Wayne Durham

Aircraft Flight Dynamics and Control

Aerospace Series

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For Fred Lutze. If I got anything wrong here it's because I didn't listen to him closely enough.
For Hank Kelley. He was right. Sometimes you have to stare at the problem for a very long time before you see it. Sitzfleisch.

Contents

Series Preface

The Aerospace Series covers a wide range of aerospace vehicles and their systems, comprehensively covering aspects of structural and system design in theoretical and practical terms. This book offers a clear and systematic treatment of flight dynamics and control which complements other books in the Series, especially books by McClean, Swatton and Diston.

The subject of flight dynamics and control has always been of importance in the design and operation of any aircraft, much of it learned by trial and error in the development of very early aircraft. It developed as an engineering science throughout succeeding generations of aircraft to support increasing demands of aircraft stability and control and it now has a major role to play in the design of modern aircraft to ensure efficient, comfortable and safe flight. The emergence of a need for unstable and highly manoeuvrable combat aircraft, and the dependence on full authority fly-by-wire software based control systems for both military and commercial aircraft together with a demand for economic automatic operation has ensured that the understanding of flight dynamics is essential for all designers of integrated flight systems. Growing trends towards unmanned air vehicles will serve to strengthen this dependency. Modern on-board sensors and computing in integrated systems offers the opportunity to sense aircraft motions and rates and to include aircraft models in the control systems to further improve aircraft performance. Engineers with an interest in these aspects will find this book essential reading.

The book has been built up from a combination of practical flying experience, the evolutionary improvement of a mentor's text and a desire that students should understand the basic concepts underlying modern modelling practices before applying them – an excellent way to evolve a text book to provide a real teaching experience. Much of the content has been validated by use in a teaching environment over a period of years.

This is a book for all those working in the field of flight control systems and aircraft performance for both manned and unmanned flight control as well as auto-flight control for real time applications in aircraft and high fidelity simulation.

Peter Belobaba, Jonathan Cooper and Allan Seabridge

Glossary

Greek symbols

α Angle of attack. The aerodynamic angle between the projection of the relative wind onto the airplane's plane of symmetry and a suitably defined body fixed x-axis.

n/α The change in load factor n resulting from a change in angle-of-attack α, or more properly the partial derivative of the former with respect to the latter. A parameter used in the determination of short-period frequency requirements in flying qualities specifications, often called the 'control anticipation parameter'.

β Sideslip angle. The aerodynamic angle between the velocity vector and the airplane's plane of symmetry.

$\boldsymbol{\omega}, \omega$ As a vector (**bold**), usually signifies angular velocity. As a scalar, often subscripted, a component of such a vector.

χ Tracking angle. One of three angles that define a 321 rotation from inertial to the wind reference frames.

δ_ℓ A generic control effector that generates rolling moments L. It is often taken to be the ailerons, δ_a.

δ_a The ailerons, positive with the right aileron trailing-edge down and left aileron trailing-edge up.

δ_e The elevator, positive with trailing-edge down.

δ_m A generic control effector that generates pitching moments M. It is often taken to be the elevator, δ_e, or horizontal tail, δ_{HT}.

δ_n A generic control effector that generates yawing moments N. It is often taken to be the rudder, δ_r.

δ_r The rudder, positive with trailing-edge left.

δ_T Thrust, or throttle control.

Δ Indicates a change from reference conditions of the quantity it precedes. Often omitted when implied by context.

γ Flight-path angle. One of three angles that define a 321 rotation from inertial to the wind reference frames.

λ An eigenvalue, units s^{-1}.

λ Latitude on the earth.

Λ A diagonal matrix of a system's eigenvalues.

μ Longitude on the earth.

μ Wind-axis bank angle. One of three angles that define a 321 rotation from inertial to the wind reference frames.
ω_d Damped frequency of an oscillatory mode.
ω_n Natural frequency of an oscillatory mode.
Ω Every combination of control effector deflections that are admissible, i.e., that are within the limits of travel or deflection.
ϕ Bank attitude. One of three angles that define a 321 rotation from inertial to body-fixed reference frames.
Φ The effects, usually body-axis moments, of every combination of control effector deflections in Ω. Sometimes called the Attainable Moment Subset.
ψ Heading angle. One of three angles that define a 321 rotation from inertial to body-fixed reference frames.
ρ Density (property of the atmosphere).
θ Pitch attitude. One of three angles that define a 321 rotation from inertial to body-fixed reference frames.
ζ Damping ratio of an oscillatory mode.

