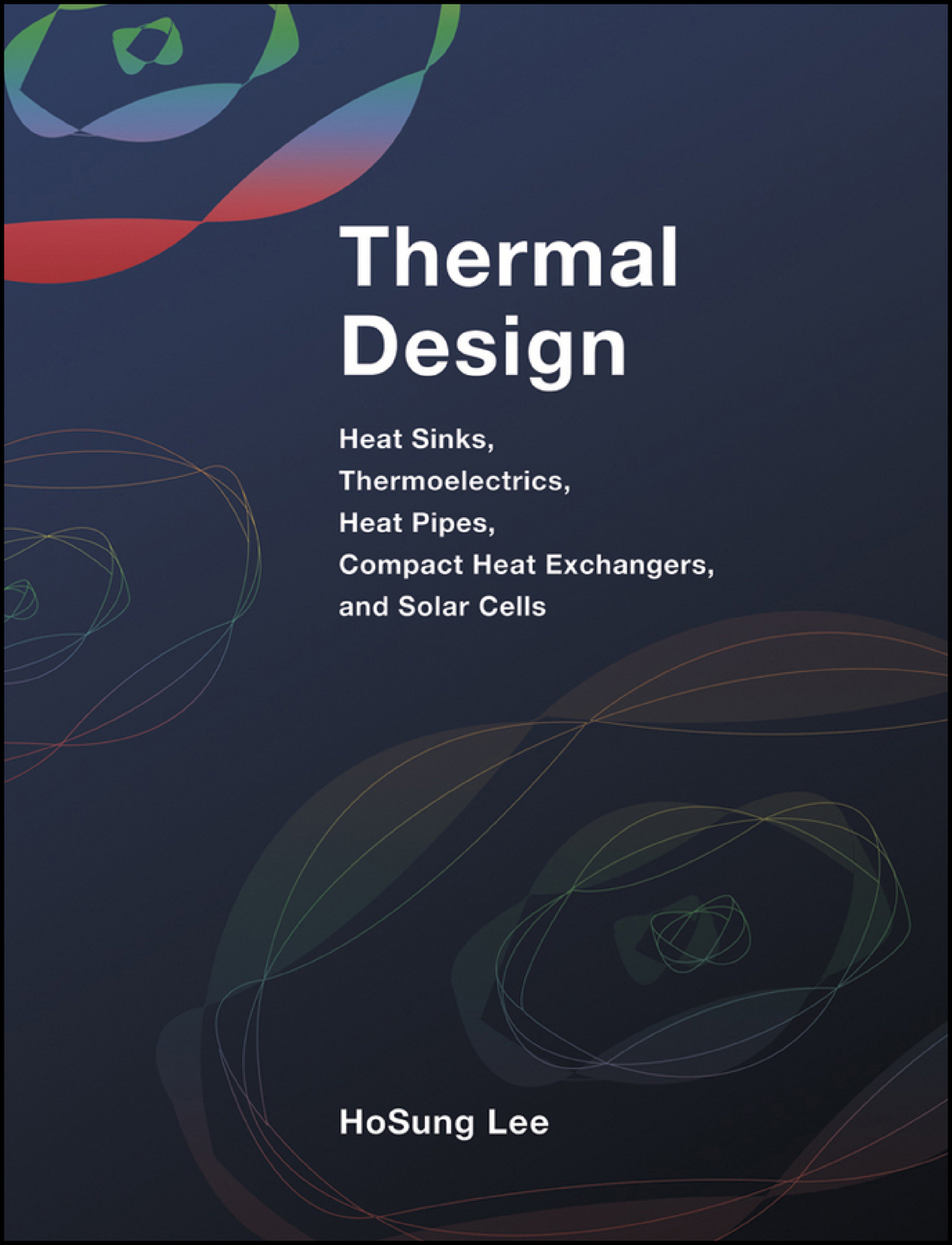

Thermal Design

Heat Sinks,
Thermoelectrics,
Heat Pipes,
Compact Heat Exchangers,
and Solar Cells

HoSung Lee

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Preface

This book is written as a senior undergraduate or a first-year graduate textbook, covering modern thermal devices such as heat sinks, thermoelectric generators and coolers, heat pipes, compact heat exchangers, and solar cells as design components. These devices are becoming increasingly important and fundamental in thermal design in such diverse areas as microelectronic cooling, green or thermal energy conversion, thermal control and management in space, and so on. However, there is no textbook available that includes these topics, which is the rationale for the writing of this book. This book may be used as a capstone design course after students have finished the fundamental courses in areas such as thermodynamics, fluid mechanics, and heat transfer. The concept of this book is to give the student first an understanding of the physical mechanisms of the devices with detailed derivations, and second, practice in designing the devices with use of mathematical modeling, graphical optimization, and occasionally computational-fluid-dynamic (CFD) simulation. This is done through pertinent design examples developed using a commercial software, MathCAD. In other words, the design concept is embodied through the sample problems. The graphical presentation generally provides designers or students with rich and flexible solutions giving the optimal design.

