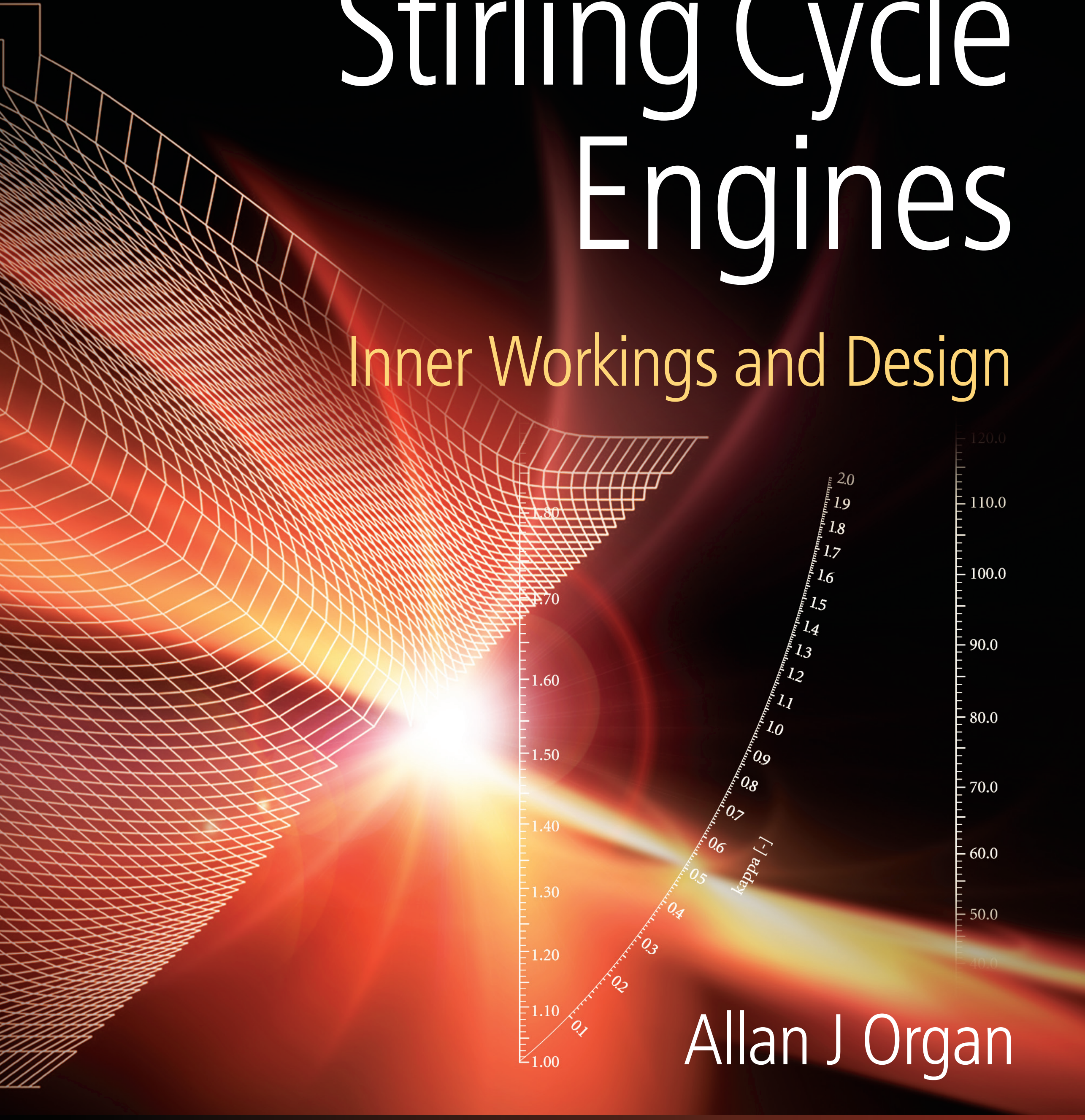
Stirling Cycle Engines

Inner Workings and Design



Allan J Organ

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About the Author

Apart from a six-year spell developing tooling for high-speed metal forming, the author's academic research career – which concluded with 20 years in Cambridge University Engineering Department where he learned more than he taught – has focused on Stirling cycle machines.

It would be gratifying to be able to claim that this commitment had resulted in his becoming an authority – perhaps *the* authority. However, it is a feature of the Stirling engine – perhaps its irresistible attraction – that it does not yield up its subtleties quite so readily. Thus the author writes, not as a self-styled expert, but as a chronic enthusiast anxious that the results of his most recent enquiries should not expire with him! It must now fall to a more youthful intellect to pursue matters.

With the 200th anniversary of the 1816 invention fast approaching, the author remains optimistic of a commercial future for a design targeting appropriate applications and undertaken with realistic expectations. Readers interested in this vision may wish to look out for a work of fiction entitled '*The Bridge*'.

