The Strongest Magnetic Fields in the Universe

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Magnetic fields are a fundamental component of the physical world on all scales—as constituents of the electromagnetic environment of all matter. The interaction between the magnetic fields and matter is at the heart of many dynamic processes that shape astrophysical objects and their environments. Regions of space threaded by magnetic fields control or at least influence the interactions that take place between them. Observing and understanding magnetic fields and the role they play in physical processes in the solar system and beyond have been the subject of a series of Workshops and publications by the International Space Science Institute since 2008.

The first three volumes covered the origin and dynamics of solar magnetism, the magnetic fields of the planets, and the magnetic field of the Earth. These were meant to constitute a trilogy of magnetic fields within the confines of the solar system. Then came the fourth volume which extended the coverage to the whole Universe, but concentrating on large-scale magnetic fields, comparable to, or smaller in magnitude than those directly measured within the solar system. The four volumes are:


The topic of the current volume contains reviews of completely new aspects of magnetic fields in the astrophysical Universe—and in many cases of aspects of physical processes and phenomena fundamentally different from those addressed in the first four volumes. The strength of the magnetic fields of the compact objects reviewed in this volume is up to 8 to 10 orders of magnitude higher than that of typical sunspots which are the strongest fields in the solar system. Large-scale astrophysical magnetic fields, such as interstellar and galactic fields are weaker still than the fields experienced in the solar system. The gap between field strengths covered in the previous volumes and those in astrophysical object and environments addressed in the present volume is illustrated in Fig. 1.

The first four volumes in the ISSI series on magnetism have left open the questions related to the generation and effects of the very strong magnetic fields found near the most compact ob-
The range of astrophysical magnetic field strengths, covering 21 orders of magnitude, effectively representative of the whole spectrum of astrophysical phenomena in the Universe. The lower part of the graph shows the topics covered by four previous volumes in the Space Science Series of ISSI; the objects with extremely high magnetic fields, neutron stars, magnetars, are the subject of the current volume.

The motivation for the current volume has been to complete the review of such magnetic fields, associated with Magnetic Stars, White Dwarfs, Neutron Stars and Active Galactic Nuclei (AGNs) and their environments. The reviews presented here describe the current understanding of how the extremely strong magnetic fields of these objects are generated, how they interact with matter and how they generate the broad range of observed phenomena, among them shock waves and many different types of radiation over a very broad spectrum. Of particular interest are the phenomena leading to the generation of astrophysical jets and their complex physics, the occurrence of Gamma Ray Bursts, the formation and extreme properties of relativistic shocks and the production of high-energy particles by the cosmic engines. Radio pulsars (highly magnetised neutron stars) have been studied for the last half century; these have magnetic fields in the range $10^{11}$ to $10^{13}$ G. In such strong fields the basic physics changes for the reason that the electron Larmor (gyro) radius drops below the Bohr (atomic) radius. Magnetars are also neutron stars, but with magnetic fields in the range $10^{13}$ to $10^{16}$ G. The two or more orders of magnitude difference in the magnetic fields of these objects leads to differences in their energetic output. In particular, the explosive energy in the observed “giant flares” from magnetars, powered by the dissipation of magnetic energy, is of order $10^{44}$ ergs, about 12 orders of magnitude more energetic than the largest of solar flares. We are indeed in the presence of extreme physical phenomena, ascribed to the ability of these exceptional astrophysical objects to generate the exceptionally strong magnetic fields.

There are fifteen review papers in this volume, providing a comprehensive and up-to-date coverage of a field that remains in a state of rapid evolution. With this volume, the ISSI series on astrophysical magnetic fields concludes an ambitious review of topics that are central to the progress of astrophysics and our understanding of the Universe.

The ISSI Workshop on the Strongest Magnetic Fields in the Universe was held in ISSI, Bern, Switzerland on 3–7 February 2014. The Convenors of the Workshop (André Balogh, Vasily Be-skin, Maurizio Falanga, Maxim Lyutikov, Sandro Mereghetti, Tsvi Piran and Rudolf Treumann) and the Editors of this volume are greatly indebted to all the participants of the Workshop who brought their broad range of expertise and interest in astrophysics to deepen our understanding of the issues related to the extreme magnetic fields in the Universe and their parent objects. The resulting collection of review papers was the outcome of the exchanges and fruitful collaboration among the participants; we thank them for their successful efforts to integrate the lessons learned in the different topics, as the reviews in the volume testify. Thanks are also due to the reviewers of the papers; in all cases the reviews were thorough and constructive and the volume bears witness to their contribution. We thank the staff of ISSI for their dedicated support: Prof. Rafael
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Part I
Introduction
Magnetic Fields at Largest Universal Strengths: Overview

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Abstract A brief review is given about the role strong magnetic fields play in the universe. We list the main observational and theoretical achievements treated in the following chapters including a number of open questions which future research is going to attack. Strong fields in the universe exceed any large scale fields by several orders of magnitude, at first glance suggesting that their generation mechanisms would be different. However, it is believed that gravitational collapse and magnetic flux conservation is responsible for the amplification of fields generated in the progenitors to the observed strengths. In this sense the extremely strong fields are mainly fossil, and their variety confirms the different masses and stages where the collapse comes to rest, at the lightest in white dwarfs and at the strongest in magnetars, which are a particular class of neutron stars with strongly inhomogeneous particularly structured crust. Various effects related to the detection of such fields, radiation generation and consequences for the environment are pointed out and referred to the relevant chapters in this volume.

Keywords Magnetic stars: white dwarfs, magnetars, pulsar · Neutron stars · Accretion, outflow and jets

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1 Introduction

That magnetic fields in the universe are abundant was unknown until the first half of the past century, when sporadic radio emission was discovered from the sun and later also from more distant objects. Such emissions were attributed to synchrotron radiation requiring the presence of moderately strong magnetic fields. Magnetic fields on the sun were estimated in the range of several in the corona to several thousand gauss (G) in sunspots and below. More remote large scale fields turned out to be substantially weaker, of the order of $\mu G < B < mG$. It is now established that magnetic fields in the universe can be as weak as $\mu G$ on the large galactic and intergalactic scales (Beck et al. 2012) and possibly as strong as $10^{17}$ G in magnetars. Obviously, there is a profound difference in scales between these limits. Interstellar fields range in the somewhat stronger (up to mG) field strengths, earth-like planets have fields from fractions of one to several G, the remainder from ten G up to, say, $10^5$ G are reserved for stars, non-degenerate stars in the first place. The strongest fields are found in degenerate stars: white dwarfs, neutron stars, including pulsars (see Trümper et al. 1977 for the first detection of fields the order $\sim 10^{13}$ G in the HerX1 pulsar) and, ultimately, magnetars. Moderately strong magnetic fields exist in parts of accretion disks, while strong fields are believed to be involved in the compact objects relatively close to their horizons as well as in the inner parts of their outflow jets. Though it is not known what the ultimate upper limit is for any magnetic fields in the universe, it has been argued (Duncan 2000; Treumann et al. 2014) that such fields are limited by quantum electrodynamics to strengths quite far above the strongest observed magnetar fields. Field strengths could, theoretically, go up to $\sim 10^{32}$ G or so (see Fig. 1). However, in the absence of magnetic monopoles, no mechanism is known which could produce fields of such strengths. Today it is believed that magnetic fields are generated in magnetic dynamo and also battery processes which feed mechanical energy of a turbulent flow of electrically conducting matter into current filaments the integrated effect of which gives rise to observed magnetic fields. Magnetic dynamos of this kind act in the magnetohydrodynamic fluid within magnetic stars. In earthlike planets and neutron stars they act in the metallic-fluid conducting and turbulent matter flow in their interiors. In magnetars, the strongest magnetised objects known so far, some version of neutron stars, it has been shown (Duncan and Thompson 1992; Thompson and Duncan 1995) that neutron star fields have been amplified a bit by further dynamo action during collapse. Concentration of the field in crustal inhomogeneities cares for locally very strong fields responsible for strong magnetic loops reaching out into the magnetar environment. In the environment of highly compact objects, jets and the close part of accretion disks, it seems that dynamo action and magneto-rotational instability (MRI) as well sign responsible for fields substantially stronger than on large scales. Additionally, strong fields can be produced by general relativistic effects known as the Znajek–Blandford (Blandford and Znajek 1977) mechanism acting near rotating black holes and generating magnetic funnels. In all those cases we are in the range of fields much stronger than on large scales though substantially weaker than the above mentioned theoretical limits. Magnetic fields of these comparably high strengths affect the behaviour of matter exposed to them. Once the electron cyclotron energy $\hbar \omega_{ce} \sim e^2 / 4\pi \varepsilon_0 \mu_0 a_0$ exceeds the Coulomb energy of an electron on an atomic Bohr radius, which happens for fields $B > B_0 \approx 2 \times 10^5$ T $= 2 \times 10^9$ G, the magnetic field affects the atomic properties and fundamentally changes the behaviour of matter. This will necessarily happen in the interiors of neutron stars, magnetars, pulsars. It also affects the matter in the magnetospheres of those objects. On the other hand, when for weaker fields the magnetic energy becomes comparable to the gravitational binding energy it would affect the orbits of bound objects. Such fundamental properties of strong magnetic fields
Magnetic Fields at Largest Universal Strengths: Overview

