Electromagnetic Well Logging
Electromagnetic Well Logging

Models for MWD/LWD Interpretation and Tool Design

Wilson C. Chin, Ph.D., MIT
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Preface

Electromagnetic wave resistivity methods in Measurement-While-Drilling and Logging-While-Drilling applications, or simply MWD/LWD, are now approaching their fourth decade of practice. They are instrumental in anisotropy determination, dip angle analysis, bed boundary detection, fluid identification, and so on, and are important to economic analysis, stimulation planning, geosteering, unconventional resources and numerous exploration challenges. Essentially, phase delays and amplitude changes measured at (one or more) coil receivers relative to (one or more) transmitters are interpreted using Maxwell’s equations to provide clues related to vertical and horizontal resistivities $R_v$ and $R_h$. That said, the objectives are well-defined and easily understood. However, the general modeling problem is difficult and mathematical challenges persist.

Fifty years ago, induction logging practice and interpretation were straightforward. Formations were thick and homogeneous. Wells were vertical. Tools were concentrically placed. Azimuthal symmetry was the rule. Coils wound around fiberglass mandrels, with their planes perpendicular to the axis, implied that only $R_h$ was available from measurements. But that was fine – fluid flowed only radially toward the well so only horizontal (or radial) properties mattered. Like everything else back then, life was simple in the slow lane, and well logging and math modeling were no exception. The simple dipole model taught in physics sufficed for most purposes and log analysis was elementary.

Deviated and horizontal well drilling have redefined the problem. Coils are now wrapped around steel mandrels whose planes need not intersect tool axes at right angles. Diameters are typically several inches, greater than the thin layer thicknesses they were designed to evaluate. Drill collars navigate through narrow pay zones bounded by beds with contrasting electrical properties. Charges (acting as secondary transmitters that are responsible for polarization horns) are induced at their interfaces whose
strengths depend on conductivity differences, frequency, coil orientation and dip. Transmitters and receivers are closely situated. Needless to say, the dipole model as generations of practitioners have appreciated, is history, at least in MWD/LWD applications. A completely different approach is required. But even in recent wireline triaxial induction applications, which pose less of a challenge, dipole models may apply but not without major reformulation. Complications due to dip, layering and anisotropy still impose limits on rigor, accuracy and speed. But without good math models for these new physical phenomena, well logs cannot be properly interpreted and hardware improvements will remain on the sidelines.

Many readers know of me as a researcher with broad interests in managed pressure drilling, MWD design and telemetry, formation testing, annular flow for drilling and completions, reservoir flow analysis, and other areas related to fluid mechanics. As an engineer, I have been challenged by “things that I can see,” and this prior work has led to nine books, over forty domestic and international patents, and about one hundred papers. After all, I earned my Doctorate at the Massachusetts of Institute of Technology in aerospace engineering, and its flying vehicles and robots personified everything that an engineer would and should dream about. But on finishing my thesis and happily preparing for my grand exit, I was asked that fateful day, “What about your minor?” My minor? I thought it was Applied Math. “No, an M.I.T. education means broadening yourself. You can’t do that with something you’re good in.”

And with that comment, my Committee had me enroll in the school’s Course 8, its reputable but notoriously difficult Physics Department, one known for Nobel Prize winners, string theorists, relativity and quantum physicists, people responsible for things that I could neither see nor feel. I studied electrodynamics and I was challenged. I dreamed electric and magnetic fields instead of fluid streamlines. I thought the Navier-Stokes equations were bad, but Maxwell’s equations were worse. Nonetheless, I survived, and lived to join Boeing, where I worked in Aerodynamics Research. And thank goodness, no more electrodynamics. But the company’s powerful tools and their connection to “e/m” would lay dormant until, like sleeping giants, they would awaken and change my world and the way I thought. All of which goes to show how life works in strange ways. Nothing is predictable, but at least electrodynamics is.