This book is unique as a textbook of *thermal design* with the present topics and design methodology. It has been developed from the author's lecture notes. Since this book exhibits the fundamental framework of thermal design using modern thermal devices, the applications to the thermal systems associated with these devices are unlimited.

This book is self-contained. For example, an introduction to thermal radiation was added prior to the section of *fin design in space* for the readers who are not familiar with this subject. Many appropriate charts and tables were attached in the appendices so that readers need not look at other reference books. Detailed tutorials appropriate for use in CFD and MathCAD homework problems are also included in the appendices to help students.

Particular effort was made to create figures representing key concepts of the subject matter, keeping in mind that "one good figure is better than a thousand words." Needless to say, figures are important learning tools, focus attention and stimulate curiosity and interest.

In the past decade, a good deal of attention has been given to critically assessing traditional pedagogy and to exploring means by which students' learning may be enhanced. With respect to the development of educational tools and curricula, this assessment has stimulated serious consideration of *learning objectives* and the means of determining the extent to which prescribed objectives are being met. This textbook has three learning objectives:

1. The students should delineate physical mechanisms and transport phenomena for any process or system particularly associated with thermal devices such

as heat sinks, thermoelectric generators and coolers, heat pipes, compact heat exchangers, and solar cells.

2. The students should be able to develop mathematical models and graphical optimization for any process or system associated with the thermal devices or systems.
3. The students should be able to professionally design the thermal devices or systems using design tools.

As mentioned before, this book is self-contained. An attempt was made to include necessary background in thermodynamics, fluid mechanics, and heat transfer. Chapter 1 focuses on material needed in later chapters. Students can use this chapter as reference or review. The first and second laws of thermodynamics, internal and external convection flow, and heat transfer mechanisms were presented with essential formulas and empirical correlations.

Chapter 2 is devoted to *heat sinks*, which are the most common thermal devices for use in the electronics industry. They are used to improve the thermal control of electronic components, assemblies, and modulus by enhancing their exterior surface area through the use of fins. The governing formulas on heat dissipation and efficiency for single and multiple fins are derived and incorporated into the modeling and optimization of fin design. Particular effort was given to creating appropriate examples to reflect the design concept, which involves mathematical modeling and graphical optimization. Also in the chapter, fin design with thermal radiation (in space) was explored with two design examples.

Chapter 3 provides the fundamentals of the design of *thermoelectric generators* and *coolers*. The field of thermoelectrics has grown dramatically in recent years, but in spite of this resurgence of interest, there are very few books available. This may be the first book to deal with the design of thermoelectrics and heat pipes, providing the physical principles and fundamental formulas, which lead to mathematical modeling and graphical optimization. This chapter may prompt students to look for the waste energy recovery from the exhaust gases in automotive vehicles and power systems for spacecrafts using radioisotope thermoelectric generators (RTG). A design example at the end of the chapter was conceptualized and developed from a commercial product, which consists of two heat sinks, two fans and a thermoelectric cooler (TEC) module as a thermal system.

Chapter 4 is devoted to the design of *heat pipes*, which have been recently employed in numerous applications ranging from temperature control of the permafrost layer under the Alaska pipeline to the thermal control of optical surfaces in spacecraft. Today every laptop computer has a heat-pipe related cooling system. This book gives a clear understanding of the fundamentals of heat pipes, including the formulas, which allow modeling and optimization in design. This chapter deals with various heat pipes such as variable conductance heat pipe, loop heat pipe, micro heat pipe, and heat pipe in space. The end-of-chapter design example discusses the detailed design aspects: selecting materials and working fluid, sizing the heat pipe, selecting the wick, and performance map.

Chapter 5 discusses the design of *compact heat exchangers* including plate heat exchangers, finned-tube heat exchangers, and plate-fin heat exchangers. In order to discuss these complex exchangers, simpler heat exchangers such as a double-pipe heat exchanger and a shell-and-tube heat exchanger are also introduced. Usually, it takes a semester to cover the entire material of this chapter. However, the complex geometry and time-consuming work are incorporated into the illustrated models. This saves a lot of the students' time. Rather, students can put their efforts either into improving the model or implementing into the system design. Design tools such as MathCAD enable easy and precise minimizing of human errors in calculations.

Chapter 6 is devoted to *solar cell design*. It is often said that solar energy will be the energy in the future. Solar cells need to be developed in order to meet the formidable requirements of high efficiency and low cost. A solar cell is a technology-dependent device, wherein efficiency is a key issue in performance and design. The author believes that solar cells should be dealt with in undergraduate programs, when we consider their importance and huge demand in the future. Unfortunately, a solar cell involves many disciplines: physics, chemistry, materials, electronics, and mechanics (heat transfer). This book was written to provide the fundamentals of solar cells including both the physics and design with a ready-to-use model so that we may achieve our goal in a minimum of time.