Fig. 1 Log-Log plot scaling of the maximum possible magnetic field strength, $B_c$, normalized to the (fictitious) Planck-magnetic field, $B_{Pl}$, as function of fundamental length scales based on Eq. (1). Length scales $\ell$ on the abscissa are normalized to the Planck length $\ell_{Pl}$. The dotted red cross indicates the crossing point of the Compton length with the Aharonov–Bohm critical magnetic field line at the so-called quantum limit field $B_q \approx 10^9$ T, the critical field of magnetized neutron stars (pulsars) in agreement with observation of the strongest cyclotron lines. Horizontal lines indicate the relation between other length scales and critical magnetic fields under the assumption of validity of the Aharonov–Bohm scaling. Space magnetic fields correspond to scales of $\sim 1$ mm. Strongest detected magnetar fields correspond to the first order relativistic correction on the lowest Landau level energy $E_{LLL}$ (shown as graph on the right with $\tilde{\alpha} \approx \alpha/2\pi$ the reduced fine structure constant). Inclusion of higher order corrections would allow for fields of up to $B_{qed} \sim 10^{28}$ T deep in the (shaded) relativistic domain which have not been observed. It is interesting that this limit coincides approximately with the measured (Gabrielse et al. 2006) absolute upper limit on the electron radius (vertical blue dashed line). The black dashed curve indicates a possible deviation of the Aharonov–Bohm scaling near the quantum electrodynamic limit. At GUT scales, fields could theoretically reach values up to $\sim 10^{45}$ T, according to simple Aharonov–Bohm scaling (adapted from Treumann et al. 2014).

are reviewed in the introductory paper Lai (2015) of this volume. The following chapters provide a summary of magnetic fields in various astrophysical candidates of strong fields: magnetic stars, white dwarfs, pulsars, magnetars, their observational evidence, effects and theories.

2 Physics in Very Strong Fields

Quantum mechanics provides an easy way to obtain a first limit on the magnetic field from solution of Schrödinger’s equation, first found by Landau (1930), of an electron orbiting in a straight magnetic field. The physical interpretation of this solution was given much later in the Aharonov–Bohm theory (Aharonov and Bohm 1959) of quantization of magnetic flux $\Phi$. From the requirement that the magnetic flux confined in an electron gyration orbit must be single valued, Aharonov and Bohm found that the magnetic flux $\Phi = v\Phi_0$ in a magnetic field $B$ is quantized with flux element $\Phi_0 = 2\pi \hbar e$, $e$ the elementary charge, and

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\( v = 1, 2, \ldots \) Since \( v = \Phi / \phi_0 \) is the number of elementary fluxes carried by a magnetic field, and \( B = \Phi / \pi \ell^2 \), putting \( v = 1 \) defines a smallest magnetic length

\[
\ell_B = \left( \frac{\phi_0}{\pi B} \right)^{\frac{1}{2}} = \left( \frac{2 \hbar}{e B} \right)^{\frac{1}{2}}
\]

This magnetic length, which is the gyroradius of an electron in the lowest lying Landau energy level, can be interpreted as the radius of a magnetic field line in the magnetic field \( B \). Field lines become narrower the stronger the magnetic field. On the other hand, rewriting Eq. (1) yields an expression for the magnetic field

\[
B_c = \frac{2 \hbar}{e \ell_c^2}
\]

from which, for a given shortest "critical" length \( \ell_B \equiv \ell_c \) the maximum magnetic field \( B_c \) corresponding to \( \ell_c \) can, in principle, be estimated. Since the Planck length is the shortest scale where quantum electrodynamics ceases to be valid, the last expression yields the theoretical limit for magnetic field strengths for a very wide range. Putting, for instance, \( \ell_c = 2\pi \hbar / mc \) equal to the electron Compton length \( \lambda_0 = \hbar / mc \), one obtains the critical pulsar (neutron star) magnetic field strength \( B_{\text{ns}} \approx 3 \times 10^{13} \text{ G} \). Approximately this field strength was indeed inferred from observation of the fundamental \( (v = 1) \) electron cyclotron harmonic X-ray line detected from the HerX1 pulsar (Trümper et al. 1977), roughly two decades after Aharonov and Bohm’s, and half a century after Landau’s theory. However, magnetic fields require a mechanism of generation, i.e. current flow. Since currents are based on lepton (electron, positron) dynamics (including their spin dynamics) current flow is limited once the lepton radius is reached. Its currently best estimate is \( r_e \sim 10^{-22} \text{ m} \) (Gabrielse et al. 2006) which yields the above mentioned theoretical estimate of \( B \sim 10^{32} \text{ G} \) far above any observed field strength.

In addition to current flow, magnetic fields may affect the behaviour of matter. It is known that under classical conditions fields fluid containing oppositely directed magnetic field can annihilate their fields when contacting, a process well known as magnetic reconnection. It causes redistribution of fields and currents, magnetic energy dissipation, plasma jetting and heating as well as violent acceleration of small groups of resonant particles. Such processes, observed and confirmed in near-Earths space also work under astrophysical conditions where in strong field relativistic effects come into play. It has even suggested that this mechanism of magnetic energy annihilation forms the main mechanism of dissipation in strong turbulence of magnetised media. There the injection of mechanical energy at the largest scales causes eddies which themselves cause current flow. Nearly dissipationless cascading of the turbulence down to ever shorter eddy scales leads to the formation of narrow current filaments. Once their width drops below the lepton gyro-radius inertial effects take over, and the current filaments enter into the reconnection regime. The mechanical turbulent energy is then violently dissipated, a mechanism which seems to be important already on the classical scale of moderately strong magnetic fields in stellar environments and coronas and is also expected in stellar interiors.

Much stronger magnetic fields may affect processes on the molecular and atomic scales. This happens when the magnetic energy density becomes comparable to the binding energy density. There is a wealth of phenomena which magnetic fields cause under those conditions, several of them are reviewed in Lai (2015) in the first chapter of this volume. Further aspects of the effects of very strong fields on matter (Ruderman 1974) are discussed in connection with the structure of neutron stars in Pothekhin et al. (2015a, 2015b).
In the following we present a brief overview of the various classes of objects in the universe which support strong magnetic fields. We do not necessarily follow the content of this volume. Instead we just point on the main important and generally interesting facts of interest for the reader who may consult the different chapters for details.

3 Stellar Magnetism

Except for extended and extremely massive objects that will be considered below, strong magnetic fields are a property of a few different classes of stars. From the point of view of generation of such strong fields one may decide between ordinary stars, that is non-degenerate stars and degenerate stars. It is by now known that magnetic fields in either of them result originally from dynamo actions in the stellar interiors even when not acting anymore in which case the fields are ‘fossil’ remainders of original dynamos.

