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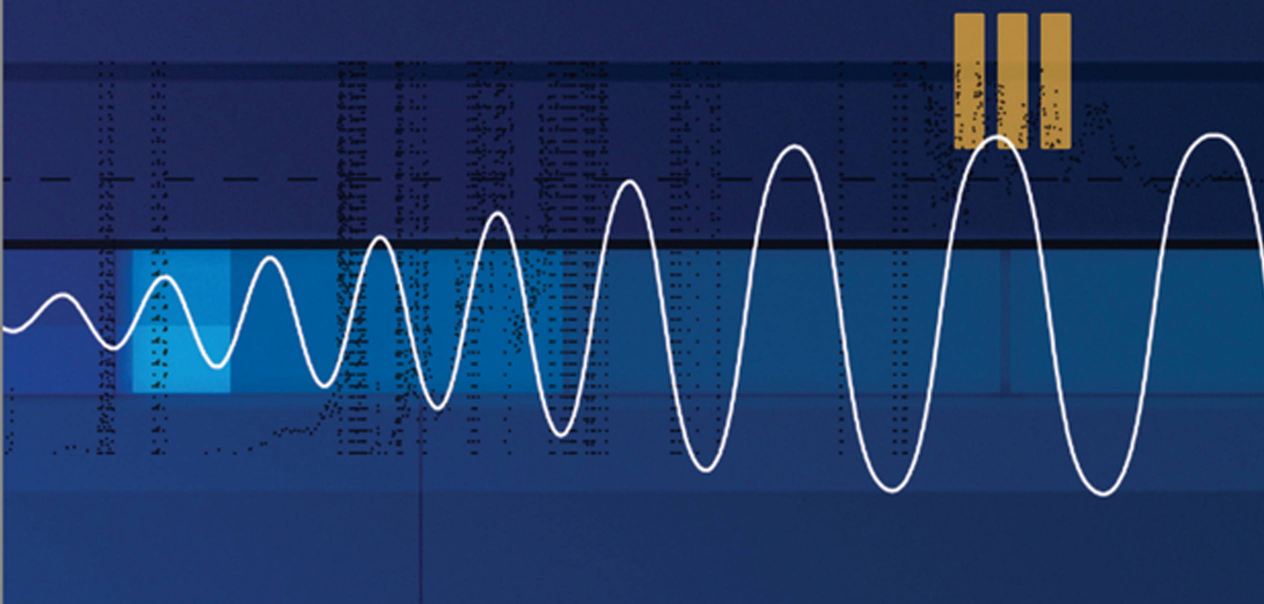

Algebraic identification and estimation methods in feedback control systems

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This book is dedicated to our families, friends, colleagues, and students. Also, to our beloved countries: Venezuela, Mexico, and Colombia

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Series Preface

Electromechanical Systems permeate the engineering and technology fields in aerospace, automotive, mechanical, biomedical, civil/structural, electrical, environmental, and industrial systems. The Wiley Book Series on dynamics and control of electromechanical systems will cover a broad range of engineering and technology these fields. As demand increases for innovation in these areas, feedback control of these systems is becoming essential for increased productivity, precision operation, load mitigation, and safe operation. Furthermore, new applications in these areas require a reevaluation of existing control methodologies to meet evolving technological requirements. An example involves distributed control of energy systems. The basics of distributed control systems are well documented in several textbooks, but the nuances of its use for future applications in the evolving area of energy system applications, such as wind turbines and wind farm operations, solar energy systems, smart grids, and energy generation, storage and distribution, require an amelioration of existing distributed control theory to specific energy system needs. The book series serves two main purposes: 1) a delineation and explication of theoretical advancements in electromechanical system dynamics and control, and 2) a presentation of application driven technologies in evolving electromechanical systems.

This book series will embrace the full spectrum of dynamics and control of electromechanical systems from theoretical foundations to real world applications. The level of the presentation should be accessible to senior undergraduate and first-year graduate students, and should prove especially well-suited as a self-study guide for practicing professionals in the fields of mechanical, aerospace, automotive, biomedical, and civil/structural engineering. The aim is an interdisciplinary series ranging from high-level undergraduate/graduate texts, explanation and dissemination of science and technology and good practice, through to important research that is immediately relevant to industrial development and practical applications. It is hoped that this new and unique perspective will be of perennial interest to students, scholars, and employees in aforementioned engineering disciplines. Suggestions for new topics and authors for the series are always welcome.

Mark J. Balas
John L. Crassidis
Florian Holzapfel
Series Editors

Preface

This work has been made possible thanks to Professor Michel Fliess's professional mathematical vision of real engineering problems. Without his convincing and precise mathematical formulation of fundamental problems in control theory of uncertain systems and signal processing, this book would never have existed.