Foreword

What is the source of the Stirling cycle's siren song? Why has it attracted so many for so long (including yours truly over some 30-plus years)? In part, it is simply that Stirling engines are not in common use, so that each idealist who stumbles upon some brief description of such, perceives that he has stumbled upon hidden gold, a gift to the world that needs only to be unwrapped and presented to a grateful society that will receive its promise of clean, quiet, reliable power. No valves, no timing, no spark, no explosions – what could be simpler? Arthur C. Clarke said "Any sufficiently developed technology is indistinguishable from magic". Surely, then, with just a bit of development, or better materials, or the keen insight of one who sees what others have missed, the Stirling engine will blossom into the fruition of its magical promise.

And yet. …

The first such machines were built two centuries ago, though only retrospectively identified with their eponymous creator, Rev. Robert Stirling. That naming came mostly by virtue of his invention of the "economizer" (what we now recognize as the regenerator in a reciprocating machine of this type), and was promoted most effectively by Rolf Meijer, who led the change from air-charged machines to light-gas, pressurized devices at Philips in the 1940s. Dr. Meijer might be said to be the father of the modern Stirling engine, as the earlier work did not have the benefits of thermodynamics as a science or modern materials like stainless steel.

And yet. …

It is remarkable that some 70 years after the Philips Stirling generator sets were produced, then abandoned as unprofitable, they remain one of the benchmarks against which novel attempts at Stirling engines of practical utility are measured. Few have succeeded in bettering their technical performance, and fewer still have achieved any greater commercial returns. Countless hobbyists, dozens of corporate ventures, and even a few large-scale government projects have come and gone.

And yet. …

Not one living person in a million today has seen, used, or been empowered by a Stirling engine. Why then, do we persist in our apparently sisyphean pursuit of this esoteric system? And commensurately, why is the present book important?

It has been noted that most diversity exists at the interfaces among ecosystems: that the junctions of field and forest, sea and shore, or sky and soil support more life in more forms than the depths of any one such domain. It is sure that these are the sites of evolution. As an inventor–instructor–entrepreneur, it has been apparent to me that a similar effect applies to intellectual pursuits: the greatest opportunities for development are to be found at the intersections of disciplines, rather than at their cores. Recent advances in combination fields such as mechatronics, evolutionary biology, and astrophysics might be evidence of some truth in this observation. Let us consider the Stirling engine in this light, the better to appreciate the value of this book.

Thermodynamics, heat transfer, fluid science, metallurgy, structural mechanics, dynamics of motion: all these and more are essential elements in Stirling embodiments and their mastery serves as arrows in the quiver of the developer aiming for Stirling success. Perhaps this is one reason why we are attracted to the Stirling – each finds his own expertise essential to it. And perhaps this is one reason why success is so hard to achieve, for who has all these skills at hand? And if success demands such polymath capacities, where is one to begin?

There are many published works on the broad topic of Stirling cycles, engines, and coolers, especially if the reader seeks descriptive or historical information. A selection of analytical texts can be found for those seeking guidance on the first-principles design of aspects of a new engine, including some worthy treatises on the numerical simulation of complete engines (or coolers). Yet none provides a technically sound, computationally compact path to buildable, valid engines.

This new work builds on the author's earlier focus on the essential regenerator and the application of similarity principles, validated by well-documented machines that serve here as a basis for scaling rules and the design of new engines for applications and operating conditions that superficially differ greatly from prior examples. It must be at those new conditions, perhaps at the intersections of conventional mechanics with micro-, or bio-, or other technologies, that new and evolved implementations of Stirling technology will arise and become, perhaps, successful. In this offering, Dr. Organ does the world of Stirling developers (and would-be Stirling magnates) a great service. For many times, new energy has been brought to this field and applied without reference to the experiences, successes, and failures of the past, here applied to great effect.

That tendency to dive in without a thorough grounding in the prior art is due only in part to the aforementioned siren song of Stirling and its addling effect on the newly captured. Such repetitive waste is also driven by the relative inaccessibility of much of the greater body of Stirling technical literature (e.g., I watched the published output of one famous free-piston company go through several 2-year cycles of re-inventions, as successive tiers of graduate students rotated through!).

The challenge is that, even when accessible as correct content, much of what is published in Stirling literature is either uselessly facile or excruciatingly partial in scope, so as to preclude its ready application to new designs. Tools to fit that job have not heretofore been available to those not willing or able to amass and absorb a gargantuan (if dross-filled) library of publications and apply that through associated years of experimental training. This book, through the author's elegant nomographic presentation – fully sustained by clear text and mathematical underpinnings – provides just that holistic entry point, presented with wit and minimal pain in calculations.

