

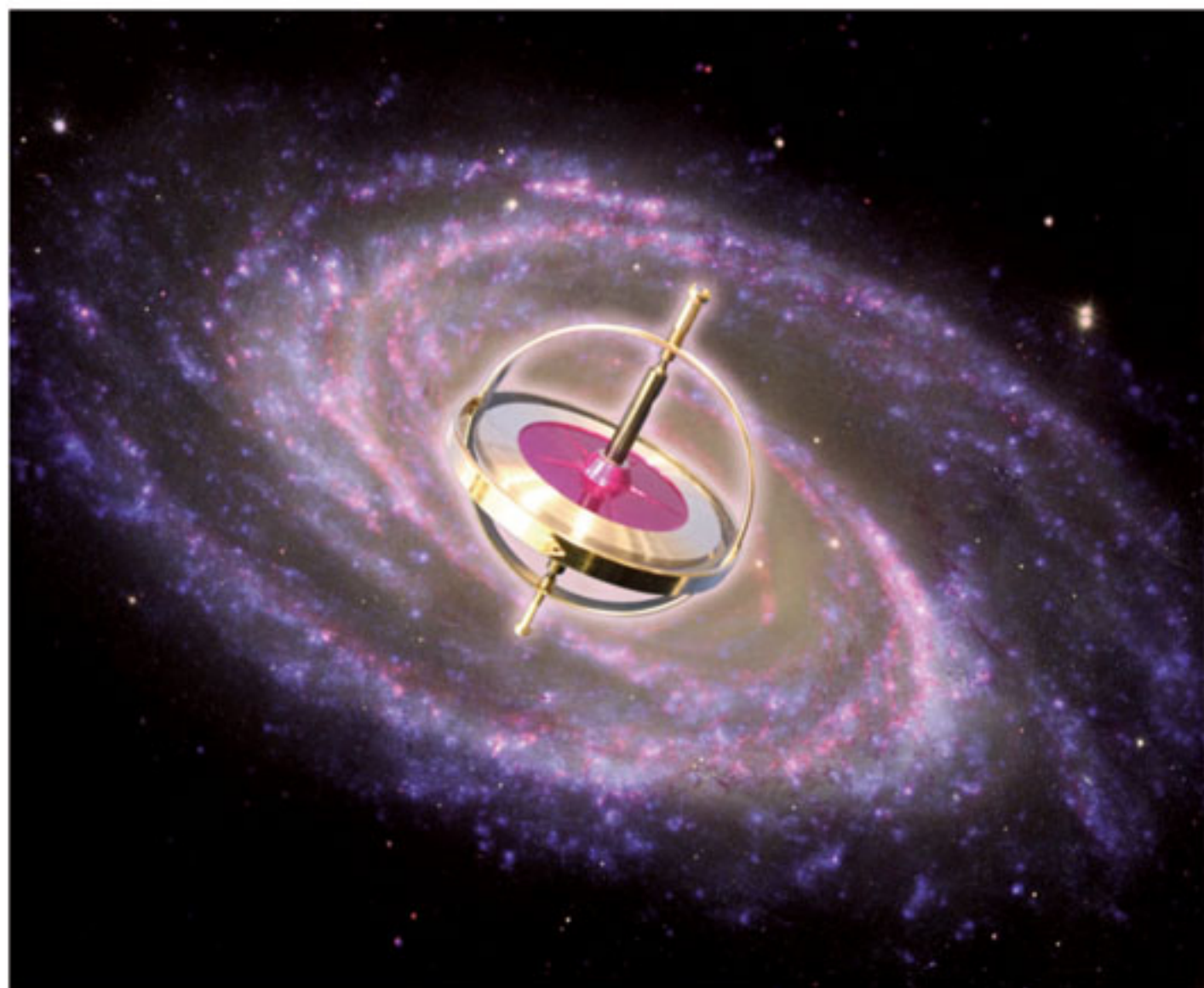
Yukio Ishida and Toshio Yamamoto

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A Modern Treatment with Applications

Second, Enlarged and Improved Edition



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Dedicated to the memory of Professor Yamamoto

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Foreword to the First Edition

The dynamics of rotating machinery has been extensively investigated during the past century. Many people in England, Germany, and the United States, in the late nineteenth and early twentieth centuries, studied the fundamental concepts associated with rotordynamic systems and investigated the effects of many types of mechanisms on these systems. The published material in this area diminished significantly between the start of World War I and the end of World War II. With the development of the gas turbine as a commercially viable engine after World War II, the need to better understand the dynamics of high-speed rotating systems became critical. Subsequent development of the digital computer assisted the development of many highly sophisticated procedures for analyzing, simulating, designing, and testing rotor systems. Many talented and dedicated people, throughout the world, have contributed to a better understanding of the dynamics of high-speed machinery during the last half of the twentieth century.

