

Kedar Nath Das · Kusum Deep  
Millie Pant · Jagdish Chand Bansal  
Atulya Nagar *Editors*

# Proceedings of Fourth International Conference on Soft Computing for Problem Solving

SocProS 2014, Volume 2



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Janusz Kacprzyk, Polish Academy of Sciences, Warsaw, Poland  
e-mail: [kacprzyk@ibspan.waw.pl](mailto:kacprzyk@ibspan.waw.pl)

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Editors

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*Editors*

Kedar Nath Das  
Department of Mathematics  
National Institute of Technology Silchar  
Silchar, Assam  
India

Jagdish Chand Bansal  
Department of Mathematics  
South Asian University  
New Delhi  
India

Kusum Deep  
Department of Mathematics  
Indian Institute of Technology Roorkee  
Roorkee  
India

Atulya Nagar  
Department of Computer Science  
Liverpool Hope University  
Liverpool  
UK

Millie Pant  
Department of Paper Technology  
Indian Institute of Technology Roorkee  
Roorkee  
India

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# Preface

SocProS, which stands for ‘Soft Computing for Problem Solving’, is entering its fourth edition as an established and flagship international conference. This particular annual event is a joint collaboration between a group of faculty members from institutes of repute like NIT Silchar, IIT Roorkee, South Asian University, Delhi and Liverpool Hope University, UK.

The first in the series of SocProS started in 2011 and was held from 20 to 22 December at the IIT Roorkee Campus with Prof. Deep (IITR) and Prof. Nagar (Liverpool Hope University) as the General Chairs. JKLU Jaipur hosted the second SocProS from 28 to 30 December 2012. Coinciding with the Golden Jubilee of the IIT Roorkee’s Saharanpur Campus, the third edition of this international conference, which has by now become a brand name, took place at the Greater Noida Extension Centre of IIT Roorkee during 26–28 December 2013.

Like earlier SocProS conferences, the focus of SocProS 2014 is on Soft Computing and its applications to real-life problems arising in diverse areas of medical and healthcare, supply chain management, signal processing and multimedia, industrial optimisation, image processing, cryptanalysis, etc. SocProS 2014 attracted a wide spectrum of thought-provoking articles. A total of 103 high-quality research papers were selected for publication in the form of this two-volume proceedings.

We hope that the papers contained in this proceeding will prove helpful towards improving the understanding of Soft Computing at the teaching and research levels and will inspire more and more researchers to work in the field of Soft Computing.

The editors express their sincere gratitude to the SocProS 2014 Patron, Plenary Speakers, Invited Speakers, Reviewers, Programme Committee Members, International Advisory Committee, and Local Organizing Committee; without whose support, the quality and standards of the Conference could not be maintained. We express special thanks to Springer and its team for this valuable support in the publication of this proceedings.

Over and above, we express our deepest sense of gratitude to ‘National Institute of Technology (NIT) Silchar’ for facilitating the hosting of this conference. Our sincere thanks to all the sponsors of SocProS’ 2014.

Silchar, India  
Roorkee, India  
Roorkee, India  
New Delhi, India  
Liverpool, UK

Kedar Nath Das  
Kusum Deep  
Millie Pant  
Jagdish Chand Bansal  
Atulya Nagar

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# About the Editors

**Dr. Kedar Nath Das** is now working as an Assistant Professor in the Department of Mathematics, National Institute of Technology Silchar, Assam, India. Over the last 10 years, he has contributed immensely towards research in ‘soft computing’. He has many papers to his credit in national and international journals of repute. His areas of interest include Evolutionary and Bio-inspired algorithms for optimization.

**Prof. Kusum Deep** is working as a Full-time Professor in the Department of Mathematics, Indian Institute of Technology Roorkee, Roorkee, India. Over the last 25 years, her research is increasingly well-cited making her a central International figure in the area of Nature Inspired Optimization Techniques, Genetic Algorithms and Particle Swarm Optimization.

**Dr. Millie Pant** is an Associate Professor at the Department of Paper Technology, Indian Institute of Technology Roorkee, Roorkee, India. She has to her credit several research papers in journals of national and international repute and is a well-known figure in the field of Swarm Intelligence and Evolutionary Algorithms.

**Dr. Jagdish Chand Bansal** is an Assistant Professor with the South Asian University, New Delhi, India. Holding an excellent academic record, he is an excellent researcher in the field of Swarm Intelligence at the national and international level, having several research papers in journals of national and international repute.

**Prof. Atulya Nagar** holds the Foundation Chair as Professor of Mathematical Sciences and is the Dean of Faculty of Science, at Liverpool Hope University, Liverpool, UK. Professor Nagar is an internationally recognised scholar working at the cutting edge of theoretical computer science, applied mathematical analysis, operations research and systems engineering and his work is underpinned by strong complexity-theoretic foundations.