Acronyms, abbreviations, and other terms

$\cdot$ Placed above a symbol of a time-varying entity, differentiation with respect to time.
$\hat{}$ Placed above a symbol to indicate that it is a non-dimensional quantity.
$\{\mathbf{v}_a^b\}_c$ A vector $\mathbf{v}$ that is some feature of a (position, velocitiy, etc.) relative to b and represented in the coordinate system of c.
$\mathbf{f}$ A vector of scalar functions, or a function of a vector.
$\mathbf{F}$ A vector usually signifying force. See $X,\ Y,\ Z$ and $L,\ C,\ D$.
$\mathbf{h}$ A vector usually signifying angular momentum.
$\mathbf{M}$ A vector usually signifying body-axis moments. See $L,\ M,\ N$.
$\mathbf{q},\ q$ As a vector (**bold**), usually signifies the transformed states of a system, such transformation serving to uncouple the dynamics. As a scalar, a component of such a vector.
$\mathbf{r}, r$ As a vector (**bold**), usually signifies position. As a scalar, often subscripted, a component of such a vector.
$\mathbf{T}$ A vector usually signifying thrust.
$\mathbf{u}$ Vector of control effector variables.
$\mathbf{v}, v$ As a vector (**bold**), usually signifies linear velocity. As a scalar, often subscripted, a component of such a vector.
$\mathbf{W}$ A vector usually signifying weight.
$\mathbf{x}$ Vector of state variables.
$\mathcal{A}$ Aspect ratio.
$\mathcal{L}$ LaPlace transform operator.
$\tilde{}$ Placed above a symbol to indicate that it is an approximation or an approximate quantity.
A, B Matrices of the linearized equations of motion, as in $\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$. A is the system matrix, B is control-effectiveness matrix.
C_{x_y} The non-dimensional stability or control derivative of x with respect to y. It is the non-dimensional form of X_y, q.v.

c_i, r_j	The ith column, jth row of a matrix.
Comp	Complementary. A superscript to certain dynamic responses.
Cont	Controllable. A superscript to certain dynamic responses.
$D(\cdot)$	Non-dimensional differentiation.
$d(s)$	The characteristic polynomial of a system. The roots of the characteristic equation, $d(s) = 0$, are the systems eigenvalues.
d	Desired. A subscript to a dynamical response.
d	Subscript identifying the Dutch roll response mode.
DR	Subscript identifying the Dutch roll response mode. In flying qualities specifications the subscript is d.
F_B	Body-fixed reference frames.
F_E	Earth-fixed reference frame.
F_{EC}	Earth-centered reference frame.
F_H	Local-horizontal reference frame.
F_I	Inertial reference frame.
F_P	Principal axes.
F_S	Stability-axis system.
F_W	Wind-axis system.
F_Z	Zero-lift body-axis system.
$G(s)$	A matrix of transfer functions.
g	Acceleration of gravity. As a non-dimensional quantity g is the load factor n, q.v.
I	Identity matrix.
I	With subscripts, moment of inertia.
j	Imaginary number, $j = \sqrt{-1}$. Preference for j rather than i often stems from a background in electrical engineering, where i is electrical current.
Kine	Kinematic. A superscript to certain dynamic responses.
L, C, D	Lift, side force, and drag. Wind-axis forces in the $-x$-, $-y$- and $-z$-directions, respectively.
L, M, N	Body-axis rolling, pitching, and yawing moments, respectively.
L	Lift, or rolling moment, depending on context.
LD	Lateral–directional. Sometimes *Lat–Dir*.
Long	Longitudinal.
M	A matrix whose columns are the eigenvectors of a system.
M	Mach number.
m	Mass.
$N_{1/2}, N_2$	Number of cycles to half or double amplitude.
n	Load factor, the ratio of lift to weight, $n = L/W$. Measured in gs.
p_W, q_W, r_W	Wind-axis roll rate, pitch rate, and yaw rate, respectively.
p, q, r	Body-axis roll rate, pitch rate, and yaw rate, respectively.
P	A pseudo-inverse of a matrix B. $BPB = B$ and $PBP = P$, with appropriate dimensions.
Ph	Subscript identifying the phugoid response mode.
$q_0 \dots q_3$	Euler parameters.
$q, \overline{q}$	The pitch rate is q. The dynamic pressure is $\overline{q}$, Kevin.
R	Subscript identifying the roll subsidence response mode.

