

# **SLURRY TRANSPORT USING CENTRIFUGAL PUMPS**

**Third Edition**

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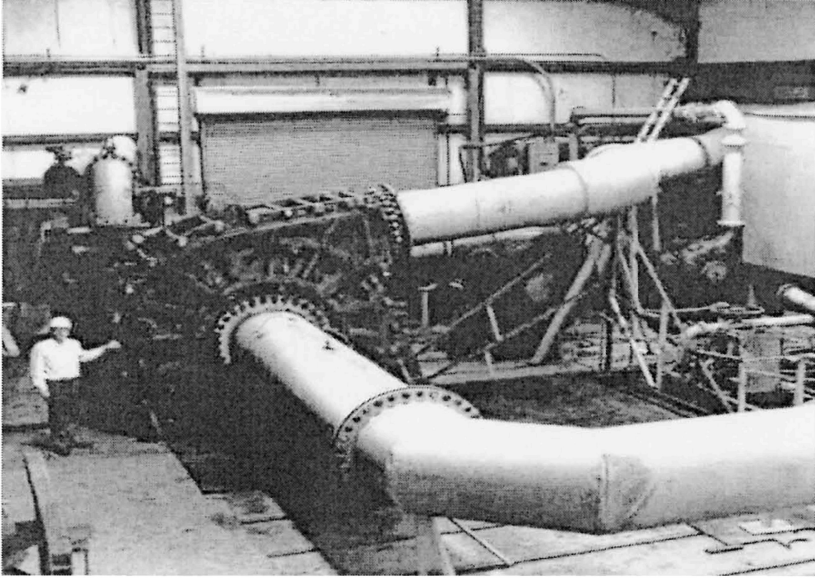
# Chapter 1

## INTRODUCTION

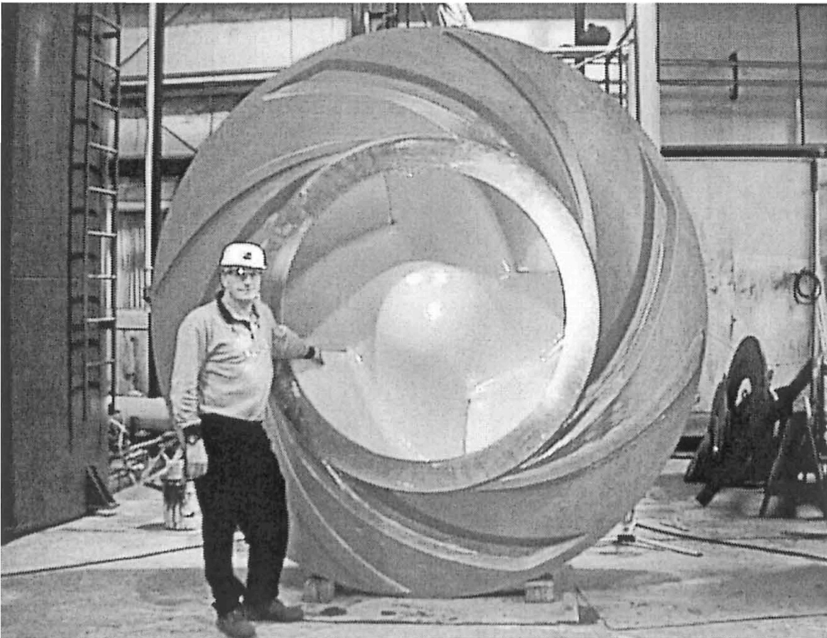
### 1.1 Applications of Slurry Transport

Vast tonnages are pumped every year in the form of solid-liquid mixtures, known as slurries. The application which involves the largest quantities is the dredging industry, continually maintaining navigation in harbours and rivers, altering coastlines and winning material for landfill and construction purposes. As a single dredge may be required to maintain a throughput of 7000 tonnes of slurry per hour or more, very large centrifugal pumps are used. Figures 1.1 and 1.2 show, respectively, an exterior view of this type of pump, and a view of a large dredge-pump impeller (Addie & Helmley, 1989).