Hints of whole-physics participation in Stirling analysis abound here, not least in the dismissal of Schmidt models for their gross errors and oversimplifications that have led to conceptual misunderstandings and hampered many development efforts. I am particularly pleased at the refutation of long-standing shibboleths such as the "evil" of dead space; shattered here with clear and concrete constructions of the actual effects of dead space, and its value in the right places and amounts. In my own work of recent years, which has merged a long

Stirling experience with more recently developed thermoacoustic science, the key has been full consideration of the inertial properties of the cycle fluids, which is ignored by most Stirling models and simulations. Here, those effects are illustrated and their contributions to actual Stirling device behaviors discussed in a unique bridge between closed-form, analytical methods, and the full physics of numerical simulation, including a proper dressing-down and reformulation of the steady-flow correlations so often misapplied to this oscillatory system. The resulting graphics are both useful and beautiful.

Dr. Organ's offered tools, *FastTrack, FlexiScale, and ReScale* fulfill his promise of guiding the designer of a new Stirling engine to a safe island in the sea of possibilities. This traceable relation from technically successful engines of the past (although I am, of course, crushed that none of mine are among those cited), without the need to extract and refine the data from disparate sources elsewhere, opens the possibility of building a useful Stirling engine to a much larger population of aspirants. Perhaps by this means, some clever member thereof will at last find the sweet spot for commercial success; but at the very least, innumerable hours that would otherwise have been wasted in blind stabs can now be channelled into production refinements on a sturdy base. This is indeed a grand achievement, and being provided in so readable a volume is all the more so: a gift to the Stirling Community sure to be acclaimed throughout.

I am honored to call this author my friend and fellow explorer, and to introduce you, the reader, to this work with the certainty that if you have heard already that siren song of Stirling, this book by Allan Organ can lead you to safe harbor in plotting your course in response. Gentlefolk, Start Your Engines!

Dr John Corey

Preface

If the academic study of the Stirling engine began with Gustav Schmidt in 1861, then it has been more than a century and a half in the making. This might be deemed more than sufficient time to achieve its purpose – which must surely be to put itself out of a job.

A symbolic date looms: Tuesday 27 September 2016 – the bi-centenary of Stirling's application for his first patent – for his 'economizer' (regenerator). The Stirling engine will by then have been under development – admittedly intermittent development – for two centuries. Yet a would-be designer continues to be faced with the unhappy choice between (a) proceeding by trial and error and (b) design from thermodynamic first principles. Either course can be of indeterminate cost and duration.

In principle, nothing stands in the way of an approach to thermodynamic design which is both general and at the same time 'frozen' – general in the sense of coping with arbitrary operating conditions (*rpm*, charge pressure, working gas) and 'frozen' to the extent that thermodynamic design (numbers, lengths and cross-sectional dimensions of flow channels, etc.) are read from graphs or charts, or acquired by keying operating conditions and required performance into a lap-top computer or mobile phone 'app'. The possibility arises because, from engine to engine, the gas process interactions by which heat is converted to work have a high degree of *intrinsic similarity*. Physical processes which are formally similar are *scalable*: once adequately understood and rendered in terms of the appropriate parameters, no compelling reason remains for ever visiting them again.

Market prospects must surely be improved by relegating the most inscrutable – and arguably most daunting – aspect of the design of a new engine to a few minutes' work.

The present account is motivated by a vision along the foregoing lines. Utopia remains on the horizon, but there is progress to report:

> Wherever possible, working equations are reduced to three- and four-scale nomograms. The format affords better resolution and higher precision than the traditional $x - y$ plot, and allows a range of design options to be scanned visually in less time than it takes to launch equivalent software on a computer.
>
> New, independently formulated algorithms for thermodynamic scaling endorse the original method (*Scalit*) and increase confidence in this empirical approach to gas path design. The scaling sequence now reduces to the use of nomograms.
>
> There is a novel – and unprecedentedly simple – way of inferring loss per cycle incurred in converting net heat input to indicated work.
>
> Steady-flow heat transfer and friction correlations appear increasingly irrelevant to conditions in the Stirling engine. Attention shifts to the possibility of correlating *specific thermal load* per

> tube against a *Reynolds paramete*r for the multi-tube exchanger assembly tested *in situ*. Results are promising. Given that they derive from Stirling engines under test, relevance to the unsteady flow conditions is beyond question.
>
> Progress has resulted from noting that the context does not call for a comprehensive picture of regenerator transient response: the interests of the mathematician do not coincide with the realities and requirements of satisfactory engine operation. Focusing on the relevant margin of the potential operating envelope allows respective temperature excursions of gas and matrix to be explored independently.
>
> The kinetic theory of gases is mobilized in an attempt to dispense once for all with the suspect steady-flow flow correlations. To convey the resulting insights calls for animated display. This is not yet a feature of the conventional hard-copy volume. Selected still frames give an impression.