# Face Recognition Using 2DPCA and ANFIS Classifier

Hitesh Shah, Rahul Kher and Ketan Patel

**Abstract** With the growth of information technology coupled with the need for high security, the application of biometric as identification and recognition process has received special attention. The biometric authentication systems are gaining importance, and in particular, face biometric is more preferred for person authentication because of its easy and non-intrusive method during acquisition procedure. Face recognition is considered to be one of the most reliable biometric, when security issues are taken into concern. Various methods are used for face recognition. To recognize the face, feature extraction becomes a critical problem. In this paper, two-dimensional principle component analysis (2D-PCA) has been applied for feature extraction. The feature vectors are then applied to adaptive neuro-fuzzy inference system (ANFIS) classifier. The result indicates that ANFIS classifier yields 97.1 % of classification accuracy.

**Keywords** Face recognition · 2D-PCA · Feature extraction · ANFIS

## 1 Introduction

BIOMETRICS or biometric authentication is the identification of humans by their characteristics or traits. Biometrics is used to describe a characteristics or a process. Various biometric traits can be utilized for the purpose of human recognition like

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H. Shah (✉) · R. Kher

Department of Electronics and Communication Engineering, G.H. Patel College of Engineering & Technology, Vallabh Vidyanagar, Gujarat 388120, India  
e-mail: hiteshshah@gcet.ac.in

R. Kher

e-mail: rahulkher@gcet.ac.in

K. Patel

Department of Electronics and Communication Engineering, B & B Institute of Technology, Vallabh Vidyanagar, Gujarat 388120, India  
e-mail: ketan00in@gmail.com

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fingerprint, palm print, hand geometry, iris, face, speech, gaits, signature, and key strokes. The problem with fingerprint, iris palm print, speech, and gaits is they need active cooperation of a person, while face recognition is a process that does not require active cooperation of a person, so without instructing the person, one can recognize the person. So face recognition is much more advantageous compared to the other biometrics.

The face is the primary focus of attention in the society, playing a major role in conveying identity and emotion. Although the ability to infer intelligence or character from facial appearance is suspect, the human ability to recognize faces is remarkable. A human can recognize thousands of faces learned throughout the lifetime and identify familiar faces at a glance even after years of separation. This skill is quite robust, despite large changes in the visual stimulus due to viewing conditions, expression, aging, and distractions such as glasses, beards, or changes in hair style. Face recognition has become an important issue in many applications such as security systems, credit card verification, and criminal identification. Face recognition system consists of mainly two parts: feature extraction and classification. This paper presents a technique that utilizes two-dimensional principle component analysis (2D-PCA) and adaptive network-based fuzzy inference system (ANFIS)-based classification.

The most important problem in face recognition is the curse of dimensionality problem. The face image having very high dimension, and it contains so much irrelevant or non-informative data, and that makes much difficulty for making decision. Feature extraction can act as a powerful dimension reduction agent. So, it is desirable to select smaller number of relevant and important feature with the help of dimension reduction techniques. High dimension also pose problem in computation, so it is also desirable to reduce the dimension. There are several methods available for dimensionality reduction, which may be linear or nonlinear. We used linear method of feature extraction.

Principle component analysis (PCA), also known as Karhunen–Loeve expansion, is a classical feature extraction and data representation technique widely used in the areas of pattern recognition and computer vision. Sirovich and Kirby [1] and Kirby and Sirovich [2] first used PCA to efficiently represent pictures of human faces. They argued that any face image could be reconstructed approximately as a weighted sum of a small collection of images that define a facial basis (eigen-images), and a mean image of the face. Within this context, Turk and Pentland [3] presented the well-known eigenfaces method for face recognition in 1991. Since then, PCA has been widely investigated and has become one of the most successful approaches in face recognition [4–6]. Sirovich also discussed the problem of the dimensionality of the “face space” when eigenfaces are used for Representation. In [7–14], authors have proposed face recognition algorithms using ANFIS, neural networks (NN), support vector machine (SVM), etc. For feature extraction methods like PCA, Radon transform, Gabor transform, etc., have been used.