The quest for an algebraic approach to parameter identification started one lovely summer afternoon in 2002, while having lunch and lively discussions on automatic control matters at Ma Bourgogne restaurant, La Place des Vosges, Paris. I was in the company of Michel and Richard Marquez, an outstanding doctoral student of Michel's in Paris, who had only recently defended his thesis and who had also been a superb Master's student of mine a few years back in Mérida (Venezuela). The discussion ended later that night at Michel's apartment with the distinctive feeling that a "can of crazy worms" had just been opened. Michel and myself worked feverishly over the next few months and years. Sometimes across the Atlantic Ocean, via internet; sometimes in Paris, and on other occasions in the gardens of Cinvestav, in Mexico City. Our colleagues Mamadou Mboup, Hugues Mounier, Cedric Join, Joachim Rudolph, and Johann Reger joined Michel's efforts and quickly found applications and outstanding results of the innovative theory in new and challenging areas such as communications systems, failure detection, and chaotic systems synchronization. As set out by Michel from the beginning, the theory, of course, does not need probability theory and for that reason neither do we.

The approach to parameter estimation, state estimation, and perturbation rejection adopted in this book is radically different from existing approaches in three main respects: (1) it is not based on asymptotic approaches, (2) it does not require a probabilistic setting, and (3) it does not elude the need to compute iterated time derivatives of actual noise-corrupted signals. The fact that the computations do not lead to asymptotic schemes is buried deep in the algebraic nature of the approach. We exploit the system model in performing valid algebraic manipulations, leading to sensible schemes yielding parameters, states, or external perturbations. Naturally, our scheme rests on the category of *model-based* methods. We should point out, however, that the power of the algebraic approach is of such a nature that it also allows complete reformulation of non-model-based control schemes. One of the crucial assumptions that allows us to free ourselves from probabilistic considerations is that of "high-frequency" noises, or, more precisely, "rapidly varying perturbations." A complete theory exists nowadays, based on non-standard analysis, for the rigorous characterization and derivation of the fundamental properties of such noises. White noises, characteristic of the existing literature, are known to constitute a "worst-case" idealization, which allows for a comfortable mathematical treatment of the expressions but one that is devoid of any physical reality.

The research work contained in this book has primarily been supported by the Centro de Investigación y Estudios Avanzados del Instituto Politécnico Nacional (Cinvestav-IPN), Mexico City and the generous financial assistance of the Consejo Nacional de Ciencia y Tecnología (CONACYT), Mexico, under Research Projects No. 42231-Y and 60877-Y. Generous financial assistance of the CNRS (France) and of the Stix and Gage Laboratories of the École Polytechnique is gratefully acknowledged. The first author would like to thank Marc Giusti, Emmanuel Delaleau, and Joachim Rudolph for their kind invitations to, respectively, Palaisseau, Brest, and Dresden on several enjoyable occasions. Back at Cinvestav, the generous friendship of Dr. Gerardo Silva-Navarro is sincerely acknowledged and thanked, in many laboratory undertakings by students and challenging academic discussions. We also acknowledge his special administrative skills to produce "ways and means," as materialized in equipment and infrastructure.

The work gathered by the first author over the years has benefited enormously from the wisdom, patience, and determination of the three co-authors of this book, who set out to clean examples from many mistakes, perform the required computer simulations, and carry out successful laboratory experiments. Carlos García Rodríguez is credited for having obtained, for the first time in the world, an actual experimental application of parameter and derivative estimation, from an algebraic standpoint, in the control of an oscillatory mechanical system. His initial contribution in ordering of the material, finding useful variants, and his remarkable ability to recreate lost simulation files and carry out experiments put us on the trail of pursuing the writing of a book on the subject of algebraic parameter and state estimation. Alberto Luviano-Juárez and John Cortés-Romero, two extraordinary PhD students, joined the venture, giving the available material a definite positive push toward completion. Through countless discussions and revisions of the material, weekly projects involving lots of nightly, and weekend, work on their part, real-life laboratory implementations under adverse conditions, and contagious enthusiasm, they are credited with generously driving the book project to the point of no return. My deepest appreciation to all of them for having endured the difficult times and for the "mountains" of workload involved.

The first author is indebted to Professor Vicente Feliu-Battle of the Universidad de Castilla La Mancha (UCLM), Ciudad Real, Spain and his superb PhD students Juan Ramón Trapero, Jonathan Becedas, and Gabriela Mamani for having put the theory to a definite test with many challenging laboratory experiments that gave us the chance to publish our results in credited journals and conferences. Such an interaction was made possible thanks to Professor Feliu's administrative skills, resulting in a full sabbatical year spent in Ciudad Real. The more recent interaction with Dr. Rafael Morales, of UCLM, has proven to be most fruitful in the use of this theory in some other challenging laboratory applications.

H. Sira-Ramírez dedicates his work in this book to his friend Professor Michel Fliess, for his constant support, kind advice, and encouragement in the writing of this book and in many other academic matters.

