The Unreal Life of Oscar Zariski

Oscar Zariski, 1960 (courtesy of Yole Zariski)

"Geometry is the real life"

—Oscar Zariski

Carol Parikh

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Preface

Oscar Zariski transformed the foundations of algebraic geometry. The powerful tools he forged from the ideas of modern algebra allowed him to penetrate classical problems with a clarity and depth that brought a new rigor to the way algebraic geometers carry out proofs. The strength of his work was matched by his forcefulness as a teacher, and the students he trained at Johns Hopkins and later at Harvard have made essential contributions to many areas of mathematics.

A man who called geometry "the real life," Zariski lived intensely in the world of mathematics, and it was here that his temperament had its most free expression. Curious, optimistic, arrogant, stubborn, demanding, he was in some ways the embodiment of intellectual romance—the boy genius marked out for greatness by his teachers, the idealist torn between his love for Russia and his devotion to mathematics, the student who surpassed his masters, a precursor of the great influx of European talent that would transform academic and artistic life in America.

Neither a prodigy like Gauss nor the victim of an early death like Galois, he saw himself as having chosen mathematics. Aware from an early age of his mathematical talent, he only later discovered how much his character had contributed to the development of his gifts. As his boyhood interest in algebra ripened into love, his pragmatism drew him to geometry; the tenacity with which he attacked fundamental problems was already evident in the intensity of his early studies at the University of Kiev. "A faithful man," as he termed himself, he remained totally committed to algebraic geometry for more than sixty years.

His commitment led him safely through the turbulence of the twentieth century. Having left Kobrin to attend the gymnasium in Chernigov as a child in 1910, he went on leaving places for the sake of mathematics until he settled at Harvard in 1947. He was an undergraduate in Kiev during the 1917 revolution, a graduate student in Rome during Mussolini's rise to power, an assistant professor in Baltimore during the Depression, and a visiting professor at the University of São Paulo in 1945 when he learned that his family in Kobrin had been murdered by the Nazis. In his eighty-seven years he contributed to the radical transformation not only of algebraic geometry, but also of what it meant to be a Jew, a communist, and a university professor.

While his "real life" is recorded in almost a hundred books and papers, this story of his "unreal life" is based upon his memories and the recollections of his family, colleagues, and students. Whenever it was possible I supplemented oral accounts with letters and journals. I have used outside sources only to provide a historical context and to resolve the inevitable inconsistencies of a remembered past.

Carol Parikh

Acknowledgments

This book grew out of the loving efforts of Oscar Zariski's family and students to create a record of his life. Because the Zariski Archives in Widener Library are sealed until 2001, my account is based almost entirely on my interviews with those who knew him and on his own memories as they were preserved in a series of tape-recorded interviews made a few years before his death.

I would like, first of all, to thank Yole Zariski, Zariski's wife of more than sixty years, and his daughter, Vera DeCola, for having so generously shared their recollections with me, and for joining two of Zariski's students, Heisuke Hironaka and David Mumford, in asking me to write this biography. I would also like to acknowledge my indebtedness to Ann Kostant who, with the help of Heisuke Hironaka, Wakaiko Hironaka, and David Mumford, conducted the series of interviews with Oscar Zariski between 1979 and 1981 that form an important part of this book.

David Mumford's help with all aspects of this project, especially the sections on algebraic geometry, was invaluable, as were the corrections and suggestions of Michael Artin and André Weil. George Mackey's knowledge of the history of the Department of Mathematics at Harvard and Ray Zariski's understanding of the intricacies of the Russian Revolution added other important dimensions to my account. I would also like to thank all the friends and colleagues of Zariski who spoke to me at length and who almost all gave their permission to be quoted in this book, and to express my indebtedness to Erna Alfors, Karin Tate, Elizabeth Walsh, and Vera Widder for their lively descriptions of the Harvard community in the forties, fifties, and sixties.

Zariski's remarkable success as a teacher was evident in the enthusiasm with which his students and protégés contributed material for this book. I am grateful to Daniel Gorenstein and Maxwell Rosenlicht for their memories of Zariski's first years at Harvard; to Shreeram Abhyankar and Jun-ichi Igusa, who provided letters and stories about the middle fifties; to Heisuke Hironaka for his autobiographical writings; and to Peter Falb and Robin Hartshorne, who seem to have forgotten nothing. The accounts and letters provided by Zariski's last students, Steven L. Kleiman and Joseph Lipman, and by his French protégé, Bernard Teissier, were indispensable to my description of the final years of his research career.

