PERIODIC STRUCTURES

Mode-Matching Approach and Applications in Electromagnetic Engineering

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In Periodic Structures, Hwang gives readers a comprehensive understanding of the underlying physics in meta-materials made of periodic structures, providing a rigorous and firm mathematical framework for analyzing their electromagnetic properties. The book presents scattering and guiding characteristics of periodic structures using the mode-matching approach and their applications in electromagnetic engineering.

- Provides an analytic approach to describing the wave propagation phenomena in photonic crystals and related periodic structures
- Covers guided and leaky mode propagation in periodic surroundings, from fundamentals to practical device applications
- Demonstrates formulation of the periodic system and applications to practical electromagnetic / optical devices, even further to artificial dielectrics
- Introduces the evolution of periodic structures and their applications in microwave, millimeter wave and THz
- Written by a high-impact author in electromagnetics and optics
- Contains mathematical derivations which can be applied directly to MATLAB® programs
- Solution Manual and MATLAB® computer codes available on Wiley Companion Website

The book is primarily intended for graduate students in electronic engineering, optics, physics, and applied physics, or researchers working with periodic structures. Advanced undergraduates in EE, optics, applied physics applied math, and materials science who are interested in the underlying physics of meta-materials, will also be interested in this text.
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My objective in writing this book is twofold. The first objective is to build up a firm and rigorous mathematical framework, namely a mode-matching approach and transmission-line network representation, for analyzing the typical problem of wave processes involved in periodic structures ranging from the one-dimensional to the three-dimensional. The second objective is to allow the reader to understand that most of the interesting phenomena occurring in contemporary periodic structures can be clarified using existing classical electromagnetism, such as coupled-mode theory, phase-matching condition, and so on. I believe that some people will question the mode-matching method in handling periodic structures in regard to two disadvantages: the slow convergence rate for metallic structures and the infinite structure under consideration. I confess that how to solve the electromagnetic fields in a finite periodic structure is not my major concern in this book; after all, there are many well-developed commercial software packages available nowadays based on several numerical methods (e.g., the finite-difference time (frequency) domain, the finite-element method, the integral equation with moment method, the finite-integration method) for dealing with real-world electromagnetic problems in the microwave and optical communities.

As to the convergence problem, the modal transmission-line approach to be elucidated in this book can tackle the task. The mode-matching method has its own advantages in facilitating the understanding of electromagnetic fields using the concept of the modal solution; for example, the eigenwave solution in a periodic medium can reveal information concerning the mode phase- and dispersion-relation; the mode dispersion relation in a gratings-assisted waveguide can be directly determined by solving the generalized eigenvalue equation rather than by extracting from electromagnetic fields.

Some of the subject matter in this book has been presented for several years as a one-semester course in the Graduate School of National Chiao-Tung University, Hsinchu, Taiwan. The prerequisites for the course are a knowledge of linear algebra and electromagnetic theory. I have not attempted the task of referring to all relevant publications. The lists of books and journal and conference articles in the reference sections at the ends of each chapter are representative, but are by no means exhaustive.

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Introduction

1.1 Historical Perspective on the Research in Periodic Structures

The class of periodic structures has been a subject of continuing interest in the literature. The main effort in the past has been on the scattering and guiding of waves by one-dimensional (1D) periodic structures. In particular, the microwave field has employed periodic structures in many different applications, of which a few examples are linear accelerators, slow-wave structures in microwave tubes, filters, artificial dielectrics, slot arrays, phase-array antennas, frequency-selective structures, leaky-wave antennas, and so on. On the other hand, the 1D periodic structure also has its own applications in optical engineering; for example, in dielectric gratings used in integrated-optics applications (Tamir 1975, 1979) (diffraction gratings for beam splitting, grating couplers, and leaky-wave structures).