Students or teachers should feel free to copy the ready-to-use models to suit their own purposes. This book was designed to provide profound theories and derivation of formulas so that students can easily make modifications or improvements according to their ability. For example, a student who is not strong in physics but is creative can produce a novel design using the ready-to-use models presented in this book.

Except for Chapter 1 ("Introduction"), each chapter is independent from other chapters so that it may be taught separately. Considering the volume of material in each chapter, any combination of three chapters would be appropriate for a semester's material. Chapter 1 may be skipped. For example, a thermal-oriented course would use Chapter 2, "Heat Sinks," Chapter 4, "Heat Pipes," and Chapter 5, "Compact Heat Exchangers." An electronic cooling–oriented course would use Chapter 2, "Heat Sinks," Chapter 3, "Thermoelectrics," and Chapter 4, "Heat Pipes." Intensive design can be sought with Chapter 5, "Compact Heat Exchangers," and Chapter 6, "Solar Cells." This book can be taught for one or two semesters. Note that many problems at the end of each chapter may require at least a week of work by students.

I would like to acknowledge the many suggestions, the inspiration, and the help provided by undergraduate/graduate students in the thermal design classes over the years. I also thank the College of Engineering and Applied Sciences at Western Michigan University for providing me the opportunity to teach the thermal design courses. Particular thanks to the Wiley staff for their support and their editing of the material. I also wish to thank anonymous reviewers for their suggestions and critiques that greatly improved the quality of this book. I am immensely indebted to Professor Emeritus Herman Merte, Jr. for his advice and support in the preparation of this book. My sincere appreciation is extended to Dr. Stanley L. Rajnak for his suggestions and his review in preparation of the manuscript. He reviewed the entire manuscript with his invaluable

endeavors particularly finding errors in mathematics. Lastly I am truly indebted to my beloved wife, Young-Ae, for her support and forbearance during the preparation of this book, and also to my lovely daughter, Yujin, for her encouragement and assistance particularly in drawings. Without the supports mentioned above, this book would not be possible.

Kalamazoo, Michigan HoSung Lee

1

Introduction

1.1 INTRODUCTION

Thermal design is a branch of *engineering design*, a counterpart to *machine design*, typically involving energy, fluid flow, thermodynamics, and heat transfer. Traditionally, *thermal design* has been developed as *thermal system design*, which deals with *modeling, simulation*, and *optimization*, and proper selection of the components. It typically avoids the design of the components themselves [1–7]. The components would be pumps, fans, heat exchangers, and refrigeration units, for example. This forms a definite field of thermal design with a great deal of industrial applications. However, often the traditional system design could not meet the needs of a radically changing technology. For example, traditional fan-cooling methods of electronics in computers no longer meet today's formidable heat duty. Today, every laptop computer has *heat-pipe* cooling systems. Traditional refrigeration units are too large and noisy for small systems. These are typically replaced by thermoelectric coolers, which have no moving parts and are easily controlled by electric power. A *compact heat exchanger* in a *fuel cell* is an essential component because of its high efficiency. Since it is not independent of the system, it should be custom designed to meet the requirements and the constraints of the system. *Solar cells* will require compact and efficient coolers. As a result, a new concept of *thermal design* with the component design of the novel devices briefly addressed becomes inevitable and essential. This book studies thermal design in light of the problems mentioned here.

1.2 HUMANS AND ENERGY

At the early Stone Age (about 2 millions year ago), our ancestors wandered from one place to another to find food and shelter. One of the discoveries by humankind was fire, which gave warmth and light to their shelters and protected them from dangerous animals. This fire would be the first use of energy by humankind and also a turning point in early human life, probably comparable to the discovery of electricity in the nineteenth century. About 5,000 years ago, people began using the wind to sail from one place to another. It was only 2,500 year ago that people began using windmills and waterwheels to grind grain. About 2,300 years ago, the ancient Greek mathematician Archimedes supposedly burned the ships of attacking Syracuse by using mirrors to concentrate the sun's rays on their sails. This would have been the first recorded use of focused solar energy in history. More recently, we learned to use resources such as fossil fuels (coals, oil, and natural gas), renewable energy (sunlight, wind, biofuel, tide, and geothermal heat), and nuclear energy. Solar energy is the primary source of energy for wood growing on Earth and for algae and plankton multiplying in the sea through photosynthesis. Wood turned into coal and algae and plankton turned into oil and natural gas over hundreds of millions of years, resulting in solar energy storage for use today.

Energy consumption has been drastically increased since the discovery of fossil fuels and electricity. Fossil fuels in 2007 produce 81 percent of the world energy consumption, while nuclear power and renewable energy produce the rest (from the Energy Information Administration (EIA) Annual Energy Review 2007). The emissions from fossil fuel have unfortunately resulted in global warming, global weather change, and air pollution. Nations are advised to reduce the hazardous emissions from the fossil fuel. An alternative energy (nonhydrocarbon fuel) is in demand, of which renewable energy is the most viable resource based on technology-dependent devices that require high efficiency and low cost of manufacturing. Solar cells, fuel cells, and wind turbines are examples.

