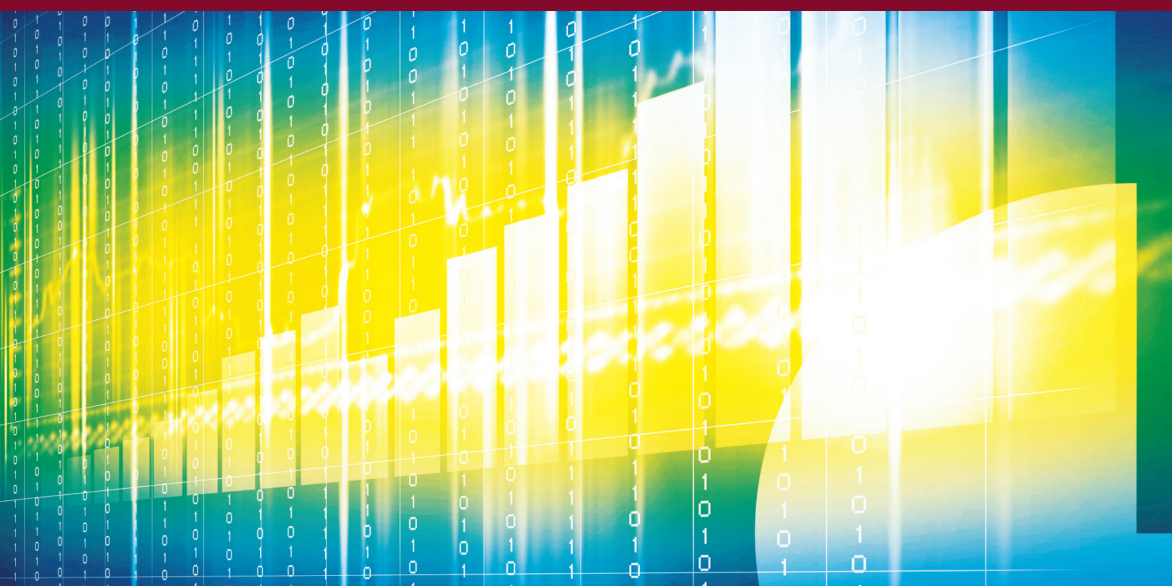
DIGITAL SIGNAL AND IMAGE PROCESSING SERIES

Mathematical Foundations of Image Processing and Analysis 2

Jean-Charles Pinoli



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To
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Preface

The era of imaging sciences and technologies

The important place of ***images*** in the modern world is undeniable. They are intimately integrated into our organic life ("visual perception" is particularly well developed in human beings). They are frequently involved in our daily life (magazines, newspapers, telephones, televisions and video games, etc.), personal life (medical imaging, biological imaging and photographs, etc.), professional life (plant control, office automation, remote monitoring, scanners and video conferencing), etc. They are not confined to the various technological sectors, but they are vectors of observations and investigations of matter at very small scales (electron microscopes and scanning probe microscopes, etc.), or of the universe at very large scales (telescopes and space probes, etc.), sometimes leading major scientific discoveries. Mankind is now able to see images of other worlds without going there (e.g. distant planets, stars and galaxies, or the surface terrain of the Earth) and worlds within (e.g. human organs, geological imaging, or atomic and molecular structures at the nanoscale level). From a technological point of view, this importance is enhanced by the performance of the systems of investigation by imaging and the powers of calculation of computers, which expanded considerably in the second half of the 20th Century, and that are still progressing, with both hardware and software advances.

The scope of ***Imaging Sciences and Technologies*** is broad and multidisciplinary. It involves all the theories, methods, techniques, devices, equipment, applications, software and systems, etc. relating to images in order to obtain information and qualitative and/or quantitative knowledge, in order to investigate, analyze, measure, understand, interpret and finally to decide. The range of applications is broad in contemporary sciences and technologies. The scientific and technical disciplines that are concerned or that use it are numerous: Astronomy, Biology, Electronics, Metallurgy, Geology, Medicine, Neurology, Optics, Physics, Perceptual Psychology and Robotics, etc. and others too numerous to name, and of course Mathematics, with their strengths and their limitations.