Although the book is living history, there were a great many facts to be checked and rechecked, and I was always readily assisted by the archivists at Columbia, Harvard, Johns Hopkins, and MIT. I would also like to thank Everett Pitcher at the American Mathematical Society, and to express my gratitude for the editorial comments of Judith B. Herman and Eliza Wyatt, for the many practical suggestions of Barbara Solomon, and for the support and helpfulness of Klaus Peters, Susan Gay, and Camille Pecoul at Academic Press.

I would, finally, like to acknowledge my debt to my husband, Rohit Parikh, whose experience as a research mathematician and former student in the Harvard Mathematics Department contributed an important dimension to this book, and to thank my children, Vikram and Uma, for occasionally lending me their perspectives.

A Foreword for Non-Mathematicians

When I first met Oscar Zariski, I was a lowly and invisible undergraduate, and he was a commanding figure preaching about a seductive world of which he was the master. Later I came to know him as a colleague, and as I gained confidence in my own strength as a mathematician, I could look at his work and see him as a fellow human being, struggling to shape half-glimpsed truths into tangible reality. Through the years he became a close friend and, as he declined physically, a friend in need of support in facing the losses that all people eventually face. It is very exciting for me to see how Carol Parikh has been able to bring to life the full development of Zariski as a person, from his youthful dreams, through his eager days as a student, to the central period in which he doubted his own teachers and found how to correct them and penetrate further into his beloved field of geometry.

I hope that this book will make the mathematical endeavor itself clearer to those readers who have always wondered what on earth mathematicians do. Zariski was a man caught up in many of the central conflicts of the twentieth century. He was torn between his early dedication to communism and his later, more sober, reflections on the success of capitalism. He was torn between an allegiance to an intellectual world that ignored the politics of race and his emotional need to find safety for those members of his family who escaped the Holocaust. Intellectually, he was torn between a love of the free-spirited, creative Italian vision of geometry and his appreciation of the need for strict logical rigor which he found in the Bauhaus-like school of the abstract German algebraists.

Unfortunately, like all working mathematicians, I have led my life with the realization that most of what I care about so passionately is nearly impossible to explain to the educated layman. "What do you mean," they say, "when you say this theorem is *beautiful* or that theorem is *deep*?" One cannot appreciate what drove Zariski and why his colleagues were so excited by his contributions without having some idea of the intellectual world in which he moved. Is it possible within the confines of this foreword to convey some idea of this world and why it is so vital for the dedicated group of mathematicians who pursue it? I won't try to explain all the terms needed to state Zariski's deepest theorems, but I think something of what draws people to his subject can actually be explained in two fairly easy illustrations.

Before I embark, I have to make one thing clear about the way mathematicians think about their world. Everyone knows that physicists are concerned with the laws of the universe and have the audacity sometimes to think they have discovered the choices God made when He created the universe in thus and such a pattern. Mathematicians are even more audacious. What they feel they discover are the laws that God Himself could not avoid having to follow. Now some would say all such laws must be obvious, that you can find nothing truly new beyond what you assumed in the beginning. But this isn't what mathematicians find. They find that by following the thread of logic, just as you would follow a river to its source, at every bend you find things that are totally unexpected. Because these things follow by logic, they have to be true in any world God creates, and yet there is no way in which they are evident on first sight. Or at least so it seems until some mathematician finds a way of rephrasing or recasting the facts; then, by some sleight of hand, they appear immediately evident. That's one of the things mathematicians mean by a beautiful proof. Yet other theorems continue to fascinate mathematicians because they have never been fully reduced to something intuitively obvious. Such theorems live on in a state of tension between seeming new and surprising and seeming clear and evident.

To be a mathematician is to be an out-and-out Platonist. The more you study mathematical constructions, the more you come to believe in their objective and prior existence. Mathematicians view themselves as explorers of a unique sort, explorers who seek to discover not just one accidental world into which they happen to be born, but the universal and unalterable truths of all worlds.

My first illustration will attempt to show in the simplest possible way how algebra and geometry come together in the field Oscar Zariski made his own, algebraic geometry. We want to go back to what was perhaps the first and arguably still one of the deepest mathematical truths—Pythagoras's theorem. We start with a right triangle *A, B, C* (see Figure 1), with a right angle at *B*, the side *AC* being the longest, the so-called hypotenuse. Pythagoras's theorem states that the square of the length *AC* equals the sum of the square of the length *AB* and the square of the length *BC*.