In the early days of aerodynamics, point vortexes were used to model lifting airfoils. Faster flow on top meant lower pressure per Bernoulli’s equation; slower flow beneath meant higher pressure, hence net lift. These simple models eventually gave way to distributions of vortexes, sources, sinks and other singularities. These were in turn supplemented
by numerical methods solving partial differential equations, initially using *staircase grids* which modeled wing sweep, and later, less noisy *boundary conforming* mesh systems.

My interest in borehole electromagnetics was sparked by the plethora of methods that acquiesced to the demands of the general MWD/LWD problem. Models with respectable names, e.g., Born approximation, hybrid method, integral equations, magnetic dipole and geometric factor, lent an air of credibility, but nonetheless conveyed the impossibility for modeling the physical problem in its reality on its terms. About a decade ago, I observed parallels with aerodynamics methods. Why not replace point dipole models with *distributions* of current source singularities? Why not replace the *staircase grids* used to model dipping bed interfaces with *boundary conforming* meshes? Why not replace the industry’s simulators for $B$ and $E$, which gave way to nightmares associated with fictitious currents and “staggered grids,” with simpler equivalent Poisson models for vector and scalar potentials $A$ and $V$ used in aerodynamics?

The strategy was two-fold: improve geometric description, while utilizing “off the shelf” partial differential equation solvers that were sophisticated, available and highly validated. The idea was more than just practical. Nobel Prize winner Richard Feynman, at Caltech where I studied earlier, had asked why one would employ $B$ and $E$ models when $A$ and $V$ seemed more intuitive. And as it would turn out, when transmitter coils are excited harmonically, the equations for the transformed variables would turn out simpler and look just like the complex Helmholtz equations Boeing solved to model unsteady flows!

There was, however, one catch. One reputable geophysicist had attempted a similar approach to obtain unphysical results. The problem turned out to be inappropriate use of finite difference formulas. In physics, a property may be continuous and its normal derivative not, and conversely. For instance, for heat transfer in a two-medium system, temperature and heat flux continuity at the interface implies that the derivative is double-valued. In Darcy flows past thin shales, the normal derivative is continuous but the pressure is not. When discontinuities are properly modeled, and stable iterative “relaxation” methods are used to solve the transformed Maxwell equations, the key physical features inherent in borehole electrodynamics are all accounted for. In this book, we develop our methods from first principles and validate our algorithms with every model accessible in the literature to demonstrate physical consistency.

Engineering correctness is paramount, but without rapid computing and numerical stability, the best of methods are not practical. As recently as last year, one consortium known for its three-dimensional models
reported efficiency gains that reduced computing times from *three hours to two!* We have done much better. Our calculations require just *ten seconds* on typical Intel Core i5 systems and at most one minute for difficult problems. We have used every possible means to reduce our need for computing resources. For instance, variable grids mean low memory requirements, smart “in place” relaxation methods eliminate many array access issues, “finite radius coils” imply less singular fields (than point dipoles) and are associated with faster convergence, and direct zeroing of electric fields at drill collar nodes when applicable eliminates needless equation access and solution. Our algorithms, which also target thinly laminated sand-shale sequences or potential laminated pay reservoirs, are optimized for stable and fast convergence for high $R_v/R_h$.

To this, we added automated three-dimensional color graphics to display all coordinate components of real and imaginary quantities, for all $E$, $B$, $A$ and $V$ fields, plus interfacial surface charge when dealing with deviated and horizontal wells that penetrate layered media. We have provided “point summaries” in both rectangular (geology focused) and cylindrical (tool-oriented) coordinates for logging and hardware design applications. We’ve developed simple dipole, Biot-Savart, interpolation and apparent resistivity “apps” for fast comparisons, log analysis and validation. Our powerful but portable numerical engine is written in Fortran and is easily ported to other operating environments.

But through it all, we have not lost sight of the physics and the need for new hardware in a downhole environment that continually seeks greater challenges. We’ve avoided “canned” voltage formulas and opted for more general $\int_a^b E \cdot dl$ approaches to facilitate innovative receiver design. We’ve provided voltage responses automatically in our post-processing and included receiver design interfaces allowing the user to design his own antennas. And our transmitter coils need not be circular; for example, they may be oriented at any angle relative to the tool axis. Our discrete current source approach, in fact, supports alternative antenna concepts, e.g., elliptic coils, open coils and nonplanar coils which do not necessarily wrap around the collar.