In recent years, considerable attention has been focused on the numerical and experimental studies of wave phenomena associated with two-dimensional (2D) and three-dimensional (3D) structures, and many applications have been demonstrated. Most of the potential applications were found and developed in the optics community, such as the photonic crystal. A photonic crystal contains dielectric or metallic inclusions periodically arranged in a 2D or 3D lattice pattern; these mimic a natural crystal with a small and basic building block of atoms or molecules repeated in space. The periodic nature of the dielectric function results in the simultaneous reflection of waves from each period, producing a stop-band where the wave propagation is forbidden. Such behavior is analogous with the electronic band gap in electronic materials, which is caused by introducing a gap into the energy band structure of a crystal so that electrons are forbidden to propagate with a certain energy in a certain direction (Kittle 1986). A semiconductor is the best representative having a complete band gap between
the valence and conduction energy bands. Therefore, engineering an artificial crystal with a complete stop-band, which can extend its gap to all possible directions, becomes a hot spot of research interest. With the complete stop-band (or photonic band gap), we may design a photonic crystal waveguide to guide light in a channel surrounded by photonic crystals, which are operated in the stop-band or below-cutoff condition, even in a tight corner (Joannopoulos et al. 1995). Additionally, the complete stop-band can also be employed to design a planar and linear defect. By tailoring the size of defect in a photonic crystal, the single mode can be pinned to the defect, producing the so-called photonic crystal microcavities (Joannopoulos et al. 1995; Soukoulis 2001).

More recently, researchers found that through engineering the commonly used dielectric materials – for example, constructing resonators and wire arrays (with subwavelength period) made of metallic strip lines printed on a dielectric substrate (Shelby et al. 2001; Smith et al. 2000) – one may obtain artificial materials that have properties that may not be found in nature, such as having simultaneously negative $\varepsilon_{\text{eff}}(\omega)$ and $\mu_{\text{eff}}(\omega)$ in a certain frequency band. Such an artificial material is called a metamaterial, which gains its electromagnetic properties from the structure instead of the chemical composition. Interestingly, the “unusual” electromagnetic property of negative refraction (negative refractive index) caused by the simultaneous existence of negative permittivity and permeability were demonstrated numerically (Engheta and Ziolkowski 1964) or experimentally (Eleftheriades and Balmain 2005).

Regarding the mathematical method for analyzing the electromagnetic field in periodic structures (e.g., microwave periodic structures, optical gratings, photonic crystals, and metamaterials), several numerical methods are developed in the literature. Sigalas et al. (2001) calculated the transmission and absorption of electromagnetic waves in 2D and 3D metallic band-gap structures by using the transfer-matrix method. The finite-difference method was employed by Smith et al. (1995) to study the defect-mode resonant cavity in a 2D metal photonic lattice. The plane-wave expansion of the fields and dielectric function is commonly employed to calculate the band structure within an infinite structure (Sakoda 2004). However, its convergence becomes poor when the contrast of the dielectric constant is large, particularly for the metallic system. Nicorovici and McPhedran (1994) and Chin et al. (1994) employed an analytical formulation using Green’s function based on lattice sums to calculate the scattering characteristics of 2D photonic crystals consisting of an array of circular metallic cylinders. They used the cylindrical harmonics, which inherently satisfy the boundary conditions at the interfaces, to expand the fields in the periodic structure. Thus, highly accurate results with relatively low computation resources are permitted to investigate the large structure. Moreover, the analytical, numerical, and computational methods used in the modeling of scattering and guiding problems in modeling photonic crystals, which include the scattering matrix methods, multipole theory, mode-matching technique, the finite-difference frequency-domain mode solution, and the finite-difference time-domain method, were reported by Yasumoto (2005).
1.2 From 1D Periodic Stratified Medium to 3D Photonic Crystals: An Overview of this Book

As mentioned in the Preface of this book, I do not attempt to teach you any novel methods to solve the electromagnetic fields in complex periodic structures or to improve the numerical accuracy and convergence rate. Contrarily, the structures under consideration in each chapter are simple but essential; they can be regarded as building blocks in periodic structures. In fact, for a structure with a complex unit cell pattern, the staircase approach can be exploited to partition them into a cascade of multiple 1D or 2D periodic layers.

On the basis of the building-block approach used in this book, it is worth noting that a structure to be discussed in an ensuing chapter is, in fact, evolved from that being demonstrated in the previous chapter; for instance, the 2D periodic structure under the situation of in-plane propagation depicted in Chapter 4 can be regarded as a stack of 1D dielectric grating layers illustrated in Chapter 3, and so on. A brief introduction to each chapter is now given in the following subsections.