We're not going to prove this theorem; rather, we'll use it to build a fundamental link between algebra and geometry. To do this, we first need to use an idea of Descartes: we can name points in the plane by means of pairs of numbers, called their *x* and *y* coordinates. That is to say, to each point, we can assign two numbers,

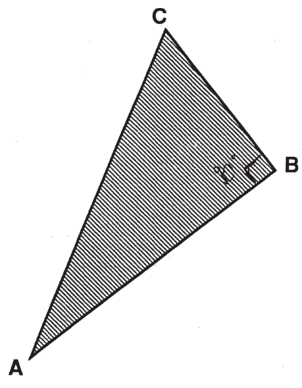

Figure 1. A right triangle

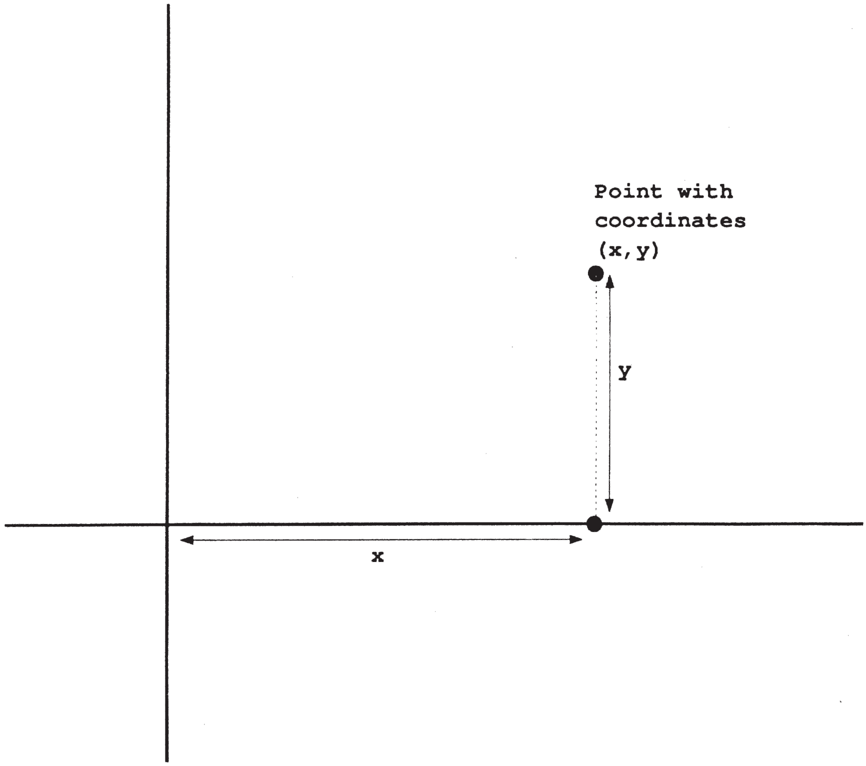

Figure 2. Cartesian coordinates

and conversely to any two numbers, we assign a single point (see Figure 2). This idea, although commonplace to anyone who has taken high school math, was an amazing step for Descartes; it was a step that the Greeks never took. In fact, the Greeks had terrible techniques for doing simple arithmetic, and they would never have thought of the reduction of geometry to arithmetic by means of coordinates as any sort of simplification (which was perhaps why they didn't think of it).

Now take Pythagoras's triangle and put point *A* at the origin of Descartes' coordinates and make side *AB* horizontal. This makes side *BC* vertical. Also let x be the length of *AB* and let y be the length of *BC*. Then we see that the pair of numbers x,y is simply Descartes's coordinates for the point *C* (see Figure 3). Finally, consider the circle whose center is the origin and whose radius is one. If *C* lies on that circle, then the length of *AC* is one, and Pythagoras's theorem tells us that the sum of the square of x and the square of y is one:

$$x^2 + y^2 = 1.$$

On the other hand, if *C* doesn't lie on that circle, then $x^2 + y^2$ is the square of some other number, less than one or greater than one, so $x^2 + y^2$ does not equal one. In other words, we have shown that the set of solutions of the equation