In the PCA-based face recognition technique, the 2D face image matrices must be previously transformed into 1D image vectors. The resulting image vectors of faces usually lead to a high-dimensional image vector space, where it is difficult to

evaluate the covariance matrix accurately due to its large size and the relatively small number of training samples. However, this does not imply that the eigenvectors can be evaluated accurately in this way since the eigenvectors are statistically determined by the covariance matrix, no matter what method is adopted for obtaining them. As opposed to conventional PCA, 2DPCA is based on 2D matrices rather than 1D vector [15]. That is, the image matrix does not need to be previously transformed into a vector; instead, an image covariance matrix can be constructed directly using the original image matrices. In contrast to the covariance matrix of PCA, the size of the image covariance matrix using 2DPCA is much smaller. As a result, 2DPCA has two important advantages over PCA. First, it is easier to evaluate the covariance matrix accurately. Second, less time is required to determine the corresponding eigenvectors. The remainder of this paper is organized as follows: In Sect. 2, 2DPCA method has been discussed, and in Sect. 3, ANFIS algorithm has been discussed; Sect. 4 discusses our simulation result, and final section discusses the conclusion.

## 2 Two-Dimensional Principle Component Analysis (2D-PCA)

Image representation and feature extraction are pervasive techniques that are commonly used for face recognition process. When the input data to an algorithm are too large to be processed and it is suspected to be notoriously redundant, the input data will be transformed into a reduced representation set of features. Transforming the input data into the set of features is called feature extraction. If the features extracted are carefully chosen, it is expected that the features set will extract the relevant information from the input data in order to perform the desired task using this reduced representation instead of the full-size input.

Yang et al. [15] proposed a new approach—two-dimensional PCA—for face recognition. We used same algorithm for feature extraction of face database. In two-dimensional PCA, the image matrix  $A$  is transformed to the  $X$  matrix using linear transformation given by

$$Y = AX \tag{1}$$

So,  $Y$  is a projected vector of image  $A$ .  $Y$  is also called as a projected feature vector. The total scatter of the projected samples can be characterized by the trace of the covariance matrix of the projected feature vectors. So the idea is to maximize the following:

$$J(X) = \text{tr}(S_x) \tag{2}$$

where  $S_x$  denotes the covariance matrix of the projected feature vectors of the training samples and  $\text{tr}(S_x)$  denotes the trace of  $S_x$ . The physical significance of

maximizing the criterion in Eq. (2) is to find a projection direction  $X$ , onto which all samples are projected, so that the total scatter of the resulting projected samples is maximized. The covariance matrix  $S_x$  is given by

$$\begin{aligned} S_x &= E(Y - EY) * E(Y - EY)^T \\ &= E[(A - EA)X] * E[(A - EA)X]^T \end{aligned} \quad (3)$$

Hence,

$$J(x) = X^T E[(A - EA) * (A - EA)^T] X \quad (4)$$

For given a set of training image  $A(1), A(2), \dots, A(M)$ ,

$$J(X) = X^T \left[ \sum_{n=1}^M (A(i) - \bar{A})(A(i) - \bar{A})^T \right] X \quad (5)$$

where  $\bar{A}$  is the average of the training images. Now, let

$$G_t = E[(A(i) - \bar{A})^T (A(i) - \bar{A})] \quad (6)$$

The matrix  $G_t$  is called the image covariance (scatter) matrix. It is easy to verify that  $G_t$  is an  $n \times n$  nonnegative definite matrix from its definition; here,  $G_t$  is computed directly from the training image. Suppose that there are  $M$  training image samples in total, the  $j$ th training image is denoted by an  $m \times n$  matrix  $A_j (j = 1, 2, 3, \dots, M)$ , and the average image of all training samples is denoted by  $\bar{A}$ . Then,  $G_t$  can be evaluated by

$$G_t = \frac{1}{M} \sum_{j=1}^M (A(j) - \bar{A})^T (A(j) - \bar{A}) \quad (7)$$

Alternatively, (2) can be written as

$$J(X) = X^T G_t X \quad (8)$$

where  $X$  is a unitary column vector. This criterion is called the generalized total scatter criterion. The unitary vector  $X$  that maximizes the criterion is called the optimal projection axis. Intuitively, this means that the total scatter of the projected samples is maximized after the projection of an image matrix onto  $X$ . The optimal projection axis  $X_{\text{opt}}$  is the unitary vector that maximizes  $J(X)$ , i.e., the eigenvector of  $G_t$  corresponding to the largest eigenvalue. In general, it is not enough to have only one optimal projection axis. We usually need to select a set of projection axis,  $X_1, \dots, X_d$ , subject to the orthonormal constraints and maximizing the criterion  $J(x)$ , that is,

$$\{X_1, X_2, \dots, X_d\} = \arg \max(J(X)) \quad (9)$$

and

$$X_i^T X_j = 0; \quad i \neq j, \quad i, j = 1, 2, \dots, d. \quad (10)$$

In fact, the optimal projection axis,  $\{X_1, X_2, \dots, X_d\}$  are the orthonormal eigenvectors of  $G_r$  corresponding to the first  $d$  largest eigenvalues. The optimal projection vectors of 2DPCA,  $\{X_1, X_2, \dots, X_d\}$ , are used for feature extraction. For a given image sample  $A$ , let

$$Y_k = AX_k \quad (11)$$

Then, we obtain a family of projected feature vectors,  $Y_1, \dots, Y_d$ , which are called the principle component of the sample image  $A$ . The principle component vectors obtained are used to form an  $m \times d$  matrix  $B = [Y_1, \dots, Y_d]$ , which is called the feature matrix or feature image of the image sample  $A$ . The feature vector generated is used to form a feature matrix, and same will be used for classification.