Hebertt Sira-Ramírez

1

Introduction

One of the main obstacles related to key assumptions in many appealing feedback control theories lies in the need to perfectly know the system to be controlled. Even though mathematical models can be derived precisely for many areas of physical systems, using well-established physical laws and principles, the problems remain of gathering the precise values of the relevant system parameters (or, obtaining the information stored in the inaccessible-for-measurement states of the system) and, very importantly, dealing with unknown (i.e., non-structured) perturbations affecting the system evolution through time. These issues have been a constant concern in the feedback control systems literature and a wealth of approaches have been developed over the years to separately, or simultaneously, face some, or all, of these challenging realities involved in physical systems operation. To name but a few, system identification, adaptive control, energy methods, neural networks, and fuzzy systems have all been developed and have tried out disciplines that propose related approaches, from different viewpoints, to deal with, or circumvent, the three fundamental obstacles to make a clean control design work: parameter identification, state estimation, and robustness with respect to external perturbations.

This book deals with a new approach to the three fundamental problems associated with the final implementation of a nicely justified feedback control law. We concentrate on the ways to handle these obstacles from an algebraic viewpoint, that is one resulting from an algebraic vision of systems dynamics and control. As for the preferred theory to deal with the ideal control problem, we emphasize the fact that the methods to be presented are equally applicable to any of the existing theories. We propose examples where sliding-mode control is used, others where passivity-based control methods are preferred, and yet others where flatness and generalized proportional integral controllers are implemented. The algebraic approach is equally suitable when dynamic observers are used. The book therefore does not concentrate on, or favor, any particular feedback control theory. We choose the controller as we please. Naturally, since the theoretical basis of the proposed algorithms and techniques stems from the differential algebraic approach to systems analysis and control, we often present the background material in detail at the end of chapters, so that the mathematically inclined reader has a source for the basics being illustrated in that chapter through numerous physically oriented examples.

Algebraic Identification and Estimation Methods in Feedback Control Systems, First Edition.
Hebertt Sira-Ramírez, Carlos García-Rodríguez, Alberto Luviano-Juárez, John Alexander Cortés-Romero.
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1.1 Feedback Control of Dynamic Systems

The control systems presented here are designed using an algebraic estimation methodology in combination with well-known control design techniques, which are applicable to linear and nonlinear systems. Since the algebraic estimation methodology is independent of the particular controller design method being used and, furthermore, it is quite easy to understand, it will be profitable for the reader to combine this tool with his preferred controllers or with conventional control techniques. This book introduces a wide variety of application examples and detailed explanations to illustrate the use of the algebraic methodology in identification, state estimation, and disturbance estimation. However, this work is not an introductory control textbook. A basic control course is a prerequisite for a deeper understanding of these examples.

1.1.1 Feedback

A control system is one whose objective is to positively influence the performance of a given system in accordance with some specific objectives. A *control law* or *controller* is a set of rules that allows us to determine the commands to be sent to the governed plant (via an actuator) to achieve the desired evolution. These rules can be described as either *open-loop* control or *closed-loop* (feedback) control. Figure 1.1 shows both control strategies, where $y(t)$ and $u(t)$ are, respectively, the output and input of the plant, and $x(t)$ represents a finite collection of internal variables (called state variables) that completely characterize, from the present, the system behavior in the future provided the future control inputs are available. The first scheme, an *open-loop control system*, does not measure (or feed back) the output to determine the control action; its accuracy depends on its calibration, and thus it is only used when there are no perturbations and the plant is perfectly known. A *perturbation* (or *disturbance*) is an unknown signal that tends to adversely affect the output value of the plant. This undesired signal may be internal (*endogenous*) or external (*exogenous*) to the system. Figure 1.1(b) shows a *closed-loop control system*, where a sensor is used to obtain an error signal. This signal is processed by the controller to determine the action necessary to reduce the difference between the output and its desired value.

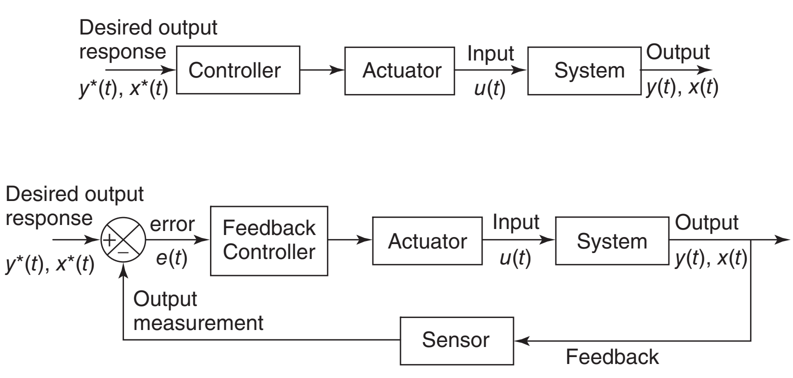

Figure 1.1 Typical control scheme

Feedback is a mechanism to command a system to evolve in a desired fashion so that the states, and outputs, exhibit a desired evolution (e.g., to track a *reference trajectory*) or stay at a prescribed equilibrium. Feedback enables the current state or the current outputs (*feedback signal*) to be measured, determining how far the behavior is from the desired state or desired outputs (i.e., to assess the *error signal* and then automatically generate a suitable *control signal* to bring the system closer and closer to the desired state). Feedback can be used to stabilize the state of a system, while also improving its performance.