(Full exploitation of the kinetic theory formulation awaits the next generation of computers, so another book looms. Another book – another preface!)

> Certain entrenched tenets of thermodynamic design have been found to be faulty. These are remedied.
>
> Versatility and utility of the design charts (nomograms) have been enhanced: The range of equations susceptible to traditional methods of construction is limited. The technique of 'anamorphic transformation' has been corrected relative to published accounts, and now allows display of functions of the form $w = f(u,v)$ in nomogram form. Function w can be the result of lengthy numerical computation. (The display remains confined to the range over which the variation in w is monotonic.)
>
> The 'hot-air' engine receives a measure of long-overdue attention.

The writer has benefited from long hours of dialogue with Peter Feulner, with Geoff Vaizey, with Camille van Rijn, with Peter Maeckel and with R G 'Jimmy' James. The influence of Ted Finkelstein endures.

Constructive criticism is always welcome at *allan.j.o<at>btinternet.com*.

This material appears in print thanks largely to a unique combination of persistence, patience and diplomacy applied over a period of 18 months by Eric Willner, executive commisioning editor at John Wiley & Sons. The project has since become reality in the hands of Anne Hunt, associate commissioning editor and Tom Carter, project editor.

Notation

Variables Having Dimensions

A_{ff}	free-flow area	m^2
A_{w}	wetted area	m^2
b	width of slot	m
c_{v}, c_{p}	specific heat at constant volume, pressure	J/kgK
d_{x}	internal diameter of heat exchanger duct	m
D	inside diameter of cylinder, or outside diameter of displacer, as per context	m
e	désaxé offset	m
f	cycle frequency ($= \omega/2\pi$)	s^{-1}
g	radial width of annular gap	m
h	coefficient of convective heat transfer	$\mathrm{W/m^2\,K}$
	specific enthalpy	J/kgK
H	enthalpy	J
H_{C}	clearance height	m
k	thermal conductivity	W/mK
L_{d}	axial length displacer shell	m
L_{ref}	reference length $V_{\mathrm{sw}}{}^{1/3}$	m
L_{x}	length of heat exchanger duct	m
m	mass (of gas)	kg
M	total mass of gas taking part in cycle	kg
M_{w}	mass of matrix material	kg
p_{ref}	reference pressure – max/min/mean cycle value	Pa
p_{w}	wetted perimeter	m
Q	heat	J
r	linear distance coordinate in radial direction	m
	radius – e.g., of crank-pin offset	m
r_{h}	hydraulic radius	m
q'	heat rate	W
q''	heat rate per unit length of exchanger	W/m
R	specific gas constant	J/kgK
S_{p}, S_{d}	stroke of work piston and displacer respectively	m
T_{E}, T_{C}	temperatures of heat source and sink	K

T_w	temperature of solid surface	K
T	temperature of gas	K
t	time	s
	thickness in radial coordinate direction	m
u	velocity in x coordinate direction	m/s
	specific internal energy	J/kgK
U	internal energy	J
V_{sw}	swept volume	m^3
W	work	J
W'	work rate	W
X, Y	linear distances in x and y coordinate directions	m
z	linear offset in kinematics of crank-slider mechanism	m
α	thermal diffusivity $k/\rho c_p$	m^2/s
$\underline{\varepsilon}_T$	mean temperature perturbation or 'error' $\underline{T - T_w}$	K
μ	coefficient of dynamic viscosity	Pas
ρ	density	kg/m^3
ω	angular speed	s^{-1}

Dimensionless Variables

a, b, c, d	coefficients and indices as required	-
a	coefficients of linear algebraic equations	-
C	numerical constant (as required)	-
CI	'cycle invariant' defined at point of use	-
C_f	friction factor $\Delta p/\frac{1}{2}\rho u^2$	-
DG	dimensionless group defined at point of use	-
Ma	Mach number $u/\sqrt{\gamma RT}$	-
n	polytropic index: n_e expansion phase; n_c expansion phase	-
n_{Tx}	number of exchanger tubes	-
N_B	specific cycle work: power/$fV_{sw}p_{ref}$	-
N_F	Fourier modulus $\alpha/\omega r_0{}^2$	-
N_{FL}	Flush ratio: ratio of mass of gas per uni-directional blow to instantaneous mass of gas in regenerator void volume.	-
N_{MA}	characteristic Mach number $\omega L_{ref}/\sqrt{RT_C}$	-
N_{Nu}	Nusselt number hr_h/k	-
N_{RE}	characteristic Reynolds number $N_{SG}N_{MA}{}^2$	-
N_{SG}	Stirling parameter $p_{ref}/\mu\omega$	-
N_T	characteristic temperature ratio T_E/T_C	-
N_{TCR}	thermal capacity ratio $\rho_w c_w T_C/p_{ref}$	-
NTU	$\underline{N}$umber of $\underline{T}$ransfer $\underline{U}$nits $StL_x/r_h = hT_C/\omega p_{ref}S$ in Carnot cycle study	-
P	parameter in eq'n. which relates L_x/d_x to L_x/L_{ref}	-
$P(\varphi)$, $Q(\varphi)$	consolidated coefficients of first-order differential equation	-
Pr	Prandtl number $\mu c_p/k$	-
QI	'quasi-invariant' defined at point of use	-
r_v	volumetric compression ratio (e.g., V_1/V_3 of Carnot cycle)	-
r_p	pressure ratio p_{max}/p_{min}	-