1.2.1 Chapter 2: Wave Propagation in Multiple Dielectric Layers

The field solutions in a uniform dielectric medium will be introduced first. If the wave propagation direction is designated as the $z$-axis, then the two bases in the vector space, which can expand an arbitrary electric and magnetic field (eigenfunction expansion), will be rigorously proved those relate to the propagation vector of a plane wave on the transverse ($x$–$y$) plane and are independent of the spatial position ($x$ and $y$). Such a complete set is in the so-called eigen coordinate system rather than the spatial coordinate system that we are familiar with. The electromagnetic fields are expressed in terms of the superposition of the above-mentioned basis along the transverse plane; their expansion coefficients are $z$-dependent functions. Furthermore, the expansion coefficient is referred to as the voltage-wave function for the tangential electric field, while the current-wave function is for the tangential magnetic field (Felsen and Marcuvitz 1973). As a consequence, the transmission-line network representation is developed for expressing the general field solution of electromagnetic fields in a uniform medium (Oliner 1963).

From the electromagnetic boundary condition, the tangential components of the electric and magnetic fields are continuous at the interface between two dielectric media, resulting in the voltage and current waves being continuous at the interface. In doing so, the electromagnetic fields in a structure consisting of multiple parallel dielectric layers can be expressed in terms of a cascade of transmission-line sections (Figure 1.1). The scattering characteristics of a multilayered structure can be easily determined by successively cascading the input–output relation (recursive-impedance method, transfer-matrix method and scattering-matrix method) of each layer. Moreover, the transverse resonance technique will also be introduced to solve the mode
dispersion relation of an open or closed\textsuperscript{1} wave-guiding structure consisting of multiple parallel dielectric layers.

\textbf{1.2.2 Chapter 3: One-Dimensional Periodic Medium}

Specifically, the 1D periodic structure with the unit cell containing several parallel dielectric layers can be regarded as a special case in the above-mentioned problem category. As will become clear later on, the reflection due to the stop-band in a 1D periodic structure with finite periods will be demonstrated.

The Bloch–Floquet (periodic) boundary condition for an infinite periodic medium will then be introduced. The electromagnetic fields problem in an infinite 1D dielectric periodic medium, which contains a holographic grating with the dielectric function $\varepsilon(x) = \varepsilon_g\{1 + 2\delta \cos[(2\pi x)/d]\}$ and the parallel dielectric layers shown in Figure 1.2, subject to the periodic boundary condition, will be converted into an electromagnetic boundary-value problem presented in the form of an eigenvalue equation. Furthermore, the eigenvalue and its associated eigenvector stand for the propagation constant\textsuperscript{2} of the wave supported in the periodic medium and the source-free electromagnetic fields in the unit cell, respectively. Regarding the mathematical details for resolving the

\textsuperscript{1} A n open structure means that the cross-section (or transverse plane), which is perpendicular to the wave-guiding (longitudinal) direction, is unbound, while the closed structure is bound.

\textsuperscript{2} The propagation constant mentioned here may be that of the Bloch wave along the periodicity direction or that along the tangential (interface) direction.
Figure 1.2 A 1D periodic medium consisting of two parallel dielectric layers in a unit cell; the direction perpendicular to the periodicity is assumed to be infinite in extent.

eigenvalue problem mentioned previously, two commonly used approaches\(^3\) including the modal transmission-line (Peng 1989) and Fourier-series expansion (Moharam and Gaylord 1981) will be illustrated. Both approaches have their respective pros and cons; for instance, for the periodic medium having a extremely large dielectric contrast in the constituent media or consisting of negative permittivity or permeability, the modal transmission-line can obtain the exact eigenvalues; however, the Fourier-series-based approach needs a considerably large number of Fourier bases for expanding the periodic dielectric function but obtaining a solution with some errors. Although the error can be reduced by increasing the number of Fourier bases, it certainly needs much more computation resources in dealing with the problem and inevitably degrades the computational efficiency. Contrarily, the eigenvalue in the Fourier-series-based approach can be directly obtained by solving the matrix eigenvalue equation without resorting to complex roots searching; however, it is inevitable in the modal transmission-line approach. For the commonly seen structure in optical engineering, the dielectric contrast is moderate (excluding the case with a plasma medium). Therefore, the Fourier-series-based approach can handle most of the problems.