In non-degenerate stars which are moderately massive objects, the dynamos are believed of acting continuously. Since all information on the fields in those stars (members of the HR-diagram) obtained relies on spectral observations of polarised/unpolarised radiation, the reliability of such measurements is a key problem. Due to limitations in the spatial and temporal resolutions observations provide mostly averages of the real fields which can be polluted by cancellation of oppositely directed magnetic fluxes and distribution of magnetic fields over the stellar surfaces and environments. Average fields can be order s of magnitude smaller than the real spatially resolved magnetic field strengths. Since magnetic fields control also the stellar environment, the various layers of stellar atmospheres, heating rates, chemistry, radiation transport, and even some mass and angular momentum loss, information about magnetic fields is also obtained from observation of such processes.

A critical compilation of the effects of magnetic fields in the different HR-diagram stars, all the various methods inferring about magnetic fields, their strengths and complexity, and the various uncertainties of estimates and interpretation in given here Linsky and Schöller (2015). The paradigm of a ‘normal’ main sequence star is the Sun. Its average longitudinal surface field is moderate, $\sim \pm 2\;\text{G}$ only, still belonging to strong fields when compared with interstellar values though much weaker than any spatially better resolved fields of $\sim 1.5\;\text{kG}$ the surface filling factor of which is $f < 5\%$. Pre-main sequence stars generally have substantially stronger average longitudinal fields $< 4\;\text{kG}$ which channels the disk-accreting matter into small areas on the stellar surface causing hot gas observed in bright He I emission lines. Outside this shocked the field is a multipole field of higher order. Evolution towards the main sequence and higher mass causes weaker fields still of complex structure. Main sequence FM stars cooler than the Sun remain having complex field structures. Zeeman splitting suggests field strength $\sim 3\;\text{kG}$ and relatively high filling factors of $f \sim 0.5$. Fields in A- and B-stars on the other hand are lower multipole-like and oblique with respect to their rotation axes, in agreement with dynamo models though chemically peculiar stars seem to be slow rotators only. CP stars among them have rather strong fields $< 50\;\text{kG}$ which exert strong control of their stellar winds. Weaker fields $< 1\;\text{kG}$ are found in B stars. Hot main sequence O stars do not have convective zones and therefore, probably, no dynamos. Their fields are believed to be of fossil or just tidal dynamo origin, having strengths $< 400\;\text{G}$. Finally post-main sequence stars which ultimately are slow rotators have dynamo generated weaker $\sim \text{kG}$ fields which sometimes show rapid (monthly) changes in their magnetic morphology.
4 Generation of Stellar Fields

What concerns the generation mechanisms of stellar magnetism, there is still no complete agreement. Above we stressed that the main mechanism would be the dynamo either ongoing in pre- and main sequence stars or conserved ‘fossil’ in post-main sequence stars and, as commonly believed, in degenerate stars (white dwarfs, neutron stars, . . .). The stellar dynamo remains to be the general paradigm since its first proposal, having been very successful in application to the Sun but substantially less successful in application, for instance, to planetary magnetism. Excluding planets which are not treated in this volume mainly for the reason that observation of their magnetic fields from distance is so far technically impossible, and reference to the magnetic fields of the solar system planets is therefore not necessarily representative, the magnetic dynamo maintains its status of the dominant field generation, mostly assumed to be convection driven though one could also refer to other mechanical sources like tides or general turbulence, accretion driven dynamos and dynamos driven by merging protostellar objects. Indeed, stellar collisions are not as rare as believed. In this case incidence of magnetic fields should increase with stellar mass, which indeed seems to be the case.

The central problem is the question why only a fraction, not even the larger one, of main sequence stars is magnetic. A number of mechanisms, in particular the dynamo and fossil mechanism scenarios are critically reviewed in Ferrario et al. (2015a), mainly in view of their relevance in the generation of the magnetic fields in magnetic white dwarfs and neutron stars, both degenerate stars for which the field must have been either generated in their non-compact progenitors or, as believed, further attributed to accretion from a non-degenerate companion. The latter, however, seems not being feasible for the apparently complete absence of such companions in magnetic white dwarfs. Fossil fields have not just to form during contraction of a star to become a white dwarf (or neutron star), they have also to survive the complex phases of evolution during and after contraction. These processes are indeed nontrivial as they include the decoupling of the convection zone of a contracting star from the core, retardation of the core by magnetic forces and other effects. Thus fields becoming fossil is easy to predict from rough estimates of flux conservation but not easy to demonstrate by taking into account all the physical processes taking place during contraction and collapse, including destruction and dissipation of the progenitor field by magnetic instabilities on all scales including reconnection, and/or reconstruction by magneto-rotational instability effects or turbulence. Extension of dynamo action during contraction of the progenitor and for some time after contraction has therefore been proposed and cannot be discarded as well.

It thus seems that a single universal deterministic process like a convective dynamo is insufficient to stand for all the different magnetic fields attributed to stars from pre-main sequence to neutron stars/magnetars.

5 Degenerate Stars

The two cases of stable degenerate stars which have formed by contraction and collapse are white dwarfs and neutron stars. Among both of them just a fraction is strongly magnetised. Magnetic white dwarfs are reviewed in Ferrario et al. (2015b) which also contains extended tables of essentially all known white dwarfs, their parameters and magnetic fields. The fields cover the range of $10^3 < \langle B \rangle < 10^9$ G. The morphology of the stronger field is fairly complex with presence of higher multipoles. In agreement with estimated ohmic
dissipation times of $\sim 10^{11}$ years no relevant decay of their magnetic fields has been found. Indeed, high magnetic fields in white dwarfs correlate with higher masses of the white dwarf thus suggesting that accretion of matter and field is a cause of stronger fields. Weaker fields $\langle B \rangle < 3 \times 10^8$ G are found in binaries (magnetic Cataclysmic Variables) with less detectable multipoles. What concerns the generation of fields in magnetic white dwarfs the fossil hypotheses is paralleled with the assumption of accretion by wind from a hot companion star in the above binaries also explaining their variabilities. At least, the accretion hypothesis of strengthening the field gains increasing theoretical and observational support.

Production of neutron stars, as is well known, proceeds along similar lines as production of white dwarfs with neutron stars being heavier. The gross difference is that they are kept stable not by the degeneration pressure of electrons but of the nuclear matter, neutrons. For reasons of equilibrium and stability of neutrons they consist of a mixture of neutrons, protons and electrons. Neutron stars are heavier than white dwarfs, therefore contraction and collapse provide them with smaller radii, higher densities, and substantially stronger magnetic fields as noted in the introduction. Average fields can become as strong as $\langle B \rangle \sim 10^{14}$ G. Observation of magnetic fields is a clue for gathering information about their internal structure, consistence of their cores and crusts, and their dynamics. Their interiors are superdense, essentially resembling huge atomic nuclei, possibly being superfluid/superconducting, crustal matter being of substantially more complicated structure, their “atmospheres”, i.e. their close spatial environments, in highly magnetised relativistic plasma state. Information about any of those regions is gathered solely by observation of radiation in different parts of the electromagnetic spectrum.

Which radiation? At finite surface and atmospheric temperatures $T \sim 10^2$ eV, neutron stars emit thermal radiation in X rays, independent of their magnetisation (Pothekhin et al. 2015a). This is indeed the case for isolated neutron stars as well as for neutron stars in binary systems though the number of such neutron stars with an unambiguously identified thermal component is not large. Once they are magnetised with fields of the order of $B \sim 10^{12}$ G, neutron stars become radio emitters (Beskin et al. 2015), emitting at much lower frequencies in different modes, synchrotron radiation or pulsed radio signals from the obliquely rotating neutron star with beamed radiation. Such radio pulsars are almost precise clocks in the universe; they moreover provide an important and observationally well accessible laboratory for the investigation of high energy classical and quantum problems in electrodynamics. Almost 90 % of the known pulsars are isolated; the remaining 10 % are found in binaries with mostly negligible mass transfer from the companion. Though pulsar physics is by now half a century old, and many aspect of their radiation emission have been understood there remain many unsolved problems, among them the very mechanism of generation of coherent radiation.