During the approximately 20-year interval defined by the end of World War II and the beginning of the digital computer revolution, three people stand out as significant contributors to the rotordynamic literature. F. M. Dimentberg in Russia documented his work in 1961 with the publication of a book entitled *Flexural Vibrations of Rotating Shafts*. In Czechoslovakia, Alex Tondl documented his studies in 1965 with the publication of a book entitled *Some Problems of Rotor Dynamics*. During the same period, Toshio Yamamoto worked independently and conscientiously at his Nagoya University laboratory on numerous topics related to rotordynamic systems. His work focused on fundamental concepts related to the dynamics of high-speed machinery and included meticulous laboratory test rigs to back up his analytical predictions. All of this work was done without the benefit of a high-speed digital computer or sophisticated electronic test equipment. Some of his work was documented in 1954 and 1957 with the publication of two *Nagoya University Memoirs* entitled "On the Critical Speed of a Shaft" and "On the Vibrations of a Rotating Shaft." These works and other publications of Dr Yamamoto, were not, however, circulated widely outside Japan. Thus it has taken the engineering world a little longer to recognize the genius of this dedicated and talented researcher. This book includes presentations on several of the original topics investigated by Dr Yamamoto and Dr Ishida, and also many important

contributions by several other rotating machinery analysts and researchers around the world.

In 1975, Yukio Ishida graduated as one of Toshio Yamamoto's doctoral students at Nagoya University. Since that time, Dr Ishida has both independently and in collaboration with Dr Yamamoto made many additional and significant contributions to the area of rotordynamics. In particular, he and Dr Yamamoto have paid special attention to the effects of various nonlinear mechanisms on the dynamic behavior of rotor systems. Dr Ishida has also extensively investigated the use of modern digital signal processing as a valuable tool for the analytical and experimental investigation of vibrations in rotordynamic systems. Several topics associated with these more recent studies are included in this work. It is highly fitting that Dr Yamamoto and Dr Ishida are finally documenting a significant portion of their half a century of rotordynamic expertise with the publication of this book.

Professor and Chair
Department of Engineering
Texas Christian University
1991–1998

Harold D. Nelson, Ph.D.

Preface to the First Edition

Rotating machinery, such as steam turbines, gas turbines, internal combustion engines, and electric motors, are the most widely used elements in mechanical systems. As the rotating parts of such machinery often become the main source of vibrations, correct understanding of the vibration phenomena and sufficient knowledge of rotordynamics are essential for considering adequate means to eliminate vibrations. However, compared to rectilinear vibrations, the whirling motions of rotors seem difficult for students and engineers to understand in the beginning.

Studies of rotordynamics started more than 100 years ago. At that early stage, the primary concern was prediction of the resonance rotational speeds called *critical speeds*. As the normal operation speed increased above the critical speeds, engineers encountered various kinds of new problems, and rotordynamics developed through their efforts to overcome these technical difficulties. It is a very difficult task to master the entire range of rotordynamics by reading various technical papers. Although many standard books on vibrations contain a chapter explaining rotordynamics, the content is insufficient for practicing engineers. Recently, many excellent books on rotordynamics have been published by distinguished researchers. However, some of them are too practically oriented and others contain only recent technical topics.

The scope of this book includes most branches of rotordynamics. But it is intended especially to provide a detailed explanation of the basic concepts of rotordynamics because correct understanding becomes a strong tool with which to tackle vibration problems.

In Chapter 1, a classification of rotating shaft systems and a historical perspective are given.

In Chapter 2, the fundamentals of rotordynamics are explained using rotor models consisting of a massless elastic shaft and a rigid disk. The key ideas common to all branches of rotordynamics, such as critical speed, gyroscopic moment, whirling motion, and frequency diagrams, are explained. Balancing a rigid rotor is also discussed.

In Chapter 3, the dynamic analysis of a rotor with distributed mass is presented. Gyroscopic moment inertia and rotary are considered in the equation of the motion. Balancing a flexible rotor is also explained.

In Chapter 4, vibrations of an asymmetrical shaft with unequal stiffness and an asymmetrical rotor with unequal moments of inertia are discussed. Such systems with rotating asymmetry in stiffness or inertia belong to the category of parametrically excited systems, and the coefficients of the equations of motion are functions of time. The most distinguished feature of these systems is the appearance of an unstable zone at the major critical speed. Analysis of free vibrations, forced vibrations, and unstable vibrations is explained in this chapter.

In Chapter 5, various types of nonlinear resonances are considered. Rotating shaft systems have many elements that can cause nonlinearity in the shaft restoring forces. Several methods to obtain solutions for subharmonic resonances, combination resonances, and chaotic vibrations are presented. Nonlinear phenomena, such as jump phenomena, hysteresis phenomena, and period doublings, are shown. The dynamic behavior of a cracked shaft, which has attracted the interest of researchers in the field of vibration diagnosis, is also explained.