Our methodology need not represent the final product, but instead, provides the highly documented foundation for more powerful and versatile tools for borehole electrodynamic analysis. However, the software in its present form is intended for petrophysicists who wish to acquire more detailed perspectives about their logging runs. Readers anxious for “hands on” results are encouraged to browse through Chapters 8 and 9 first, written to convey ideas rapidly and to facilitate applications; all of the examples
shown, in fact, were completed and documented in a single work day, with all calculations running quickly and stably the first and every time. Efficiency is enhanced by a user-friendly graphical Windows interface designed about typical petroleum workflows. A quick perusal of Chapter 9, in fact, may be useful in understanding how easily the detailed numerical results of Chapters 1-7 were created and how our claims for rapid simulation are realized in practice.

Stratamagnetic Software, LLC, was formed in 1999 to develop and commercialize this approach, “strata” conveying the subtleties associated with layering and “magnetic,” well, recalling my dreaded minor in graduate school. But as luck would have it, we worked for more than a decade in other interesting fluid-dynamics areas, e.g., formation testing, annular flow, MWD telemetry, and so on, engineering challenges that literally paid the bills. However, our vision and obsession to develop the general borehole model presented in this book have never faltered. With fast and accurate logging interpretation demand driving offshore evaluation, rapid geosteering and the hunt for unconventional energy resources, and with fluids modeling (I think, for the time being) finally behind us, the time for uncompromised borehole electrodynamics is now ... and the simulator and its complete underlying technology are yours.

Wilson C. Chin, Ph.D., M.I.T.
Email: wilsonchin@aol.com
Phone: (832) 483-6899
Acknowledgements

Our novel approach to “general three-dimensional electromagnetic models for non-dipolar transmitters in layered anisotropic media with dip,” first published in Well Logging Technology Journal, Xi’an, China, August 2000 more than a decade ago, was subject to more than the usual reviews. Wondering whether the problem I had addressed was so trivial that no one cared, or too difficult, that others would not consider it, I turned to two well known M.I.T. physicists adept at the subject.

I expressed this concern to Professor John Belcher, my former electromagnetics teacher, and he honestly replied, “To me it sounds like a very difficult problem that I would have no idea of how to approach.” That, coming from a Professor of Astrophysics, the Principal Investigator for the Voyager Plasma Science Experiment, a two-time winner of NASA’s Exceptional Scientific Achievement Medal, plus other well-deserved honors, was unsettling as it attested to the difficulty of this innocuously looking problem.

Professor Belcher would refer me to another M.I.T. colleague, Markus Zahn, Professor of Electrical Engineering, affiliated with the school’s prestigious Laboratory for Electromagnetic and Electronic Systems, and author of the classic book Electromagnetic Field Theory: A Problem Solving Approach (John Wiley & Sons, 1979). Professor Zahn’s reply is reproduced below.

“I enjoyed reading your paper because as far as I could tell everything was correct in it. By the way depending on the reciprocal frequency with respect to the dielectric relaxation time, ε/σ, or the magnetic diffusion time, σμL², the problem can be considered electro-quasistatic or magneto-quasistatic and decouples the vector and scalar potentials, generally allowing a simpler set of approximate Maxwell equations to be solved.

About fifteen years ago I did a similar but simpler analysis for Teleco using a Fourier series method under magneto-quasistatic conditions to develop a downhole method for transmitting measurable signals to the surface. This was to be an electromagnetic replacement for the pressure
pulse method. Your numerical method lets you treat great complexities in geometry.’

These comments, in Clint Eastwood’s words, would “make my day.” The method was designed to handle geometric complexity and it did: general coil and antenna topologies, arbitrary layers at dip, interfacial charge, the complete frequency spectrum, plus steel mandrels, all without the “decoupling” that Professor Zahn alluded to.