Ref	Subscript, 'evaluated in reference conditions'.
RS	Subscript identifying the coupled roll–spiral response mode.
$S, \bar{c}, b$	Wing area, chord, and span, respectively.
s	Complex variable in LaPlace transformations.
S	Subscript identifying the spiral mode.
SP	Subscript identifying the short-period response mode.
ss	Subscript signifying steady state.
$t_{1/2}, t_2$	Time to half or double amplitude, seconds.
$T_{a,b}$	A transformation matrix that transforms vectors in coordinate system b to their representation in system a.
T	The period of an oscillatory response, seconds.
t	Time, seconds.
V_C	Magnitude of the velocity of the center of mass.
x_W, y_W, z_W	Names of wind axes.
X_y	Where X is a force or moment and y is a state or control, a dimensional derivative, $\partial X/\partial y$. It is the dimensional form of C_{x_y}, q.v. Note that the definition does *not* include division by mass or moment of inertia in this book.
X, Y, Z	Body-axis forces in the x-, y- and z-directions, respectively.
x, y, z	Names of axes. With no subscripts usually taken to be body axes.
8785C	Short for MIL-F-8785C, 'Military Specification, Flying Qualities of Piloted Airplanes'.
ACTIVE	Advanced Control Technology for Integrated Vehicles. A research F-15 with differential canards, axisymmetric thrust vectoring, and other novel features.
AMS	Attainable Moment Subset. See Φ.
ARI	Aileron–Rudder Interconnect. Normally used to reduce adverse yaw due to aileron deflection.
BIUG	Background Information and User's Guide, companion to Military Specifications for Flying Qualities.
CAS	Control Augmentation System.
CHR	Cooper–Harper Rating; sometimes HQR.
Control effector	The devices that directly effect control by changing forces or moments, such as ailerons or rudders. When we say 'controls' with no qualification, we usually mean the control effectors. The sign convention for conventional flapping control effectors follows a right-hand rule, with the thumb along the axis the effector is designed to generate moments, and the curled fingers denoting the positive deflection of the trailing edge.
Control inceptor	Cockpit devices that control, through direct linkage or a flight-control system or computer, the control effectors. Positive control inceptor deflections correspond to positive deflections of the effectors they are connected to, barring such things as aileron–rudder interconnects (ARI, q.v.).
E	The capital letter in Euler's name, not lowercase. Like the capital V in the Victorian era. 'Euler' is pronounced 'Oh e ler' by Swiss Germans, or 'Oiler' by many English speakers, but never 'Yuler'.

FBW	Fly By Wire. The pilot flies the computer, the computer flies the airplane.
Ganged	Said of mechanical devices linked so that they move in fixed relation to each other, such as ailerons and the rudder.
HARV	High Angle-of-Attack Research Vehicle.
HQR	Handling Qualities Rating.
Kt	Abbreviation for knot, a nautical mile per hour.
Lat–Dir	Lateral–directional.
OBM	On-Board Model. A set of aerodynamic data for an aircraft stored in a computer in the aircraft's flight control computer.
PA	Powered Approach. One of several flight phases defined in flying qualities specifications. See Section 11.2 for a complete list.
PIO	Pilot-Induced Oscillation. There's a more politically correct term that removes the onus from the pilot.
PR	Pilot Rating; sometimes HQR.
SAS	Stability Augmentation System.
SSSLF	Steady, Straight, Symmetric, Level Flight.
SVD	Singular-Value Decomposition.
TEU, TED, TEL, TER	Trailing-Edge Up, Down, Left, Right. Terms used to describe the deflection of flapping control surfaces.

1

Introduction

1.1 Background

This book grew out of several years of teaching a flight dynamics course at *The Virginia Polytechnic Institute & State University,* more commonly known as *Virginia Tech,* in Blacksburg, Virginia, USA. That course was initially based on Bernard Etkin's excellent graduate level text *Dynamics of Atmospheric Flight* (Etkin, 1972). There is a newer edition than that cited, but the author prefers his copy, as it can be relied on to fall open to the desired pages.