Mathematical Imaging

When dealing with image processing and analysis, the most surprising point at first glance, not only for many engineers or scientists, but also for academics and mathematicians, is the key role of Mathematics. Although the image processing and analysis field was historically largely applied and still partly remains so, it is not limited to an engineering field. Indeed, it has attracted the attention of many scientists during the past three decades, and the fundamentals that it requires are becoming strong and of high-level, in particular from a mathematical viewpoint.

The so-called ***Mathematical Imaging*** is currently a rapidly growing field in applied Mathematics, with an increasing need for theoretical Mathematics. More and more mathematicians are interested in carrying out their research into image processing and analysis. In fact, image processing and analysis have created tremendous opportunities for Mathematics and mathematicians. The contemporary field of image processing and analysis is very attractive because it has very interesting application issues, is closely related to the fascinating Human Vision and requires advanced mathematical bases.

Historically, input from mathematicians has had a fundamental impact on many scientific, technological and engineering disciplines. When accurate, robust, stable and efficient models and tools were required in more traditional areas of science and technology, Mathematics often played an important role in helping to supply them. No doubt, the same will be true in the case of imaging sciences. Mathematical Imaging has become a critical, enthusiastic and even exciting, but still in-progress, branch in contemporary sciences.

Author claims

Nowadays, there exist several good books or monographs, each dealing with one or some mathematical fundamentals for image processing and analysis purposes, but a textbook completely focused on the mathematical foundations of image processing and analysis does not currently exist.

The proposed textbook is intended:

– to fill a niche by providing a self-contained, (relatively) complete and informative review of the mathematical foundations of image processing and analysis;

– to emphasize with an (as far as possible) accessible style, the role of Mathematics as a rigorous basis for imaging sciences;

– to be a review of mathematics that are necessary for imaging sciences, often existing only in the (generally hidden) background for non-mathematicians;

– to help mathematicians to become more familiar with image processing and analysis;

– to be a mathematical companion for image processing and analysis students, scientists, researchers, scholars, engineers and even practitioners.

Textbook aims

This textbook aims to provide a comprehensive and convenient overview of the key mathematical concepts, notions, tools and frameworks involved in the various fields of gray-tone and binary image processing and analysis. It establishes a bridge between pure and applied mathematical disciplines, and the processing and analysis of gray-tone and binary images. It is accessible to readers who have neither extensive mathematical training, nor peer knowledge in image processing and analysis. The notations will be simplified as much as possible in order to be more explicative and consistent throughout the textbook. The explanations provided will be sufficiently accurate for one such statement. The mathematical aspects will systematically be discussed in the image processing and analysis context, through practical examples or concrete illustrations. Conversely, the discussed applicative issues allow the role held by Mathematics to be highlighted.

The author would greatly appreciate if the present textbook could help mathematicians to become more familiar with image processing and analysis, and likewise, image processing and image analysis scientists and engineers to get a better understanding of mathematical notions and concepts.

The proposed book is not:

– an introductory book, treatise, or textbook on image processing and analysis;

– a long textbook with extensive treatments on Mathematical Imaging;

– a monograph or a textbook on some mathematical aspects for image processing and analysis;

– a mathematical book with too heavy a jargon and detailed technical developments or complete proofs.

The proposed book is:

– a two-volume, self-contained textbook on the mathematical notions, concepts, operations, structures and frameworks that constitute the foundations of image processing and analysis, emphasizing the role of Mathematics as a rigorous basis for imaging sciences.

Organization of the textbook

This textbook is organized into an introduction, a concluding discussion with perspectives, a textbody, appendices with two tables and three indexes and a detailed bibliography.

The textbook is split over two volumes, made up of 7 main parts divided into 40 chapters and sub-divided into 207 sections.

Part 1 entitled "An Overview of Image Processing and Analysis (IPA)" presents the basic terms and notions for gray-tone and binary imaging (Chapters 1 and 3, respectively), a first overview dealing with the main image processing and image analysis fields and subfields for gray-tone images (Chapter 2), and a second overview dealing with the main image processing and image analysis fields and subfields for binary images (Chapter 4). Then, the key notions and concepts for image processing and analysis are exposed, followed by comments on how and why mathematical imaging frameworks are presented in this textbook (Chapters 5 and 6, respectively).