The manufacture of fertiliser is another process involving massive slurry-transport operations. In Florida, phosphate matrix is recovered by huge draglines in open-pit mining operations. It is then slurried, and pumped to the wash plants through pipelines with a typical length of about 10 kilometres. Each year some 34 million tonnes of matrix are transported in this manner. This industry employs centrifugal pumps that are generally smaller than those used in large dredges, but impeller diameters up to 1.4 m are common, and drive capacity is often in excess of 1000 kW. The transport distance is typically longer than for dredging applications, and



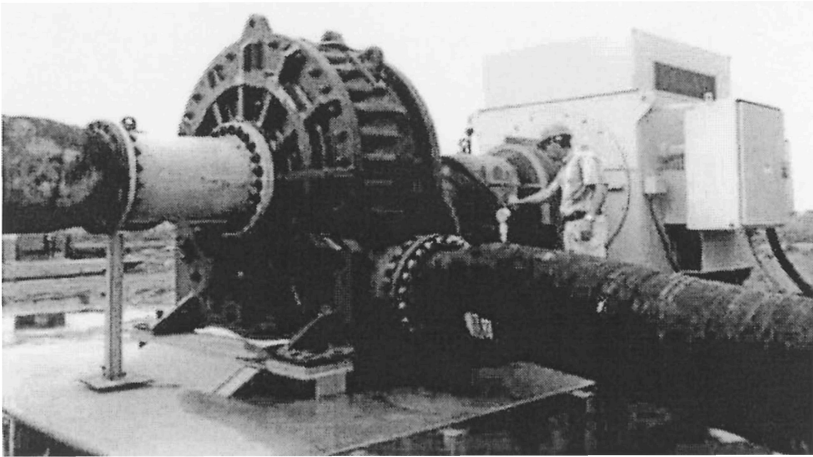
*Figure 1.1.* Testing a dredge pump at the GIW Hydraulic Laboratory



*Figure 1.2.* Impeller for large dredge pump

hence a series of pumping stations is often used. Figure 1-3 shows a booster-pump installation in a phosphate pipeline.

Many other types of open-pit mining use slurry transport, and the number of such applications is increasing as it becomes clear that, for many short-haul and medium-haul applications, slurry transport is more cost-effective than transport by truck or conveyor belt. Partially-processed material from mining and metallurgical operations and other industries is often already in slurry form, facilitating pipe transport. Much of this is carried out using relatively small lines. For example newly-mixed concrete is a slurry and is sometimes piped from the mixing plant to other parts of a construction site.



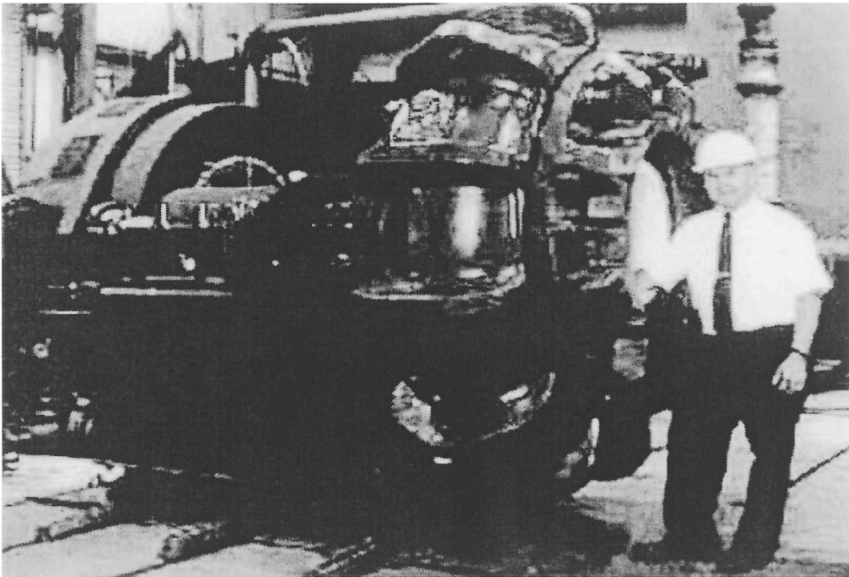
*Figure 1.3.* Booster pump in a phosphate-matrix pipeline