The author was taken on at Virginia Tech after a full career in the U.S. Navy as a fighter pilot and engineering test pilot. They taught an old dog new tricks, awarded him his PhD, and put him to work.

The author's background crept into the course presentation and Etkin's treatment became more and more modified as different approaches were taken to explaining things.

A sheaf of hand-written notes from a mentor who had actually designed flight control systems at Northrop; course material and flight experience from two different test-pilot schools; the experience of thousands of hours of flight in aircraft at the leading edge of the technology of their time; the precise and clear-minded approach to the analysis of flight dynamics problems that Fred Lutze demanded: all these things and more overlaid the tone and style of the course.

Then, one day, the author's course notes were so different from Etkin's work that it made no sense to continue using that book, and this book was born.

The course as taught at Virginia Tech was intended for first-year graduate students in aerospace engineering. The students all had previous course work in engineering mathematics. For purposes of the current treatment, multi-variable calculus, and a good understanding of ordinary differential equations and their solutions in the time domain and using LaPlace transforms are needed.

The undergraduate preparation at Virginia Tech also included a sound course in aircraft performance as in, for example, Anderson's excellent text (Anderson, 1989). Our undergraduates also had an award-winning sequence of courses in aircraft design taught by Bill Mason. While that course undoubtedly gave the students a better feel for what makes airplanes fly, such background is in no way essential to the understanding of this book.

The undergraduates had also studied intoductory flight stability and control, most often using another of Etkin's books (Etkin and Reid, 1995). Once again, previous exposure

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to this subject matter is by no means essential to mastering the material in this book. The author has seen mechanical engineering students who had *no* previous course-work involving airplanes or *any* airplane experience stand at the top of their classes. These students often had small models of airplanes that they brought with them to class.

The chapters on automatic flight control were not part of the course as originally taught. The major thrust of the book is airplane flight dynamics, but it was felt that some discussion of control was desirable to motivate future study. It seemed unlikely that anything as comprehensive as, for example, Stevens and Lewis (1992) could be included. Therefore just a basic introduction to feedback control is presented, but with some examples that are probably not often found in flight control design.

The last chapter was motivated by the author's pride in the accomplishments of many of his past students in real-world applications of flight dynamics and control.

The method of choice in current flight control system design appears to have settled on to dynamic inversion, and the associated problem of control allocation, and so a brief introduction to these disciplines is offered. Enough material is presented that the reader will be comfortable in the midst of modern flight control system engineers, and may even know something they do not.

Finally, almost all references to MATLAB® will be new to previous students. The author's approach to flight dynamics and control has always been to learn the basics, then adopt the modern tools and software to implement the basics. It is not expected that any reader will often use Fedeeva's algorithm in his work, but understanding it does afford one a singular look inside the minds of the men and women who solved these problems with pencil and paper, and who later went on to develop the algorithms that underlie the simple looking MATLAB® commands. But the dimension of the problems kept getting bigger and bigger, so some MATLAB® tools are now used.

1.2 Overview

The study of aircraft flight dynamics boils down to the determination of the position and velocity of an aircraft at some arbitrary time. This determination will be developed as the *equations of motion* of the airplane. The equations of motion consist of nonlinear ordinary differential equations in which the independent variables are the *states* of the airplane–the variables that fully describe the position and velocity.

The results of the analysis of flight dynamics–the equations of motion–are important in several related studies and disciplines. Chief among these are:

- Aircraft performance. Items of flight performance typically include stall speeds, level flight performance (range, endurance, etc.), excess power and acceleration characteristics, turn performance and agility, climb performance, descent performance, and takeoff and landing performance. Each of these items is governed by the equations of motion. The equations of motion are analyzed to determine the relevant parameters of the performance item, and these parameters are used to devise flight-test techniques to measure performance, or alternatively, to modify aircraft design to improve a particular area of performance.
- Aircraft control. Aircraft control is a very broad discipline, with primary sub-disciplines of manned control, automatic control, and optimal control.