### 3 ANFIS Classifiers

Fuzzy logic and artificial NN are complementary technologies in the design of intelligent system. The combination of these two technologies into an integrated system appears to be a promising path toward the development of intelligent systems capable of capturing qualities characterizing the human brain. However, fuzzy logic and NN generally approach the design of intelligent systems from quite different angles.

NN are essentially low-level, computational algorithms that sometimes offer a good performance in pattern recognition tasks. On the other hand, fuzzy logic provides a structural framework that uses and exploits those low-level capabilities of NN. Both NN and fuzzy logic are powerful design techniques that have their strengths and weaknesses. NN can learn from data sets, while fuzzy logic solutions are easy to verify and optimize. The main power of artificial NN lies in their ability to correctly learn the underlying function or distribution in a data set from a number of samples. This ability can be expressed in terms of minimizing the estimation error of the neural network, on previously unseen data. Table 1 shows a comparison of the properties of these two technologies. The combination of this technology can produce better results.

The ANFIS concept was first introduced by Prof. J.S. Roger Jang in National Tsing Hua University [16]. The model represents a neural network approach combined with fuzzy inference system based on Takagi–Sugeno inference model. ANFIS is a hybrid learning algorithm which uses the learning ability of NN to adjust the membership function parameters in a fuzzy inference system in order to

**Table 1** Comparison of neural network and fuzzy logic technology

	Neural network	Fuzzy logic
Knowledge representation	Implicitly, the system cannot be easily interpreted or modified	Explicitly, verification and optimization are very easy and efficient
Trainability	Train itself by learning from data set	Everything must be defined explicitly

build the adaptive system. Fuzzy inference can provide rule-based generation from human expert's knowledge, whereas the neural network approach supports tuning of membership function parameters from input–output data pairs.

The ANFIS uses scaled conjugate gradient methods to train the network. Learning in a neural network is to reduce the error function. Success of algorithm depends on its convergent rate. SCG has the highest convergent rate than other algorithm. It uses Gaussian membership function to make the member of an input data.

The ANFIS uses Sugeno-type fuzzy interface system. There are two types of interface system: Mamdani and Sugeno. When compared to Mamdani, Sugeno is computationally efficient. It works well with linear technique. It works well with optimization and adaptive techniques. It is well suited to mathematical analysis. So ANFIS is the combination of two best methods available in neural network and fuzzy logic.

Figure 1 shows the system architecture of ANFIS system. It consists of five layers. The node in the layer is of two types: adaptive and fixed. First and forth nodes are adaptive, and rest three are fixed nodes. In the first layer, all the nodes are adaptive nodes.

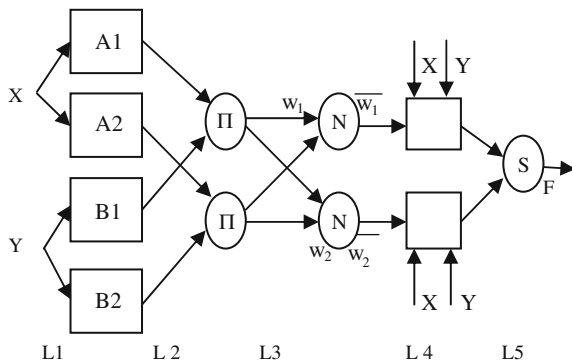
Layer-1: Every node  $i$  in this layer is a square node with a node function. The outputs of layer 1 are the fuzzy membership grade of the inputs, which are given by

$$\begin{aligned} O_i^1 &= \mu A_i(x), \quad i = 1, 2 \\ O_i^1 &= \mu B_{i-2}(x), \quad i = 3, 4 \end{aligned} \quad (12)$$

where  $\mu A_i$ ,  $\mu B_{i-2}$  are any membership functions, where  $x$  and  $y$  are inputs to node  $i$ , and  $A_i$  and  $B_i$  are linguistic labels for inputs. If the bell-shaped membership function is employed,

$$\mu A_i = \frac{1}{1 + \left\{ \left( \frac{x-c_i}{a_i} \right)^2 \right\}^{b_i}} \quad (13)$$

where  $a_i$ ,  $b_i$ , and  $c_i$  are the parameter of the membership function.