Feedback control has ancient origins, as noted by Mayr (1970). Throughout history, many examples of ingenious devices, based on feedback, can be found. In ancient Greece, China, during the Middle Ages, and in the Renaissance, many examples have been recovered and explained in modern terms. These artifacts were improved and specialized to become pressure regulators, float valves, and temperature regulators. One of these devices was the fly-ball or centrifugal governor, used to control the speed of windmills; it was later adapted by James Watt, in 1788, to control steam engines.

It was only in the 1930s that a theory of feedback control was fully developed by Black and Nyquist at Bell Laboratories. They studied feedback as a means to vastly improve the amplifier performance in telephone lines. They had to face the (well-known) *closed-loop instability problem* when the feedback gain was set too high, transforming the amplifier into an oscillator.

1.1.2 Why Do We Need Feedback?

Fundamentally, feedback is necessary for the following reasons.

- Counteracting *disturbance* signals affecting the plant: A controller must reject the effects of unknown undesirable inputs and maintain the output of the plant within desired values.
- Improving system performance in the presence of model *uncertainty*: Uncertainty arises from two sources – unknown or unpredictable inputs (disturbance, noise, etc.) and uncertain or unmodeled dynamics.
- Stabilizing an *unstable plant*.

This book addresses only *additive perturbations*, which will be divided into structured and unstructured perturbations. *Structured perturbations* are generated by the initial conditions of the system and unknown exogenous inputs that can be modeled as families of time polynomials. The *unstructured perturbations* are considered as highly fluctuating, or oscillating, phenomena affecting the behavior of the system. A common unstructured perturbation is the case of a zero-mean noisy signal. Notwithstanding this, it is still possible to build an algebraic estimator that takes into account these disturbances and mitigates their effects.

The main objectives of feedback control are to ensure that the variables of interest in the system either track prescribed reference trajectories (*tracking problem*) or are maintained close to their constant set-points (*regulation problem*).

1.2 The Parameter Identification Problem

A model is a mathematical representation of the essential characteristics of an existing system. When a system model can be defined by a finite number of variables and parameters, it is called a parametric model. Examples of parametric models are the transfer function of an electrical circuit, the equations of motion of a mechanical suspension system, and so on. The whole family of functions and equations that integrate a model is called the model structure. In general, this structure can be linear or nonlinear. The models used in modern control theory are, with a few exceptions, parametric models in terms of linear and nonlinear state equations. To implement a model-based controller, it is necessary to know precisely the structure of the model of the system and its associated parameters. Therefore, if parameters are initially unknown, the process of parameter identification is quite important for the design of the control system.

1.2.1 Identifying a System

According to Eykhoff (1974), model building begins with the application of basic physical laws (Newton's laws, Maxwell's laws, Kirchhoff's laws, etc.). From these laws, a number of relations are established between variables describing the system (e.g., ordinary differential or difference equations or, sometimes, partial differential equations). If all external and internal conditions of the system are known, and if our physical knowledge about the plant is complete, then in principle the numerical value of all the parameters in those relations may be determined. Eykhoff (1974) recalls that model building consists of four steps: (a) selection of a model structure based on physical knowledge, (b) fitting of parameters to available data (identification), (c) verification and testing of the adopted model, and (d) application of control theory to the model to achieve a desired purpose. The word *process* is just another term for referring to a given system.

The identification problem is sometimes tackled via the inverse problem of system analysis; *given an input and output time history, determine the equations and parameters that describe the system behavior*. Zadeh (1956) defines *identification* as the determination, on the basis of inputs and outputs of a system, of a system model which produces an equivalent behavior to the real plant. *Parameter estimation*, in contrast, is the experimental determination of values of parameters that govern the dynamic behavior of the system, assuming that the structure of the process is well known. The method implies comparing the actual process output, contaminated with measurement noise, and the response of the hypothesized model output when both are subject to the same input signal. Because of the uncertainty introduced by noise and neglected dynamics, it is then necessary to find an adjustment of the model according to a certain *criterion*.

1.3 A Brief Survey on Parameter Identification

The problem of parameter estimation in dynamical systems, from available measurements, dates back to 300 B.C. when the motion of celestial bodies was characterized by six parameters (see Sorenson, 1980).

Estimation, based on the minimization of an error function, can be attributed to Galileo Galilei (see Favier, 1982). One of the most important early contributions in this area was the statistical view of the parameter identification problem, proposed by Gauss (1809) and known as the "inverse problem of computing the response of a system with known characteristics," and the maximum likelihood procedure introduced by Fisher (see Aldrich, 1997).

The term “identification” was coined by Zadeh (1956), as the problem of determining the input–output relation by experimental means, considering the linear class of models as a “black-box” model.

Linear estimation for stochastic systems was introduced in the works of Wiener (1949) and Kolmogorov (1941). These works placed the identification problem in the context of important engineering problems that remained open mathematical problems. An excellent survey paper on this theory can be found in Kailath (1974).