As this slurry has a high resistance to flow, and as considerable static lifts are also common in construction, the pumps employed are usually of the positive-displacement type. Such pumps are also used in other high-head applications; and the best-known of these is the Black Mesa pipeline which transports a partially-processed coal slurry from the mine to a electrical generating station more than 400 km distant. This line, which involves heads at each pumping section of 75 atmospheres or more, began operation in 1970. Since that time many similar, but larger, coal pipelines have been proposed, but none has yet been built.

Recent decades have seen a great increase in the transport of waste materials, in slurry form, to suitable deposit sites. The concept is an old one and the earliest known use was by Hercules, who removed a decade's accumulation of animal droppings from King Augeas' stables by diverting two rivers to form an open-channel slurry transport system. Waste-disposal

problems are even more severe at present, and maintenance of the environment requires that wastes be conveyed to dedicated and monitored disposal sites, either underground or on the surface. This requirement can often be satisfied by backfilling mines (either deep or open-pit), and slurry transport is the favoured placing method. Such backfilling operations are usually characterised by significant tonnages with short to medium hauls, and hence centrifugal pumps are the natural choice for these applications.

Large backfilling operations are found in Alberta, where petroleum is extracted from oil sands which are obtained by open-pit mining operations. After the extraction operations are completed, the sand (actually a mixture of sand, fine particles and some residual organics) is used as backfill in areas previously mined. An example of a high-pressure pump in an oil-sands tailings pipeline (Addie *et al.*, 1995) is shown on Figure 1.4.



*Figure 1.4.* High-pressure pump for a tar-sands tailings pipeline

## 1.2 Topics Covered in this Book

Much of the material presented in this book has been developed in connection with a short course presented annually since the early 1970's under the joint sponsorship of GIW Industries and Augusta State University. Three of the authors of the book have been associated with the course since

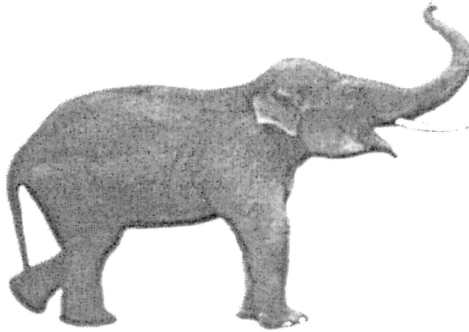
its inception, and the fourth has been involved for many years. The course, and thus this book, also draws from the work of many other contributors, and only the major contributions could be recognized in the Acknowledgements.

Over the years the course has laid stress on the need for experimentation, and the students have always participated in slurry testing at the GIW Hydraulic Laboratory. The data obtained from the experiments carried out for the course are at state-of-the-art level and have often been included in papers presented at international conferences or published in technical journals (from Clift *et al.*, 1982; to Whitlock *et al.*, 2004). Points arising during the course also led to other research (for example Clift & Clift, 1981; Sellgren & Addie, 1989; Wilson, 2004). Such results were incorporated into subsequent offerings of the course, and also in the successive editions of the present text.

The book falls into two basic sections. The first half, up to the end of Chapter 8, is concerned primarily with various sorts of slurry behaviour including scale-up from one pipe size to another and prediction of particle deposition and pressure gradients. The second portion of the book is concerned with the behaviour of centrifugal pumps handling slurries, and with how pumps and pipelines interact as a system.