It is not very difficult to predict that solar energy will be the energy of the future. Some day, every unit of community or family will have its own inexpensive power station producing electricity, heat, and nonhydrocarbon fuel (hydrogen) from either natural gases or solar-cell panels. Each automobile will run on a fuel cell powered with hydrogen-delivering water that will be recycled (zero air pollution), and will have its own small power station so that an appreciable amount of green fuel (hydrogen) can be produced during working hours. No grid and large cables will be seen on the streets. Deserts, for example, in the Middle East and North Africa, will be huge sources of green fuel (hydrogen) and electricity from solar energy. This book is intended to put a first step toward the thermal design of the novel devices for our future energy solutions.

1.3 THERMODYNAMICS

1.3.1 Energy, Heat, and Work

Energy is the capacity to perform work. Energy can exist as a numerous forms such as kinetic, potential, electric, chemical, and nuclear. Any form of energy can be transformed into another form, but the total energy always remains the same. This principle, *the conservation of energy*, was first postulated in the early nineteenth century. It applies to any isolated system. *Heat* (or *heat transferred*) is *thermal energy* that is transferred between two systems by virtue of a temperature difference. *Work* is the energy transfer associated with a force acting through a distance. Work can exist as a numerous forms such as piston work, shaft work, and electrical work. The rate of work is called *power*.

1.3.2 The First Law of Thermodynamics

The first law of thermodynamics, also known as *the conservation of energy*, provides a sound basis for studying the relationship among the various forms of energy, total energy, heat, and work. This states that energy can be neither created nor destroyed

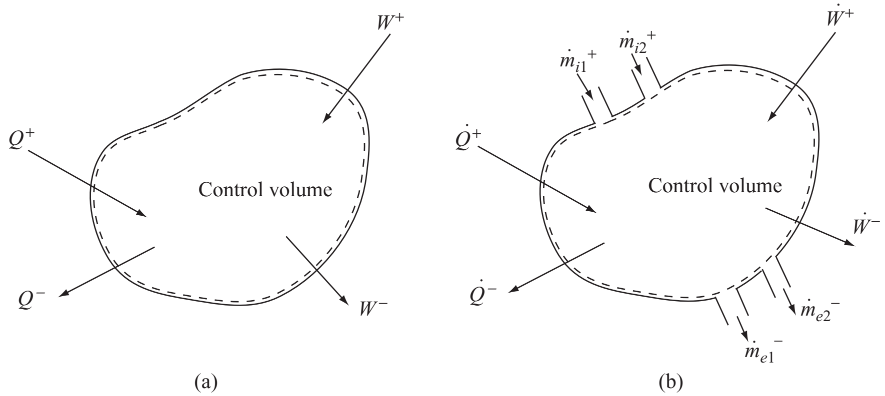

Figure 1.1 Control volumes and sign convention for (a) closed system and (b) open system

during a process; it can only change forms. The sign convention is illustrated in Figure 1.1(a).

$$Q > 0 \quad \text{Heat transferred to system} \tag{1.1a}$$

$$Q < 0 \quad \text{Heat transferred from system} \tag{1.1b}$$

$$W > 0 \quad \text{Work done on system} \tag{1.1c}$$

$$W < 0 \quad \text{Work done by system} \tag{1.1d}$$

The first law of thermodynamics for a closed system in Figure 1.1(a) is given by

$$\Delta E = Q + W \tag{1.2}$$

where ΔE is the *total energy* change in the system, Q the *heat* transferred, and W the *work* done. Caution is given to the sign of work in Equations (1.1c) and (1.1d); some books define work as positive when it is done by the system. The reason is that many engineering applications focus on the work done by a particular heat engine, and so it is helpful to define that as positive. In that case, Equation (1.2) becomes that $\Delta E = Q - W$.

Total energy, E, is the extensive property and presents all the energy such as kinetic, potential, thermal, latent, chemical, nuclear, and others. All energy except kinetic and potential energy is called *internal energy*, U. The *kinetic energy*, KE, is associated with the motion of the system as a whole. The potential energy, PE, is associated with the position of the system as a whole. In the case of a stationary system, $\Delta E = \Delta U$. The *potential energy* may be present in a variety of fields (gravity, electric, or magnetic). The total energy is expressed as

$$E = U + KE + PE \tag{1.3}$$

The internal energy is

$$U = \text{Thermal} + \text{Latent} + \text{Chemical} + \text{Nuclear} + \text{Others} \tag{1.4}$$