**Fig. 1** Architecture of ANFIS system

Layer 2: Every node in this layer is a circle node labeled  $\Pi$  which multiplies the incoming signals and sends the product out. In the second layer, all the nodes are the fixed. The output of second layer is

$$O_i^2 = w_i = \mu A_i(x) \mu B_i(y), \quad i = 1, 2 \quad (14)$$

$w_i$  is called the firing strengths of the rules.

Layer 3: Every node in this layer is a circle node labeled  $N$ . The  $i$ th node calculates the ratio of the  $i$ th rules firing strength to the sum of all rule's firing strengths. In this layer, the nodes are fixed nodes. The label  $N$  indicates that they are used for normalization. The output of this layer is given by

$$O_i^3 = \bar{w}_i = \frac{w_i}{w_1 + w_2}, \quad i = 1, 2 \quad (15)$$

which are called the normalized firing strengths.

Layer 4: In the fourth layer, the nodes are adaptive nodes. The output of each node in this layer is simply the product of the normalized firing strength and a first-order polynomial. Thus, the outputs of this layer are given by

$$O_i^4 = \bar{w}_i f_i = \bar{w}_i (p_i x + q_i y + r_i), \quad i = 1, 2 \quad (16)$$

Layer 5: In the fifth layer, there is only one single fixed node labeled with  $S$ . This node performs the summation of all incoming signals. Hence, the overall output of the model is given by

$$O_i^5 = \sum_{i=1}^2 \bar{w}_i f_i = \frac{\sum_{i=1}^2 w_i f_i}{w_1 + w_2} \quad (17)$$

It can be observed that there are two adaptive layers in this ANFIS architecture, namely the first layer and the fourth layer. In the first layer, there are three modifiable parameters  $\{a_i, b_i, c_i\}$ , which are related to the input membership functions. These parameters are the so-called premise parameters. In the fourth layer, there are also three modifiable parameters  $\{p_i, q_i, r_i\}$ , pertaining to the first-order polynomial. These parameters are so-called consequent parameters.

## 4 Simulation Results

The proposed algorithm (2D-PCA + ANFIS) has been tested on well-known, publically available ORL database. The database was generated in Olivetti Research laboratory in April 1992. The ORL or AT&T database contains 400 images of 40 persons, each person having 10 images. The database having images of  $112 \times 92$  pixel size, and all are monochrome image. The face image having different times, varying lighting, facial expressions (open/closed eyes, smiling/not smiling), and facial details (glasses/no glasses). All the images were taken against a dark homogeneous background. Before applying our algorithm, we have gone through preprocessing steps.

### 4.1 Preprocessing Steps

In our experiments, the input image has been resized. The original image is of size  $112 \times 92$  pixels, and after preprocessing steps, it is converted to  $56 \times 46$  pixels. The advantage of preprocessing is that it consumes less time for computation and increases speed of recognition and requires less memory for storage. Figure 2 shows the original image and resized image.



**Fig. 2** Original image and resized image

## 4.2 Feature Extraction

First, an experiment was performed using the first five image samples per class for training, and the remaining images for test. Thus, the total number of training samples and testing samples were both 200. The 2DPCA algorithm was first used for feature extraction. Here, the covariance matrix size is  $46 \times 46$ . We choose 3 eigenvectors corresponding to 3 largest eigenvalues. After having 3 projection axes, we project our image onto the projection axis and we have obtained three principal component vectors. Using this vector, we form feature matrix. The size of the feature matrix is of  $200 \times 168$ .

## 4.3 Recognition

For recognition, ANFIS classifiers has been used. The parameter set for experiments are Training epoch number (100), Training error goal (0.0001), Initial step size (0.01), Step size decrease rate (0.9), Step size increase rate (1.1), sigma ( $5.0e-5$ ), lambda ( $5.0e-7$ ), success (1), Lambda\_bar(0), Lambda\_k (lambda), num\_X(1).

In simulation, we test our algorithm with different number of epoch and calculate the recognition rate. Table 2 shows the comparison of epoch versus recognition rate. From this table, we can say that as the number of epoch increases, the recognition rate also increases. The epoch represents the number of time the algorithm learns. This experiment was conducted on 400 images of the database. The number of feature selected for one image is 168. Table 2 shows the recognition rate for test as well as train image also. From Table 2, we can say that after 10 epochs, we are able to get 100 % recognition rate for train images and 94.0 % of recognition rate for test images. After 100 epochs, we are able to get 100.5 for train images and 97.1 % of recognition rate for test images.

