Gerhard K. Ackermann and Jürgen Eichler

Holography

A Practical Approach
Gerhard K. Ackermann
and Jürgen Eichler

Holography
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Holography

A Practical Approach

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Technically the hologram can best be described as an off axis Fourier transform lens matrix, holographic optical element (H.O.E.). This particular technique produces a white light viewable hologram of pure dimensional light alone, and allows for a greater degree of spontaneity in the process of making a hologram. The holographic imagery appears as a kinetic form of pure light, instead of reflected light from a given object.

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Peter Hesse, Berlin

“MATRIX 18R”, 1985, silver halide emulsion on glass is one of a series of Kinetic Yantras by Fred Unterseher.

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Preface

More than 10 years ago the authors published a first book on holography. Since then holography evolved in many areas. The new possibilities of holography, the free design of colors and forms within a hologram inspired many artists to impressive works and installations. In addition making portraits is one of the professional areas using pulse holography.

The technical application in the field of counterfeiting and in other security devices is of a high standard. Holograms are found in everyday life: credit cards, bills, visa, logos and trademarks are secured against counterfeiting by holograms. Holography penetrates technology in many areas like nondestructive testing, holographic optical elements (HOE), optoelectronic devices, holographic storages, and digital displays.

In order to incorporate the new developments and changes in holography the authors published this book on Practical Holography. In the field of holographic material many new companies are on the market and well known have given up. Computer generated holograms are used widely in the scientific community because of the high diffraction efficiency. The book takes into consideration these new developments.

The textbook is based on laboratory courses, which were offered since two decades at our University. It is designed for students and newcomers as well as for all professionals in holography. On over 300 pages it gives all necessary information to do and to understand holography. It contains more than 100 figures and more than 100 problems including the solutions. Some mathematical more complex details are handled in the three appendices.

The 60th anniversary of holography in this year is the best motivation to publish a book on this fascinating subject. We hope, that many students and interested people will enjoy reading this book. It is made to assist in first steps in holography as well as in more advanced applications.

We are very indebted to the publishing staff of the Wiley-VCH company.
We dedicate this book to Ursula, Evelyn and Sascha’s family.

June 2007

G. Ackermann and J. Eichler
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Part 1  Fundamentals of Holography
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1

Introduction

With its many applications holography is one of the most interesting developments in modern optics. Its scientific importance is emphasized by awarding the 1971 Nobel prize to its inventor Denis Gabor. The term “holography” is a compound of the Greek words “holos = complete” and “graphein = to write.” It denotes a procedure for three-dimensional recording and displaying of images and information without the use of lenses. Therefore holography opens up completely new possibilities in science, engineering, graphics and arts. Fields of applications are interferometric measurement techniques, image processing, holographic optical elements and memories as well as art holograms.

1.1

Photography and Holography

1.1.1

Object Wave

To see an object it has to be illuminated. In doing so light is scattered and a so-called object wave is created. This wave contains the complete optical information of the object. The light wave is characterized by two parameters: the amplitude, which describes the brightness, and the phase, which contains the shape of the object. In Fig. 1.1 two waves of different objects are shown which have the same amplitudes but different phases. The objects have the

Fig. 1.1 Illustration of two light waves with same amplitudes but different phases.
same brightness but a different shape. For most holograms the color of the objects is not important, so the first chapters only deal with light waves of one wavelength. This changes for color holography which uses several wavelengths.

1.1.2 
Photography

During the process of vision an object is imaged by the eye lens onto the retina. The optical path in a camera is similar: the objective creates an image on the film. For observation or to photograph an object it has to be illuminated. The scattered light, i.e., the object wave, carries the information of the object. The light wave can be made visible in a plane of the optical path, for example using a screen. The object wave appears as a very complex light field (Fig. 1.2) which results from the superposition of all waves emerging from the individual object points. If this light field could be recorded on a screen and displayed again, an observer (or a camera) would see an image that is not discriminable from the object [27].

If there is a photographic film at the position of the screen, the object wave will cause a darkening distribution during the following processing of the film. But only the light intensity is recorded; all information of the phase in the plane of the screen is lost. This loss of phase also happens if the object is imaged onto a film by a lens. Therefore the object wave can never be completely restored from a normal photographic image. A two-dimensional image is the result.

