corrupted by measurement noise. A second procedure is to consider a small frequency interval and try to estimate the average height of spectrum over that interval.

A second example of estimation of process parameters arises in such diverse areas as radio astronomy, spectroscopy, and passive sonar. The source generates a narrow-band random process whose center frequency identifies the source. Here we want to estimate the center frequency of the spectrum.

A closely related problem arises in the radio astronomy area. Various sources in our galaxy generate a narrow-band process that would be centered at some known frequency if the source were not moving. By estimating the center frequency of the received process, the velocity of the source can be determined. The received waveform may be written as

$$r(t) = s(t, v) + n(t), \qquad T_i \leq t \leq T_f, \tag{11}$$

where $s(t, v)$ is a sample function of a random process whose statistical properties depend on the velocity v.

These examples of detection and estimation theory problems correspond to the third level in the hierarchy that we outlined in Chapter I-1. They have the common characteristic that the information of interest is imbedded in a random process. Any detection or estimation procedure must be based on how the statistics of $r(t)$ vary as a function of the hypothesis or the parameter value.

In Chapter 2, we formulate a quantitative model of the simple binary detection problem in which the received waveform consists of a white Gaussian noise process on one hypothesis and the sum of a Gaussian signal process and the white Gaussian noise process on the other hypothesis. In Chapter 3, we study the general problem in which the received signal is a sample function from one of two Gaussian random processes. In both sections we derive optimum receiver structures and investigate the resulting performance.

In Chapter 4, we study four special categories of detection problems for which complete solutions can be obtained. In Chapter 5, we consider the M-ary problem, the performance of suboptimum receivers for the binary problem, and summarize our detection theory results.

In Chapters 6 and 7, we treat the parameter estimation problem. In Chapter 6, we develop the model for the single-parameter estimation problem, derive the optimum estimator, and discuss performance analysis techniques. In Chapter 7, we study four categories of estimation problems in which reasonably complete solutions can be obtained. We also extend our results to include multiple-parameter estimation and summarize our estimation theory discussion.

The first half of the book is long, and several of the discussions include a fair amount of detail. This detailed discussion is necessary in order to develop an ability actually to solve practical problems. Strictly speaking, there are no new concepts. We are simply applying decision theory and estimation theory to a more general class of problems. It turns out that the transition from the concept to actual receiver design requires a significant amount of effort.

The development in Chapters 2 through 7 completes our study of the hierarchy of problems that were outlined in Chapter I-1. The remainder of the book applies these ideas to signal processing in radar and sonar systems.

1.3 SIGNAL PROCESSING IN RADAR–SONAR SYSTEMS

In a conventional active radar system we transmit a pulsed sinusoid. If a target is present, the signal is reflected. The received waveform consists of the reflected signal plus interfering noises. In the simplest case, the only source of interference is an additive Gaussian receiver noise. In the more general case, there is interference due to external noise sources or reflections from other targets. In the detection problem, the receiver processes the signal to decide whether or not a target is present at a particular location. In the parameter estimation problem, the receiver processes the signal to measure some characteristics of the target such as range, velocity, or acceleration. We are interested in the signal-processing aspects of this problem.

There are a number of issues that arise in the signal-processing problem.

1. We must describe the reflective characteristics of the target. In other words, if the transmitted signal is $s_t(t)$, what is the reflected signal?
2. We must describe the effect of the transmission channels on the signals.
3. We must characterize the interference. In addition to the receiver noise, there may be other targets, external noise generators, or clutter.
4. After we develop a quantitative model for the environment, we must design an optimum (or suboptimum) receiver and evaluate its performance.

In the second half of the book we study these issues. In Chapter 8, we discuss the radar–sonar problem qualitatively. In Chapter 9, we discuss the problem of detecting a slowly fluctuating point target at a particular range and velocity. First we assume that the only interference is additive white Gaussian noise, and we develop the optimum receiver and evaluate its performance. We then consider nonwhite Gaussian noise and find the optimum receiver and its performance. We use complex state-variable theory to obtain complete solutions for the nonwhite noise case.

In Chapter 10, we consider the problem of estimating the parameters of a slowly fluctuating point target. Initially, we consider the problem of estimating the range and velocity of a single target when the interference is additive white Gaussian noise. Starting with the likelihood function, we develop the structure of the optimum receiver. We then investigate the performance of the receiver and see how the signal characteristics affect the estimation accuracy. Finally, we consider the problem of detecting a target in the presence of other interfering targets.

The work in Chapters 9 and 10 deals with the simplest type of target and models the received signal as a known signal with unknown random parameters. The background for this problem was developed in Section I-4.4, and Chapters 9 and 10 can be read directly after Chapter I-4.

In Chapter 11, we consider a point target that fluctuates during the time during which the transmitted pulse is being reflected. Now we must model the received signal as a sample function of a random process.