Part 2 entitled "Basic Mathematical Reminders for Gray-Tone and Binary Image Processing and Analysis" is devoted to basic elements in Mathematics, mainly in set theory, algebra, topology and functional analysis, that can possibly be skipped by the reader well-versed in Mathematics.

Part 3 entitled "The Main Mathematical Notions for the Spatial and Tonal Domains" focuses on the first-level mathematical notions for the spatial and tonal domains (Chapters 9 and 10).

Parts 4, 5, 6 and 7 present the functional and geometrical mathematical frameworks for image processing and analysis, and comprise a total of 30 chapters.

Part 4 entitled "Ten Main Functional Frameworks for Gray Tone Images" focuses on the main mathematical (functional) frameworks for gray-tone image processing and analysis, detailed in 10 chapters.

Part 5 and 6, entitled "Twelve Main Geometrical Frameworks for Binary Images" and "Four Specific Geometrical Frameworks for Binary Images", respectively, focus on the main mathematical (geometric) frameworks for binary image processing and analysis, detailed in 12 chapters and 4 chapters, respectively.

Part 7, entitled "Four 'Hybrid' Frameworks for Gray-Tone and Binary Images", is a further extension and supplementation focusing in 4 chapters on four mixed functional and geometric mathematical frameworks for gray-tone or/and binary images.

The textbook will be organized following two main entries:

– "*The Imaging entry*": from an image processing and analysis viewpoint, the straightforward way to read this textbook is to start from Part 1 and then Part 3.

– "*The Mathematics entry*": the reading of Part 2 is not required. The reader can refer to it if necessary. Part 4 is primarily based on the concepts and tools of functional analysis. Parts 5 and 6 rely primarily on the concepts and tools of geometry. The reading of Parts 5 and 6 are (almost) independent. Part 7 is mathematically advanced and needs the readings of Parts 4, 5 and 6.

The mathematical frameworks for image processing or analysis purposes are presented in separate chapters following a "*generic organization form*", with four sections appearing successively: (1) paradigms, (2) mathematical notions and structures, (3) main approaches for image processing or analysis and (4) main applications to image processing or analysis.

Most chapters end with a section entitled "*additional comments*", in which readers will find some historical comments, several main references: introductory or overview journal articles, seminal and historical articles, textbooks and monographs, bibliographic notes and additional readings, suggested further topics and recommended readings, and finally (often) some references on applications to image processing and analysis, all with short comments.

Important lists or tables are presented in the appendices as follows:

– a detailed and extended appendix on notation is organized in 23 *tables of notations and symbols*; special effort has been put into alleviating the notations and symbols, making them easier to read and understand, promoting genericity and declination, and avoiding confusion and inconsistencies;

– a *table of acronyms*;

– a *table of Latin phrases*;

– a complete *list of referenced authors*, with a few pieces of information (dates of birth and death, nationality, main discipline(s) of expertise). This list is of more cultural interest and will allow the readers to locate in time and space the cited scientists;

– a detailed and extended *list of subjects and keyterms*; this list will often be a real entry for any reader, who wants to search the meaning and use of a particular subject or keyterm.

A large *bibliography* is also proposed, including as far as possible historical references and seminal papers, current reviews, and cornerstone published works.

Intended audiences

This textbook is written for a broad audience: students, mathematicians, image processing and analysis specialists, and even for other scientists and practitioners.

The author hopes that the individual reader should come up with his or her own comfortable usage of the textbook.

Students

This textbook is primarily intended for 3rd/4th year undergraduate, graduate, post-graduate and doctorate students in image processing and analysis, and in Mathematics who are interested in the mathematical foundations of image processing and analysis. These students will be provided with a comprehensive and convenient summary of the mathematical foundations, that they should use or refer to throughout undergraduate, Master of Science (MSc), Master of Engineering (MEng), or PhD courses.