Additionally, the phase diagram, which draws the relation among the propagation constants along the \(x\)- and \(z\)-axes and \(k_o\), is demonstrated for understanding the coupling between space harmonics, and also the propagation characteristics concerning

\(^3\) The rigorous coupled-wave analysis approach was developed by Moharam and Gaylord. The modal transmission line approach was developed by Peng et al. (1975). More than a decade later, Peng extended the original mathematical formulation, which can only deal with the single polarization problem, into one able to handle the oblique incident of plane wave, simultaneously considering the hybrid TE–TM modes.
the phase and group velocities of the wave in a 1D periodic medium. Another tremendously useful plot, the Brillouin diagram (also termed a $kd$ versus $\beta d$ diagram), is also introduced to allow us to understand the electromagnetic coupling between space harmonics in the frequency domain. Specifically, the stop-band (band-gap or forbidden band), wherein the wave experiences a strong reflection, caused by the contra-flow interaction, will be addressed.

In Figure 1.2, the structure is assumed to be infinite in extent and, therefore, the general field solutions are obtained by solving the eigenvalue problem. If we truncate the original structure into a finite thickness along the $z$-axis but retain the infinite periods along the $x$-axis, the finite-thickness periodic structure will become a so-called grating, which is commonly used in optical engineering. The grating is usually mounted on the top of a multilayered structure, as depicted in Figure 1.3. Such a structure can be employed as a diffraction grating, which splits and diffracts the incident wave into several beams traveling in different directions, or serve as a grating-assisted waveguide, which is able to couple the incident light into the dielectric waveguide beneath the grating later, as shown in this figure; reciprocally, the waveguide mode, such as a surface wave, supported in the uniform dielectric layer can be converted into the space (leaky) wave radiating into the surrounding medium. Additionally, the reflector or deflector can also be designed based on the grating waveguide. In this book, the general electromagnetic-field solutions (hybrid TE–TM modes) will be taken into account and the electromagnetic coupling between the two polarizations will also be addressed.

1.2.3 Chapter 4: Two- and Three-Dimensional Periodic Structures

We stack up and interleave the finite-thickness 1D periodic structure (or grating) – which is on the top of the multiple dielectric layers depicted in Figure 1.3 – and uniform separator to form a 2D periodic structure, as shown in Figure 1.4.

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4 Here, the 2D periodic structure means that it has periodicities along two directions.
Figure 1.4  By stacking up and interleaving a 1D grating layer (which is sketched in Figure 1.3) and a uniform separator, a 2D periodic structure can be achieved. In this figure, the periods along the horizontal and vertical directions are denoted as $a$ and $b$, respectively. The period along the horizontal direction is assumed to be infinity. The number of 1D grating layers along the vertical direction is also infinity.

In-plane propagation is considered,\textsuperscript{5} the input-output relation of the unit cell consisting of a 1D periodic layer and a uniform separator can be readily determined by cascading the generalized scattering matrices (Hall et al. 1988) of the 1D grating and uniform dielectric layer. Furthermore, for an infinite stack of 1D periodic layers (becomes a 2D periodic medium), we can find the eigenwave (source-free) solution supported in such a medium of infinite extent. By imposing the the periodic boundary condition (along the $z$-axis) on the input-output relation of the unit cell, we can obtain a generalized eigenvalue equation with the eigenvalue relating to the propagation constant along the $z$-axis. As usual, the phase diagram and dispersion (Brillouin) diagram will be demonstrated. Notably, in comparison with the 1D periodic case, the mode coupling in a 2D periodic structure is much more complex, since the space harmonics along the two directions, say the $x$- and $z$-axes, have to be taken into account, at the same time. Specifically, the slanted stop-band, which is caused by the coupling between the space harmonics along the two periodicities, will be presented and elucidated.

For the above-mentioned 2D infinite periodic structure, the wave propagation direction is limited to the $x$-$z$ plane. The mathematical formulation demonstrated there cannot treat the problem that the energy propagates along the $y$-axis; namely, the

\textsuperscript{5} In-plane indicates that the wave is propagating along the $x$-$z$ plane.
out-of-plane propagation. Let us now redraw the 2D periodic structure as shown in Figure 1.5; the structure is infinite in extent along both $x$- and $z$-directions. The dimension along the $y$-axis can be a finite number. Under this situation, the propagation constant along the $y$-axis, $k_y$, does not vanish. If we consider a plane wave is incident from the top of the structure, the energy propagation along the $y$-axis shall be of considerable concern. Moreover, since the propagation constants along the transverse plane, $k_x$ and $k_z$, are prescribed by the incident plane wave, the problem will be one of how to figure out $k_y$. In view of the problem description, the mathematical procedures will be totally different from those mentioned previously. To this end, the double Fourier-series expansion approach will be developed. The electric and magnetic fields in a 2D periodic medium will be decomposed into the superposition of plane-wave solutions invoking the periodicities along the two axes; therefore, the infinite number of space harmonics along the $x$- and $z$-axes must be taken into account. The input-output relation for a finite-thickness 2D periodic layer (let us imagine the structure shown in Figure 1.5 is confined along the $y$-axis) along the $y$-axis, as usual, will be presented in the form of generalized scattering matrix. As a consequence, we are able to evaluate the scattering properties of a 2D grating-assisted waveguide, as well as its guidance characteristics, by invoking the transverse resonance technique.