Re	Reynolds number $4\rho\lvert u\rvert\mu/r_h$	-
RE$_\omega$	Reynolds number characteristic of exchanger operation over a cycle	-
S	linear scale factor $L_d/L_p = L_{derivative}/L_{prototype}$	-
Sg	Stirling number $pr_h/\mu\lvert u\rvert$ (see Stirling *parameter* above)	-
St	Stanton number $h/\rho\lvert u\rvert c_p$	-
TCR	net thermal capacity ratio (Chapter 17)	-
U( )	step function used in ideal adiabatic cycle	-
x	numerical scale factor	-
x, y, z	cartesian coordinates	-
XQ_x	specific thermal load on exchanger assembly	-
XQ_{XT}	specific thermal load on individual tube of exchanger assembly	-
Z	work quantity (e.g., loss per cycle) normalized by MRT_C or by $p_{ref}V_{sw}$	-
α	phase advance of events in expansion space over those in compression space	-
β	phase advance of displacer motion over that of work piston	-
γ	isentropic index – specific heat ratio	-
Δ	any finite difference or change	-
ΔT	temperature difference $T - T_w$	-
δ	dimensionless dead space V_d/V_{sw}	-
φ	crank angle = ωt	-
θ	angular coordinate in circumferential direction	-
	angle through which coordinate frame is rotated in process of dealing with molecular collision	-
κ	thermodynamic volume ratio V_C/V_E	-
λ	ratio of volume swept by work piston to that swept displacer $\approx S_p/S_d$ in parallel-cylinder, coaxial ‘beta’ machine	-
Λ	Hausen’s ‘reduced length’ – equivalent to *NTU*	-
ν	Finkelstein’s dimensionless dead space $2\Sigma\upsilon_i T_i/T_C$	-
ψ	specific pressure p/p_{ref}	-
Π	Hausen’s ‘reduced period’ hA_wL_r/fM_wc_w	-
ρ	composite dimensionless function arising in Finkelstein’s formulation of Schmidt analysis: $\surd\{N_T^{-2} + \kappa^2 + 2\kappa\cos(\alpha)/N_T\}/(N_T^{-1} + \kappa + \nu)$	-
σ	specific mass $mRT_C/p_{ref}V_{sw}$	-
σ'	specific mass rate $d\sigma/d\varphi = mRT_C/\omega p_{ref}V_{sw}$	-
ς	dimensionless length x/L_{ref}	-
τ	Finkelstein’s temperature ratio $T_C/T_E = N_T{}^{-1}$	-
υ	Finkelstein’s proportional dead space V_{di}/V_E	-
Σ	total inventory of specific mass $MRT_C/p_{ref}V_{sw} = \sigma_e + \sigma_c + \nu\psi$ – assumed invariant	-
$\P_v$	volume porosity	-

Subscripts

comb	combustion
C	relating to compression or to compression space
d	relating to displacer

env	‘envelope’ of overlapping displacements of piston and displacer in coaxial, ‘beta’ machine
exp	experimental
E	relating to expansion or to expansion space
f	relating to flow friction
ff	free-flow (area)
i	in or inlet
j,k	array subscripts
L	lost – as in lost available work
o	out or outlet
p	relating to piston
q	relating to heat transfer
r	regenerator
ref	reference value of variable
rej	(heat) rejected
sw	swept (volume)
w	wetted (area), wall or wire, as per context
x	exchanger: $_{\mathrm{xe}}$ – expansion exchanger; $_{\mathrm{xc}}$ – compression exchanger
∞	free-stream

Superscripts

deriv	derivative design of scaling process
prot	relating to specification of prototype of scaling process
T	relating to the new (transposed) reference frame
+	extra or additional (dead space)

1

Stirling myth – and Stirling reality

1.1 Expectation

'*Stirling's is a perfect engine, and is the first perfect engine ever to be described.*' (Fleeming Jenkin, 1884). '*offering silence, long life.*' (Ross, 1977). '*... thus enabling the thermal efficiency of the cycle to approach the limiting Carnot efficiency.*' (Wikipedia 2013). *A silent, burn-anything, mechanically simple, low-maintenance, low-pollution prime mover with potential for the thermal efficiency of the Carnot cycle*. Here, without doubt, is a recipe for run-away commercial success which Lloyds of London would surely be happy to underwrite.