The change of internal energy U can be expressed thermodynamically:

$$\Delta U = \begin{cases} mc_p \Delta T & \text{for liquids and solids} \\ mc_v \Delta T & \text{for gases or air} \end{cases} \tag{1.5}$$

where m is the mass and c_p and c_v are the specific heat at constant pressure and at constant volume, respectively. ΔT is the temperature change during the process ($\Delta T = T_2 - T_1$). In a rate form, the first law of thermodynamics can be given by

$$\Delta \dot{E} = \frac{dE_{sys}}{dt} = \dot{Q} + \dot{W} \tag{1.6}$$

where $\Delta \dot{E}$ is the rate of change of total energy in the system (being the same for both the system and the control volume for a closed system). In steady-state conditions (no change with time), the rate of change of energy in the system is zero ($\Delta \dot{E} = 0$). Equation (1.6) then shows that the work $\dot{W}$ becomes equal to the heat $\dot{Q}$. The relationship between work and heat was not clear before the discovery of the first law of thermodynamics.

Now consider the *mass flow rate*, $\dot{m}$, into and out of the system, as shown in Figure 1.1(b). The mass flow internally involves pressure work, and the enthalpy (flow energy) is the sum of the internal energy (u) and the pressure work (pv), which is called enthalpy ($h = u + pv$). For a control volume, CV, in Figure 1.1(b), the first law of thermodynamics with mass flow rates is expressed by

$$\Delta \dot{E}_{CV} = \frac{dE_{CV}}{dt} = \dot{Q} + \dot{W} + \sum_{in} \dot{m} \left(h + \frac{v^2}{2} + gz \right) - \sum_{out} \dot{m} \left(h + \frac{v^2}{2} + gz \right) \tag{1.7}$$

This is equivalent to the *energy balance* in heat transfer, which is expressed as

$$\Delta \dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g \tag{1.7a}$$

where $\Delta \dot{E}_{st}$ is the energy stored and equal to $\Delta \dot{E}_{CV}$. $\dot{E}_{in}$ is the energy entering into the control volume, $\dot{E}_{out}$ the energy exiting from the control volume and $\dot{E}_g$ the heat generation (electricity or nuclear reaction). This equation is particularly useful in heat transfer calculations. Mass must be conserved. Hence, *the conservation of mass* is expressed as

$$\frac{dm_{\mathrm{CV}}}{dt} = \sum_{\mathrm{in}} \dot{m} - \sum_{\mathrm{out}} \dot{m} \tag{1.8}$$

In the case of a steady flow from Equations (1.7) and (1.8), we note that

$$\frac{dE_{CV}}{dt} = 0 \quad \text{and} \quad \frac{dm_{CV}}{dt} = 0 \tag{1.9}$$

Equation (1.7), *the first law of thermodynamics*, is the basis of fluid-thermal engineering and is widely used. However, there is an important aspect to the equation. Consider a steady-state process in a container where electrical work $\dot{W}_e$ is applied while allowing heat transfer on the surface of the container, as shown in Figure 1.2(a). You may consider a heating element for the electrical work. In accordance with the first law of thermodynamics, $\dot{W}_e$ must turn completely into $\dot{Q}$. Intuitively, this process

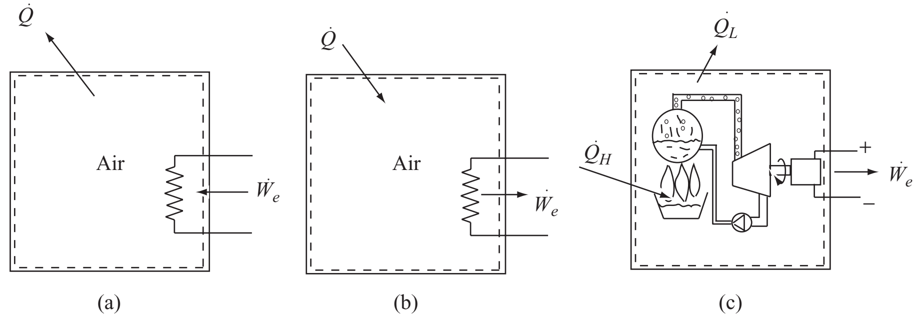

Figure 1.2 Steady-state systems with heat and electrical work, (a) possible process (100 % efficiency), (b) impossible process, and (c) steam power plant (about 60 % efficiency)

is possible. Now consider the opposite process, as shown in Figure 1.2(b). Heat flows into the system and the electrical work is produced. Do you think that this process is possible? The answer is no. Heat cannot be readily converted to the useful mechanical work or electricity. It is an impossible process. We find here a very important aspect that the first law of thermodynamics provides the quantity but not necessarily the quality. Thermodynamically, it is an *irreversible process* that typically involves *heat transfer* and *friction*. Hence, a reversible process is ideal and possesses the maximum efficiency in devices. The irreversibility is treated with a quantitative property that is referred to as *entropy* (degree of disorder). *The second law of thermodynamics* deals with the irreversibility, the quality of process, or the entropy.