$$x^2 + y^2 = 1$$

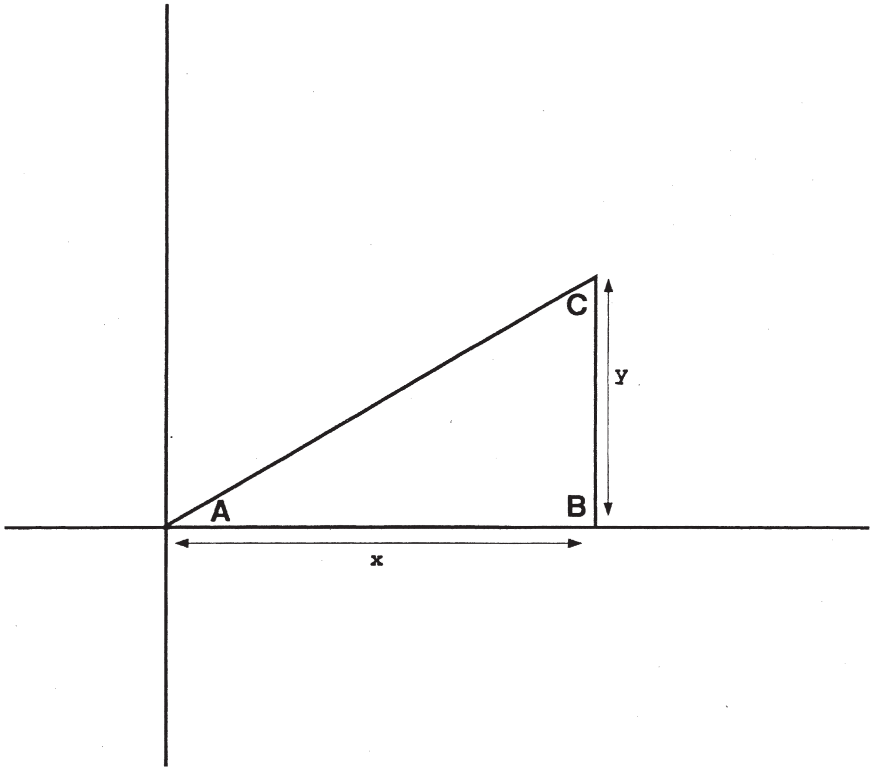

Figure 3. Pythagoras' theorem in Cartesian coordinates

is the same as the set of coordinates (x,y) of the points on our circle! We have an equation, and a simple one at that, for the most basic object of geometry. We have reduced the circle, one of the great building blocks of geometry, to a polynomial $x^2 + y^2$.

This idea, of taking equations of any kind and plotting their set of solutions using Cartesian coordinates, is the secret to the link between algebra and geometry, and the origin of algebraic geometry. What happens with other equations? We can take any equation made up by adding, subtracting, and multiplying x and y and ordinary numbers and out of it get a curve, which is called an algebraic curve. The curve is the set of points whose coordinates x,y solve the equation. In Figure 4, we have drawn three such curves to give you an idea what can happen. Clearly the algebra can produce a whole lot of geometry.

What sort of rules apply to this dictionary between equations and curves? We need some terminology. The equations are built by adding and multiplying the coordinates x,y by various numbers and by each other, and we call x and y the "variables" in the equation because they can be given any value. Some rules are easy: for instance, if the equation is linear (it doesn't multiply variables by each other, but only adds them