The reality of the modern Stirling engine is in terms of tens of thousands of 'one-offs' – prototypes or designs of different degrees of sophistication, only a tiny handful of which have been followed up by a degree of further development.

The outcome of a technological venture has much to do with expectation: Where this is unreasonably or irrationally high, the outcome falls short. The verdict – by definition – is failure. The *identical* technological outcome based on realistic expectation can amount to success.

The Stirling engine is not silent – but can be quiet relative to reciprocating internal combustion engines of comparable shaft power. There is not the remotest chance of approaching the so-called Carnot efficiency, but claims for brake thermal efficiencies comparable to those achieved by the diesel engine appear genuine. Nominal parts count per cylinder of the multi-cylinder Stirling is, indeed, lower than that of the corresponding four-stroke IC engine. On the other hand, many individual components pose a severe challenge to mass manufacture.

Supposing this more sober view to be correct, a world of limited resources and increasing environmental awareness probably has room for the Stirling engine. If so, responding to the need is going to require a lowering of expectation. This may be helped by the shedding of a substantial body of myth.

Stirling Cycle Engines: Inner Workings and Design, First Edition. Allan J Organ.

1.2 Myth by myth

1.2.1 That the quarry engine of 1818 developed 2 hp

This can be traced back to an article in *The Engineer* of 1917 celebrating rediscovery of the patent specification – but no further. A back of envelope calculation will indicate whether further enquiry is necessary.

All power-producing Stirling engines of documented performance yield approximately the same value of Beale number N_B:

$$N_B = \frac{\text{shaft power [W]}}{\text{charge pressure [Pa]} \times \text{swept volume [m}^3\text{] and speed [cps} = rpm/60]}$$

The value is dimensionless and is typically 0.15 [–] – the 'Beale number'.

Charge pressure p_{ref} of Stirling's engine was 1 atm or 10^5 Pa. Linear dimensions cited in the 1816 patent convert to swept volume V_{sw} of 0.103 m^3. Rotational speed is not on record, but on the basis of hoop stress, the *rpm* capability of the 10-ft diameter composite flywheel has been estimated with some confidence (and with subsequent corroboration – Organ 2007) at 27.

On this basis the Beale arithmetic suggests a power output of 695 W – or 0.93 hp. The figure of 0.15, however, derives from engines operating with expansion-space temperatures T_E at or above 900 K (627 °C). Stirling is specific about a much lower value of 480 °F, consistent with limitations of materials available in 1818 (wrought iron). This converts to 297 °C, or 570 K. Performance is very much a function of temperature ratio, $N_T = T_E/T_C$, where T_E and T_C are absolute temperatures, K. Where N_T departs from the norm of $N_T = N_T{}^* = 3$, the definitive parametric cycle analysis (Finkelstein 1960a) justifies a temperature correction factor of $(1 - 1/N_T)/(1 - 1/N_T{}^*)$. The factor is 0.345/0.666 in the present instance, or 0.518, reducing the shaft power prediction to 0.482 hp, or 360 W.

The figure is corroborated by a forensic study (Organ 2007) based on experiments with a full-size replica (Figure 1.1) of furnace and displacer cylinder, backed up by computer simulation.

Chapter 1 of the 2007 text describes the construction of the replica furnace and flue, the stoking experiments and temperature measurements. At maximum stoking rate on coal, peak temperature measured at the rectangular exhaust outlet is 200 °C. It is possible to hold a hand against the upper inner surface of the expansion cylinder, suggesting that the internal temperature of the metal surface remains below 60 °C.

If the inevitable conclusion fails to convince, then it may help to recall the last time an egg was successfully boiled with the heat source some 10 feet (3 m) from the saucepan.

The simulation re-created the cyclic volume variations generated by the crank mechanism of the 1816 patent drawings. The start-point for exploratory simulation runs was the earlier estimate of flywheel-limiting speed of 27 *rpm* derived from considerations of hoop stress. Interpretation of the results erred in favour of best possible performance: Wire diameter d_w and winding pitch (essentially inverse of mesh number m_w) settled on for the regenerator were those which caused the simulation to indicate optimum balance between loss due to pumping and loss due to heat transfer deficit.

On this basis peak power occurred at 28 *rpm*. The simulation could coax no more than ½ hp (373 W) from the crankshaft.