Those neutron stars who reside in binaries and accrete matter from their companion star emit in X-rays. Such neutron stars possess magnetic fields up to $B \sim 10^{13}$ G at strengths where the cyclotron resonance line was first detected. Observing this lines provides a simple and precise way of estimating the magnetic field strength though only at the highest fields. At weaker magnetic fields one relies on different and less precise techniques (Revnivtsev and Mereghetti 2015). Being strongly magnetised and in a binary system their magnetospheres are highly distorted by the accretion processes which power their X ray emissions. Geometrical deformation of the magnetosphere sign responsible for their spectral X ray and timing properties. In particular accretion affects the spin of such neutron stars.
6 Magnetars

Strongest magnetic fields, in fact the strongest magnetic fields ever inferred in any natural system, were found in a particular class of neutron stars called magnetars. Such neutron stars have been found to possess surface fields $B \leq 10^{15} \text{ G}$ and internal magnetic fields up to roughly $B \leq 10^{17} \text{ G}$ which, originally, occurred to be surprising. Meanwhile it is known that such field strengths are not excluded by fundamental physics (Treumann et al. 2014), and a number of models and mechanisms for generation of such strong fields have been proposed in the recent past (Duncan and Thompson 1992; Duncan 2000; Thompson and Duncan 1995). A critical collection of such models and the properties of magnetars is provided in Mereghetti et al. (2015) for the persistent emission in various regions of the electromagnetic spectrum (spanning the range from radio to X rays) from magnetars and what can be learned from its observation. About thirty magnetars and magnetar candidates are today known in the galactic vicinity, most of them appearing as highly variable X ray emitters suggesting that a substantial amount of matter accretes while the high temporal variability indicates that accretion is either not a permanently stable process or that the magnetic field strength at these high values undergoes some temporal restructuring. Most of this magnetic activity is attributed to processes in the neutron star crust while the core of the star is generally believed to be a type II superconductor which occasionally may expel magnetic flux tubes thus contributing to the temporal variability.

There are many very interesting questions and problems concerning the structure of the crust and core in the presence of the very strong magnetic fields as inferred for magnetars. Such problems have become the subject of most recent investigations beyond those discussed in the present book. Crustal matter seems not to be as simply structured as fluid models including relativistic magnetohydrodynamics assume. There are indications that the strong magnetic fields in such neutron stars cause the matter in the crust to be not only layered but in the layers just beneath the very thin outer crust becoming “pasta-like” vertically (to the surface) smeared out structures of matter that may become mixed with superfluid cells, broken by magnetic flux tubes. Such structures are subject to plastic deformation which may generate magnetic activity in magnetars (Lyutikov 2015). It is not known what effect such a structure might have on the magnetic field distribution in the crust and the neutron star environment. Recent three-dimensional numerical fluid investigations (Wood and Hollerbach 2015)—including Hall effects but ignoring this particular structuring—suggest that small scale crustal magnetic fields may survive for long time and, similar to the butterfly structure of sunspots, occur in bands that, however, move towards the equator where the crustal currents may organise themselves into a strong equatorial electrojet (see Fig. 2).

![Magnetic field lines](image)

**Fig. 2** Magnetic field lines obtained from a fluid-like three-dimensional simulation of a neutron star crust including Hall effect (adapted from Wood and Hollerbach 2015). From its crust sources the field evolves into localised structures which concentrate around the equator indicating an equatorial jet flow.
The magnetic stresses of such fields and currents may excite magnetar quakes and related glitches and temporal variations in the magnetic field within estimated up to a million years after formation of the neutron star.

7 The Environments of Strongly Magnetised Objects

Naturally magnetic fields leak out from their body source extending far into the surrounding space and thereby substantially affecting any charged matter present in the stellar environment. Regions where the magnetic field dominates the dynamics of the matter are the magnetospheres. In magnetised white dwarfs, neutron stars, pulsars and magnetars these magnetospheres are strongly controlled by the magnetic field (Beskin et al. 2015; Mereghetti et al. 2015). In rotating neutron stars their spatial extension is limited by the light cylinder, the distance up to that dilute charged matter is believed to corotate with the star as long as it is frozen to the magnetic field, items well known for long time including their problems. The light cylinder in this case acts as an almost cylindrical surface of injection of matter from the rotating neutron star into the environment. In the opposite case when the fields are not as strong, the star emits a wind which extends radially out to large distances. The best known example is the solar wind which because of the relative weakness of the solar field dominates the latter almost completely. Similar effects happen at magnetised stars causing not only winds but channelled outflows mostly from the polar-dipolar field regions of open field flux tubes. In rotating neutron stars the spatial extension is limited by the light cylinder, the distance up to that dilute charged matter is believed to corotate with the star as long as it is frozen to the magnetic field, items well known for long time including their problems. The light cylinder in this case acts as an almost cylindrical surface of injection of matter from the rotating neutron star into the environment. In the opposite case when the fields are not as strong, the star emits a wind which extends radially out to large distances. The best known example is the solar wind which because of the relative weakness of the solar field dominates the latter almost completely. Similar effects happen at magnetised stars causing not only winds but channelled outflows mostly from the polar-dipolar field regions of open field flux tubes. In binary systems the inverse of such outflows is the already mentioned accretion of matter from one star to the other. Most of the relevant questions concerning these latter cases are reviewed in depth in three comprehensive chapters in this volume (Hawley et al. 2015; Kargaltsev et al. 2015; Romanova and Owocki 2015) presenting a wealth of new material based on observation and simulation.

Pulsars, rotating neutron stars, emit most of their energy and, in particular, momentum in ejection of highly relativistic magnetised pair (electron/positron) plasmas in the form of pulsar winds (Kargaltsev et al. 2015) reaching into the region far outside the light cylinder where they become accelerated near the pulsar wind termination shock (Schlickeiser 2002; Bykov and Treumann 2011). The momentum loss causes a spin-down of the pulsar which is a measurable parameter. Spectacular observations of the Crab and Vela pulsars have confirmed this long standing hypothesis including sudden glitches in the rotation period related to flares and eruptive ejections of matter. Otherwise, speed-ups of rotations have been attributed to accretion of matter (Romanova and Owocki 2015) which, however, is not observed in isolated pulsars but restricted to binary systems. The high energy accelerated particles generated outside the termination shock where it is found to produce pulsar wind nebulae which are seen in X rays and Gamma rays. The nebulae populate essentially two morphologies: torus-like formations combined with a jet as in the Crab pulsar, and bow shocks.

In general, magnetised stars affect their environment more or less, depending on whether or not the compact magnetic star possesses an accretion disk, the normal case if it is accompanied by a weakly magnetic of non-magnetic large hot companion which, in the vicinity of a compact object usually fills its Roche-lobe. The strong compact object magnetic field truncates the accretion disk and funnels the disk plasma from along the field to the stars polar region or, in case of magnetically multipolar stars to the multipolar regions. Otherwise plasma may be ejected from the magnetospheres along the open field lines, a process that
is the more efficient the faster the compact magnetic star rotates. Fast rotators with accretion may then eject the funnelled accreting matter out along the open fields even before it hits the star's surface. Many such processes can only be treated by numerical simulations of which a representative collection is presented in Romanova and Owocki (2015), while Hawley et al. (2015) covers the knowledge of the properties and instability (mainly to the magneto-rotational instability MRI) of accretion disks that are formed in many those cases under the action of the strong gravity of the central object in combination with its fast rotation that leads to the centrifugal forces. In particular, formation and internal properties of jets and ejection of matter along the open strong magnetic field lines are an important field of research. Break-out, collimation and interaction of jets with the accretion disk and external environment are treated here as well. Some of these processes which have not yet been understood properly for the reason that jets are spatially confined and thus of highly complicated internal structure and that the jet plasma is highly relativistic and thus under some circumstances subject to strong radiation effects in addition to non-negligible interaction between high-energy particles. Processes of this kind are treated to some extent in the last part in this volume on basic processes.

8 Basic Plasma Processes

Three basic plasma processes which lie at the fundament of any understanding of highly magnetised plasma in the vicinity of compact objects in the universe are treated in Granot et al. (2015), Sironi et al. (2015), Kagan et al. (2015). The first deals with the famous gamma ray burst that have been observed since the seventies being about isotropically distributed in the sky. Most of them have been attributed to objects containing strong magnetic fields, thus being compact objects. It is also believed that highly relativistic collisionless shocks play an important role in these gamma ray bursts, in particular in their afterglow. The properties and physics of gamma ray bursts and the different scenarios and models of generation of prompt radiation and afterglow, which have been put forward, are reviewed pointing out their pros and cons. Emphasis is on the role of strong magnetic fields in generation of gamma ray bursts of shortest duration and highest luminosity. It seems that highest luminosities require the presence of very strong magnetic fields like in neutron stars and magnetars.