Regarding the 3D periodic structure shown in Figure 1.6, it can be considered as a stack of 2D periodic structures. Since we have built up the input-output relation of a finite-thickness 2D periodic structure, the scattering characteristics of a 3D periodic structure consisting of finite 2D periodic layers can be readily determined by successively cascading the generalized scattering matrix of each layer.

Additionally, if the layer number along the $y$-axis is infinity, we have a 3D periodic medium of infinite extent. Therefore, how to obtain the source-free solution (eigen-
value problem) will be a major concern. As usual, by imposing the periodic boundary condition along the $y$-axis over a period, we can set up a generalized eigenvalue equation; however, the matrix size is much larger than that in the 2D periodic structure. The 3D phase diagram, which plots the relation among $k_x$, $k_y$, $k_z$ for a given wavelength (or $k_0$), will be conducted; the dispersion diagram will also be figured out. Specifically, the stop-band behavior caused by the cross-polarization coupling between the TE-like and TM-like space harmonics presented in the dispersion diagram will be carefully examined and elucidated by the reflection property of the corresponding 3D periodic structure, however, with a finite thickness along the $z$-axis.

1.2.4 Chapter 5: Introducing Defects into Periodic Structures

In this part we aim at studying the types of modes supported in a 1D periodic medium with defects. As shown in Figure 1.7, several uniform layers are removed from a complete 1D periodic medium. The structure is a so-called the 1D periodic structure with plane ($y$–$z$ plane) defect. Initially, such a structure was designed to serve as a waveguide for guiding the microwave signal. Significantly, the two semi-infinite periodic stratified media are operated in the stop-band or below-cutoff condition, so that the mode is bouncing back and forth between the two 1D periodic side walls. The transverse resonance technique will be employed to calculate the mode dispersion relation. The two types of modes with sinusoidal and parabolic variation along the transverse direction ($x$-axis) within the defect channel are obtained; the former is the parallel-plate-waveguide-like mode and the latter is the surface state with the electric field decaying in both directions.
When one or more 1D periodic layers are carved out from the 2D periodic medium shown in Figure 1.4, the structure becomes the one depicted in Figure 1.8. Alternatively, the structure can also be regarded as a parallel-plane channel sandwiched by a pair of 2D periodic structures. As usual, the scattering properties of the structure can be determined by successively cascading the input-output relation of the 1D periodic layer and uniform dielectric slab.

In addition, the guidance properties of the waveguide mode supported in the parallel-plate-waveguide-like structure are also taken into account. In view of the symmetry of the structure with respect to the central plane in the defect channel, the open-circuit (terminated by a perfect magnetic conductor (PMC)) and short-circuit (terminated by a perfect electric conductor (PEC)) bisection, as depicted in Figure 1.9, can be applied to simplify the mathematical analysis. Through the numerical calculation, the mode dispersion relations, which generally are complex numbers with the phase and attenuation constants respectively denoted as $\beta$ and $\alpha$, will be obtained. The attenuation of the wave during propagation means power leakage into the surrounding medium, resulting in a leaky wave. The leaky-wave and beam-steering phenomena will be systematically examined in detail.

Since such a structure is similar to a Fabry–Perot filter with the 2D periodic layers served as the partially reflection mirror, the wavelength-selective transmission is in presence behaving as creating narrow passbands in a forbidden band. In this book, we will build up a framework for relating the scattering and guiding characteristics of the defected periodic structure. Specifically, the phase-match...
Figure 1.8 We carve out a row of a 1D periodic layer from a 2D periodic medium shown in Figure 1.4 to obtain a parallel-plane waveguide (defect region) sandwiched by a pair of 2D periodic structures.

condition between the tangential-component of the phase constant of an incident plane wave and the phase constant of the defect-channel mode is carefully inspected for predicting the wavelengths of resonance transmission, which is attributed to the re-radiation of the leaky-mode excited by the incident plane wave due to phase-match condition.