1.1.3 
Holography

Holography uses the properties interference and diffraction of light which make it possible to reconstruct the object wave completely. To be able to see these effects coherent laser light has to be used. “Coherence” means that the
1.1 Photography and Holography

The laser on one hand illuminates the object and the scattered light hits the photographic film (object wave) (Fig. 1.3a). On the other hand, the film is illuminated directly with the same laser (reference wave). The object and the reference waves interfere with each other on the holographic film. This generates interference fringes in the holographic layer as are shown as a largely magnified image in Fig. 1.4. The distance of the fringes is in the region of μm which is in the order of magnitude of the light wavelength. The information of the object wave is contained in the modulation of the brightness of the fringes and in the distance of the fringes.

The photographic film is exposed and developed resulting in the hologram. The first step in holography, the recording, is made. The second step, the reconstruction or display of the object wave, is shown in Fig. 1.3b. After developing the film the hologram is illuminated with a light wave that should resemble the reference wave as best as possible. This reconstruction wave is diffracted by the interference pattern of the hologram generating the object wave. An observer looking at the hologram will see a three-dimensional image of the object.
1.2 Interference and Diffraction

1.2.1 Interference During Recording

Light is an electromagnetic wave ranging from 0.4 to 0.7 µm. In the following the superposition of two constants, i.e., coherent light waves, is described. This process, known as “interference,” is responsible for the recording of holograms.

A general description of the waves emerging from the object is complicated. Therefore for simplification a plane object wave is considered. The object in this case is a single point at a large distance. According to Fig. 1.5a a plane object wave and a plane reference wave impinge on the photographic layer. The superposition of the waves creates equally spaced interference fringes, i.e., parallel bright and dark areas. Dark areas occur when the waves cancel out each other by superposition of a maximum and a minimum. Bright areas occur when maxima (or minima) of the waves are superimposed. After exposing and developing the photographic layer a grating is created where exposed areas appear dark.

![Fig. 1.5 Hologram with a plane object wave: (a) recording of the hologram (fabrication of a diffraction grating) and (b) reconstruction of the object wave (diffraction by the grating) [27].](image)
1.2.2

Diffraction During Reconstruction

The image is displayed by illuminating the grating with a wave that closely resembles the reference wave (Fig. 1.5b). According to Huygens’ principle each point of the grating sends out a spherical elementary wave. They are shown in Fig. 1.5b for the center of the bright fringes. The superposition of the elementary waves can be shown by their envelope. Plane waves are created which represent the 0th, 1st, and $-1$st diffraction orders [1]. (Higher order of diffraction does not occur in sinusoidal gratings.) The zeroth order is the wave passing the grating in the direction of the impinging wave. The first order represents the object wave.

Through the effect of diffraction the object wave is reconstructed; this is the principle of holography. The $-1$st order is often not desirable in this simple stage of holography; it is called the “conjugate object wave.”

1.3

History of Holography

The physical basics of holography are optics of waves, especially interference and diffraction. The first achievements are that of C. Huygens (1629–1694), who phrased the following principle: every point that is hit by a wave is the origin of a spherical elementary wave. Using this statement a lot of problems of diffraction can be calculated by adding up the elementary waves. Important on the way of developing holography are also the works of T. Young (1733–1829), A.J. Fresnel (1788–1827) and J. von Fraunhofer (1877–1926). Already at the beginning of the 19th century enough knowledge was at hand to understand the principles of holography. A lot of scientist were close to the invention of this method: G. Kirchhoff (1824–1887), Lord Rayleigh (1842–1919), E. Abbe (1840–1905), G. Lippmann (1845–1921), W.L. Bragg (1890–1971), M. Wolfke and H. Boersch. But it took until 1948 when D. Gabor (1900–1979) realized the basic ideas of holography.