The Kalman filter (Kalman, 1960; Kalman and Bucy, 1961) represents a state-space-based algorithmic procedure geared to optimally solve the problem of state estimation in the presence of measurement noises and uncertain initial states. The approach is based on the minimization of an integral quadratic estimation error performance criterion. The great legacy of the Kalman filter has led to extensions of nonlinear systems and to the formulation and solution of some other related problems, such as the parameter identification problem. Åstrom and Bohlin (1965) introduced the concept of identifiability in the context of the maximum likelihood approach. The survey by Åstrom and Eykhoff (1971) presents a complete account of classical identification schemes.

An important reference text, from the early years of identification, is the book by Box and Jenkins (1976). This book is concerned with the identification of discrete-time stochastic systems, the forecasting of time series, and the estimation of parameters in transfer functions, with emphasis on applications. Other authors focused on the problem of model approximation and system order reduction (see Anderson *et al.*, 1978 and the references cited therein).

The book by Ljung (1987) is a most important reference in the context of linear systems identification. This book focuses on the “engineering approach to system identification,” as mentioned in Gevers (2006), where numerical schemes are made to play an important rôle in the identification procedure. Other important books regarding this topic are Åstrom and Wittenmark (1995), Eykhoff (1974), Goodwin and Sin (1984), Johansson (1993), Landau (1990), Sastry and Bodson (1989), Söderström and Stoica (1989), and Sorenson (1980). A more recent authoritative survey of system identification can be found in the work of Gevers (2006).

1.4 The State Estimation Problem

The *state* of a system is a minimum set of variables (state variables) whose present values, together with the values of the input signals in the future, completely determine the future behavior of the system.

Conditions and practical ways of estimating the state are generally set according to the mathematical approach being used (dynamical systems, signal theory point of view, etc.), the system nature (stochastic, deterministic, linear, nonlinear), the amount of variables to estimate, etc. Here, it will be assumed that the system to be controlled is described by differential equations in a state-space representation.

We consider the state estimation problem as the problem of determining the states from observations of the outputs. We usually seek to test the observability of the system model with respect to its output to find an answer to this problem. This property indicates how well internal states of a system can be reconstructed by knowledge of its external outputs. The concept of observability was introduced by Rudolf E. Kalman for linear systems (Kalman, 1959, 1963).

To understand the importance of state estimators in the feedback control, we need to analyze the block diagram of a control system as shown in Figure 1.2.

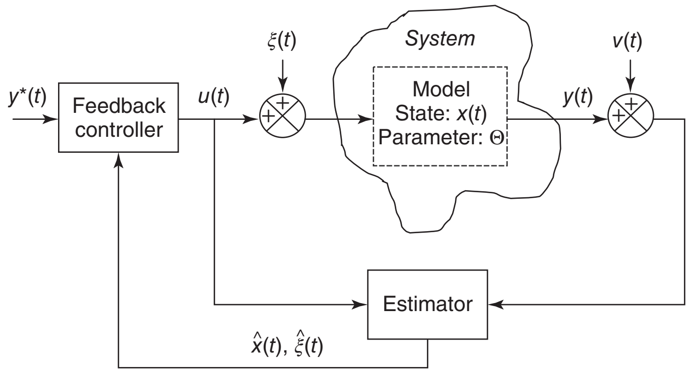

Figure 1.2 Block diagram of a control system with state and perturbation estimators

For the continuous-time case, the system model consists of a set of differential equations which describe the evolution of the state variables, $x(t)$, and a set of parameters θ. The known input signal, or control input, is denoted by $u(t)$. Sometimes, the system is also affected by an unknown external disturbance, $v(t)$. The output, $y(t)$, may frequently be contaminated by measurement noise, $\xi(t)$. The reference signal, the set point or desired output value that the control system will aim to reach, is $y^*(t)$. As Figure 1.2 shows, the block for the feedback controller requires the actual state value to produce the control signal. Since not all of the state variables are measurable, we need to build a state estimator in order to implement this feedback controller. In most practical cases, a state estimator requires an adequate noise immunity level.

In order to cancel, or at least reduce, the disturbance effects, a robust control law based on disturbance estimators can be proposed. In some cases, the disturbance estimation problem is dealt with as the estimation of an additional state.

Generally, to estimate the system state we can use two types of tools: state observers and time derivative estimators. In both approaches, the model of the system needs to satisfy a particular condition guaranteeing the feasibility of the state computation or reproduction from inputs and outputs or outputs alone. Such a property is the system observability. In the linear system case (continuous and discrete), the property may easily be assessed from the rank of a matrix called the observability matrix. Its conceptual introduction is due to Kalman.

1.4.1 Observers

The state can be generated from a dynamical system called the observer, which is based on the model of the plant, driven by measurements of the plant inputs and outputs. If the order of the observer is equal to the order of the system model, the observer is said to be a full-order observer. If the order of the observer is less than the order of the plant, then the observer is said to be a reduced-order observer. Reduced-order observers take advantage of the possibility of directly computing some state variables from the available outputs.