Mathematicians

This textbook is also intended for applied, but also ‘pure’ mathematicians. There are a still growing number of mathematicians in applied and computational Mathematics, but also in pure Mathematics, who have either little or no previous involvement in image processing and image analysis, but wish to broaden their own horizon of view, scope of knowledge, and fields of application. The author recommends that they follow the proposed logical structure of the current textbook. Those readers will find, on the one hand, an overview of image processing and analysis fields and subfields, and, on the other hand, a review of the main mathematical frameworks involved in imaging sciences.

Image processing and analysis specialists

This textbook will serve as a two-volume textbook for practitioners, researchers lecturers or scholars in image processing and analysis that aims at overviewing the mathematical foundations of image processing and analysis. It is hoped that this textbook will become the useful mathematical companion to anybody reading image processing and analysis books or articles, writing research or technical articles, preparing a lecture or a course, or for teaching.

Other scientists and practitioners

As secondary audiences, this textbook should also be of interest to many scientists of various disciplines too numerous to name who make use of images and are thus faced with image processing and analysis problems and tools. They may have an occasional need of this textbook for a better understanding of a mathematical notion.

The textbook is also intended for research and development, or industrial engineers, or project leaders, scientists, technical or scientific directors, wishing to discover or improve their knowledge of the scientific aspects of image processing and analysis, and the role of Mathematics in image processing and analysis.

Underlying matter

This textbook has been written starting from two scientific articles published in French by the Scientific and Technical Encyclopedia "Techniques de l'Ingénieur" in 2012:

– "Mathématiques pour le traitement et l'analyse d'images à tons de gris", *Techniques de l'Ingénieur*, [E6610], 25 pages, February 2012 (Jean-Charles Pinoli) [PIN 12b];

– "Mathématiques pour le traitement et l'analyse d'images binaires", *Techniques de l'Ingénieur*, [E6612], 25 pages, September 2012 (Jean-Charles Pinoli) [PIN 12c];

– Several extensions have been presented and new developments included (e.g. Parts 2, 6 and 7). Four unpublished chapters have been added, together with five important detailed and commented lists or tables: 23 tables of notations and symbols, a table of Latin phrases, a list of acronyms, a list of referenced authors and a list of subjects.

This textbook is also an outgrowth of PhD, Master of Engineering and Master of Science courses, which have been given for many years by the author.

Notes for the textbook reading

"*Italics*" will be used to mark a passage in a foreign language, including in particular Latin phrases, that are briefly defined and explained in the Table of Latin Phrases in Appendices.

Key terms and subject matters will appear in "***slanted bold***" in the body of the textbook. They are collected in the Appendices in the List of Subjects.

Quotation marks or inverted commas (informally referred to as quotes) are punctuation marks surrounding a word or phrase with a specific meaning or use. *Single quotes* '...' will be used to indicate a different meaning, or a direct, rough or even abusive speech. *Double quotes* "..." will emphasize that an instance of a word refers to the word itself rather than its associated concept. The so-called "use-mention distinction" is necessary to make a clear distinction between using a word or phrase and mentioning it.

As a rule, a whole publication (e.g. a book title) would be both slanted and double quoted, while a citation will be both italicized and double quoted.

JEAN-CHARLES PINOLI
June, 2014

Introduction

I.1. Imaging sciences and technologies

The last few decades have largely been the dawning years of the era of ***Imaging Sciences and Technologies***, which is a multidisciplinary field concerned with the (by alphabetical order) acquisition, analysis, collection, display, duplication, generation, modeling, modification, processing, reconstruction, recording, rendering, representation, simulation, synthesis and visualization, etc., of images.

From a computer science viewpoint, there are two dual fields: (1) ***Computer Vision***, which tries to reconstruct the 3D world from observed 2D images, and (2) ***Computer Graphics***, which pursues the opposite direction by designing suitable 2D scene images to simulate our 3D world. Image processing is the crucial middle way connecting the two. Image synthesis in the computer graphics field being the dual of image analysis treated in computer vision.