Can we produce electricity from heat (or fuels)? The answer is yes. If we think of a steam power plant (typically about 60 percent thermal efficiency) as shown in Figure 1.2(c), the answer is obvious: 100 percent of $\dot{Q}_H$ (fuel) enters and produces 60 percent work of the entered energy as electricity, which leads to delivering 40 percent energy $\dot{Q}_L$ as waste heat, where $\dot{Q}_H$ and $\dot{Q}_L$ are the rate of heat transferred from the high temperature source to the low temperature sink, respectively. Diesel engines typically have about 50 percent thermal efficiency. Gasoline engines have about 30 percent thermal efficiency. Solar cells have about 15 percent thermal efficiency. Thus, the gain in electrical work from heat depends on devices and technology. We know that electricity is the form of energy most easily convertible to other forms of energy. The most efficient form of energy is called *exergy*. Exergy is a measure of the quality of energy. Thus, electricity has the highest level of exergy of any form of energy. It is interesting to note that the entities in the first law of thermodynamics can be realistic or ideal, depending on whether the thermal efficiency is included in the equation or not since the thermal efficiency reflects irreversibility and the second law of thermodynamics. The determination of a device's efficiency or effectiveness is therefore essential in thermal design. For another example, the incorporation of *friction* (or friction factor) into the equation allows engineers to use the equation in their design.

1.3.3 Heat Engines, Refrigerators, and Heat Pumps

Devices that can contain the energy of heat and make it do work are called *heat engines*. Steam and gas turbine, diesel engine, gasoline engine, and ocean thermal energy conversion (OTEC) are examples of heat engines. Most of our energy comes from the burning of fossil fuels and from nuclear reactions: both of these supply energy as heat. However, we need mechanical energy to operate machines or propel vehicles. Exergy transformation from a low exergy (heat) to a high exergy (mechanical energy or electricity) is always achieved in the expense of the low thermal efficiency or vice versa. Fuel cells are a good example that the chemical energy of a fuel is directly converted to electricity without the process to heat. This is a reason why fuel cells have a higher efficiency than conventional combustion engines. A heat engine can produce mechanical work between a high temperature source and a low temperature sink, as shown in Figure 1.3.

If we apply the first law of thermodynamics to the heat engine in Figure 1.3(a), we come up with

$$\dot{W}_{net} = \dot{Q}_H - \dot{Q}_L \tag{1.10}$$

where $\dot{W}_{net}$ is the net work, $\dot{Q}_H$ the heat transferred to the heat engine from the high temperature source, and $\dot{Q}_L$ the heat transferred to the low temperature sink from the heat engine. Using Equation (1.10), the thermal efficiency in the heat engine is expressed as

$$\eta_{th} = \frac{Output}{Input} = \frac{\dot{W}_{net}}{\dot{Q}_H} = \frac{\dot{Q}_H - \dot{Q}_L}{\dot{Q}_H} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} \tag{1.11}$$

Note that the thermal efficiency is less than unity. If we apply the first law of thermodynamics to the *refrigerator* in Figure 1.3(b), we have the same relationship with Equation (1.10) as

$$\dot{W}_{net} = \dot{Q}_H - \dot{Q}_L \tag{1.12}$$

The efficiency of the refrigerator is expressed in terms of *coefficient of performance* (COP), denoted by COP_R.

$$COP_R = \frac{\text{Output}}{\text{Input}} = \frac{\dot{Q}_L}{\dot{W}_{net}} = \frac{\dot{Q}_L}{\dot{Q}_H - \dot{Q}_L} = \frac{1}{\dot{Q}_H/\dot{Q}_L - 1} \tag{1.13}$$

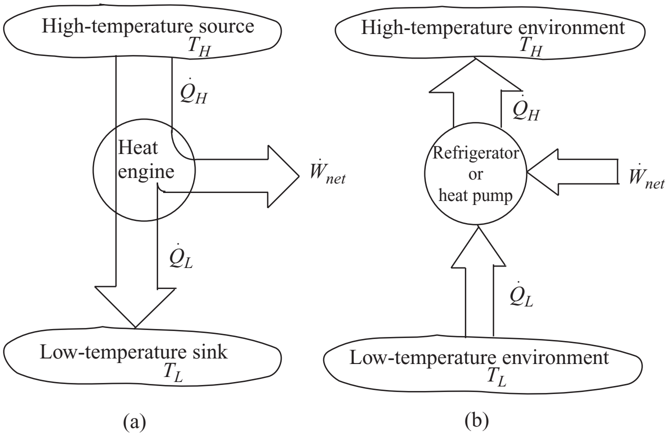

Figure 1.3 Schematic flow diagrams of (a) a heat engine and (b) a refrigerator or a heat pump

Notice that the value of COP_R can be greater than unity. That is, the amount of heat removed from the low-temperature environment can be greater than the amount of work input. Most refrigerators have COP_R of $2.5 \sim 3.0$.