**Table 2** Epoch versus recognition rate

No. of epochs	RR for train image (%)	RR for test image (%)
10	100	94.0
20	100	94.5
30	100	94.5
40	100	95.0
50	100	95.0
60	100	96.0
70	100	96.5
80	100	96.5
90	100	97.0
100	100	97.1

**Table 3** Recognition rate with different number of images

No. of image (class)	No. of epochs	No. of train images	No. of test images	RR for train image (%)	RR for test image (%)
10 (1)	10	5	5	100	100
50 (5)	10	25	25	100	100
100 (10)	10	50	50	100	100
150 (15)	10	75	75	100	100
200 (20)	30	100	100	100	100
250 (25)	30	125	125	100	99.5
300 (30)	30	150	150	100	98.5
350 (35)	80	175	175	100	97.3
400 (40)	100	200	200	100	97.1

We test our algorithm with different number of test images while keeping the number of feature for a single image as 168. Here, 168 is the length of feature for a single image. In Table 3, the first column shows the number of test image, second column shows the number of epoch required reaching the best recognition rate, third and fourth columns show the number of train and test image, and fifth and sixth columns show the recognition rate. By using our algorithm, we are able to get 100 % of recognition rate for 200 test image. But as the number of image increases, the recognition rate is decreasing. Applying the algorithm to each of the images in a database of 400 images, we are able to get up to 97.1 % of recognition rate.

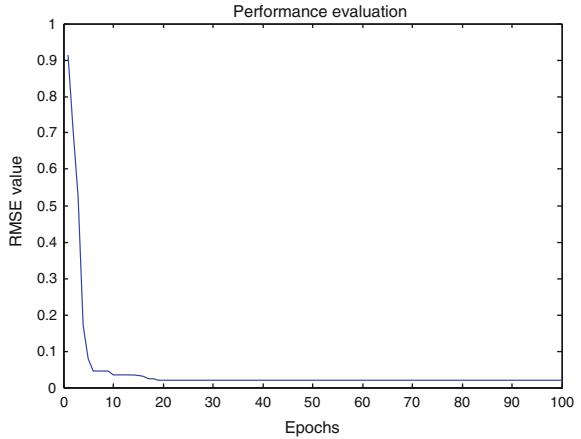
In our experiments, we use different length of feature vector for 400 images. From Table 4, we can say that as the number of feature for a single image increases, the recognition rate increases. We test our algorithm with 23, 56, 112, 135, and 168 features. We are able to get maximum of 97.1 % of recognition rate with 168 features of a single image.

Figure 3 shows the performance evaluation of our system. For performance measurements, we have used root mean square (RMSE) value. The RMSE is the square root of the mean of square of the error. The error is the difference between the “target output” and the “actual output” of the classifier. The y-axis shows the RMSE value, and x-axis shows the number of epochs. From the figure, we can say

**Table 4** Forty classes with different size of feature vectors

Feature vector size	No. of class	No. of train image	No. of test image	Recognition rate (%)
23	40	200	200	62.05
56	40	200	200	81.5
112	40	200	200	94.5
135	40	200	200	95
168	40	200	200	97.1

**Fig. 3** Performance evaluation



**Table 5** Comparison of adopted methods

Method	Recognition rate (%)
PCA + neuro-fuzzy classifiers	85
2D-PCA + ANFIS (proposed method)	97.1

as the number of epoch increases, the RMSE is going to decrease. Table 5 shows the recognition performance comparison between PCA + neuro-fuzzy classifier and 2D-PCA + ANFIS classifier. It can be seen that the 2D-PCA-based ANFIS classifier outperforms the neuro-fuzzy classifier based on PCA.

### 5 Conclusion

This paper presents a face recognition method based on 2DPCA and ANFIS. Facial features are extracted by the 2DPCA method, which reduces the dimensionality of the original face images. The extracted features are given to adaptive neuro-fuzzy classifiers. The classification was performed with several combinations of features and class of images. The overall recognition rate achieved by the method implemented over here is 97.1 %.

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# Optimisation of Mechanical Properties of Wood Dust-reinforced Epoxy Composite Using Grey Relational Analysis

Barnasree Chanda, Rahul Kumar, Kaushik Kumar and Sumit Bhowmik

**Abstract** In the present era of product development, composites are being used because of the ease in manufacturing and low weight to volume ratio. The increasing awareness towards environmental issues and requirement of more versatile polymer-based materials has led to higher interest in natural fibre/filler composites. In this paper, epoxy composite with six different filler contents (wt%) of sundi wood dust are tested at three different speeds. Tensile and flexural tests are performed according to ASTM standard. Different design parameters i.e., filler content (wt%) and speed for load, tensile stress and flexural stress values are optimised using grey relational analysis (GRA). Entropy method determines the corresponding weights to each criterion. A grey relational grade (GRG) has shown the improved performance parameter, and the best performance is observed at 10 % filler content with speed of 1 mm/min.