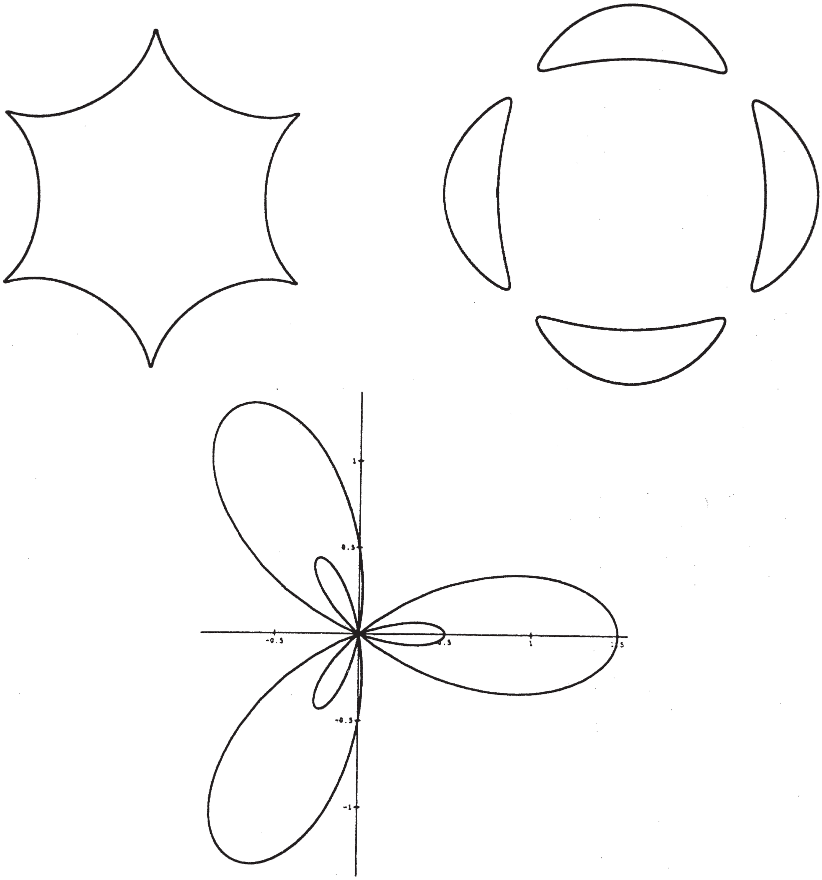

Figure 4. Some algebraic curves

up after multiplying them by known numbers), then the curve is a straight line. If the equation is quadratic, meaning that each side is a sum of pieces in which at most two variables are multiplied (i.e., x^2, xy, or y^2), then we get a circle or a stretched circle, called an ellipse, or a few other simple types (see Figure 5). Newton was the first to make a systematic study and to classify the curves obtained from cubic equations.

Now, here's our second illustration of the way mathematics works. We ask a simple question: If we start with two algebraic curves, is there a rule for predicting how many points they have in common, that is, how large is their intersection? Well, two lines always meet in exactly one point—unless they are parallel, a special case that we shall leave aside for the moment. A line and a circle can meet in two points, or in one point if they are tangent, or in no points if the line doesn't go near the circle at all (see Figure 6). Looks like a mess!

But here we can adopt another strategy that mathematicians love and that often leads to great surprises: if you find a mess in the world you start in, why not change