An observer for a linear system is readily feasible whenever the system is observable, that is, when its observability matrix has full rank. Reconstructible systems are referred to as those systems whose observability matrix is not full rank but such that the unobservable part of the system exhibits asymptotic behavior toward a certain trivial equilibrium state. Observers have been studied extensively in linear systems, via the appropriate selection of gains, showing that observers can be constructed in such a way that their states approach the true states with dynamics having prescribed stability features. If the observer gains are optimized for the noise input to the system and to the sensor(s), the observer is called a Kalman filter. If the gains are not so optimized, and the setting is deterministic, the observer is called a Luenberger observer. The concept of an observer for a dynamic process was introduced in 1966 by D. Luenberger. The generic *Luenberger observer* appeared several years after the Kalman filter.

1.4.2 Reconstructing the State via Time Derivative Estimation

Another way to estimate the state is via numerical differentiation. If a system is observable, the state-space can be reconstructed from a set of measurements of the input, the output, and a finite number of their time derivatives. However, a problem arises from estimating derivatives; a typical differentiator is greatly affected by noise. Therefore, if we choose this estimation technique, we should necessarily consider implementing an additional filtering algorithm in our estimation procedure to obtain reasonable results under noisy measurement conditions.

A systematic way to reconstruct the state-space via time derivatives is by exploiting the differential flatness property of a system. We say that a system is flat if it can be described in terms of a set of special output variables and their time derivatives up to finite order. For example, for a single-input/single-output (SISO) flat system, all state variables, including the control input, can be parameterized in terms of the flat output and a finite number of its time derivatives.

From an algebraic estimation point of view, two types of state estimators may be proposed. The first is based directly on the model of the plant, and the second is based on time-polynomial approximations of the output signals. Both schemes constitute time-derivative estimators. The flatness property of the system can be used advantageously to reconstruct the complete state. Unlike classical estimators, the algebraic estimators are robust with respect to initial conditions of the system. They have an adequate noise immunity level and include mechanisms to mitigate the effects of unknown but bounded external disturbances.

The works of Norbert Wiener on optimal filtering inspired the development of the state observation theory accomplished and completed by Rudolph Kalman, in the state-space stochastic context. A deterministic approach to the problem of estimation of inaccessible-for-measurement states, through available outputs, was developed by David G. Luenberger (1964, 1966, 1971).

Observability in nonlinear systems is developed in the work of Hermann and Krener (1977). Other important contributions to the theory of nonlinear observers are given by Brockett (1972) and Sussmann (1977, 1979). In relation to the differential geometric approach, observability was studied by Isidori (1989) and Nijmeijer and van der Schaft (1990). Another important observability concept, given in terms of the differential algebraic approach, was introduced by Diop and Fliess (1991a,b). In this approach, a system variable is said to be algebraically observable if it can be represented in terms of the input, the output, and a finite number of their time derivatives. In this particular sense, observer design is reduced to the design of a numerical differentiator (Diop *et al*., 1993, 2000). In the realm of signal differentiators, many interesting approaches have been proposed, ranging from Luenberger-like observers to sliding-mode-based differentiators (Levant, 1998). High-gain observers (Doyle and Stein, 1979; Khalil, 1999) have also become popular in recent times, in both theory and applications, as evidenced by the survey of Khalil (2008).

1.5 Algebraic Methods in Control Theory: Differences from Existing Methodologies

This identification methodology has been used to estimate parameters (Trapero-Arenas *et al.*, 2008), states (Barbot *et al.*, 2007), derivatives of signals (Villagra *et al.*, 2008), polynomial disturbances, and also to detect faults (Fliess *et al.*, 2004). In general, the algebraic technique allows us to deal with three fundamental obstacles in controller design tasks: parameter identification, state estimation, and robustness with respect to additive perturbations (such as constant, ramp, and parabolic, i.e., classical, perturbations).

This technique is applicable to linear and nonlinear systems, and is easy to understand. For the linear case, there are two general (equivalent) approaches: the time-domain approach and the frequency-domain (operational calculus) approach. In the first case, only time differentiations of expressions, multiplications by positive (suitable) powers of the time variable, and iterated integrations (with advantageous use of the integration by parts formula) are sufficient to obtain linear expressions in the parameters, clean from the influence of initial conditions and classical perturbation inputs. The second approach only uses tools such as Laplace transforms, derivation with respect to the complex variable s (also called "algebraic derivations"), and multiplication by suitable negative powers of the complex variable s. Hence, the algebraic approach is fully compatible with the concepts and tools of classic control. For the nonlinear case, one may only resort to the time-domain approach. The fundamental difference from the linear case is the need to obtain convolutions of time powers with nonlinear expressions of the measured input and output variables or, sometimes, with nonlinear expressions of state variables. For nonlinear systems where the measured outputs are naturally the outputs of the plant, the differential flatness property of the system is quite helpful. Unlike the linear case, beside the input and the output, sometimes it is necessary to also know one or several components of the state vector. Since this methodology is independent of the preferred control design technique, it can be adapted to the needs of any other model-based control law. Their design turns out to be easier, and much faster, than many adaptive identification schemes, (Gensior *et al.*, 2008).