1.2.5 Chapter 6: Periodic Impedance Surface

The Huygens principle states that the field solution in a region $V'$ is completely determined by the tangential fields specified on the surface $S'$ enclosing $V'$. Let us imagine a periodic structure with a period along the $x$-axis, as depicted in Figure 1.10; the structure is assumed to be infinite in extent along the $x$-axis. Such a periodic structure consists of metallic corrugation and metal plate in a unit cell, where the metallic corrugation can be regarded as the ppwg short-circuited at its bottom end. Since the metallic periodic structure under consideration is infinite in extent, the scattering and guidance waves only exist in the upper half-space. The Huygens principle mentioned previously can be extended to the impedance boundary condition (IBC) prescribed on an artificial surface above the structure under consideration. Referring to Figure 1.10, the IBC is defined on the top of the 1D metallic periodic structure. We will show that if the input impedance looking into the slot (the region between two metallic corrugations shown in Figure 1.11) is employed to approximate the periodic IBC (or
Figure 1.9  Owing to the symmetry of the structure with respect to the central plane in the defect channel shown in Figure 1.8, the open-circuit bisection (OCB) or short-circuit bisection (SCB) can be applied to simplify the structure redrawn in this figure. In this figure, the zigzag pattern inside half of the defect channel denotes the waveguide mode is bouncing back and forth between the two interfaces; the termination plane and waveguide side wall are made of 2D periodic structures. The wave leakage into the surrounding medium means that the mode has a complex propagation constant, which is decaying along the guiding axis while the amount of attenuating power is radiating into the air.

Figure 1.10  A periodic impedance surface with the impedance approximated by the input impedance looking into the corrugation (parallel-plate-waveguide (ppwg)-like structure); the width of the ppwg width is assumed to be electrically small (sub-wavelength width, which is smaller than operation wavelength). The corrugation length is around a quarter wavelength; the input impedance in this situation is an extremely high impedance for the TM wave considered in this example.
short-circuited transmission-line corrugated metal surface

Figure 1.11 A corrugated metal surface and its transmission-line representation: the region between two corrugations is the ppwg region with sub-wavelength width; the input impedance looking into each slot is equal to $j Z_o \tan k_o t$ for TM mode, where $k_o$ is the free-space wavenumber and $t$ is the length of the corrugation.

periodic impedance surface), their scattering results can approach the exact solutions obtained by the rigorous mode-matching approach (MMA). I would emphasize that the approach of the periodic impedance surface is rigorous; the errors are due to the approximation error of the periodic impedance surface. Additionally, the scattering analysis concerning a 2D dyadic periodic impedance surface is also considered, as the method to obtain the 2D dyadic periodic impedance surface from a real structure is beyond the scope of this book.

1.2.6 Chapter 7: Exotic Dielectrics Made of Periodic Structures

With the mathematical background for resolving those problems that contain eigenwaves (source-free solution) in a periodic medium and the scattering and guidance characteristics of a periodic structure ranging from one to three dimensions having been elucidated in the previous six chapters, no more new mathematical approaches will be presented in this chapter.

From the literature, we know that most metamaterials are made of a periodic structure with sub-wavelength periods. In microwave applications, the unit cell is usually composed of metallic strips printed on a dielectric substrate. In optics, the unit cell is made by engineering a dielectric material; the fabrication process is analogous with that of a photonic crystal. As is well known, the study of artificial dielectrics is not a new one (Kock 1948). The artificial dielectric indeed is a 2D or 3D periodic structure; however, the method of theoretical analysis only involves that of the 1D periodic
structure. As shown in Figure 1.12, an artificial dielectric (or metamaterial) made of a 2D dielectric or metallic columns array immersed in a uniform dielectric medium is presented. We will apply the well-developed Mode-Matching Approach (MMA) for investigating the wave process involved in such a medium made of periodic structures. Specifically, the essential phase relation of an eigenwave will be employed to understand the group (or energy) velocity of the Bloch wave propagating in such an effective medium. With the phase relation, we will know that the effective refractive index depends not only on frequency, but also on the angle. As will become clear later on, we may engineer the phase relation of an exotic material to manipulate the energy flow inside that material. Finally, an analytical expression regarding the effective refractive index of an artificial dielectric made of a 1D periodic structure will be illustrated.