The paper was later submitted to Petrophysics (Society of Professional Well Log Analysts) and critically reviewed by David Kennedy, who suggested numerous changes to style and focus, and then, to a senior Schlumberger colleague and friend for his expert insights on borehole electromagnetics. Confident the approach would prove useful to the industry, I formed Stratamagnetic Software, LLC to commercialize the method, but would delay publication until all of the theory, numerics, validations and software could be documented. This process, given intervening work in drilling, cementing, formation testing, MWD telemetry and other areas, consumed more than ten years but would offer the challenge of producing a unique and usable product.

With deep offshore exploration becoming routine but nonetheless more challenging by the day, and with real-time, three-dimensional imaging, and difficulties with low resistivity pay and anisotropy dominating the well logging agenda, publication of this wide body of work is now timely indeed. The author is indebted to Professors John Belcher and Markus Zahn, to SPWLA President David Kennedy, and to my Schlumberger colleague and friend, for their encouragement, support and votes of confidence. He is also grateful to his doctoral thesis advisor Professor Marten Landahl, the aerospace pioneer, for suggesting an electrodynamics minor, a critical decision that would be crucial to important methods integrating fluid mechanics and resistivity logging, to appear.

Scientific progress requires more than cursory knowledge of industry models, typically presented in advertising, and more often than not, “validated” by field usage and payzone discoveries. Until companies share their methods through unrestricted technical exchanges, true progress will not be possible. Without equations, detailed math formulations and open access to software, engineers and petrophysicists remain dependent on input and output devices. The author is especially indebted, in this regard, to Phil Carmical, Acquisitions Editor and Publisher, not just for his interest in this book and other works in progress, but for his continuing support and willingness in reporting the mundane but important technical details that really matter.
1

Motivating Ideas –
General Formulation and Results

1.1 Overview

The general, three-dimensional, electromagnetic problem in layered anisotropic media with dip is solved using a full finite difference, frequency domain solution to Maxwell’s equations that does not bear the inherent limitations behind Born, geometric factor, hybrid and linearized integral equation approaches. Several important physical capabilities are introduced. First, transmitter coils, no longer represented by point dipoles, are modeled using eight azimuthally equidistant nodes where complex currents are prescribed. The coil may reside across multiple beds, a feature useful in modeling responses from thinly laminated zones; the transmitter operates in wireline “coil alone” or Measurement-While-Drilling (MWD) “steel collar” modes, with or without conductive mud or anisotropic invasion, and with or without borehole eccentricity. Because coil size and near-field details are explicitly considered, accurate simulation of charge radiation from bed interfaces (responsible for polarization horns) and Nuclear Magnetic Resonance (NMR) sensitive volume size and orientation in layered media are both assured.

Second, dipping interfaces are importantly oriented along coordinate planes, eliminating well known numerical noise effects associated with “staircase grids.” Transmitter and layer-conforming variable mesh systems, which expand in the farfield to reduce computational overhead, are automatically generated by the simulator. Third, costly performance penalties incurred by anisotropic “staggered grid” formulations are avoided in the vector and scalar potential method, where all complex Helmholtz equations are solved by modern matrix inversion algorithms that intelligently seek high gradient fields, relaxing and suppressing their numerical residuals. Fourth, rapid computing speeds, e.g., seconds to a minute on typical personal computers, make the approach invaluable for array deconvolution, NMR applications, and rigsite log and geosteering analysis. The availability of a single, self-consistent, open-source model eliminates the uncertainties associated with different proprietary formulations solved by different methodologies at different organizations.
Benchmark studies show excellent agreement with analytical dipole solutions in uniform and layered media and with classical Biot-Savart responses for finite loop coils. Suites of results are described, for responses in complicated media, with and without steel mandrels, invasion and borehole eccentricity, for a range of dip angles. Depth-of-penetration simulations, for electric and magnetic fields, are offered, with a view towards integrated resistivity and NMR formation evaluation. The new algorithm, which is extremely stable, fast and robust, is highly automated and does not require user mathematical expertise or intervention. It is hosted by user-friendly Windows interfaces that support approximately thirty complete simulations every hour. Fully integrated three-dimensional, color graphics algorithms display electromagnetic field solutions on convergence. Receiver voltage responses are given along tool axes, together with circumferential contributions in separate plots; detailed tabulated field results are reported in both geologically-focused rectangular and tool-oriented cylindrical coordinates. Features useful to modern logging instrument design and interpretation are available. For example, users may reconfigure transmitter coils to noncircular oblique geometries “on the fly” (results for elliptical cross-sections and linear geometries used in existing resistivity designs are given later). In addition, users may dynamically “rewire” nodal outputs in order to experiment with novel transmitter, receiver and formation evaluation concepts or to interrogate problem geologies for additional formation properties.