The algebraic method enables the robust design of online parameter identifiers, and also of state estimators in the presence of noise and time-polynomial disturbances. Algebraic estimators do not need statistical knowledge of the noises corrupting the data (see, e.g., Fliess *et al.*, (2003, 2008) for linear and nonlinear diagnoses). The basic *invariant filtering* used in this work, for the treatment of noise effects, is based on the noise-attenuating properties of the integration operation. Traditional low-pass filtering and even nonlinear filtering is suitably merged with the algebraic parameter and state-estimation procedures.

Unlike the asymptotic observers, the algebraic estimators do not rely on asymptotic convergence arguments and Lyapunov stability theory. The identifications are nearly instantaneous in nature. In García-Rodríguez *et al.* (2009), the sentinel parameter was proposed as an alternative to experimentally decide the moment at which the estimated parameters have converged to acceptable values. Another approach, based on the condition number involved in the identifier equation system, is given in Trapero-Arenas *et al.* (2008). This simplifies the controller design and enables online tuning of the certainty-equivalence controllers.

The algebraic identification method has already been compared with standard recursive identification algorithms, showing that the unknown parameters are obtained in a substantially shorter time period (Garrido and Concha 2013). Given that the algebraic parameter identification is quite fast, the estimated values are accurate enough that they can readily be used by the controller in order to accomplish a given task. It may be shown that, thanks to the almost instantaneous nature of the algebraic identification procedure, it does not require the classical “persistency of excitation condition” characteristic of slow parameter convergence in traditional adaptive control schemes. This is replaced by an algebraic consistency condition which avoids singularities over small open intervals of time (see Fliess and Sira-Ramírez, 2003). Contrary to the persistency of excitation, which is known to fundamentally interfere with desired control objectives and has to be sustained typically over rather long periods of time, the algebraic requirements only need to be valid during the small time interval needed to compute the parameter via a *static formula*. After the parameters are determined, the identification process may be permanently stopped if the parameters are known to be perfectly constant. Otherwise, one may let the process be reinitialized at a later time, provided the degradation in performance can be attributed to ongoing parameter variations in time.

Algebraic parameter estimators have been used successfully in the control of DC-to-DC power converters (Gensior *et al*., 2008); Linares Flores *et al*., 2011) and, more recently, a most interesting model-free control design technique based on the algebraic approach to systems control has been proposed by Michel *et al*. (2010).

Generally speaking, there are still many aspects that require development within the algebraic methodology for parameter and state estimation. These are:

- a higher noise immunity;
- to achieve a relaxation of the conditions of model-based design to cope with dynamic uncertainties;
- an extension to estimation in time-varying systems;
- a more complete extension into the realm of nonlinear systems;
- advantageous combinations with other estimation techniques;
- new application areas and extensions to new classes of systems (hybrid, networked, etc.).

All these topics are attractive options for further investigation, given their inherent simplicity and formidable power.

1.6 Outline of the Book

This book is essentially a tutorial. Each chapter has a number of examples, which set out to illustrate and explain the use of the theory or, in some instances, to motivate it. The examples, although mostly physically oriented, are therefore quite elementary, quite low dimensional, and, frankly, rather simple. We believe that with this option at hand, the reader can readily grasp the essential features of the approach and try the techniques in their own, more complex, applications. If something fails, there is always an exposition of, or reference to, the theory to find the inherent limitations of the approach or the overlooked and unsatisfied properties of that particular example. Our experience indicates that the proposed theory always yields a correct and satisfactory answer.

Chapter 2 deals with the parameter identification problem in the context of linear systems. Whether state or input–output representations, we begin with systems undergoing structured perturbations (i.e., perturbations generated by homogeneous, linear, time-invariant systems whose initial states and parameters cannot be obtained). To this class of perturbations belong the so-called classical perturbations: constant perturbations, ramps, polynomial and sinusoidal perturbations, etc. Efforts will be geared to obtain, or identify, the system or plant parameters rather than the perturbation-defining parameters. The objective we keep in mind is that of being able to implement a feedback controller where such parameter information is lacking and deemed to be crucial. In simple terms, we adopt the certainty-equivalence control approach. We design the feedback controller as if the unknown set of parameters is at our disposal but, initially, with an arbitrary value being used in its place. The identification of the actual value of the parameter is to be carried out online, rather fast, and in the presence of noise. Once the parameter is identified with accuracy, its value is immediately replaced in the designed controller. Algebraic calculation of parameters from measured inputs and outputs must be capable of dealing with unavoidable measurement noises. In this chapter we introduce a technique known as invariant filtering. Instead of pre-filtering the involved signals, we post-filter the involved expressions from where the calculation is possible. This is shown to enhance the signal-to-noise ratio and provide accurate parameter estimations in the presence of significant measurement noise.