The origin of holography was at first connected to problems in optics of electrons. Gabor made his first groundbreaking experiments using a mercury vapor lamp. At the beginning the holographic technique was of minor importance and was forgotten for some time. It was not until the coming up of laser technology when developments in holography experienced a significant upturn. So 23 years after his experiments Gabor was awarded the Nobel prize in 1971. In 1962 the theoretical aspects of this methods were refined by E. Leith and J. Upatnieks and a year later they showed off-axis holograms. This technique marks the breakthrough for the practical application of holography.
Problems

Problem 1.1 What are the two essential elements, which describe an electromagnetic wave?

Problem 1.2 Considering the two elements mentioned in Problem 1.1, what is stored during exposure of a photograph and a hologram and what is the reason of the different results?

Problem 1.3 How is the phase of the object wave preserved during holographic exposure? Name the basic optical principles for exposure and reconstruction of a hologram.

Problem 1.4 Would it help to use coherent light in a photographic exposure in order to get a three-dimensional image?
2 General View of Holography

The basic ideas for holographic recording and reconstruction have been presented in the previous chapter in a simplified way. The next section will give a short mathematical description of holography (see [27], [3], and [66]).

2.1 Interference of Light Waves

Light is an electromagnetic wave, whereas – like within many scientific and technological applications – in holography the electrical field strength is considered only. A light wave is described by a spatial and temporal varying electrical field amplitude. The intensity $I$ of a wave is the square of the electrical field amplitude. Within this book the object wave is abbreviated with $o$, and the reference wave with $r$. The object wave $o$ and the reference wave $r$ are superimposed within holographic experiments. The superposition is called interference.

The hologram represents an interference pattern that is created by the superposition of object wave $o$ and reference wave $r$. The phenomenon of superposition will now be described in more detail.

2.1.1 Wave

A wave corresponds to a spatially propagating oscillation. The oscillation of the electrical field $E(t)$ at a given point, in this case the point of origin, can be described by the following equation:

$$E(t) = A \cos(2\pi ft + \phi) = A \cos(\omega t + \phi).$$  \hspace{1cm} (2.1)

Here $A$ is the amplitude of the oscillation. The parameter $\phi$ represents a phase factor which is determined for $t = 0$. For abbreviation the term “angular frequency $\omega$” is introduced: $\omega = 2\pi f$, where $f$ is the frequency.

The oscillation for example propagates in the $z$-direction; Fig. 2.1 shows a “snapshot” of the light wave. The shortest distance between two points that
oscillate with the same phase is called the wavelength \( \lambda \). The time a wave with the velocity \( c \) travels a distance \( \lambda \) is called the period \( T \). The reciprocal value describes the frequency \( f = 1/T \). Since a point at the distance \( z \) from the point of origin starts to oscillate with a phase shift which is proportional to the time \( t_0 = z/c \) the equation of the oscillation at this point looks like

\[
E(t) = A \cos(\omega(t - t_0) + \varphi).
\] (2.2)

Fig. 2.1 Representation of a wave (snapshot).

From the relation \( t_0 = (z/c) = z/(f \cdot \lambda) \), we obtain the equation for a plane wave, i.e., the oscillation at every point \( z \):

\[
E(z, t) = A \cos(\omega t - k z + \varphi) = A \cos(\omega t + \Phi).
\] (2.3a)

where \( k = 2\pi/\lambda \) is called the “wave number.” With this expression the plane wave is mathematically described. The phase \( \Phi = \varphi - k z \) was introduced in the equation.

Generally speaking, a wave, propagating in the \( z \)-direction, has no fixed oscillation direction within the \( x/y \) plane perpendicular to \( z \). In Fig. 2.1 the vector of the electrical field strength is oscillating in the plane of paper. Such a wave, oscillating in a fixed direction, is called linear polarized [2]. Due to technical reasons the radiation of many lasers used in holography is linear polarized.

The complex notation using Euler’s relation often has advantages (see Appendix A):

\[
e^{\pm i \varphi} = \cos \varphi \pm i \sin \varphi.
\]

For the wave (2.3a) the complex notation is used which is denoted by bold characters:

\[
E(z, t) = A e^{-i(\omega t - k z + \varphi)} = A e^{-i(\omega t - \Phi)}.
\] (2.3b)

Here only the real part is important. The frequency \( f \) of a light wave is of the order of \( 10^{14} \) Hz and cannot be observed directly. In each measurement the