1.2 Introduction

The interpretation of borehole resistivity logs in layered media with dip is complicated by anisotropy, low-to-high mandrel conductivities, nonzero transmitter coil diameter, borehole eccentricity, multiple wave scattering, and polarized interfaces and charge radiation, interacting effects which cannot be studied using simplifying dipole, geometric factor, hybrid or linearized integral equation models. These approaches restrict the physics for mathematical expediency. As such, they address only specific and narrow aspects of the complete problem, e.g., purely planar layering, axisymmetric analysis, dipping bed effects modeled by vertical and horizontal dipoles, and so on.

A comprehensive model encompassing all of the above effects has been elusive. For example, real formations are not isotropic, but an anisotropic formulation covering the complete frequency spectrum plus real layering effects is not available. Moran and Gianzero (1979), for instance, deal with the induction limit only, and do not address dipping beds and the problems associated with interfacial surface charge. Howard and Chew (1992) tackle these issues, but the numerically intensive isotropic model invokes geometric factor and Born-type assumptions.

Most computational algorithms do not converge for wide ranges of frequency or resistivity and anisotropy contrast. While models are available for induction sondes with non-conducting mandrels, specialized codes are required
to handle the high collar conductivities typical in MWD applications. It is usually not possible to simulate induction and MWD runs in the same formation with the same model, thus complicating interpretation and tool design.

And dipole models, often used in induction logging, are inappropriate to MWD because high conductivity collars and large coils preclude simple description. Furthermore, such tools typically log horizontal wells, where cross-sections often reside across multiple thin beds with thicknesses comparable to coil diameters. Point-wise models, moreover, cannot simulate near-field sensitive magnetic volumes accurately, a requirement that bears increasing importance with the acceptance of NMR logging and the need for improved tool design. As noted, vertical and horizontal dipole superpositions are used to model dipping beds, but this breaks down for the “large coils” found when tool diameters and layer thicknesses are comparable. Improved source models are necessary to good interpretation and, of course, to next-generation array and azimuthal resistivity tool design and “pinpoint rf” NMR excitation.

Additional difficulties abound. For example, the continuity law “$\sigma_{v,1} E_{1z} = \sigma_{v,2} E_{2z}$” for steady-state vertical current, is often applied incorrectly. Only in Howard and Chew (1992) is the “continuity of complex current,” as derived in the classic electrodynamics book of Stratton (1941), properly invoked in transient applications. The effects of multiple wave scattering, for instance, are implicitly ignored in Born and linearized integral equation approaches, thus restricting their usage to small conductivity contrasts and dip angles. Recent finite difference methods, despite the apparent generality, are likewise prone to uncertainty. Druskin et al (1999), a case in point, devise a sequential approach that solves the static problem to leading order; it therefore represents a “small frequency” perturbation expansion that is not necessarily convergent. Thus, speed is achieved at the expense of accuracy. Moreover, this requires that “$\sigma \mu_0 R^2 << 1$” dimensionlessly, where R is the transmitter radius, but the implications of higher conductivities, large coil radius and length scales associated with layered media are not discussed.