Chapter 3 is devoted to the algebraic approach to parameter identification in a class of nonlinear systems where the vector of unknown parameters is weakly linearly identifiable. Contrary to the case of linear systems, it is not always possible to obtain explicit formulae for the components of the parameter vector which depend only on inputs and outputs, or even nonlinear functions of inputs and outputs alone. In general, the state vector components are needed to be able to compute the unknown parameters. This important limitation will be lifted in later chapters by means of online, non-asymptotic algebraic state estimation methods. In this chapter we concentrate mainly on demonstrating that the algebraic approach to parameter estimation can equally be extended to nonlinear systems by centering the approach around the time domain. Several examples of parameter identification in nonlinear systems are presented. Also, the extension of the certainty-equivalence control method for the nonlinear case is treated, providing a method for fast adaptive control of systems of a physical nature. An important class of nonlinear systems examples where the current parameter-identification techniques fail is the class of switched systems undergoing sliding motions. The high-frequency nature of the control input, under sliding conditions, naturally clouds the possibilities of accurate parameter estimation in traditional identification schemes. In our algebraic approach, however, such an inconvenient feature of the control signal is effectively dealt with by means of invariant filtering. In this chapter, we present several examples of a certainty-equivalence sliding-mode control approach to systems with unknown parameters. We should stress and clarify the following point: customarily, in sliding-mode control schemes for switched systems, a knowledge of system parameters may be unnecessary to achieve sliding motion and an asymptotic satisfaction of the control objectives. This is particularly so in state-space representations and, perhaps more specifically, in linear time-invariant systems. However, when no system states are available, it is still possible to resort to an output sliding-mode feedback control scheme that exploits knowledge of the system and implements an average output feedback controller scheme through a $\Sigma - \Delta$ modulation circuit. Such an average feedback control law requires fast identification of the unknown parameters while the measured switch position function, acting as the only measurable input signal of the system, is a high-frequency bang–bang signal.

Chapter 4 deals with the discrete-time counterpart of the online, continuous, linear and nonlinear parametric identification approach presented in Chapters 2 and 3. The extension of the algebraic approach for parameter identification to this ubiquitous class of systems is also based on the module-theoretic vision of discrete-time linear dynamics, which has become classic. As in the continuous-time case, we achieve closed-loop identification in a relatively small time interval involving few samples and with no need for statistics. Several physically oriented case studies of linear and nonlinear nature, mono-variable and multi-variable, are discussed in detail along with their corresponding computer simulations.

Chapter 5 shifts our attention to the problem of state estimation in linear systems from the algebraic viewpoint. Algebraic manipulations, either in the time domain or in the frequency domain, of the system equations allow for explicit formulae to be developed for the efficient computation of the states of a system. In this chapter, we show how states can be computed continuously, in an online fashion, after a rather short time interval. Several examples of these manipulations are presented, along with simulation results. One may quickly realize that the possibilities for simultaneous, online, state and parameter estimations are indeed certain, although perhaps limited to some particularly nice linear systems. However, by resorting to suitable piecewise polynomial approximations, simultaneous state and parameter estimations can be pushed to the realm of nonlinear systems. This chapter dwells on several case studies that illustrate the powerful possibilities of the algebraic approach in the combined state and parameter identification task. Clearly, a clean algebraic approach to solve both problems simultaneously may easily be hampered by the richness of system nonlinearities and their possible combinations with unknown parameters. For this reason, an altogether fresh view of the state-estimation problem, for perturbed input–output models, was undertaken for the natural class of differentially flat systems with available flat outputs. The end of the chapter presents applications of the algebraic method to the handling of the popular synchronization problem in chaotic systems. This problem essentially involves the development of efficient state estimations for nonlinear systems. The algebraic approach is shown to fit the problem of state estimation suitably in chaotic systems. Several well-studied chaotic systems are used as illustrative instances in the application of the algebraic state-estimation problem via differentiations of measurable outputs. As a byproduct, the problem of encrypted message detection, and remote message identification, is readily solved using the developed viewpoint.

Every differentially flat system is an observable system from the vector of flat outputs. For this reason, the state-estimation problem for flat systems is intimately related to the problem of computing the successive time derivatives of the flat outputs. In general, however, the state-estimation problem of a nonlinear dynamical system is tied to time differentiations of the available outputs and of the input signals in a sufficiently large number. We propose, in Chapter 6, a non-asymptotic algebraic procedure for the approximate, piecewise local, estimation of a finite number of time derivatives of a general time signal that may be corrupted by measurement noise. The method is based on results from differential algebra and furnishes some general formulae for the time derivatives of any measurable signal. Naturally, the state-estimation method proposed may be combined with the notion of differential flatness to complete a feedback loop, with desirable dynamics, based on the flat output feedback. In this respect, we no longer regard the nonlinear model of the output variables as a nonlinear function of the state, but as a linear, time-invariant, homogeneous model describing a finite-order, local, polynomial approximation to the realization of the output signal as a time signal.

Lastly, Chapter 7 is devoted to presenting a series of alternatives in the application of the algebraic method for parameter and state estimation in the context of feedback control. Several options are explored, such as the possibility of using bounded exponential functions instead of powers of time variables in the parameter identification for linear systems.

The appendices at the end of the book contain background material, and tutorial introductions, in the algebraic methods of control systems. In particular, linear control systems.

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