As the human visual system has been achieved by mother nature, there is nowadays a tremendous need for developing so-called ***Artificial Vision*** systems. Such systems consist of four more or less independent stages: (1) image acquisition, (2) image processing, (3) image analysis and (4) image interpretation.

"***Image acquisition***" mainly focuses on the physical and technological mechanisms and systems by which imaging devices generate spatial observations, but it also involves mathematical and computational models and methods implemented on computers, integrated into and/or associated to such imaging systems. The term "***image processing***", is usually understood as all kinds of operations or transformations performed onto images (or sequences of images), in order to increase their quality, restore their original content, emphasize some particular aspects of the information content, optimize their transmission, or perform radiometric and/or spatial analysis. The term "***image analysis***" is usually understood as all kinds of operations or operators performed on images (or sequences of images), in order to extract qualitative and/or quantitative information content, perform various measurements, and apply statistical analysis. All these methods and techniques have of course a wide range of applications in our daily world: biological imaging, industrial vision, materials imaging, medical imaging, multimedia applications, quality control, satellite imaging, traffic control and so on. "***Image interpretation***" is roughly speaking, the inverse stage of image acquisition. The latter deals with the 2D or 3D imaging of spatial structures that are investigated. The former, however, aims at understanding the observed 3D world from generally 2D images.

I.2. Historical elements on image processing and image analysis

The first digital pictures dated back to the early 1920s [MCF 72]. Then, practical works and more theoretical research mainly focused on picture coding and compression for transmission applications, and then for television image signals (see, e.g. [MER 34, GOL 51]) [SCH 67].

Historically, the "***Image Processing and Analysis (IPA)***" field has emerged early from the 1950s (see, e.g. [KOV 55] or [KIR 57]), and mainly from the 1960s (see, e.g. [GRA 67, SCH 67, ROS 69a, ROS 69b, ROS 73c] and many references therein), in works carried out and published by researchers and engineers belonging to several academic and professional communities, and from different scientific trainings, mainly "Applied Physics" (Electrical Engineering and Signal Processing), "Computer Sciences" (Computer Vision, Pattern Recognition and Artificial Intelligence), and "Mathematics" (mainly, Statistics, Applied Functional Analysis and (generally discrete) Geometry and Topology).

The first textbook entitled "*Picture Processing by Computer*" [ROS 69a] was written in 1969 by Azriel Rosenfeld, a mathematician, who was then regarded as a pioneer, and even "the" pioneer of image processing and image analysis, and as a leading researcher in the world in the field of computer image processing and analysis. Another book appeared soon after, with a similar title "*Computer Techniques in Image Processing*" [AND 72], by Harry C. Andrews, an applied physicist and computer scientist.

Several other *pioneering textbooks* were published later in the 1970s and early 1980s, mainly: "*Digital Image Restoration*" (1977) [AND 77] by Harry C. Andrews, and Bobby R. Hunt, applied physicists and computer scientists, "Digital Image Processing" (1977) [GON 87; 1st ed., 1977], by Rafael C. Gonzalez and Paul Wintz, electrical engineering specialists, "*Digital Image Processing*" (1978) [PRA 07; 1st ed., 1978] by William K. Pratt, an applied physicist, "*A Computational Investigation into the Human Representation and Processing of Visual Information*" (1982) [MAR 82] by David Marr, a computer scientist, "*Algorithms for Graphics and Image Processing*" (1982) [PAV 12; 1st ed., 1982] by Theo Pavlidis, a computer scientist, "*Image Analysis and Mathematical Morphology*" (1982) [SER 82] by Jean Serra, an applied mathematician.