Collisionless shocks (Balogh and Treumann 2013; Treumann 2009) are one of the important natural tools for heating plasmas and accelerating particles to very high energies (Schlickeiser 2002) as present in cosmic rays. In the astrophysical context they are mostly related to winds and jets emanating from strongly magnetised objects where they become relativistic (Bykov and Treumann 2011). Recently they have also been noted in the context of gamma ray bursts when including general relativistic effects. Understanding their detailed physics is necessary for the correct interpretation of many of the observations of magnetised compact objects, including gamma ray bursts and accretion of the kind discussed in this volume. The current state of the art is summarised in Sironi et al. (2015).

Finally, since magnetic fields imply that they reorganise when contacting each other, the other most important process concerning magnetic fields is collisionless reconnection (Treumann et al. 2014). Under astrophysical conditions, reconnection becomes again relativistic which substantially complicates the process of magnetic rearrangement. Kagan et al. (2015) discusses the relevant processes in collisionless relativistic reconnection in very strong magnetic fields, its effect seen in observations and application to highly magnetic objects. Both, collisionless shocks and reconnection turn out not only to be strong accelerators of particles contributing to cosmic rays but also strong radiation sources in a broad spectrum.
9 Conclusions

The present volume aims on giving a comprehensive review of the state of our knowledge about the strongest magnetic field that have been detected in the universe. These field are many orders of magnitude higher than the average universal magnetic fields (Beck et al. 2012; Widrow et al. 2012; Durrer and Neronov 2013). Their origin, in the progenitors of the most extreme fields, is believed to be mostly dynamical, generated by kind of magnetic dynamos. These fields become substantially increased by magnetic flux conservation when the massive objects contracts to its final compact state. Dynamo action may even prevail during and a short time after collapse (Duncan and Thompson 1992; Thompson and Duncan 1995), a very important point since this prevail not only explains in a few cases the generation of over-strong magnetic field, it also gives an interpretation for the occurrence of strong fields in compact objects in general in particular when they occur in object that are isolated and have not experienced accretion to overcome the gravitational limit for contraction/collapse. The related question is: why do such objects wait until they contract in order to allow the internal dynamo to generate magnetic fields that in contraction become so strong? Such heavy objects should almost immediately collapse according to gravitational theory. Dynamo times are usually long. Hence, dynamos should work as long as the star has not been pushed beyond the limit. The extension of this working time to periods during and after collapse releases this problem. On the other hand, as demonstrated in Ferrario et al. (2015b) the fossil field assumption itself brings its problems with it. Hence the case is not completely settled yet, and more work is expected, mainly numerical and observational, to make progress. Neutron stars, magnetars, pulsars and their environments including magnetospheres, winds, outflow, inflow, accretion, jets and radiation are well treated as far as they concern very strongly magnetised objects. No magnetic strange stars (Chau 1997) have yet been observed. Hence, the processes leading to generation of magnetic fields have probably not to be extended to the inclusion of other new ones which would lead to vastly stronger fields than those observed so far.

References


Physics in Very Strong Magnetic Fields

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Abstract This paper provides an introduction to a number of astrophysics problems related to strong magnetic fields. The first part deals with issues related to atoms, condensed matter and high-energy processes in very strong magnetic fields, and how these issues influence various aspects of neutron star astrophysics. The second part deals with classical astrophysical effects of magnetic fields: Even relatively “weak” fields can play a strong role in various astrophysical problems, ranging from stars, accretion disks and outflows, to the formation and merger of compact objects.

Keywords Magnetic fields · Stars · Accretion

1 Introduction

The subject “Physics in Very Strong Magnetic Fields” is a very broad one, and the title is also somewhat ambiguous. The first question one may ask is: How strong a magnetic field is “Strong”? The answer to this question will depend on the objects one is dealing with, on the issues one is interested in, and on whom one is talking to.

In the following, we will first review issues of strong magnetic fields from a general physics point of view and discuss how these issues may relate to some aspects of neutron star astrophysics. This focus on neutron stars reflects that fact that neutron stars are endowed with the strongest magnetic fields in the universe where fundamental strong-field physics can play an important role. It also reflects the author’s own research interest on the subject. For most other astrophysics problems, covering a wide range of sub-fields of astrophysics, magnetic fields are essentially classical, i.e., we are essentially dealing with Maxwell equations. We will discuss why such “weak” magnetic fields can be considered strong, and how such fields play an important role in various astrophysics contexts, ranging from stars and star formation, to disks and outflows, and to stellar mergers.
2 Atomic and Molecular Physics

When studying matter in magnetic fields, the natural (atomic) unit for the field strength, \( B_0 \), is set by equating the electron cyclotron energy \( \hbar \omega_{ce} \) to the characteristic atomic energy \( e^2/a_0 = 2 \times 13.6 \text{ eV} \) (where \( a_0 \) is the Bohr radius), or equivalently by \( \hat{R} = a_0 \), where \( \hat{R} = (\hbar c/eB)^{1/2} \) is the cyclotron radius of the electron. Thus it is convenient to define a dimensionless magnetic field strength \( b \) via

\[
b \equiv \frac{B}{B_0}; \quad B_0 = \frac{m_e^2 e^3 c}{\hbar^2} = 2.3505 \times 10^9 \text{ G}.
\]  

For \( b \gg 1 \), the cyclotron energy \( \hbar \omega_{ce} \) is much larger than the typical Coulomb energy, so that the properties of atoms, molecules and condensed matter are qualitatively changed by the magnetic field. In such a strong field regime, the usual perturbative treatment of the magnetic effects (e.g., Zeeman splitting of atomic energy levels) does not apply. Instead, the Coulomb forces act as a perturbation to the magnetic forces, and the electrons in an atom settle into the ground Landau level. Because of the extreme confinement (\( \hat{R} \ll a_0 \)) of the electrons in the transverse direction (perpendicular to the field), the Coulomb force becomes much more effective in binding the electrons along the magnetic field direction. The atom attains a cylindrical structure. Moreover, it is possible for these elongated atoms to form molecular chains by covalent bonding along the field direction. Interactions between the linear chains can then lead to the formation of three-dimensional condensates (see Lai 2001; Harding and Lai 2006 for review).

(i) Atoms: For \( b \gg 1 \), the H atom is elongated and squeezed, with the transverse size (perpendicular to \( B \)) \( \sim \hat{R} = a_0/b^{1/2} \ll a_0 \) and the longitudinal size \( \sim a_0/(\ln b) \). Thus the ground-state binding energy \( |E| \simeq 0.16 \ln(b)^2 \) (au) (where 1 au = 27.2 eV; the factor 0.16 is an approximate number based on numerical calculations). Thus \( |E| = 160540 \text{ eV} \) at \( B = 10^{12}, 10^{14} \text{ G} \) respectively. In the ground state, the guiding center of the electron’s gyro-motion coincides with the proton. The excited states of the atom can be obtained by displacing the guiding center away from the proton; this corresponds to \( \hat{R} \rightarrow R_s = (2s + 1)^{1/2} \hat{R} \) (where \( s = 0, 1, 2, \ldots \)). Thus \( E_s \sim -0.16(\ln[(b/(2s + 1))]^2 \) (au).

We can imagine constructing a multi-electron atom (with \( Z \) electrons) by placing electrons at the lowest available energy levels of a hydrogenic ion. The lowest levels to be filled are the tightly bound states with \( v = 0 \) (zero node in the wavefunction along the field direction). When \( a_0/Z \gg \sqrt{2Z - 1} \hat{R} \), i.e., \( b \gg 2Z^3 \), all electrons settle into the tightly bound levels with \( s = 0, 1, 2, \ldots, Z - 1 \). Reliable values for the energy of a multi-electron atom for \( b \gg 1 \) can be calculated using the Hartree-Fock method or density functional theory, which takes into account the electron-electron direct and exchange interactions in a self-consistent manner.