A device that transfers heat from a low temperature body to high temperature body is called a *heat pump*. The *heat pump cycle* is identical to a *refrigerator cycle* in principle, but differs in that the primary purpose of the heat pump is to supply heat rather than to remove it from an enclosed space. The measure of performance is also expressed in terms of coefficient of performance COP_{HP}, defined as

$$COP_{HP} = \frac{\text{Output}}{\text{Input}} = \frac{\dot{Q}_H}{\dot{W}_{net}} = \frac{\dot{Q}_H}{\dot{Q}_H - \dot{Q}_L} = \frac{1}{1 - \dot{Q}_L/\dot{Q}_H} \tag{1.14}$$

Most heat pumps have COP_{HP} of $3.0 \sim 5.0$.

1.3.4 The Second Law of Thermodynamics

As mentioned previously, the second law of thermodynamics is associated with the irreversibility, the quality, or the entropy of the process. The classical statements for *the second law of thermodynamics* are restated here in several different ways.

Statements:

1. Heat is naturally transferred always from a high temperature body to a low temperature body, not vice versa.
2. Work is required in order to make heat transfer from a cold temperature body to a high temperature body.
3. It is impossible to construct a heat engine that operates 100 percent thermal efficiency.
4. It is impossible to construct a heat engine without a heat loss to the environment.

For example, if a person claims to invent a machine that produces 100 percent work from the energy of heat entered, this satisfies the first law of thermodynamics but violates the second law of thermodynamics. Perpetual motion machines touted in the nineteenth and early twentieth centuries violated the second law of thermodynamics. Of course, the machines were never functional.

1.3.5 Carnot Cycle

If the efficiency of all heat engines is less than 100 percent, what is the most efficient cycle we can have? The *Carnot cycle* is the most efficient cycle. A *heat engine* that operates in the Carnot cycle was proposed in 1824 by French engineer *Sadi Carnot*. Heat engines are cyclic devices and the working fluid of a heat engine returns to its initial state at the end of each cycle.

The *Carnot cycle* is ideal, so the ideal gas is used as a working fluid and the entire process is reversible. There is no friction and no heat transfer except the isothermal processes (heat transfer). The most efficient heat-transfer process is known to be an *isothermal process* where the high- or low-source temperature remains constant during the process. And the most efficient process between two-temperature

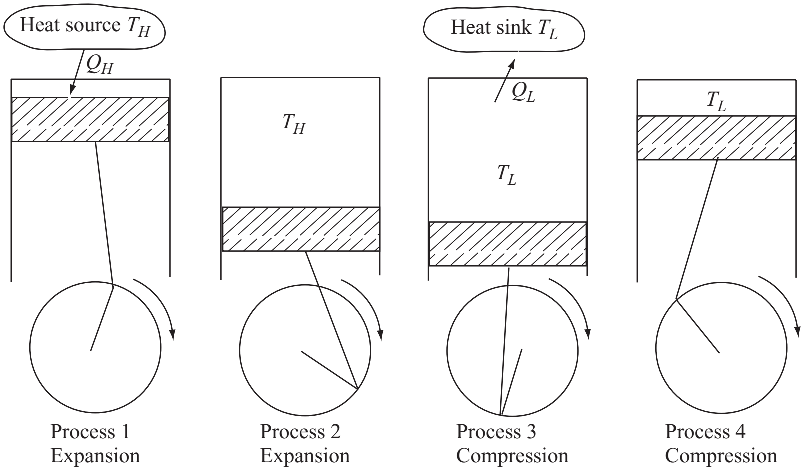

Figure 1.4 Execution of the Carnot cycle in a closed system

sources is a reversible adiabatic process. We want to construct a Carnot cycle based on these two hypotheses. A simple piston cycle giving the Carnot cycle is shown in Figure 1.4.

Process 1–2 (reversible isothermal expansion): During this process, the gas in the cylinder expands while heat Q_H is transferred from a high temperature source at T_H, to the cylinder where T_H is the temperature at the high temperature source and T_L is the temperature at the low temperature sink. Hence, the gas temperature remains constant at T_H. During this process, heat must be added in order to compensate the temperature drop due to the expansion. State 2 is determined inasmuch as the amount of work done by the piston equals the amount of heat Q_H.

Process 2–3 (reversible adiabatic expansion): During this process, the gas continues to expand until the gas temperature T_H turns into T_L while the supply of heat is stopped. The cylinder is insulated. State 3 is determined when the gas temperature reaches T_L.

Process 3–4 (reversible isothermal compression): During this process, the gas is compressed by the piston while heat Q_L is allowed to transfer from the cylinder to the low-temperature sink at T_L. Hence, the gas temperature remains constant at T_L. During this process, heat must be removed in order to compensate the temperature rise due to the compression. State 4 is determined inasmuch as the amount of work done on the piston equals the amount of heat Q_L.