Concerning technical, engineering and scientific journals, deserving of special mention are two journals that early on published papers on picture processing. One of these journals, the "*Proceedings of the IRE*" (the journal of the "*Institute of Radio Engineers*"), was founded in 1913 and was renamed in 1963 as the "*Proceedings of the IEEE*" (the journal of the "*Institute of Electrical and Electronics Engineers (IEEE)*"), when the "*American Institute of Electrical Engineers (AIEE)*" and the "*Institute of Radio Engineers (IRE)*" merged to form the "*Institute of Electrical and Electronic Engineers (IEEE)*"). The other journal, "*Pattern Recognition*" (the journal of the "*Pattern Recognition Society*"), was founded in 1968. In this connection, *The Journal of the ACM* (the journal of the *Association for Computing Machinery (ACM)*, established in 1954) should also be mentioned, which published several papers on image processing and analysis in the 1960s and 1970s. The series of volumes on "*Machine Intelligence*", initiated in 1967, and of the journal "*Artificial Intelligence*", founded in 1970, should also be noted.

The *first scientific journals* dedicated to, completely or partially, image processing and analysis were published during the 1970s (e.g. "*Computer Graphics, Vision and Image Processing*" in 1972 and "*IEEE Transactions on Pattern Analysis and Machine Intelligence*" in 1979). After that period of pioneers, the field of image processing and analysis started its growth from about the middle of the 1980s. In Europe, "*Acta Stereologica*" was founded in 1982 by the "*International Society for Stereology*" and was renamed "*Image Analysis and Stereology*" in 1999. Many papers dealing with image analysis were and still are currently published.

In addition, significant contributions to image processing and even more to image analysis were also made by researchers or practitioners from other disciplines, such as for example the cytometrists, geologists, metallographs and mineralogists, just to name a few (e.g. [COS 86, WEI 81, RIG 89]).

The first *international scientific conferences* focusing only on image processing and analysis appeared at end of the 1980s (i.e. "*International Conference on Computer Vision (ICCV)*" in 1987) and at the beginning of the 1990s (i.e. "*International Conference on Image Processing (ICIP)*" in 1994).

The first *mathematical imaging journal* explicitly on both Mathematics and Image Processing and Analysis only appeared in the early 1990s (i.e. "*Journal of Mathematical Imaging and Vision*" in 1992). Very recently, the SIAM society ("*Society for Industrial and Applied Mathematics*") published its first mathematical journal in Mathematical Imaging (i.e. "*SIAM Journal on Imaging Sciences*") in 2008.

However, although presented in this short introductory, historical discussion under the joint name “Image Processing and Image Analysis”, it is important to note that on one side “Image Processing”, and on the other side “Image Analysis” have been addressed by researchers and engineers generally from different scientific communities. This is still often the case even if an interpenetration of the two fields is in progress. Earlier, some mathematicians focused on Image Analysis in the 1960s and 1970s. More mathematicians became interested in Image Processing from the 1980s, and even more in the 1990s. One of the main scientific reasons, if not the most important, is that image analysis required knowledge of geometry and topology, that were and still are often too poorly taught in MSc courses, and therefore are less prevalent than those most used in mathematical analysis, especially due to the strong interest in Mathematical Physics in general, during the 1980s, and in particular for image problem modeling using partial differential equations and their numerical resolutions. The following statement then appears as a logical consequence:

There exist nowadays a (relatively) large number of books dealing with image processing, but mainly on a or some particular field(s), and often in the form of edited books rather than monographs. On the contrary, only a small number of books are dealing with image analysis.

I.3. Mathematical Imaging

Early mathematical contributions and/or reviews were authored by researchers of the Electrical Engineering and Signal Processing community (see, e.g. [JAI 81]), and Discrete Geometry community (see, e.g. [ROS 66, GRA 71]).

Several areas of Mathematics have contributed to and in fact increasingly contribute to essential progress of Image Processing and Image Analysis. Mathematics provide the fundamentals for image processing and image analysis frameworks, operations, models, techniques and methods.

However:

– there is no single “mathematical theory of image processing and image analysis”. Quite often, different approaches exist to model the same problem, using notions coming from different disciplines of Mathematics. Those disciplines underlying and/or involved in Image Processing and Analysis range from Algebra to Analysis, from Set Theory and Topology to Geometry, from Functional Analysis to Calculus of Variations, from Probability Theory to Statistics, and so on;

– the ties between Image Sciences and Mathematics are still not strong enough. International conferences are very often organized by a specific scientific community. Very few symposiums are organized to promote interaction between researchers of image sciences and mathematicians.