Finally, practical problems associated with large equation systems persist. In the general case, eight coupled unknowns describe each spatial point, and their efficient inversion is needed in formation evaluation and geosteering applications. Issues related to fast problem setup, user-friendliness, robust solvers, low hardware cost, software licensing and efficient color graphics displays, also arise in any attempt to address the overall problem. Fortunately, all of the theoretical and practical problems discussed have been solved, and we are pleased to report the elements of a new computational approach and its implications. Our Stratamagnetic Software emXplorerTM modeling system is available for complimentary download from our cloud servers and further information may be obtained directly from the publisher or author.
1.3 Physical Model and Numerical Formulation

Our Stratamagnetic Software emXplorer™ modeling system will be explained in its entirety, but in order to facilitate its description, physical ideas are developed first, and later reinforced by mathematical analysis. This section is followed by a comprehensive one describing validation procedures and practical real-world examples. The application of the model to new ideas in hardware design and formation evaluation is undertaken, and details related to overall software specification and implementation are given.

1.3.1 Design philosophy.

The fully three-dimensional model is designed to honor most of the geometrical and geological details of the logging tool and formation. As such, it may run slower than “zero” or “one-dimensional” models, but faster than many three-dimensional models, requiring at most one minute or so per simulation on typical computers. Its objective is easily stated: allow users to test assumptions about the formation, perhaps simplifying ones that justify the use of more elementary electromagnetic models. Much of this book is devoted to results that support our physical validations which are of interest to instrument designers. However, the equally important needs of petrophysical analysts are also addressed; for example, receiver responses both axially and azimuthally can be plotted, and log generation capabilities are available.

1.3.2 New discretization approach.

A comprehensive model in which all of the salient physical features are retained was developed within a finite difference, frequency domain framework. Here, the “numerical noise” associated with the grid sizes and aspect ratios of conventional “staircase” approximations to dipping interfaces does not appear, because coordinate surfaces aligned with local bedding planes define special oriented grids (these are not to be confused with the “staggered grids” discussed later).

![Staircase grids vs Boundary conforming grids](image)

*Figure 1.1.* “Staircase” versus “boundary conforming” grids for layered media.
“Staircase grids,” originally used in computational aerodynamics in the 1970s, have fallen in favor relative to “boundary conforming” meshes, which resolve derivatives parallel and perpendicular to boundaries more precisely. In other words, classical boundary conditions cannot be accurately implemented despite grid refinement although, on a case-by-case basis, acceptable agreement with analytical solutions can be achieved by trial and error. At the same time, crude point source models of airfoils were replaced by distributed line singularities; these were more stable than point singularities, since they are “less infinite.” Taken together, these two methods streamlined analysis and reduced computing, vastly improving performance and predictive capabilities. This “grid plus source point” technology is adapted to the present modeling work.

In our approach, local “vertical, z planes” are aligned with bed interfaces. Consistent with aerodynamic practice, the transmitter coil is modeled by eight azimuthally equidistant points where current is specified. Although we assume constant frequency and real currents, the underlying iterative model actually allows “worst case” complex excitation, so that the numerically stable model applies to Fourier components of pulsed transient systems as well.

To enhance resolution, the coil is always discretized taking “six constant meshes across” in the near-field, as in Figure 1.2; this mesh is geometrically expanded in the farfield to conserve memory and reduce unnecessary computation as in Figure 1.3. Distributed sources are less singular than point sources; they promote stability and increase convergence speed. The “six across” (or, “eight around”) source model resolves transmitter coil geometry well and provides the framework for approximate steel drill collar modeling. As shown in Figures 1.4 and 1.5, the wireline “coil alone” or MWD “drill collar” nature of any tool is modeled by twenty-one internal points, which may or may not reside across multiple formation layers. This provides the resolution needed for resistivity modeling in thinly laminated zones and NMR magnetic volume simulation in layered media. These models apply to tools of all diameters.

1.3.3 Analytical formulation.

The theory behind Maxwell’s equations appears in Stratton (1941). His work can be specialized to transversely isotropic media, leading to the anisotropic model of Moran and Gianzero (1979). Different authors solve different forms of these equations, and in order to understand the differences, we derive all underlying relationships from first principles. Because much of the derivation is generally available, we will sketch the overall approach, and reserve detailed commentary only to original research results.