(ii) Molecules and Chains: In a strong magnetic field, the mechanism of forming molecules is quite different from the zero-field case. The spins of the electrons in the atoms are aligned anti-parallel to the magnetic field, and thus two atoms in their ground states do not bind together according to the exclusion principle. Instead, one H atom has to be excited to the \( s = 1 \) state before combining (by covalent bond) with another atom in the \( s = 0 \) state. Since the “activation energy” for exciting an electron in the H atom from \( s \) to \( (s + 1) \) is small, the resulting H\(_2\) molecule is stable. Moreover, in strong magnetic fields, stable H\(_3\), H\(_4\) etc. can be formed in the similar manner. The dissociation energy of the molecule is much greater than the \( B = 0 \) value: e.g., for H\(_2\), it is 40350 eV at \( 10^{12}, 10^{14} \text{ G} \) respectively. A highly magnetized molecule exhibits excitation levels much different from a \( B = 0 \) molecule.
Neutron Star Atmospheres and Radiation: An important area of research where the atomic physics in strong magnetic fields plays an important role is the study of neutron star (NS) atmospheres and their radiation (see Potekhin et al. 2014 for more details). Thermal, surface emission from isolated NSs can potentially provide invaluable information on the physical properties and evolution of NS (equation of state at super-nuclear densities, superfluidity, cooling history, magnetic field, surface composition, different NS populations). In recent years, considerable observational resources (e.g., Chandra and XMM-Newton) have been devoted to such study. For example, the spectra of a number of radio pulsars (e.g., PSR B1055-52, B0656+14, Geminga and Vela) have been observed to possess thermal components that can be attributed to emission from NS surfaces and/or heated polar caps. Phase-resolved spectroscopic observations are becoming possible, revealing the surface magnetic field geometry and emission radius of the pulsar. A number of compact sources in supernova remnants have been observed, with spectra consistent with thermal emission from NSs, and useful constraints on NS cooling physics have been obtained. Surface X-ray emission has also been detected from a number of SGRs and AXPs. Fits to the quiescent magnetar spectra with blackbody or with crude atmosphere models indicate that the thermal X-rays can be attributed to magnetar surface emission at temperatures of \((3–7) \times 10^6\) K. One of the intriguing puzzles is the absence of spectral features (such as ion cyclotron line around 1 keV for typical magnetar field strengths) in the observed thermal spectra. Clearly, detailed observational and theoretical studies of surface emission can potentially reveal much about the physical conditions and the nature of magnetars.

Of particular interest are the seven isolated, radio-quiet NSs (so-called “dim isolated NSs”; see van Kerkwijk and Kaplan 2007; Haberl 2007). These NSs share the common property that their spectra appear to be entirely thermal, indicating that the emission arises directly from the NS atmospheres, uncontaminated by magnetospheric processes. Thus they offer the best hope for inferring the precise values of the temperature, surface gravity, gravitational redshift and magnetic field strength. The true nature of these sources, however, is unclear at present: they could be young cooling NSs, or NSs kept hot by accretion from the ISM, or magnetars and their descendants. Given their interest, these isolated NSs have been intensively studied by deep Chandra and XMM-Newton observations. While the brightest of these, RX J1856.5-3754, has a featureless spectrum remarkably well described by a blackbody, absorption lines/features at \(E \simeq 0.2–2\) keV have been detected in six other sources, The identifications of these features, however, remain uncertain, with suggestions ranging from cyclotron lines to atomic transitions of H, He or mid-Z atoms in a strong magnetic field (see Ho and Lai 2004; Ho et al. 2008; Potekhin et al. 2014). Another puzzle concerns the optical emission: For four sources, optical counterparts have been identified, but the optical flux is larger (by a factor of 4–10) than the extrapolation from the black-body fit to the X-ray spectrum. Clearly, a proper understanding/interpretation of these objects requires detailed NS atmosphere modeling which includes careful treatments of atomic physics in strong magnetic fields.

3 Condensed Matter Physics

Several aspects of condensed matter physics in strong magnetic fields play an important role in neutron star astrophysics.

(i) Cohesive Property of Condensed Matter: Continuing our discussion of atoms/molecules in strong magnetic fields, as we add more atoms to a H molecular chain, the energy per atom in a \(H_n\) molecule saturates, becoming independent of \(n\). We then have a 1D
metal. Chain-chain interactions then lead to 3D condensed matter. The binding energy of magnetized condensed matter at zero pressure can be estimated using the uniform electron gas model. Balancing the electron kinetic (zero-point) energy and the Coulomb energy in a Wigner-Seitz cell (containing one nucleus and Z electrons), we find that the energy per unit cell is of order \( E \sim -Z^{9/5}b^{2/5} \). The radius of the cell is \( R \sim Z^{1/5}b^{-2/5} \), corresponding to the zero-pressure density \( \rho \sim 10^3AZ^{3/5}B_1^{6/5} \) g cm\(^{-3} \) (where \( A \) is the mass number of the ion).

Although the simple uniform electron gas model and its Thomas-Fermi type extensions give a reasonable estimate for the binding energy for the condensed state, they are not adequate for determining the cohesive property of the condensed matter. In principle, a three-dimensional electronic band structure calculation is needed to solve this problem. The binding energies of 1D chain for some elements have been obtained using Hartree-Fock method (Neuhauser et al. 1987; Lai et al. 1992). Density functional theory has also been used to calculate the structure of linear chains in strong magnetic fields (Jones 1986; Medin and Lai 2006a, 2006b). Numerical calculations carried out so far indicate that for \( B_{12} = 1 - 10 \), linear chains are unbound for large atomic numbers \( Z \gtrsim 6 \). In particular, the Fe chain is unbound relative to the Fe atom; therefore, the chain-chain interaction must play a crucial role in determining whether the 3D zero-pressure Fe condensed matter is bound or not. However, for a sufficiently large \( B \), when \( a_0/Z \gg \sqrt{2Z + 1}R \), or \( B_{12} \gg 100(Z/26)^3 \), we expect the Fe chain to be bound in a manner similar to the H chain or He chain (Medin and Lai 2006a, 2006b). The cohesive property of magnetized condensed matter is important for understanding the physical condition of the “polar gap” and particle acceleration in pulsars (Medin and Lai 2007).

(ii) Phase Diagram and Equation of State: Given the energies of different bound states of a certain element, one can determine the phase diagram as a function of the field strength \( B \) and temperature. This is relevant to the outmost layer of neutron stars (NSs). For a given \( B \), there is a critical temperature below which the phase separation will occur, and the NS surface may be in a condensed state, with negligible gas above it. Some isolated NSs with low surface temperatures may be in such a state (see van Adelsberg et al. 2005; Medin and Lai 2007).

Beyond zero-pressure density, the Coulomb interaction can be neglected, and the effects of magnetic field on the equation of state of matter depend on \( B \), \( \rho \) and \( T \). We can define a critical “magnetic density”, below which only the ground Landau level is populated (at \( T = 0 \)), given by

\[
\rho_B = 0.802Y_e^{-1}b^{3/2} \text{ g cm}^{-3} = 7.04 \times 10^3Y_e^{-1}B_1^{3/2} \text{ g cm}^{-3},
\]

where \( Y_e = Z/A \) is the number of electrons per baryon. We can also define critical “magnetic temperature”,

\[
T_B \simeq \frac{\hbar\omega_{ce}}{k_B} \left( \frac{m_e}{m_e^*} \right) = 1.34 \times 10^8 B_1(1 + x_F^2)^{-1/2} \text{ K},
\]

where \( m_e^* = \sqrt{m_e^2 + (p_F/e)^2} = m_e\sqrt{1 + x_F^2} \). There are three regimes characterizing the effects of Landau quantization on the thermodynamic properties of the electron gas:

(a) \( \rho \lesssim \rho_B \) and \( T \lesssim T_B \): In this regime, the electrons populate mostly the ground Landau level, and the magnetic field modifies essentially all the properties of the gas. The field is sometimes termed “strongly quantizing”. For example, for degenerate, nonrelativistic electrons (\( \rho < \rho_B \) and \( T \ll T_F \ll m_e^* c^2/k_B \), where \( T_F \) is the Fermi temperature), the pressure is \( P_e = (2/3)n_e E_F \propto B^{-2}\rho^3 \). This should be compared with the \( B = 0 \) expression \( P_e \propto \rho^{5/3} \). Note that for nondegenerate electrons (\( T \gg T_F \)), the classical ideal gas equation of state, \( P_e = n_e k_B T \), still holds in this “strongly quantizing” regime.
(b) $\rho \gtrsim \rho_B$ and $T \lesssim T_B$: In this regime, the electrons are degenerate, and populate many Landau levels but the level spacing exceeds $k_B T$. The magnetic field is termed “weakly quantizing”. The bulk properties of the gas (e.g., pressure and chemical potential) are only slightly affected by such magnetic fields. However, the quantities determined by thermal electrons near the Fermi surface show large oscillatory features as a function of density or magnetic field strength. These de Haas–van Alphen type oscillations arise as successive Landau levels are occupied with increasing density (or decreasing magnetic field). With increasing $T$, the oscillations become weaker because of the thermal broadening of the Landau levels; when $T \gtrsim T_B$, the oscillations are entirely smeared out, and the field-free results are recovered.

(c) $T \gtrsim T_B$ or $\rho \gg \rho_B$: In this regime, many Landau levels are populated and the thermal widths of the Landau levels ($\sim k_B T$) are higher than the level spacing. The magnetic field is termed “non-quantizing” and does not affect the thermodynamic properties of the gas.

(iii) Transport Properties: A strong magnetic field can significantly affect the transport properties and thermal structure of a neutron star crust. Even in the regime where the magnetic quantization effects are small ($\rho \gg \rho_B$), the magnetic field can still greatly modify the transport coefficients (e.g., electric conductivity and heat conductivity). This occurs when the effective gyro-frequency of the electron, $\omega_{ce}^* = eB/(m^*_e c)$, where $m^*_e = \sqrt{m^2_e + (p_F/c)^2}$, is much larger than the electron collision frequency $1/\tau_0$. When $\omega_{ce}^* \tau_0 \gg 1$, the electron heat conductivity perpendicular to the magnetic field, $\kappa_\perp$, is suppressed by a factor $(\omega_{ce}^* \tau_0)^{-2}$. In this classical regime, the heat conductivity along the field, $\kappa_\parallel$, is the same as the $B = 0$ value. In a quantizing magnetic field, the conductivity exhibits oscillatory behavior of the de Haas–van Alphen type. On average, the longitudinal conductivity is enhanced relative to the $B = 0$ value due to quantization. The most detailed calculations of the electron transport coefficients of magnetized neutron star envelopes are due to Potekhin (1999), where earlier references can be found (see Potekhin et al. 2014 for more details).

The thermal structure of a magnetized neutron star envelope has been studied by many authors (see Potekhin et al. 2014 for review). In general, a normal magnetic field reduces the thermal insulation as a result of the (on average) increased $\kappa_\parallel$ due to Landau quantization of electron motion, while a tangential magnetic field (parallel to the stellar surface) increases the thermal insulation of the envelope because the Lamor rotation of the electron significantly reduces the transverse thermal conductivity $\kappa_\perp$. A consequence of the anisotropic heat transport is that for a given internal temperature of the neutron star, the surface temperature is nonuniform, with the magnetic poles hotter and the magnetic equator cooler (see, e.g., Shabaltas and Lai 2012 for a recent application).

4 High Energy Physics: QED in Strong Magnetic Fields

In superstrong magnetic fields, a number of quantum-electrodynamic (QED) processes are important. A well-known one is single-photon pair production, $\gamma \rightarrow e^+ + e^-$. This process is forbidden at zero-field, but is allowed for $B \neq 0$, and is one of the dominant channels for pair cascade in pulsar magnetospheres (Sturrock 1971; Medin and Lai 2010). Another process is photon splitting, $\gamma \rightarrow \gamma + \gamma$, which can attain appreciable probability for sufficiently strong fields. The critical QED field strength is set by $\hbar \omega_{ce} = m_e c^2$, i.e.,

$$B_Q = \frac{m_e^2 c^3}{e\hbar} = 4.414 \times 10^{13} \text{ G.}$$

Above $B_Q$, many of these QED effects become important.
A somewhat surprising strong-field QED effect is vacuum polarization, which makes even an “empty” space birefringent for photons propagating through it. This can significantly affect radiative transfer in neutron star atmospheres and the observed spectral and x-ray polarization signals even for modest field strengths. We discuss this issue below.

The magnetized plasma of a NS atmosphere is birefringent. An X-ray photon, with energy $E \ll E_{ce} = \hbar \omega_e = 1.16 B_{14} \text{ MeV}$ (where $B_{14} = B/(10^{14} \text{ G})$), propagating in such a plasma can be in one of the two polarization modes: The ordinary mode (O-mode) has its electric field $E$ oriented along the $B$-$k$ plane ($k$ is along direction of propagation), while the extraordinary mode (X-mode) has its $E$ perpendicular to the $B$-$k$ plane. Since charge particles cannot move freely across the magnetic field, the X-mode photon opacity (e.g., due to free-free absorption or electron scattering) is suppressed compared to the zero-field value, $\kappa_X \sim (E/E_{Be})^2 \kappa(B=0)$, while the O-mode opacity is largely unchanged, $\kappa_O \sim \kappa(B=0)$ (e.g., Meszaros 1992). Vacuum polarization can change this picture in an essential way. In the presence of a strong magnetic field, vacuum itself becomes birefringent due to virtual $e^+e^-$ pairs. Thus in a magnetized NS atmosphere, both the plasma and vacuum polarization contribute to the dielectric tensor of the medium. The vacuum polarization contribution is of order $10^{-4} (B/B_Q)^2 f(B)$ (where $B_Q = m_e^2 c^3/e \hbar = 4.414 \times 10^{13} \text{ G}$, and $f \sim 1$ is a slowly varying function of $B$), and is quite small unless $B \gg B_Q$. However, even for “modest” field strengths, vacuum polarization can have a dramatic effect through a “vacuum resonance” phenomenon. This resonance arises when the effects of vacuum polarization and plasma on the polarization of the photon modes “compensate” each other. For a photon of energy $E$ (in keV), the vacuum resonance occurs at the density $\rho_V \simeq 0.964 Y_e^{-1} B_{14}^2 E^2 f^{-2} \text{ g cm}^{-3}$, where $Y_e$ is the electron fraction (Lai and Ho 2002). Note that $\rho_V$ lies in the range of the typical densities of a NS atmosphere. For $\rho \gtrsim \rho_V$ (where the plasma effect dominates the dielectric tensor) and $\rho \lesssim \rho_V$ (where vacuum polarization dominates), the photon modes are almost linearly polarized—they are the usual O-mode and X-mode described above; at $\rho = \rho_V$, however, both modes become circularly polarized as a result of the “cancellation” of the plasma and vacuum polarization effects. When a photon propagates outward in the NS atmosphere, its polarization state will evolve adiabatically if the plasma density variation is sufficiently gentle. Thus the photon can convert from one mode into another as it traverses the vacuum resonance. The conversion probability $P_{\text{conv}}$ depends mainly on $E$ and atmosphere density gradient; for a typical atmosphere density scale height ($\sim 1 \text{ cm}$), adiabatic mode conversion requires $E \gtrsim 1–2 \text{ keV}$ (Lai and Ho 2003a). Because the O-mode and X-mode have vastly different opacities, the vacuum polarization-induced mode conversion can significantly affect radiative transfer in magnetar atmospheres. In particular, the effect tends to deplete the high-energy tail of the thermal spectrum (making it closer to blackbody) and reduce the width of the ion cyclotron line or other spectral lines (Ho and Lai 2003, 2004; Lai and Ho 2003a; van Adelsberg and Lai 2006). It is tempting to suggest that the absence of lines in the observed thermal spectra of several AXPs is a consequence of the vacuum polarization effect at work in these systems.