In Chapter 6, the effects of internal damping on shaft stability are explained. Owing to internal friction in the shaft material and dry friction between rotating components, self-excited vibrations appear in a wide speed range above the major critical speed. Expressions for such internal damping forces and the characteristics of self-excited vibrations are investigated.

In Chapter 7, nonstationary vibrations during transition through critical speeds are explained. Such nonstationary phenomena become a matter of great interest when the rated speed of a rotating machine is above the major critical speeds. Nonstationary phenomena are highly dependent on the magnitude of the driving torque of a motor. Interaction of the oscillating system with the energy source occurs, especially when the driving torque is small.

In Chapter 8, vibrations due to various machine elements are outlined. Ball bearings, bearing pedestals, universal joints, and couplings are the elements widely used in rotating machinery and may cause vibrations. In practical operations, an understanding of the characteristics of these machine elements is indispensable.

In Chapter 9, various kinds of flow-induced vibrations are explained. Oil whip in journal bearings, steam whirls in turbines, and vibrations of a hollow rotor partially filled with liquid are typical phenomena caused by liquid flow.

In Chapter 10, the finite element method is explained. In the analysis of a complex-shaped rotor in a practical machine, theoretical derivation of the equations of motion is impossible. Therefore, numerical procedures suitable for computer calculation are adopted instead. One of the most widely used means in the analysis of complex structures is the finite element method. This chapter describes how to apply this method to analyze rotating shaft systems.

In Chapter 11, another representative computational method called the *transfer matrix method* is explained. This method is especially suitable for the analysis of rotor systems.

In Chapter 12, a digital signal processing technique called complex fast Fourier transform (complex-FFT) is explained. In contrast to the ordinary FFT, this technique can distinguish between forward and backward whirling motions. The

complex-FFT method is useful especially for the experimental study of rotor whirling motions.

Nagoya, Japan
January 2001

Toshio Yamamoto
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Preface to the Second Edition

Ten years have passed since the first edition of this book was published in 2001. I was very pleased to learn that the first edition has come to be used as a text in some graduate schools, and I am grateful to have received valuable comments and questions from many readers about the contents. The highly constructive advice from readers has helped us to polish the interpretations and explanations contained in this text and motivated us to publish this second edition. The major objective of our revisions is to first discuss additional subject matter of interest to engineers in the field of rotordynamics. Furthermore, we also decided to include additional case histories in order to illustrate the importance of the contents; exercises have also been included to aid readers' understanding of the text. The additional chapters cover topics such as balancing (Chapter 4), vibration suppression (Chapter 11), steam turbines and windmills (Chapter 12), cracked rotors (Chapter 13), and magnetic bearings (Chapter 17). In addition, Chapter 9 on vibrations of mechanical elements is extensively revised. It is hoped that these revisions will assist readers in their endeavor to develop a holistic understanding of rotordynamics.

In the second half of the twentieth century, the power of turbines increased very rapidly, and accordingly, the theory of rotordynamics developed remarkably during this time. Now, many of the researchers and engineers who first engaged in the design of these turbines are gradually retiring. I am afraid that their technology and "know-how," which have been accumulated through many failures and successes, may be lost over time. Owing to the commercial demand in recent times for the shortening of the development period, many engineers have come to depend on commercial software, and thus opportunities to learn from practice seem to me to be decreasing. It is our greatest wish that this book should help to transfer the contributions of our predecessors to the next generation of rotordynamists.

Finally, Professor Toshio Yamamoto, the coauthor of the first edition of this book, regrettably passed away in 2007 at the age of 86. I believe that he would be pleased at the publication of this new edition, although he was unable to make these changes by his own hand. With this in mind, I dedicate this book to Professor Yamamoto with respect to and in memory of his great achievements in the field of rotordynamics.

Nagoya, Japan
January, 2012

Yukio Ishida

Acknowledgments

The first edition and this second edition were developed out of lecture notes used in our undergraduate and postgraduate courses. The information contained in this book is based not only on our own work but also on the work of many excellent pioneers and leaders in the field of rotordynamics. We hereby record our sincere gratitude to all these distinguished scholars. We also wish to express our special thanks to Professor Harold D. Nelson, formerly of Texas Christian University, for writing the Foreword and to Professor Ali H. Nayfeh, of Virginia Polytechnic Institute and State University, for recommending and acting as series editor for this publication. Further, we would like to thank Professor Takashi Ikeda of Hiroshima University, Professor Imao Nagasaka of Chubu University, Professor Takao Torii of Shizuoka University, Professor Tsuyosi Inoue of Nagoya University and Dr. Kentaro Takagi of Nagoya University for their review and kind suggestions. Finally, we would like to thank our former and present students for their enthusiasm and contributions to our research work at Nagoya University.