- Manned control refers to a human operator, manipulating control inceptors in the cockpit to drive external control effectors to modify the state and change the trajectory of the aircraft. The relative ease with which a pilot can control the aircraft is described as the aircraft's *flying qualities*. Here, the equations of motion are analyzed to determine the factors that influence the pilot's workload in controlling the aircraft. These factors drive flight-test techniques and provide design guidance to improve flying qualities.
- Automatic control ranges from stability augmentation systems, which modify the aircraft's response to manned control, to autopilots, to full fly-by-wire systems. In fly-by-wire systems, the pilot effectively flies a computer, and the computer decides how to drive the external control effectors to best satisfy what its program thinks the pilot wants to do. Most forms of automatic control are typically based on a special, linearized, form of the equations of motion. The linearized equations of motion are used to design feedback systems to achieve the desired aircraft responses.
- Optimal control is often analyzed open-loop, that is, with no human operator. Representative objectives of optimal control are to minimize (or maximize) some performance measure, such as minimum time to climb, maximum altitude, or minimum tracking error in the presence of external disturbances. The complexity of the solution techniques used to determine optimal control usually means that some simplification of the equations of motion is required. Some ways in which the equations of motion are simplified include treating the aircraft as a point-mass with no rotational dynamics, or reduction in order (reducing the number of states) of the equations of motion.

• Flight simulation. In flight simulation the full nonlinear equations of motion, as ordinary differential equations, are programmed into a computer that integrates the equations (often using high-speed computers in real-time) using numerical integration algorithms. The forces that act upon the airplane are often provided as data in tabular form. The data may be aerodynamic (from wind tunnel or flight tests), or any other description of external forces (thrust, landing gear reactions, etc.)

The information that describes an airplane's position and velocity is usually expressed as relative to some external reference frame, for instance:

• Position and velocity relative to the earth, required for navigation. A pilot navigating from city to city needs to maintain altitude, track, and airspeed within parameters dictated by air-traffic control, and aircraft performance requirements. During terminal flight phases, especially approach and landing, the position and velocity must be maintained within very close tolerances.
• Position and velocity relative to another aircraft, required for rendezvous, formation flying, in-flight refueling, air-to-air combat, and so on. Military aircraft often fly within a few feet of one another; only the relative position and speed are important to the wingman, so that position and velocity relative to the earth are relatively unimportant.
• Position and velocity relative to the atmosphere determines the aerodynamic forces and moments. Students of aircraft performance immediately recognize the relationship of an aircraft's angle-of-attack to its lift coefficient, and of lift coefficient and dynamic pressure to the lift force itself. These simple relationships are the tip of the iceberg:

angular velocities, as well as other aerodynamic relationships, are needed for the most basic understanding of the forces and moments acting on a maneuvering aircraft.

- The last example offered is less intuitive than the others. We consider the position and velocity of an aircraft relative to some reference condition. If the position tends back to the reference position, and the change in velocity tends to zero, then we have described a stable flight condition. The changes in position and velocity might be induced by some external influence, such as a gust or turbulence. An aircraft that is stable in response to such disturbances will be easier to fly. On the other hand, the changes in relative position and velocity might be created by the pilot redefining the reference condition, say by changing from straight, level flight to turning flight. In this case the response of the aircraft determines the ease with which pilot is able to control it.

The approach taken in this book is essentially an application of Newton's Second Law,

$$F = ma \tag{1.1}$$

Simplistically, we assume some knowledge of the mass m and the applied forces F, solve for the acceleration $a = F/m$, and integrate the result with respect to time twice to yield velocity and position, respectively. There are, however, several considerations that will make our analysis non-trivial:

- Newton's Second Law correctly stated is that the external forces acting on a particle of infinitesimal mass are proportional to the time rate of change of the inertial momentum of the particle. We will show that the formula $F = ma$ can be applied to the aircraft as a whole, so long as we are talking about the acceleration of the instantaneous center of mass. The problem of finding an inertial reference frame in which to measure the accelerations will be dealt with by determining a reference frame that is almost inertial, and hopefully showing that the approximation does not introduce too great an error.
- The preceding mention of Newton's Second Law dealt with linear accelerations and can be expected to yield inertial position and linear velocity. An aircraft's angular position and velocity is also of great interest and must be determined through the extension of Newton's Law that relates externally applied moments (torque) to the time rate of change of angular momentum. This will yield the angular accelerations, and twice integrating we obtain angular position and velocity. One major problem with this formulation is that the various reference frames of interest generally rotate with respect to one another, which will require consideration of the various rotational accelerations.
- The mass of what we consider to be the aircraft may become redistributed and change, for example as fuel is burned or external stores are released. The redistribution of mass within the aircraft can arise from many sources and may be easy to consider (rotating machinery) or very difficult to formulate (aeroelastic flutter). It is impractical and undesirable to keep track of the various particles of mass that are moving around or are no longer attached to the aircraft, so we will look for reasonable approximations that allow us to neglect them.
- The various motions, forces, and moments we will consider all have coordinate systems in which they are most naturally characterized. For example, in a suitably defined coordinate system gravity always points 'down' or in the z-direction. For another, our