Within the first half of the book, Chapter 2 leads into the main topics by providing a review of the basic principles of fluid mechanics. This is needed not only as a ready reference source for points on which some readers may wish to brush up, but also to establish a common basis for readers of different backgrounds, e.g., Civil, Mechanical and Chemical engineers. (A common list of symbols is also required, of course. This appears at the end of Chapter 1). Chapter 2 also deals with the settling of solid particles in liquids, which forms a necessary background for slurry flow analysis. As mentioned above, engineers dealing with slurry transport come from various backgrounds. These reflect not only educational discipline, but also different experiences with design or operation of slurry transport systems. The effect of this wide difference in backgrounds has been noted since the earliest offerings of the short course. In fact, although the technical content of the course has advanced considerably over the years, one point has remained constant, and that is the invocation of the old tale of the blind men and the elephant (Saxe, 1872). In this fable six blind men, who had always wanted to know what elephants were like, were finally allowed to spend a few minutes in contact with one of these beasts. Since each man had touched a different part of the animal, they later found themselves in greater disagreement than before, comparing the elephant to everything from a house (based on its side) to a rope (based on its tail); see Figure 1.5.

In order to avoid the mistakes of the blind men it is necessary to elucidate the different aspects of slurry flow. Chapter 3 deals with homogeneous flow of non-Newtonian slurries. It considers both laminar and turbulent flow, mentioning various rheological models and presenting the laminar scaling laws based on the Rabinowitsch-Mooney transform. Equivalent affinity laws are derived for turbulent flow. The effects of solids concentration are outlined, and the laminar-turbulent transition is investigated. This chapter closes with a case study which demonstrates many of the points discussed.



*Figure 1.5.* The elephant

Examination of the various sorts of slurry flow of settling particles begins in Chapter 4, where the basic distinction between pseudo-homogeneous flow and settling-slurry flow is introduced, and the underlying physical mechanisms are outlined. Significant aspects of the literature are also reviewed here, to provide background for the following four chapters, which present recent work on the analysis of slurry flow.

Chapters 5 and 6 deal with settling slurries in horizontal pipes. In line with the analysis of Chapter 4, the distinction is made between particles supported by granular contact and those supported by the fluid, i.e. contact load and suspended load. Chapter 5 is concerned with contact load, which is analysed by setting up a force balance. The pressure gradient provides the driving force which tends to move the solids, while the resisting force, which tends to hold the particles in place as a deposit, is associated with the submerged weight of the contact-load particles. The force-balance analysis is used to determine the velocity at the limit of stationary deposition (a condition that must be known so that deposition can be avoided). A nomographic chart based on the analysis provides a convenient method of estimating the deposition point, and this is complemented by equations

which show the effect of solids concentration and other variables (a variable not included here is pipe slope; it is considered later, in Chapter 8). The force balance of Chapter 5 is also applied to cases where all the particles are in motion, giving rise to a method of calculating pressure drop for coarse-particle transport. This method is illustrated by a case study of large clay balls moving through the pipeline from a dredge.

The heterogeneous flow dealt with in Chapter 6 involves both contact load and suspended load. The fraction of contact load, indicated by the stratification ratio, is found to be very important in determining hydraulic gradients and specific energy consumption for flows of this type. Using a method introduced previously, it is shown how the stratification ratio can be obtained from experimental tests of slurry behaviour and scaled to prototype pipelines. If results of flow tests are not available for the slurry of interest, the parameters in the scaling relations can be estimated from fluid and particle properties, on the basis of recent developments in the understanding of particle support mechanisms. Case studies demonstrate the application of the methods developed in this chapter to pipeline flow of slurries of sand and coal.

Chapter 7 deals with the flow of complex slurries. It first presents an analysis of slurries with very broad particle grading, and then considers the commercially-important case of coarse particles travelling in a non-Newtonian carrier medium, and develops scaling relations for cases of this type.

Chapter 8 considers slurry flow in pipes which are not horizontal. The simplest case to analyse is vertical flow, which is employed in a U-tube instrument for determining solids concentration, and also has direct applications, particularly in the mining industry. In addition to vertical flow, the flow in inclined pipes is covered in this chapter, which includes sections pertaining to the effect of inclination angle on deposition limit and on friction loss. These effects can be very important in inclined suction pipes, as shown in the case study of a suction dredge.