Process 4–1 (reversible adiabatic compression): During this process, the gas continues to be compressed until the gas temperature T_L turns into T_H while the transfer of heat is stopped and the cylinder is insulated. State 1 is determined when the gas temperature T_L reaches T_H.

The *Carnot cycle* is an ideal cycle and also the most efficient heat cycle. The four processes mentioned here are plotted in a P-V diagram in Figure 1.5. It is interesting to note that the Carnot cycle must have a heat loss Q_L, although the cycle is reversible. Actually, this satisfies *the second law of thermodynamics*. In order words, no heat

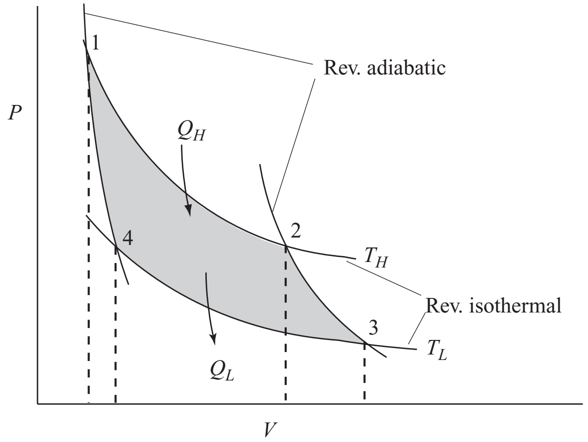

Figure 1.5 P-V diagram of the Carnot cycle

engine can be constructed without Q_L. Therefore, no 100 percent efficient heat engine is possible.

From Equation (1.11), we find that the thermal efficiency η_{th} is a function of the ratio of Q_L to Q_H since Q_L/Q_H is the same as $\dot{Q}_L/\dot{Q}_H$. The ratio is examined further here. The ideal gas law for an isothermal process at T_H gives

$$P = \frac{mRT_H}{V} \tag{1.15}$$

where P is the pressure, V the volume, and R the *gas constant.* If we apply the first law of thermodynamics to process 1–2, the isothermal process ($\Delta T = 0$), we have

$$\Delta E = \Delta U = mc_{\mathrm{v}}\Delta T = Q_{\mathrm{H}} + {}_1W_2 \tag{1.16}$$

Then we come up with the fact that the heat transferred must be equal to the work done by the system as

$$Q_H = -{}_1W_2 = -\int_1^2 PdV = -\int_1^2 \frac{mRT_H}{V}dV = -mRT_H \ln\frac{V_2}{V_1} \tag{1.17}$$

Similarity for process 3–4,

$$Q_L = {}_3W_4 = \int_3^4 PdV = \int_3^4 \frac{mRT_L}{V}dV = mRT_L \ln\frac{V_4}{V_3} = -mRT_L \ln\frac{V_3}{V_4} \tag{1.18}$$

The ratio of the two quantities is thus

$$\frac{Q_L}{Q_H} = \frac{-mRT_L \ln \dfrac{V_3}{V_4}}{-mRT_H \ln \dfrac{V_2}{V_1}} = \frac{T_L}{T_H} \frac{\ln \dfrac{V_3}{V_4}}{\ln \dfrac{V_2}{V_1}} \tag{1.19}$$

From process 1–2 and process 3–4, we have

$$P_1 V_1 = P_2 V_2 \tag{1.20}$$

$$P_3 V_3 = P_4 V_4 \tag{1.21}$$

Since both process 2–3 and process 4–1 are reversible and adiabatic (isentropic), we can apply the polytropic relation with $k = 1.4$ to the ideal gas in the cylinder as

$$PV^k = \text{constant} \tag{1.22}$$

Therefore, we have

$$P_2 V_2^k = P_3 V_3^k \tag{1.23}$$

$$P_4 V_4^k = P_1 V_1^k \tag{1.24}$$

Multiplying Equations (1.20), (1.21), (1.23) and (1.24) and canceling the factor $P_1 P_2 P_3 P_4$ gives

$$V_1 V_2^k V_3 V_4^k = V_2 V_3^k V_4 V_1^k \tag{1.25}$$

Rearranging,

$$(V_2 V_4)^{k-1} = (V_3 V_1)^{k-1} \tag{1.26}$$

Finally, we have

$$\frac{V_2}{V_1} = \frac{V_3}{V_4} \tag{1.27}$$

From Equation (1.19), we derive an important relationship between the ratio of heat and the ratio of temperature.

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} \tag{1.28}$$

From Equation (1.11), the thermal efficiency η_c for a Carnot cycle engine is

$$\eta_c = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 1 - \frac{T_L}{T_H} \tag{1.29}$$

This surprisingly simple result says that the efficiency of a Carnot cycle engine depends only on the temperatures of the two heat sources. When the difference of the two is large, the efficiency is nearly unity; when it is small, the efficiency is much less than unity. Equation (1.28) can be also applied to Equation (1.13) for refrigerators or Equation (1.14) for heat pumps for its Carnot cycle efficiency.