study of airfoils yields characterizations in which lift is perpendicular to the mean airflow and drag is parallel to it, suggesting a coordinate system in which some axis points in the direction of the relative wind. We will therefore define several coordinate systems that typically are rotating relative to one another. The problem then becomes one of describing the orientation of one system with respect to another, and determining how the orientation varies with time.

- The externally applied forces and moments may be hard to calculate accurately. Only for the force of gravity do we have a reasonably accurate approximation. Propulsive forces depend on a variety of factors that are difficult to predict, such as propeller efficiency or duct losses. Aerodynamic forces and moments in particular create difficulties because they are dependent in complex ways upon the various quantities we are trying to determine. Our understanding of those dependencies relies on empirical and analytical studies, the most extensive of which capture only the most salient relationships. Our study of flight dynamics will typically ignore the uncertainties in aerodynamic and propulsive data, marking these uncertainties for future consideration.
- Assuming we can find tools and reasonable approximations to deal with the aforementioned difficulties and can formulate $a = F/m$ (and its rotational counterpart) then we have the problem of solving these equations for positions and velocities. While it is easy to state that the velocity is the first integral of acceleration, and that position is its second, it is quite another thing to actually solve these equations. The complex relationships among the variables involved guarantee that we will have to find solutions to coupled, nonlinear ordinary differential equations for which analytical solutions are generally unavailable. We will overcome this difficulty by first considering unaccelerated motion of the aircraft, and then asking the question: how does the aircraft behave following small disturbances from this motion, or in response to control inputs? The result of this analysis will be systems of coupled, linear ordinary differential equations for which solutions are available.
- The solutions to the systems of linear ordinary differential equations (equations of motion) will tell us a good deal about how the aircraft behaves as a dynamical system. Unanswered is the question of what the behavior should be in order for the aircraft to be a ‘good’ aircraft. The problem is that most aircraft are piloted, and the human pilot does not want to spend all the time making corrections to keep the aircraft pointing in the right direction. The response and behavior of a piloted aircraft is called flying qualities (sometimes, handling qualities). The difficulty here is that how a pilot would like an aircraft to handle varies from pilot to pilot, and depends on what the pilot is trying to get the aircraft to do. Our study of flying qualities will be based on statistical analyses of a range of pilots’ opinions that give rise to criteria and guidance for the design of aircraft.
- In the event an aircraft does not have inherently good flying qualities, it may be necessary to modify certain dynamical response characteristics. Structural modifications to an aircraft late in the design or in its operational phase are costly. It is relatively inexpensive to incorporate electronic control systems that can be tuned to provide the desired characteristics. The approach we will first investigate utilizes classical feedback control theory. Then we will briefly examine a recent development in automatic control called *dynamic inversion,* and examine its application to interesting control problems.

1.3 Customs and Conventions

We will attempt to proceed from the general to the specific in addressing the various aspects of flight dynamics and control. We will define certain generic quantities and operations needed to address the problem. In applying these results to the study of aircraft, custom and convention dictate that certain variables get their own names. The definitions used here are largely those of Etkin (1972), generally held in the United States to be authoritative. One should always be careful when encountering any terminology to be sure the exact definitions are understood.

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2

Coordinate Systems

2.1 Background

The need to define appropriate coordinate systems arises from two considerations. First, there may be some particular coordinate system in which the position and velocity of the aircraft 'make sense'. For navigation we are concerned with position and velocity with respect to the Earth, whereas for aircraft performance we need position and velocity with respect to the atmosphere. Second, there are coordinate systems in which the phenomena of interest are most naturally expressed. The direction of a jet engine's propulsive force may often be considered fixed with respect to the body of the aircraft.