We begin with Maxwell’s equations for \( \mathbf{E} \) and \( \mathbf{B} \) in the usual form, i.e.,

\[
\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0, \quad \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}, \quad \nabla \cdot \mathbf{B} = 0, \quad \text{and} \quad \nabla \cdot \mathbf{D} = \Theta.
\]

If we resolve \( \mathbf{J} \) into a source \( \mathbf{J}_s \) plus a conduction current \( \mathbf{\sigma} \mathbf{E} \), where \( \mathbf{\sigma} \) is a diagonal conductivity tensor [\( \sigma_h \), \( \sigma_h \), \( \sigma_v \)] and “h” and “v” are horizontal and vertical
directions in Figure 1.2, the second equation becomes $\nabla \times \mathbf{H} - \partial \mathbf{D}/\partial t - \mathbf{E} = \mathbf{J}_s$. We assume $\mathbf{D} = \varepsilon \mathbf{E}$ and $\mathbf{B} = \mu \mathbf{H}$ where $\varepsilon$ and $\mu$ are (for simplicity) isotropic inductive capacities. When $\sigma_h$ and $\sigma_v$ are equal, we recover the model of Druskin et al (1999). In that work, the authors deal with $\mathbf{E}$ and $\mathbf{B}$ directly. Thus, the Yee (1966) “staggered grid” algorithm must be used, in which the dependent variables are evaluated alternatively at grid centers and edges, so that fictitious currents do not arise.

Davydycheva and Druskin (1995) extend this formalism to anisotropic media, but the computational overhead is substantial, with workstation solutions requiring hours per run. Together, these “staggered” plus “staircase” mesh systems pose formidable obstacles to obtaining useful solutions. But the most severe limitation appears in the iterations. Druskin et al (1999) note that a static solution is solved first, that is in turn improved by frequency-dependent corrections. As such, the method is, whatever the formalism, a de facto expansion in small frequency that is not necessarily convergent. Thus, speed is achieved at the expense of accuracy. Consider, for example, homogeneous media, where the only length scale is the coil radius $R$. Because $\sigma \nu \omega R^2$ is the sole dimensionless quantity physically possible, the scheme implicitly requires $\sigma \nu \omega R^2 << 1$. Convergence is not addressed, and neither are the implications of high conductivity, larger coil radius and additional length scales arising in layered media. The only apparent benefits are solutions for multiple frequencies obtained with minimal effort, but this is possible with any perturbation scheme taken in powers of $\sigma \nu \omega R^2$.

1.3.4 An alternative approach.

In problems excited by external currents, Feynman et al (1964) note that it is more natural to solve the potential form of Maxwell’s equations. This is especially true in practice, because the formulation involves classical differential operators (Courant and Hilbert, 1989) whose solution algorithms are widely available. These do not require staggered meshes, nor do they produce “spurious solutions” associated with direct field approaches. But complications related to boundary conditions do arise, which we have identified and addressed.

A completely equivalent alternative to “$\mathbf{E}$ and $\mathbf{B}$” is “$\mathbf{A}$ and $\mathbf{V}$,” where the latter represent, respectively, well known vector and scalar potentials. As before, the basic derivation is standard, and can be extended to anisotropic conductivity by introducing straightforward changes. The idea is to represent, without loss of generality, the magnetic and electric fields in the form $\mathbf{B} = \nabla \times \mathbf{A}$ and $\mathbf{E} = -\partial \mathbf{A}/\partial t - \nabla \mathbf{V}$. When the Lorentz gauge $\nabla \cdot \mathbf{A} = -\mu \varepsilon \partial \mathbf{V}/\partial t - \mu \sigma_h \mathbf{V}$ is used, the equations governing $\mathbf{A}$ and $\mathbf{V}$ are found as $\nabla^2 \mathbf{A} - \mu \varepsilon \partial \mathbf{A}/\partial t - \mu \varepsilon \partial^2 \mathbf{A}/\partial t^2 - \mu (\mathbf{A} - \sigma_h \nabla \mathbf{V}) = -\mu \mathbf{J}_s$ and $\nabla^2 \mathbf{V} - \mu \sigma_h \partial \mathbf{V}/\partial t - \mu \varepsilon \partial^2 \mathbf{V}/\partial t^2 = -\Theta/\varepsilon$. 