Figure 1.1 Furnace, flue and upper displacer cylinder re-constructed full-size to the drawings and dimensions of the Edinburgh patent

Stirling's own hand-written description cites temperature *difference* (between upper and lower extremes of the cylinder) of 480 °F – or 297 °C. If ambient temperature were 30 °C this would require the hottest part of the cylinder to be at 337 °C. The quarry engine doubtless functioned – but not in the elegant configuration of the patent drawing.

A drawing of an engine which, by contrast, is readily reconciled with brother James Stirling's retrospective (1852) account is shown at Figure 1.2: flywheel, link mechanism, cylinder, piston, and displacer are re-used. However, the entire assembly now operates upside down relative to the patent illustrations, with cylinder head immediately above the furnace and the flame in direct contact.

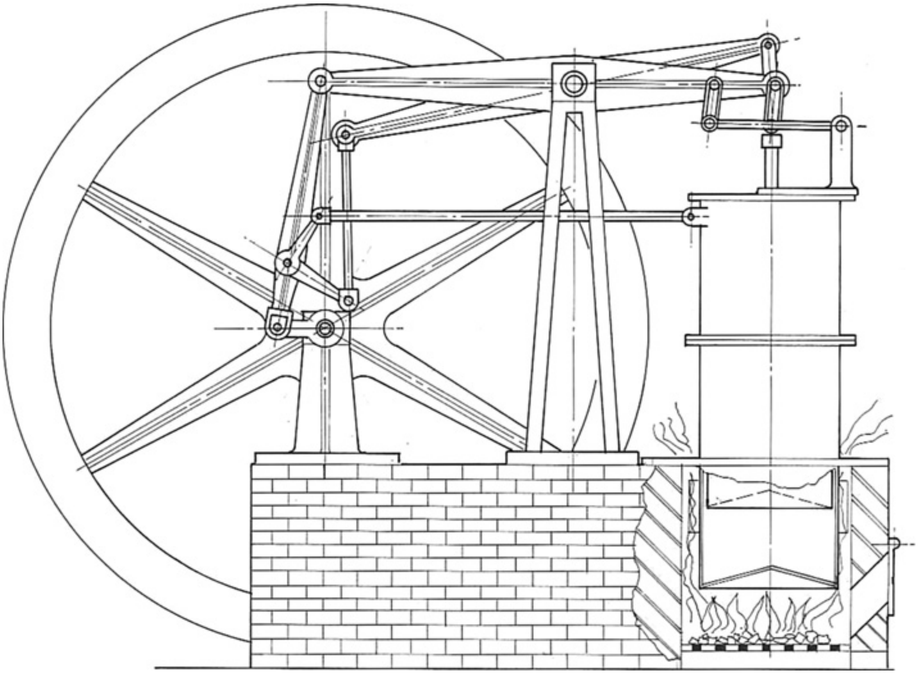

Figure 1.2 How the quarry engine might have appeared after inversion of the cylinder unit to allow heating to a viable operating temperature

Achieving ½ hp no longer requires Stirling to have optimized the thermal and flow design of the regenerator. Brother James' account of the eventual failure of the engine now makes sense: '*… the* ***bottom*** *of the air vessel became over-heated.*'

1.2.2 That the limiting efficiency of the stirling engine is that of the Carnot cycle

Nothing could be further from the truth! The Stirling engine functions by virtue of an irreversible process – that of of forced, convective heat transfer. It does so in spite of two further irreversibilities: thermal diffusion and viscous fluid flow. The *engine* – the hardware as distinct from the academic distraction – no more aspires to the Carnot efficiency than does the cement mixer.

The spurious comparison raises important matters which will need re-visiting at a later stage:

> The Carnot cycle appears to be imperfectly understood – not least by the man himself, Sadi Carnot.
> A part-understood criterion is not one which gets applied logically.

(There is something mildly fraudulent afoot when a principle hailed as an unsurpassed ideal on the blackboard promises dismal failure when converted to hardware. A Carnot 'engine' would be crippled by inadequate thermal diffusivity of the working gas, or by sealing problems – or by both. When *any* Stirling engine turns a crank, thermal efficiency and specific power surpass those of the hypothetical Carnot embodiment.)

Used appropriately, however, the ideal cycle has unexpected insight to offer: the element of net work W [J] when heat Q_E [J] is admitted to the cycle at T_E [K] is $Q_E(T_E - T_C)/T_E$ [J]. The value reflects the absence of losses of any kind. When the practical Stirling engine takes in Q_E per cycle at T_E, the resulting W is less than the Carnot value to the extent of net losses. For a given engine the numerical total of the loss per cycle varies with charge pressure, *rpm*, and so on. The point is, however, that the value of that net loss is given by simple numerical subtraction, and that a figure is available corresponding to each documented combination of charge pressure, *rpm* and working fluid.

Chapter 3 exploits this approach. The sheer magnitude of the loss relative to the useful work element will probably come as a revelation.