All coordinate systems will be right-handed and orthogonal. Coordinate systems will be designated by the symbol F with a subscript intended to be a mnemonic for the name of the system, such as F_I for the inertial reference frame. The origin of the system will be denoted by O and a subscript (e.g., O_I). If we speak of where a coordinate system is we mean where its origin is. Axes of the system are labled x, y, and z with the appropriate subscript. Unit vectors along x, y, and z will be denoted **i**, **j**, and **k**, respectively, and subscripted appropriately.

It is customary in flight dynamics to omit subscripts when speaking of certain body-fixed coordinate systems. If this is not the case then the lack of subscripts will be taken to mean a generic system.

The definition of a coordinate system must state the location of its origin and the means of determining at least two of its axes, the third axis being determined by completing the right-hand system. The location of the origin and orientation of the axes may be arbitrary within certain restrictions, but once selected may not be changed. Following are the main coordinate systems of interest.

2.2 The Coordinate Systems

2.2.1 *The inertial reference frame, F_I*

The location of the origin may be any point that is completely unaccelerated (inertial), and the orientation of the axes is usually irrelevant to most problems so long as they too are fixed with respect to inertial space. For the purposes of this book the origin is at the Great Galactic Center.

Aircraft Flight Dynamics and Control, First Edition. Wayne Durham.

2.2.2 *The earth-centered reference frame, F_{EC}*

As its name suggests this coordinate system has its origin at the center of the Earth (Figure 2.1). Its axes may be arbitrarily selected with respect to fixed positions on the surface of the Earth. We will take x_{EC} pointing from O_{EC} to the point of zero latitude and zero longitude on the Earth's surface, and z_{EC} in the direction of the spin vector of the Earth. This coordinate system obviously rotates with the Earth.

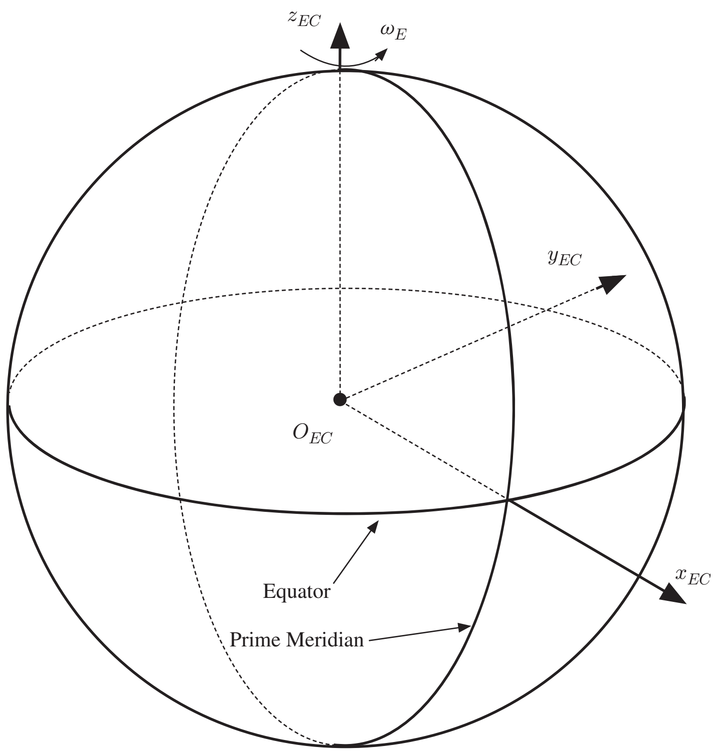

Figure 2.1 Earth-centered reference frame.

2.2.3 *The earth-fixed reference frame, F_E*

This coordinate system (Figure 2.2) has its origin fixed to an arbitrary point on the surface of the Earth (assumed to be a uniform sphere). x_E points due north, y_E points due east, and z_E points toward the center of the Earth.

2.2.4 *The local-horizontal reference frame, F_H*

This coordinate system (Figure 2.3) has its origin fixed to any arbitrary point that may be free to move relative to the Earth (assumed to be a uniform sphere). For example, the origin may be fixed to the center of gravity (*CG*) of an aircraft and move with the *CG*. x_H points due north, y_H points due east, and z_H points toward the center of the Earth.