We also note that even for “ordinary” NSs (with $B \sim 10^{12–10^{13}} \text{ G}$), vacuum resonance has a profound effect on the polarization signals of the surface emission; this may provide a direct probe of strong-field QED in the regime inaccessible at terrestrial laboratories (Lai and Ho 2003b; Wang and Lai 2009; see Lai 2010 for a review). Such polarization signals will be of interest for future X-ray polarimetry detectors/missions.

Finally, magnetic fields can modify neutrino processes that take place in neutron stars. For example, in proto-neutron stars with sufficiently strong B-fields, the neutrino cross sections and emission rates, as well as their angular dependences, can be affected, and these can
contribute to the natal velocity kick imparted to the neutron star (e.g., Arras and Lai 1999a, 1999b; Maruyama et al. 2014).

5 “Classical” Astrophysics

For most areas of astrophysics, magnetic fields are “classical”. That is, we are dealing with Maxwell’s equations, MHD and classical plasma physics. The quantization, microscopic effects discussed previous sections are not relevant. Nevertheless, these classical magnetic field effects are important, interesting and rich. We will highlight some of these in the following.

5.1 Clouds, Stars and Compact Objects

The first effect of “classical” magnetic fields is that they can influence the equilibrium of bound objects via the so-called magnetic Virial theorem. For a spherical cloud or star of mass $M$ and mean radius $R$, static equilibrium requires that the ratio of the magnetic energy and gravitational energy be less than unity, i.e.,

$$\frac{E_{\text{mag}}}{E_{\text{grav}}} \sim \frac{B_{\text{in}}^2 R^3}{6GM^2/R} \sim \frac{1}{6\pi^2 G} \left( \frac{\Phi}{M} \right)^2 \lesssim 1,$$

(5)

where the second equality assumes that the dominant internal magnetic field takes form of a large-scale poloidal field, and $\Phi = \pi R^2 B_{\text{in}}$ is the magnetic flux threading the cloud.

In the context of star formation, clouds (cores) with $E_{\text{mag}}/E_{\text{grav}} \gtrsim 1$ cannot collapse on a dynamical timescale, but require ambipolar diffusion to eliminate the magnetic flux. This process is perhaps relevant for the formation of low-mass stars (e.g., Shu et al. 1999), although in recent years the roles of turbulence in the molecular clouds have been recognized (McKee and Ostriker 2007).

For neutron stars (with $M \simeq 1.4M_\odot$ and $R \simeq 10$ km), equation (5) implies $B_{\text{in}} \lesssim 10^{18}$ Gauss. This is the maximum field strength achievable in all astrophysical objects.

What do we know observationally about magnetic fields of isolated neutron stars? For radio pulsars, the dipole magnetic fields are inferred indirectly from the measured $P$ and $\dot{P}$ (rotation period and period derivative), and the assumption that the spindown is due to magnetic dipole radiation. For most “regular” pulsars, the magnetic fields thus derived lie in the range of $10^{12-13}$ G. A smaller population, so-called “millisecond pulsars”, have fields in the range of $10^{8-9}$ G. How such a “weak” field evolves from the regular field of $10^{12-13}$ G remains unclear (see Payne and Melatos 2004). In recent years, a number of “High-B Radio Pulsars” have also been found: these have $B \sim 10^{14}$ G, comparable to magnetars.

Magnetars are neutron stars powered by energy dissipation of magnetic fields. They usually have dipole fields (as inferred from $P$ and $\dot{P}$ based on x-ray timing) of $B \gtrsim 10^{14}$ G. Interestingly, a number of low-field ($\sim 10^{13}$ G) magnetars have also been found recently (Rea et al. 2010), although the internal fields could be higher. Indeed, there is growing evidence that there exist hidden magnetic fields inside neutron stars. This is the case for the neutron star in Kes 79 SNR: It has a dipole field of $3 \times 10^{10}$ G, but the internal field buried inside its crust could be larger than $10^{14}$ G, based on its observed large x-ray pulse fraction of 60 % (Halpern and Gotthelf 2010; Shabaltas and Lai 2012; Viganò et al. 2013). In the case of SGR 0418+5729, the dipole field is less than a few times $10^{12}$ G, but internal field could be much stronger (Turolla et al. 2011).

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Another way to assess whether a magnetic field is “strong” is to look at the energetics. For magnetars, even in quiescence, the x-ray luminosity is \( L \sim 10^{34–36} \text{ erg s}^{-1} \), much larger than the spindown luminosity \( I \Omega \dot{\Omega} \). The giant flares of the three SGRs indicate that a much larger internal field is possible. For example, the December 2004 flare of SGR 1806-20 has a total energy of \( 10^{46} \text{ erg} \), suggesting an internal field of at least a few times \( 10^{14} \text{ G} \).

What is the origin of such strong magnetic fields? It is intriguing to note that (Reisenegger 2013) for upper main-sequence stars (radius \( 10^{6.5} \text{ km} \)), white dwarfs (\( 10^4 \text{ km} \)) and neutron stars (10 km), the maximum observed magnetic fields (\( 10^{4.5} \text{ G}, 10^9 \text{ G} \) and \( 10^{15} \text{ G} \) respectively) all correspond to similar maximum magnetic flux \( \Phi_{\text{max}} = \pi R^2 B_{\text{max}} \sim 10^{17.5–18} \text{ G km}^2 \). This seems to suggest a fossil origin of the strongest magnetic fields. However, recent observations of magnetic white dwarfs (and their populations in binaries) indicate the strong magnetic fields (\( \gtrsim \) a few MG) of white dwarfs originate from binary mergers (Wickramasinghe et al. 2014). So perhaps the strongest magnetic fields found in magnetars is the result of dynamo action in the proto-neutron star phase (Thompson and Duncan 1993). In any case, since \( E_{\text{mag}}/E_{\text{grav}} \lesssim 10^{-6} \) (assuming no significant hidden magnetic fields), these magnetic fields have a negligible effect on the global static equilibrium of the star.

5.2 Stellar Envelopes and “Outside”

Although astrophysically observed magnetic fields have a negligible effect on the global equilibrium of a star, they can strongly influence the local “static” equilibrium of stellar envelopes. A notable example is neutron star (NS) crust. Because of the evolution of crustal magnetic fields due to a combination of Hall drift and Ohmic diffusion, the NS crust can break (e.g. Pons and Perna 2011). This occurs when \( B^2/(8\pi) \gtrsim \mu \theta_{\text{max}} \) (where \( \mu \) is the shear modulus and \( \theta_{\text{max}} \) is the maximum strain of the crust), or \( B \gtrsim 2 \times 10^{14}(\theta_{\text{max}}/10^{-3})^{1/2} \text{ G} \). The consequences of the crustal breaking (and its manifestations such as magnetar flares) are not clear. They depend on whether the breaking is fast or slow. The energy release and whether the energy can get out of the NS are also uncertain (see Link 2014; Beloborodov and Levin 2014).

Of course, outside the star, even a “weak” magnetic field can be quite “strong” and dominates the dynamics of the flow. Such magnetically dominated region is relevant to the magnetic braking of stars. In the case of radio pulsars, the electrodynamics and physical processes in the magnetosphere are ultimately responsible for most of the observed phenomena of pulsars. In recent years, there has been significant progress in ab initio calculations of pulsar magnetospheres (e.g. Tchekhovskoy et al. 2013), although it remains unclear whether the current theoretical approach can adequately explain some of the enigmatic pulsar phenomena (such as mode-switching in radiation; e.g. Hermen et al. 2013). The magnetospheres of magnetars have also been studied: Unlike radio pulsars, the closed field line regions play an important role (e.g. Thompson et al. 2002; Beloborodov 2013).

Finally, further away from pulsars, we have pulsar wind nebulae, where pulsar wind impinges upon a supernova remnant, creating a broad spectrum of non-thermal radiation (from radio to gamma rays). The ultimate source of this radiation is the pulsar’s rotational energy, and magnetic field plays an important role in making such a “transfer of energy” possible (e.g. Amato 2014).

5.3 Accretion Disks

Magnetic fields play a number of important roles in accretion disks. First, we have magnetically dominated disks. These occur when \( B^2/(8\pi) \gtrsim \rho v_k^2/2 \), where \( v_k \) is the Keplerian