The second half of the book begins by shifting the focus from the slurry to other components of the pipeline transport system, beginning in Chapter 9 with the performance and testing of centrifugal pumps. This chapter discusses characteristic curves, affinity laws, specific speed and suction performance. The basics of hydraulic and mechanical design are considered, indicating some of the differences between slurry pumps and water pumps. Finally, performance testing is discussed, including layout and instrumentation of test loops, and test procedures for determining pump characteristics for total dynamic head, efficiency, and net positive suction head.

Chapter 10 continues the study of pumps by describing the way in which the presence of solid particles influences the head produced by a pump, and its efficiency. These effects tend to become more severe for higher solids concentrations and for larger or denser particles, indicating the basic similarity between the influence of solids on pump performance and on pipeline friction. Scaling equations for pumps show how the solids effect diminishes with increasing machine size. All of these relations are brought together in a generalised solids-effect diagram, and illustrated by worked examples.

Many of the mechanisms by which the solids decrease pump performance also act to promote erosive wear, a subject which is considered in Chapter 11. This chapter introduces the two major wear mechanisms of sliding abrasion and particle impact, showing how they are influenced by features of fluid and particle motion which were considered in previous chapters. From wear mechanisms the discussion turns to wear-resistant materials (hard metals, elastomers and ceramics) and then to experimental methods for wear testing. The numerical modelling of flow and wear patterns is dealt with next, introducing computer techniques and evaluating some of their strengths and weaknesses. The results of practical experience with slurry system design and operation are then presented, and particle attrition is also considered.

Chapter 12 introduces other components of slurry transport systems. The configuration of suction piping merits considerable attention, since difficulties with cavitation can usually be eliminated by careful design of this component. Pump-house layout and spacing are also considered, along with pump drive trains. Accurate instrumentation is required to provide the information needed for system control, and thus the instruments used for monitoring slurry flow are considered in some detail.

Chapter 13 shows how the pump and pipeline characteristics interact. It considers the general form of the characteristics, and the question of the stability of their intercept, which defines system operability, and thus determines pump selection and other aspects of system design. Both the pump characteristic and that for the pipeline are affected by particle size and concentration, and it is shown how these quantities affect system operability, and also how temporal variation of the characteristics of slurry entering the system can cause operating difficulties.

Chapter 14 deals with pump selection and cost considerations, bringing together materials, wear and capital and maintenance costs in terms of the concepts of Life-Cycle Cost and Total Cost of Ownership. Chapter 15 uses the knowledge developed in the previous chapters to deal with practical problems in the design and operation of slurry systems, including both series



and parallel pumping configurations. A case-study approach is used in dealing with these problems, together with others involving difficulties with pump suction, start-up transients, and water hammer.

Chapter 16 discusses the environmental aspects of slurry systems. There is a brief introduction to the role of hydraulic transport in mining and mineral processing systems: integration of operations from mine to mill output; comparison of hydraulic conveying with belt conveying and trucks, in terms of energy consumption and other environmental impacts. Current emphasis on tailings management as a way of reducing the environmental impact of mining and mineral processing emphasizes the importance of hydraulic transport at high concentrations and forms the conclusion of the book.

A number of worked examples are included in the text. These require only a hand calculator. The case studies are presented both in SI units and in traditional U.S. units. The SI versions are located in the text, and the versions in U.S. Customary Units are gathered together in the Appendix.

### 1.3 List of Symbols

The principal symbols used in this book are listed here, together with brief definitions. The basic units of each quantity in terms of mass [M] length [L] and time [T] are given in square brackets. Dimensionless quantities are indicated by [-].