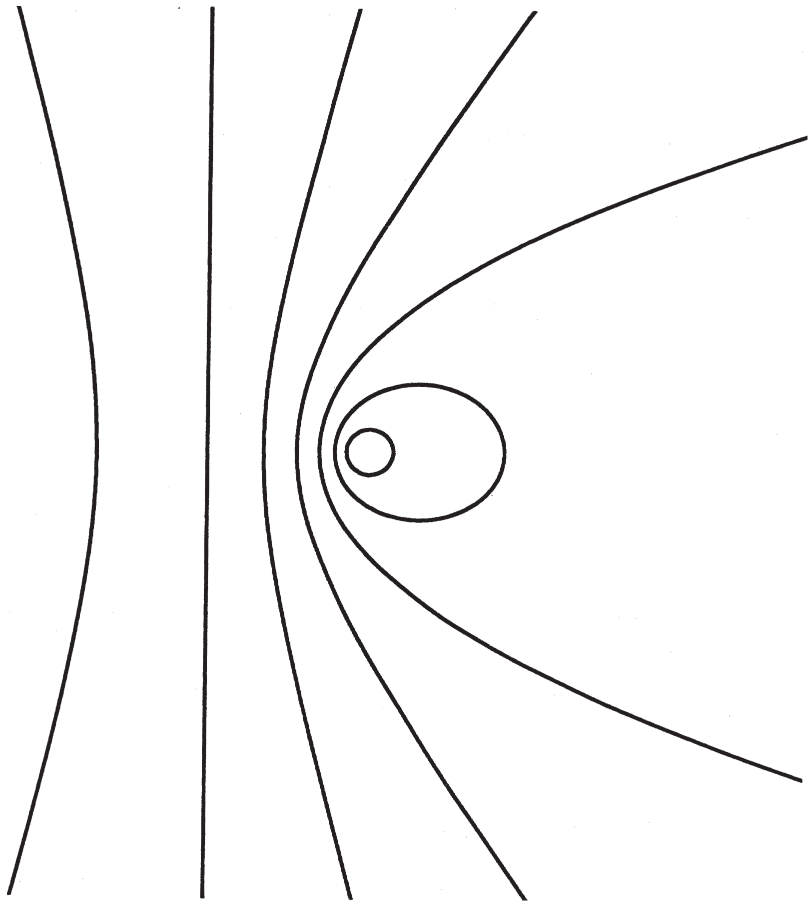

Figure 5. Some curves defined by quadratic equations

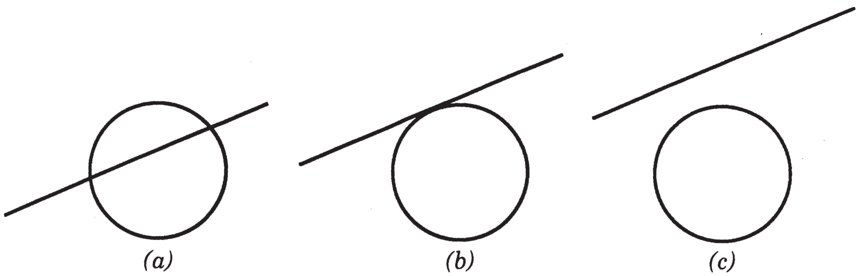

Figure 6. (a) A line and a circle that meet twice; (b) A line and a circle that meet once and are tangent; (c) A line and a circle that never meet

the world? Invent a new and better world, a castle in the sky, in which you can make your theorem come true; looking back at the dreary reality with which you started, maybe you can understand your more complex reality as a departure from this simpler picture. If you plunge ahead like this, now really pretending to be God, one of two things happens. You may find that the reality you want contains the seeds of its own self-destruction: it leads to a contradiction. Or you may find it holding up, and if you are lucky, you eventually prove that it is consistent. In either case, you have understood the original situation more *deeply*.

For the case of the parallel lines, this leap of faith, this audacious idea of altering the rules of the game, was one of the great inventions of the Renaissance, when it was declared that *parallel lines meet at infinity!* Painters realized that, in order to accurately draft rectangular buildings, they should draw the horizon on their canvasses, even where it was obscured behind nearer objects. Then the parallel lines would be drawn correctly if, when extended, they met on the horizon. Mathematicians realized that these points on the horizon depicted places that didn't literally exist in the real world because they would have to be infinitely far away. But why not say they do exist somewhere? Increase the stock of points in the plane by adding new ones, which we then call ideal or infinite points. Don't treat them as second-class citizens either, because on the canvas they appear just like real points, and the canvas can be treated as a kind of map showing points at and near infinity all at once. The new points are where the train tracks meet, where the lines of Leonardo's drawing intersect (see Figure 7). We come up with a richer geometry, in which there is more elbow room. In fact, if you go out on a line in one direction, you actually reach infinity, pass it, and then re-enter the finite world from infinity but now at the other end of the line. This way of thinking is called "projective geometry."

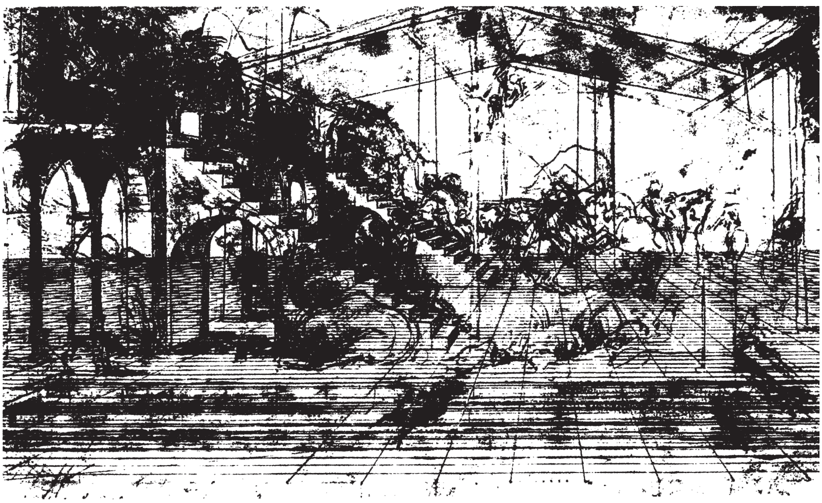

Figure 7. Leonardo da Vinci: Perspective study

Now how about the circle and the line? Ignore for a while the case where the circle is tangent to the line, as it is a special case. The two basic cases are where they meet twice and where they don't meet at all. We don't want to lose any points, so we are forced to add points again until a line totally outside a circle still "meets" it somewhere. Here is where some old ideas that originated in the Middle Ages come to our help: the square root of −1, called *i*, and the complex numbers built up from it, e.g., $2 + 3i$ or $-4.5 - 5i$. It had been known for a long time that solving polynomial equations seemed to work better if you allowed complex numbers in, either as the solutions themselves or, even if you only wanted the usual real roots, as intermediate steps in calculating the solutions. Such numbers had had an air of mystery and black magic about them at first, but gradually it was realized that there was nothing inconsistent about them; if you suspended your disbelief and admitted them for the sake of the game, you didn't reach any contradiction. A beautiful way of describing them by points in the plane, due to C. F. Gauss, the founder of the modern era of mathematics, made it totally clear that they were a perfectly consistent rigorous construction.