**Keywords** Wood dust · Reinforced epoxy composite · Mechanical properties · Entropy method · Grey relational analysis

## Nomenclature

$X'_i(k)$  Comparability sequence  
 $X'_0(k)$  Reference sequence  
 $X_i^*(k)$  Comparability sequence after normalising

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B. Chanda (✉) · R. Kumar · S. Bhowmik  
Department of Mechanical Engineering, National Institute of Technology,  
Silchar 788010, India  
e-mail: barna\_chanda@yahoo.com

R. Kumar  
e-mail: rahul.oec@gmail.com

S. Bhowmik  
e-mail: bhowmiksumit04@yahoo.co.in

K. Kumar  
Department of Mechanical Engineering, Birla Institute of Technology,  
Mesra, Ranchi 835215, India  
e-mail: kaushik.bit@gmail.com

$m$	Number of experimental data items
$n$	Number of parameters
$\Delta'_0$	Deviation sequence of the reference sequence and comparability sequence
$\xi$	Identification coefficient
$\gamma$	Grey relational coefficient
$\zeta$	Grey relational grade
$D_j$	Degree of divergence
$Z_j$	Weight of entropy

## 1 Introduction

The rising concern towards environmental problem and the need for more multifarious polymer-based materials have led to increasing interest about polymer composites filled with natural organic fillers, i.e., fillers coming from recyclable sources. The natural fibre composites, also known as green composites, have shown a growth of interest because of their recyclability, biodegradability and abundant availability. In past few decades, natural fibre composite has got immense exposure. Natural fibres are hair-like materials primarily composed of cellulose, hemicelluloses and lignin [1]. These natural fibres provide a substitute for glass fibres and other constituent synthetic fibres (e.g., nylon, polyester, acrylic and polyolefin). Natural fibres are superior to glass fibres as they are less polluting, light in weight resulting improved fuel efficiency than that of glass fibres [2]. The most important advantage of natural fibre is that they are renewable and bio degradable. Not only from nature's point of view but also they are profitable for providing high strength, low weight, corrosion resistance and low processing cost. The major disadvantages of natural fibres are including moisture absorption, thermal degradation, fire and UV (ultraviolet). But this can be deal by maintaining proper blend ratio of chemical additives, using UV stabilizer and fire retardants. [3]. Various properties and application of many natural fibres including bamboo, sugarcane, curaua, date palm, jute, sisal, hemp and wood are studied by different researchers.

Bamboo fibre-reinforced composites have shown better mechanical properties (tensile and flexural strength) than the glass fibre-based composites [4, 5]. Curaua fibres have also shows higher mechanical property [6]. The study of Arundo Donax fillers-epoxy composites shows that the size and content of Arundo Donax fillers yields higher tensile moduli, flexural moduli and lower strength properties of the matrix [7]. The date palm wood flour/glass fibre-reinforced hybrid composites of recycled polypropylene show improved mechanical properties as well as thermal properties [8, 9]. The flexural modulus of coconut fibre polypropylene matrix composite can be improved by using adequate fibre granulometry and extruder

screw speed and that of agave fibre-reinforced epoxy composite were significantly high due to alkali treatment of the fibre [10, 11]. The flexural strength and tensile strength of date wood palm flour-based polyethylene composite were decreased by increasing the filler content, while the flexural modulus was increased [12]. The tensile behaviour of jute epoxy laminated composite shows that the tensile strength of jute fibre influences the property of composite [13].

Almost all of the commonly available natural plant fibres that are cheap and abundant in nature are being used for reinforcement in combination with non-biodegradable matrix materials such as unsaturated polyester, epoxy resin, polyethylene and polypropylene [14]. Among these, epoxy resins are very versatile in nature. They are one of the most important classes of thermosetting polymers which are widely used as matrices for fibre-reinforced composite materials and as structural adhesives. They are amorphous, highly cross-linked polymers and this structure results in these materials possessing various desirable properties such as greater tensile strength and modulus, uncomplicated processing, fine thermal and chemical resistance, and dimensional stability [15].

For the natural fibre composite, most of the researches were focusing on experimental test of mechanical properties of natural composites. The correlation between the mechanical properties and the characteristic parameters, e.g., the composition of the composite and the operating conditions, is of prime importance for designing proper composites in order to satisfy various functional requirements. Optimisation of various influencing parameter is very much important. In this study, an attempt is made to analyse the effect of characteristic parameters on mechanical properties of wood dust-reinforced epoxy composites and to find out the optimal combination of parameters. For optimisation process, the various techniques such as genetic algorithm [16], artificial neural network [17], Taguchi method [18] and Multi-attribute decision making (MADM) [19] are available. In the present work, MADM method is used due to its ability to find the best and worst alternative from the given set of data.