References

Oliner, A. A. (1963) 1963 Short Course on "Microwave Field and Network Techniques": Radiating Periodic Structures: Analysis in Terms of $k$ vs. $\beta$ Diagrams, Electrophysics Department, Polytechnic Institute of Brooklyn, New York, NY.

Further Readings

Wave Propagation in Multiple Dielectric Layers

As is well known from the textbooks in electromagnetics, the plane-wave solutions in a uniform medium can be decomposed into two polarized modes. Therefore, each one can be analyzed with relative ease. However, in the presence of a discontinuity between periodic structures and a uniform medium, the single polarization mode can no longer satisfy the electromagnetic boundary condition when the plane wave is incident obliquely (out-of-plane propagation). The superposition of the two polarized modes is needed to determine the general solution. Consequently, our main effort in this section is to introduce a rigorous and systematic way for putting the two polarization modes back together. By invoking the mathematics of dyadic and eigenvalue problems, we will prove that the two orthogonal eigenbases correspond to TE and TM waves and the two associated eigenvalues relate to the wave (or characteristic) impedances of the two polarizations. Moreover, the wave propagating along the longitudinal direction can be interpreted using the commonly used transmission-line equation. Namely, through the rigorous mathematical process described in this chapter, the electromagnetic field problem in a uniform layer can be transformed into the electric circuit analogy using the transmission-line representation.

2.1 Plane-Wave Solutions in a Uniform Dielectric Medium

We begin with the Maxwell equations in the frequency domain, where $\mu$ and $\epsilon$ are the permeability and permittivity of the dielectric medium. Moreover, the electrical
charge and current are assumed to be absent in the medium. Thus, the four equations are given below.

\[ \nabla \times \mathbf{E} = -j\omega \mu \mathbf{H} \quad (2.1) \]

\[ \nabla \times \mathbf{H} = +j\omega \varepsilon \mathbf{E} \quad (2.2) \]

\[ \nabla \cdot \mathbf{E} = 0 \quad (2.3) \]

\[ \nabla \cdot \mathbf{B} = 0 \quad (2.4) \]

Taking the curl operator of both sides of Equation (2.1) and substituting Equation (2.2) into its right-hand side, we obtain

\[ \nabla \times \nabla \times \mathbf{E} = k^2 \mathbf{E} \quad (2.5) \]

where \( k^2 = \omega^2 \mu \varepsilon \).

From the vector identity

\[ \nabla \times \nabla \times \mathbf{E} = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} \quad (2.6) \]

Equation (2.5) becomes

\[ \nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0 \quad (2.7) \]

Equation (2.7) contains three partial differential equations with the electric field along the \( x \)-, \( y \)-, and \( z \)-axis. The general solutions to Equation (2.7) can be written as

\[ E_i(x, y, z) = (A e^{j k_x x} + B e^{-j k_x x})(C e^{j k_y y} + D e^{-j k_y y})(E e^{j k_z z} + F e^{-j k_z z}) \quad (2.8) \]

where \( i \) represents \( x \), \( y \), or \( z \). Parameters \( k_x \), \( k_y \), and \( k_z \) are respectively the propagation constants along the \( x \)-, \( y \)-, and \( z \)-axes, which satisfy the following relation:

\[ k_x^2 + k_y^2 + k_z^2 = k^2 \quad (2.9) \]

Moreover, parameters \( A \), \( B \), \( C \), \( D \), \( E \), and \( F \) are constants to be determined by the electromagnetic boundary conditions. In a multilayered environment, since the energy transmission across the layers is of primary concern, it is natural to define the \( z \)-axis as the longitudinal direction, while the \( x \)-\( y \) plane is designated as the transverse plane perpendicular to the \( z \)-axis. Additionally, owing to the phase-matching condition, the tangential components of the propagation vector in each layer remain the same as those of the incident plane wave. Here, we assume that the phase constants of the incident plane wave along the transverse plane are \( k_x \) and \( k_y \), respectively. Thus, the electric and magnetic field components can be written as

\[ E_i(x, y, z) = E_i(z) \exp(-j k_x x) \exp(-j k_y y) \quad (2.10) \]

\[ H_i(x, y, z) = H_i(z) \exp(-j k_x x) \exp(-j k_y y) \quad (2.11) \]