$a$	celerity of transient disturbance [ $LT^{-1}$ ]
$A'$	coefficient in Eq. 4.3 [-]. The case where $A' = 1.0$ is the equivalent-fluid model
$A_p$	maximum projected area of a particle, see Eq. 2.66 [ $L^2$ ]
$b$	breadth between pump shrouds [L]
$B$	coefficient in Eq. 4.6, dependent on flow stratification [-]
$c_m$	absolute fluid velocity in meridional direction [ $LT^{-1}$ ]
$c_r$	absolute fluid velocity in radial direction [ $LT^{-1}$ ]
$c_t$	absolute fluid velocity in tangential direction [ $LT^{-1}$ ]
$C_D$	particle drag coefficient, see Eq. 2.50 [-]
$C_r$	relative volumetric concentration of solids, $C_{vd}/C_{vb}$ [-]
$C_{rm}$	value of $C_r$ at which deposition velocity is a maximum [-]
$C_v$	volumetric concentration (volume fraction) of solids [-]
$C_{vb}$	volumetric solids concentration in loose-poured bed [-]
$C_{vd}$	delivered volumetric solids concentration, see Section 2.4 [-]
$C_{vi}$	<i>in situ</i> volumetric solids concentration, see Section 2.4 [-]
$C_w$	solids concentration by weight (or mass) [-]

$d$	particle diameter [L]
$d^*$	dimensionless particle diameter, see Eq. 2.57 [-]
$d_a$	area-equivalent particle diameter, see Eq. 2.66 [L]
$d_{50}$	mass-median particle diameter [L]
$d_{85}$	diameter for which 85% (by mass) of the particles are finer [L]
$D$	diameter (internal diameter of a pipe, diameter of a pump impeller) [L]
$D_s$	pump suction diameter [L]
$E$	voltage [V]
$E_i$	energy coefficient for specific impact wear ( $J/m^3$ )
$E_{sp}$	energy coefficient for specific wear ( $J/m^3$ )
$f$	Moody friction factor, see Eq. 2.29 [-]
$f_f$	value of $f$ for equal volumetric flow of fluid [-]
$f_w$	value of $f$ for equal volumetric flow of water [-]
$f_{12}$	value of $f$ at stratified-flow interface [-]
$fn()$	a function
$F_D$	drag force [ $MLT^{-2}$ ]
$F_L$	lift force [ $MLT^{-2}$ ]
$F_N$	normal intergranular force against pipe wall [ $MLT^{-2}$ ]
$F_W$	immersed weight force [ $MLT^{-2}$ ]
$g$	gravitational acceleration [ $LT^{-2}$ ]
$G$	coefficient in Eq. 3.18
$h$	height [L]
$h_a$	atmospheric head (in height of liquid being pumped) [L]
$h_v$	vapour-pressure head [L]
$H$	head, see Eq. 2.9 [L]
$H_A, H_B$	head at locations, A, B. [L]
$H_i$	theoretical head, see Eq. 9.10 [L]
$H_r$	head ratio, see Section 10.1 [-]
$H_s$	suction head [L]
$i$	hydraulic gradient [height of water/length of pipe] see Eq. 2.44
$i_f$	value of $i$ for equal volumetric flow of fluid [height of water/length of pipe]
$i_m$	value of $i$ for flow of mixture [height of water/length of pipe]
$i_{mh}$	value of $i$ for flow of homogeneous mixture [height of water/length of pipe]
$i_{pg}$	hydraulic gradient for plug flow, see Eq. 5.8 [height of water/length of pipe]
$\bar{i}_v$	average hydraulic gradient in ascending and descending vertical flows [height of water/length of pipe]