So where are the missing points, for example, where the line $x = 1.25$ and the circle $x^2 + y^2 = 1$ meet? One of them is the point $x = 1.25$, $y = 0.75i$, and the other is $x = 1.25$, $y = -0.75i$. (Just square 1.25 getting 1.5625, and square $.75i$ getting $-.5625$, which add up to 1.0.) With a little algebra, it's easy to see that this always works, so long as we let the coordinates x,y of the points in the plane be complex numbers. But what has this technique done to our geometry? In fact, it has made it much richer. Although we continue to treat it like a two-dimensional world, to specify a point requires two coordinates, and each of them, being complex, needs to have a real and an imaginary part (thus $2 + 3i$ has real part 2 and imaginary part 3). This means that we need in all four numbers of the ordinary sort to specify a point, so our geometry has now become four-dimensional. Moreover, we still have to add the line of points at infinity, including complex points at infinity. For instance, a circle, which in the ordinary sense doesn't go out to infinity at all, now can do so, provided the direction in which it goes has imaginary slope (the points at infinity on circles used to be called *I* and *J* and were nicknamed Isaac and Jacob by students in the college days of my colleague Lars Ahlfors). The whole affair is called the *complex projective plane* and is the place in which to "draw" algebraic curves and to do algebraic geometry.

To complete our story, what have we gained by these mental gymnastics? In fact, we have gained a tremendous amount, but to tell the story is to tell a large part of algebraic geometry. For this foreword, I'll only tell about Bezout's theorem—actually a theorem of Poncelet, I believe, but mathematicians are notorious for crediting things rather arbitrarily. Remember that any polynomial equation in x and y defines its curve of solutions. The degree of the equation is simply the largest number of times the variables are ever multiplied together (so 2 is the degree of $x^2 + y^2 = 1$, and 5 is the degree of $x^3 \bullet y^2 = -1$). Bezout's theorem states that two such curves, of degrees n and m, meet almost always in $n \bullet m$ points, and always in $n \bullet m$ points if special points of intersection, like a point where a line is tangent to a circle, are counted more than once in a careful way. (Finding techniques for counting these special points was, by the way, one of the principle technical accomplishments of Zariski's archrival Weil, *see* Ch. 12.) In other words, we have found a strong general link between the algebra of the polynomials on the one hand and the geometry of the curves on the other. Such links, many quite amazing on first sight, are the main concerns of algebraic geometry.

I want to touch on one more thing in this quick tour of the mathematician's world. The lay picture of the mathematician (as seen in *New Yorker* cartoons) shows a bespectacled, white-coated, rather unworldly man looking at a blackboard of bizarre equations. This man is probably dry and precise, following rules without fail; his failing to do so is cause for humor (see Figure 8). As discussed below, much of Zariski's life was devoted to seeking the right way to make precise a huge