1.2.3 That the 1818 engine operated '… on a principle entirely new'

There is no doubting Stirling's integrity in making the claim, but it conflicts with a reality of some 22 years earlier. The patent granted in 1794 to Thomas Mead describes '*Certain methods … sufficient to put and continue in motion any kind of machinery to which they may be applied.*' Mead describes a pair of mating cylinders coaxial on a common vertical axis. The cylinder diameters overlap (cf. the telescope), so axial motion of one relative to the other changes the enclosed volume. In one of his several 'embodiments', the outer ends of both are closed, except for provision for a rod to pass through a gland in the lowermost closure and to actuate a cylindrical 'transferrer'. Heating the top of the uppermost cylinder from the outside and alternately raising and lowering the 'transferrer' promises a cyclic swing in pressure. Mead anticipated that this would cause the 'telescope' alternately to shorten and lengthen, affording reciprocating motion capable of being harnessed.

The Mead specification, eventually published in printed form in 1856, is illustrated by a line drawing ('scetch') devoid of crank mechanism and making no pretence at depicting engineering reality. Whether or not Robert Stirling was aware of the patent, here nevertheless, in 1794, had been the essence of the closed-cycle heat engine with displacer.

1.2.4 That the invention was catalyzed by Stirling's concern over steam boiler explosions

Authors promoting this version of Stirling's motivation have been contacted. Those who have replied have declared themselves unable to offer evidence from written record.

Flynn et al. note that statistics on steam boiler explosions for the early nineteenth century are lacking. Figures attributed to Hartford Boiler Insurance Co. suggest a heyday of explosions in England – 10 000-plus incidents – during the period between 1862 and 1879. This is half a century *after* the 1816 invention.

The comprehensively-researched account by 1995 by Robert Sier of the life and times of Robert Stirling gains much of its scholarly impact by offering a wealth of quotation from the historical record – and by withholding speculation about motives.

The United Kingdom considers Frank Whittle to be the inventor of the jet engine. As a Royal Air Force officer, Whittle would have been aware of the history of gruesome amputations by aircraft propellors. No one has (so far) had the bad taste to offer that awareness in explanation of his pioneering work.

1.2.5 That younger brother James was the true inventor

The younger sibling would have been 16 at the date of filing – and only 15 while Robert was incubating the invention. Robert was living in Kilmarnock, while the Stirling family home (home of the un-married James) was 115 km distant as the crow flies (70 miles) in Perthshire.

This does not deter Kolin from asserting: *'… invention of the Stirling engine is generally attributed to Robert Stirling, which may be attractive, but is rather doubtful.' … 'The only written sign that Robert was engaged on the Stirling engine is his name, together with his brother James* (!) *on the patent specifications.'* (Kolin, 2000).

The statement is at odds with the meticulous attention to detail which distinguishes other work by Kolin. The patent description is in the name of Robert (alone), in his handwriting, and over his signature and seal. Evidence for a precocious contribution from James is, at the time of this writing, non-existent.

1.2.6 That 90 degrees and unity respectively are acceptable 'default' values for thermodynamic phase angle α and volume ratio κ

The false belief accompanies – and may stem from – a view that performance is more sensitive to changes in gas path specification (hydraulic radius, flow-passage length) than to changes in phase angle and volume ratio.

Systematic and long-overdue bench tests on a 'Vari-Engine' designed and built by Larque with input from Vaizey now convincingly indicate otherwise. If volume variation is simple-harmonic, $\alpha = 90$ degrees converts to $\beta = 45$ degrees and $\kappa = 1.0$ to $\lambda = 1.4$ of the 'beta' or coaxial configuration. (This is pure trigonometry – see Chapter 4 *Kinematics*. Thermodynamic considerations do not arise!) The ratio 1.4 is almost precisely the *inverse* of the value since demonstrated experimentally by Larque to give best operation of a small but 'real' engine of beta configuration (multi-channel exchangers, foil regenerator, modest pressurization on air).

Converting Larque's optimum values to the kinematics of the 'alpha' (opposed-piston) configuration yields $\alpha = 132$ degrees. and $\kappa = 0.78$. Both figures are readily embodied into a one cylinder-pair unit, but only the latter can be achieved in the four-cylinder, double-acting 'Rinia' configuration. In any case a better choice for the Rinia would now appear to be three cylinders with $\alpha = 120$ degrees – or six cylinders with an inter-connection between alternate pairs.

1.2.7 That dead space (un-swept volume) is a necessary evil

At *rpm* and pressure giving viable operation, the gas processes in the variable-volume spaces are closer to adiabatic than to isothermal. Chapter 4 confirms, amplifies and argues that, one this basis the cycle of the regenerative gas turbine (Joule cycle – adiabatic, constant pressure)