$i_w$	value of $i$ for equal volumetric flow of water [height of water/length of pipe]
$i_w'$	hydraulic gradient in return water line [height of water/length of pipe]
$I$	electric current [A]
$j_m$	friction gradient in height of mixture/length of pipe, see Eq. 2.45
$k$	coefficient in power-law rheological model
$K$	volumetric shape factor of particles (Eq. 2.67) [-] <i>or</i> McElvain's, head-reduction coefficient (Eq. 10.3) [-]
$L$	length, measured along pipe [L]
$m$	exponent in Eq. 3.18 [-]
$M$	exponent in stratification-ratio equation [-]
$n$	angular speed in revolutions per second [ $T^{-1}$ ] <i>or</i> Richardson-Zaki exponent, see Eq. 2.69 [-] <i>or</i> exponent in power-law rheological model [-]
$n'$	Rabinowitsch-Mooney exponent [-]
$n_s$	dimensionless specific speed, see Eq. 9.14 [-]
$N$	angular speed in revolutions per minute
$N_s$	specific speed in customary units, see Eq. 9.15
NPSH	net positive suction head, see Eq. 9.16 [L]
$p$	pressure [ $ML^{-1}T^{-2}$ ]
$p_A, p_B$	pressure at locations A, B. [ $ML^{-1}T^{-2}$ ]
$p_v$	vapour pressure [ $ML^{-1}T^{-2}$ ]
$P$	power [ $ML^2T^{-3}$ ]
$Q$	volumetric flow rate [ $L^3T^{-1}$ ]
$Q_{BEP}$	flow rate at best efficiency point [ $L^3T^{-1}$ ]
$Q_f$	volumetric flow rate of fluid [ $L^3T^{-1}$ ]
$Q_m$	volumetric flow rate of mixture [ $L^3T^{-1}$ ]
$Q_s$	volumetric flow rate of solids [ $L^3T^{-1}$ ]
$r$	radius [L]
$R_H$	head reduction factor, see Eq. 10.3 [-]
$R_{T3}$	representative shell radius, see Fig. 11.4 [L]
$R\eta$	efficiency reduction factor, see Eq. 10.3 [-]
$Re$	Reynolds number $VD/\nu$ [-]
$Re_*$	shear Reynolds number $U_*D/\nu$ [-]
$Re_p$	particle Reynolds number $v_t d/\nu$ [-]
$S_f$	relative density of fluid [-]
$S_m$	relative density of mixture [-]
$S_{md}$	delivered relative density of mixture [-]
$S_{mi}$	<i>in situ</i> relative density of mixture [-]
$S_s$	relative density of solids [-]

SDEC	specific delivered energy consumption, i.e. electrical energy drawn from the grid (kWh/tonne-mile)
SEC	specific energy consumption, see Eq. 4.9 (kWh/tonne-km)
t	time [T]
T	torque [ $ML^2T^{-2}$ ]
TDH	total dynamic head, see Eq. 2.10 [L]
u	local velocity or velocity of moving part [ $LT^{-1}$ ]
$u_{\max}$	maximum local velocity (at pipe centreline) [ $LT^{-1}$ ]
$u_1$	reference value of local velocity [ $LT^{-1}$ ]
$u^+$	dimensionless velocity $U/U^*$ [-]
U	velocity [ $LT^{-1}$ ]
$U^*$	shear velocity, $\sqrt{(\tau_o/\rho)}$ [ $LT^{-1}$ ]
$U_{*f}$	shear velocity for equivalent flow of fluid [ $LT^{-1}$ ]
$U_1$	mean velocity in upper portion of stratified flow
$U_2$	mean velocity in lower portion of stratified flow
$v_t$	terminal settling velocity of a single particle [ $LT^{-1}$ ]
$v_t'$	hindered settling velocity [ $LT^{-1}$ ]
$v_t^*$	dimensionless settling velocity [-]
$v_{ts}$	settling velocity of spherical particle [ $LT^{-1}$ ]
V	mean velocity, $4Q/(\pi D^2)$ [ $LT^{-1}$ ]
$V_A, V_B$	values of V at locations A, B [ $LT^{-1}$ ]
$V_f$	mean velocity of fluid [ $LT^{-1}$ ]
$V_m$	mean velocity of mixture [ $LT^{-1}$ ]
$V_N$	value of $V_m$ for equivalent Newtonian flow [ $LT^{-1}$ ]
$V_r$	relative velocity $V_m/V_{sm}$ [-]
$V_s$	value of $V_m$ at limit of deposition [ $LT^{-1}$ ]
$V_{sm}$	maximum value of $V_s$ [ $LT^{-1}$ ]
$V_T$	value of $V_m$ at laminar-turbulent transition [ $LT^{-1}$ ]
$V_{50}$	value of $V_m$ at which 50% of solids are suspended by fluid [ $LT^{-1}$ ]
w	particle-associated velocity, see Eq. 6.2 [ $LT^{-1}$ ] <i>or</i> velocity of fluid relative to impeller vane [ $LT^{-1}$ ] $w_{50}$ value of particle-associated velocity for $d_{50}$ [ $LT^{-1}$ ]
$w_{85}$	value of particle associated velocity for $d_{85}$ [ $LT^{-1}$ ]
$W_c$	wear coefficient, inverse of $E_{sp}$ ( $m^3/J$ )
x	distance, usually measured along pipe [L]
$X_f$	fraction of fine solids, see Section 7.2
$X_h$	fraction of heterogeneous solids, see Section 7.2
$X_p$	fraction of pseudo-homogeneous solids, see Section 7.2
$X_s$	fraction of stratified solids, see Section 7.2
y	distance measured perpendicular to flow boundary [L]