MADM is applied for decision-making in multi-criteria problems. MADM has many methods including analytic hierarchy process (AHP), entropy, technique of order preference by similarity to ideal solution (TOPSIS) and grey relational analysis (GRA) [19, 20]. It is very difficult to optimise all the criteria individually. In this respect, grey relational grade (GRG) provides best optimal criteria for multi-decision problem. GRA is used to arrest the correlations among various factors and parameters of a system. Grey stands for uncertain, incomplete or improper. One of the advantages of GRA is that from several factors with inadequate data, quantitative and qualitative relationship can be obtained. This leads to the study of decision analysis field-GRA [20]. Also, the weight of each evaluation criteria is assigned using entropy method. Weights are given by entropy approach. GRA solves MADM problems by combining the entire range of performance attribute values being considered for every alternative into one, single value. This reduces the original problem to a single-attribute decision-making problem. Therefore, alternatives with multiple attributes can be compared easily after the GRA process [20]. The optimisation of machinability of polyester/modified jute fabric composite

was done using GRA. Factorial design-based experiments were conducted at different levels of speed and feed rate. Analysis of variance has been used to study the influence of chemical treatment on delamination. GRA is employed to find the optimum drilling condition [21]. The end milling parameter for glass fibre-reinforced plastic using GRA gives a GRG to evaluate the multiple performance characteristics [22]. As a result, optimisation of the complicated multiple performance characteristics can be converted into optimisation of a single GRG.

In this paper, the tensile and flexural properties at different filler contents (wt%) and speed are determined for wood dust-reinforced epoxy composite. Filler content and speed are considered as process parameter in order to determine how it behaves for load, tensile stress and flexural stress. GRA is used to optimise the mechanical properties of wood dust filled epoxy composite.

## 2 Entropy Integrated Grey Approach

Grey analysis is extensively used to combine multiple decision criteria into one. Grey analysis is done in several steps including data pre-processing and calculation of GRG followed by calculation of grey relational coefficient. Steps for using GRA approach are as follows [20].

A preference sequence is set by  $X'_o(k)$ ,  $k = 1, 2, 3 \dots n$ .

In GRA, data pre-processing is done to transfer the original sequence to a comparable sequence. All the criteria are normalised by considering their values within the range of 0–1.

Normalised data with the aim of maximisation can be obtained by

$$X_i^*(k) = \frac{X'_i(k) - \min X'_i(k)}{\max X'_i(k) - \min X'_i(k)} \quad (1)$$

where,  $i = 1, 2, 3 \dots m$  and  $k = 1, 2, 3 \dots n$ .

Normalised data with the aim of minimisation can be obtained by

$$X_i^*(k) = \frac{\max X'_i(k) - X'_i(k)}{\max X'_i(k) - \min X'_i(k)} \quad (2)$$

Normalised data with the aim to obtain certain optimum value can be obtained by

$$X_i^*(k) = 1 - \frac{|X'_i(k) - Y'_i|}{\max |X'_i(k) - Y'_i|} \quad (3)$$

Difference between data are obtained by

$$\Delta'_0 = X'_0(k) - X_i^*(k) \tag{4}$$

Then  $\Delta_{\max}$  and  $\Delta_{\min}$  are also calculated.

Grey relational coefficient of 'q' in difference set 'i' is

$$\gamma(q) = \frac{\Delta'_{\min} - \xi \Delta'_{\max}}{\Delta'_i(q) + \xi \Delta'_{\max}} \tag{5}$$

$\Delta'_i(q)$ ,  $\Delta'_i$  is the  $q$  value. The value of coefficient  $\xi$  is between 0 and 1. Generally, the value of  $\xi$  is taken as 0.5 in most cases.

Finally, GRG is calculated by

$$\zeta(X'_0, X'_i) = \sum_{k=1}^n \gamma_i(X'_0(k), X'_i(k)) * w((k), X'_i(k)) \tag{6}$$

where  $w$  is the weight calculates by the entropy approach.

Entropy approach is used to provide weightage to parameters. The algorithm for entropy approach is given below.

- Decision matrix is selected.  
say  $X'_{ij}$  ( $i = 1, 2, \dots, m; j = 1, 2, \dots, n$ ) is the value of  $i$ th alternative to the  $j$ th criteria.
- Then the decision matrix is normalised.

$$W_{ij} = \frac{X'_{ij}}{\sqrt{\sum_{i=1}^m X'^2}} \tag{7}$$

- Value of  $F_j$  of  $j$ th criteria is obtained from

$$F_j = -k \sum_{i=1}^m W_{ij} \ln(W_{ij}) \tag{8}$$

where  $k = 1/\ln c$  so that  $0 \leq F_j \leq 1$  and  $c$  is the number of alternatives.

- Degree of divergence can be obtained by

$$D_j = |1 - F_j| \tag{9}$$