In Chapter 12, we consider a slowly fluctuating target that is distributed in range. Once again we model the received signal as a sample function of a random process. In both cases, the necessary background for solving the problem has been developed in Chapters III-2 through III-4.

In Chapter 13, we consider fluctuating, distributed targets. This model is useful in the study of clutter in radar systems and reverberation in sonar systems. It is also appropriate in radar astronomy and scatter communications problems. As in Chapters 11 and 12, the received signal is modeled as a sample function of a random process. In all three of these chapters we are able to find the optimum receivers and analyze their performance.

Throughout our discussion we emphasize the similarity between the radar problem and the digital communications problem. Imbedded in various chapters are detailed discussions of digital communication over fluctuating channels. Thus, the material will be of interest to communications engineers as well as radar/sonar signal processors.

Finally, in Chapter 14, we summarize the major results of the radar–sonar discussion and outline the contents of the subsequent book on *Array Processing* [3]. In addition to the body of the text, there is an Appendix on the complex representation of signals, systems, and processes.

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2

Detection of Gaussian Signals in White Gaussian Noise

In this chapter we consider the problem of detecting a sample function from a Gaussian random process in the presence of additive white Gaussian noise. This problem is a special case of the general Gaussian problem described in Chapter 1. It is characterized by the property that on both hypotheses, the received waveform contains an additive noise component $w(t)$, which is a sample function from a zero-mean white Gaussian process with spectral height $N_0/2$. When H_1 is true, the received waveform also contains a signal $s(t)$, which is a sample function from a Gaussian random process whose mean and covariance function are known. Thus,

$$r(t) = s(t) + w(t), \qquad T_i \le t \le T_f : H_1 \tag{1}$$

and

$$r(t) = w(t), \qquad T_i \le t \le T_f : H_0. \tag{2}$$

The signal process has a mean value function $m(t)$,

$$E[s(t)] = m(t), \qquad T_i \le t \le T_f, \tag{3}$$

and a covariance function $K_s(t, u)$,

$$E[s(t) - m(t))(s(u) - m(u))] \triangleq K_s(t, u), \qquad T_i \le t, u \le T_f. \tag{4}$$

Both $m(t)$ and $K_s(t, u)$ are known. We assume that the signal process has a finite mean-square value and is statistically independent of the additive noise. Thus, the covariance function of $r(t)$ on H_1 is

$$E[(r(t) - m(t))(r(u) - m(u)) \mid H_1] \triangleq K_1(t, u) = K_s(t, u) + \frac{N_0}{2}\delta(t - u),$$
$$T_i \le t, u \le T_f. \tag{5}$$

We refer to $r(t)$ as a *conditionally Gaussian* random process. The term "conditionally Gaussian" is used because $r(t)$, given H_1 is true, and $r(t)$, given H_0 is true, are the two Gaussian processes in the model.

We observe that the mean value function can be viewed as a deterministic component in the input. When we want to emphasize this we write

$$\begin{aligned} r(t) &= m(t) + [s(t) - m(t)] + w(t) \\ &= m(t) + s_R(t) + w(t), \qquad T_i \leq t \leq T_f : H_1. \end{aligned} \tag{6}$$

(The subscript R denotes the random component of the signal process.) Now the waveform on H_1 consists of a known signal corrupted by two independent zero-mean Gaussian processes. If $K_s(t, u)$ is identically zero, the problem degenerates into the known signal in white noise problem of Chapter I-4. As we proceed, we shall find that all of the results in Chapter I-4 except for the random phase case in Section I-4.4.1 can be viewed as special cases of various problems in Chapters 2 and 3.

In Section 2.1, we derive the optimum receiver and discuss various procedures for implementing it. In Section 2.2, we analyze the performance of the optimum receiver. Finally, in Section 2.3, we summarize our results.

Most of the original work on the detection of Gaussian signals is due to Price [1]–[4] and Middleton [17]–[20]. Other references are cited at various points in the Chapter.

2.1 OPTIMUM RECEIVERS

Our approach to designing the optimum receiver is analogous to the approach in the deterministic signal case (see pages I-250–I-253). The essential steps are the following:

1. We expand $r(t)$ in a series, using the eigenfunctions of the signal process as coordinate functions. The noise term $w(t)$ is white, and so the coefficients of the expansion will be conditionally uncorrelated on both hypotheses. Because the input $r(t)$ is Gaussian on both hypotheses, the coefficients are conditionally statistically independent.
2. We truncate the expansion at the Kth term and denote the first K coefficients by the vector $\mathbf{r}$. The waveform corresponding to the sum of the first K terms in the series is $r_K(t)$.
3. We then construct the likelihood ratio,

$$\Lambda(r_K(t)) = \Lambda(\mathbf{R}) = \frac{p_{\mathbf{r}|H_1}(\mathbf{R} \mid H_1)}{p_{\mathbf{r}|H_0}(\mathbf{R} \mid H_0)}, \tag{7}$$

and manipulate it into a form so that we can let $K \to \infty$.