1

Introduction

1.1

Classification of Rotor Systems

In general, rotating machinery consists of disks of various shapes, shafts whose diameters change depending on their longitudinal position, and bearings situated at various positions.¹⁾ In vibration analyses, such a complex rotor system is simplified and a suitable mathematical model is adopted. In this modeling process, we must know which parameters are important for the system.

Rotating machines are classified according to their characteristics as follows: If the deformation of the rotating shaft is negligible in the operating speed range, it is called a *rigid rotor*. If the shaft deforms appreciably at some rotational speed in the operating speed range, it is called a *flexible rotor*. We cannot determine to which of these categories the rotor system belongs by considering only its dimensions. In rotordynamics, the rotating speeds that produce resonance responses due to mass eccentricity are called *critical speeds*. The deformation of a rotor becomes highest in the vicinity of the critical speed. Therefore, the range of the rated speed relative to these critical speeds determines whether the rotor is rigid or flexible.

Figure 1.1 called a *critical speed diagram* shows variations of critical speeds and vibration modes versus the stiffness of the supports for a symmetrical rotor. The left part of this figure represents values for rotors that are supported softly. In the first and second modes, the rotors do not deform appreciably but the supporting parts deflect. In this case, the rotor is considered to be a rigid rotor. As the stiffness of the supports decreases, the natural frequency of these modes approaches zero. In the third mode, the rotor deforms and it is considered to be a flexible rotor. Depending on the type of mode to be discussed, the same system may be considered as a rigid or a flexible rotor. On the right part of this figure, deformation occurs in all three modes and therefore the rotor is considered to be flexible in every mode.

In some models, disks are considered to be rigid and the distributed mass of an elastic shaft is concentrated at the disk positions. Such a model is called a

1) Rotor is often used as the general term for the rotating part of a rotating machine. The opposite term is *stator*, which means the static part of the machine.

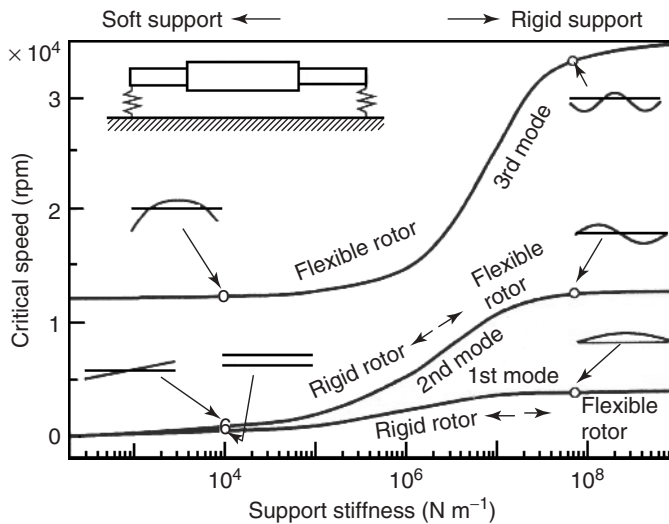


Figure 1.1 Critical speeds and mode shape versus the stiffness of the bearing support.

lumped-parameter system. If a flexible rotor with distributed mass and stiffness is considered, this model is called a *distributed-parameter system* or *continuous rotor system*. The mathematical treatment of the latter is more difficult than that of the former because it is governed by the partial differential equations.

Rotors are sometimes classified into vertical shaft systems and horizontal shaft systems. We mainly discuss the former model, however, the latter model tends to be considered in cases in which we must clarify the effect of gravity.

Rotors are sometimes classified as *high-speed rotors* or *low-speed rotors*. In this case, speed means the angular velocity or the peripheral velocity. Since high angular velocity often causes vibration, we use the term in association with the first definition in this book. The boundary between high and low is not clear and it differs depending on the situation. In some cases, the major critical speed is considered as the boundary. In ball bearing engineering, the term refers to the latter definition because it determines heating due to friction. The dimensionless parameter called *DN value* is used as an index related to the peripheral velocity. This value is defined as the product of the shaft diameter (mm) and the rotational speed (rpm). However, the unit symbol (mm · rpm) is omitted from the result of the calculation. Concerning high peripheral velocity, ball bearings and roller bearings of a main shaft of an aircraft engine have attained velocities of as much as $DN = 3 \times 10^6$ in a laboratory setting, though in practice these bearings are generally used at approximately 2.2×10^6 (Zaretsky, 1998). With regard to high angular velocity bearings, the spindle of a drill for dental use operates at approximately 50×10^4 rpm. However, since its shaft diameter is small, its *DN* value is consequently relatively small. For example, a bearing with an inner diameter of 3.175 mm used in dental drilling operates at approximately $DN = 1.6 \times 10^6$.