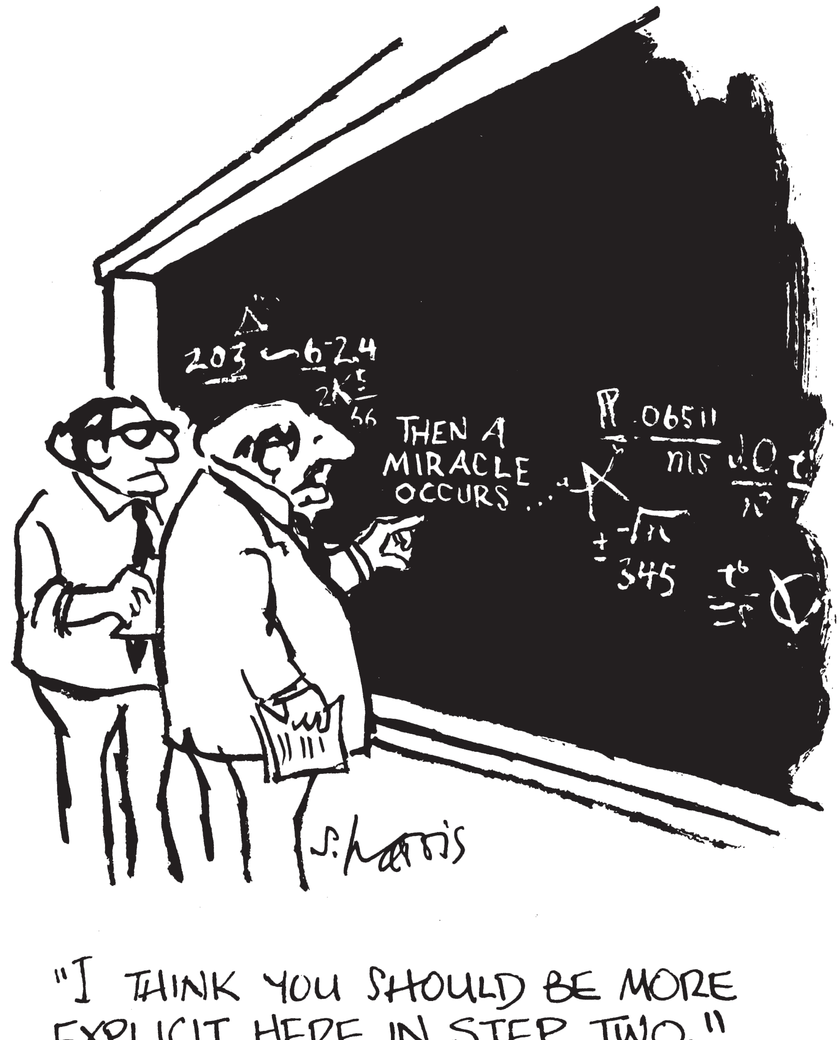

Figure 8. © 1975 by Sidney Harris, American Scientist *magazine*

amount of writing and thinking produced by other mathematicians who were anything but precise. In fact, one of them was an out-and-out romantic and another a dictatorial dramatic man with a flair for wild driving. Let the truth be known: mathematicians are as subject to human error and emotion, as subject to the fashions of intellectual trends, and as often personifications of their national characteristics, as thinkers in any other field. They do strive, or claim, to be better and more detached, but their history reveals marvelous episodes in which they have driven right off the road in pursuit of their particular vision of truth.

This book deals with one of the most colorful episodes of this type. The Italian school of algebraic geometry was created in the late 19th century by a half dozen geniuses who were hugely gifted and who thought deeply and nearly always correctly about their field. They extended its ideas over a huge new area, especially what is called the theory of algebraic surfaces (we were discussing algebraic curves; surfaces come from equations in three variables, $x, y,$ and z, instead of two). But they found the geometric ideas much more seductive than the formal details of proofs, especially when these proofs had to cover all the nasty special cases that so often crop up in geometry. So, in the twenties and thirties, they began to go astray. It was Zariski and, at about the same time, Weil who set about to tame their intuition, to find the principles and techniques that could truly express the geometry while embodying the rigor without which mathematics eventually must degenerate to fantasy.

The 20th century was, until its final decades, an era of “modern mathematics” in a sense quite parallel to “modern art” or “modern architecture” or “modern music.” That is to say, it turned to an analysis of abstraction, it glorified purity and tried to simplify its results until the roots of each idea were manifest. These trends started in the work of Hilbert in Germany, were greatly extended in France by a secret mathematical club known as “Bourbaki,” and found fertile soil in Texas, in the topological school of R. L. Moore. Eventually, they conquered essentially the entire world of mathematics, even trying to breach the walls of high school in the disastrous episode of the “new math.” Now the trend has reversed: postmodern mathematics is quite different and has reintroduced the love of the baroque; it embraces the tool of the computer and seeks out rather than shunning the complexities of applications. The theory of chaos is the best-known example of this trend, but it extends from the vast number–theoretic speculations on modular forms to the paradoxically flat yet knotted “non-standard” four-dimensional spaces. Zariski’s life is the story of a mathematician of this century, who lived with and loved and gave his soul to these struggles. He began his career with naive beliefs inherited from the nineteenth century; the middle part of his career was wholly devoted to “modern mathematics”; and in the last part, he began to look again at the richness and complexities of his material. But this is the story Carol Parikh has told so ably in the book that follows.

David Mumford