$y^+$	dimensionless form of $y$ , see Eq. 2.26 [-]
$Y$	specific value of $y$ [L]
$z$	distance measured vertically [L]
$z_A, z_B$	values of $z$ at locations A, B [L]
$\alpha$	angle defining position in pipe, see Fig. 5.1 [-]
$\beta$	angle defining interface in stratified flow, see Fig. 5.1 [-] <i>or</i> vane angle of pump [-]
$\gamma$	exponent used in Section 8.5 <i>or</i> exponent used in Section 10.2
$\delta$	sub-layer thickness [L]
$\varepsilon$	roughness height [L]
$\zeta$	relative excess pressure gradient, see Eq. 5.12 [-]
$\eta$	efficiency of a machine, see Eq. 9.5 [-]
$\eta_B$	slope of Bingham rheological model [ML <sup>-1</sup> T <sup>-1</sup> ]
$\eta_D$	efficiency of drive train [-]
$\eta_P$	efficiency of pump [-]
$\eta_r$	efficiency ratio, See Section 10.1 [-]
$\eta_t$	'tangent viscosity' of non-Newtonian [ML <sup>-1</sup> T <sup>-1</sup> ]
$\theta$	angle of inclination of pipe [-] <i>or</i> electrical phase angle [-]
$\kappa$	von Karman's coefficient for mixing length [-]
$\lambda$	exponent used in Section 10.4 [-]
$\Lambda$	lag ratio of solids, see Eq. 2.42 [-]
$\mu$	shear viscosity (dynamic viscosity) of fluid [ML <sup>-1</sup> T <sup>-1</sup> ]
$\mu_{eq}$	equivalent turbulent-flow viscosity, see Eq. 3.6 [ML <sup>-1</sup> T <sup>-1</sup> ]
$\mu_s$	mechanical friction coefficient of solids against pipe wall [-]
$\nu$	kinematic viscosity of fluid [L <sup>2</sup> T <sup>-1</sup> ]
$\xi$	ratio $v_t/v_{ts}$ , see Eq. 2.68 [-]
$\rho$	density [ML <sup>-3</sup> ]
$\rho_f$	density of fluid [ML <sup>-3</sup> ]
$\rho_m$	density of mixture [ML <sup>-3</sup> ]
$\rho_{md}$	density of mixture delivered [ML <sup>-3</sup> ]
$\rho_{mi}$	density of mixture <i>in situ</i> [ML <sup>-3</sup> ]
$\rho_s$	density of solids [ML <sup>-3</sup> ]
$\rho_w$	density of water [ML <sup>-3</sup> ]
$\sigma$	cavitation parameter for hydraulic machinery [-]
$\sigma_f$	component of $\sigma_{10}$ due to fluid turbulence, see Eq. 6.6 [-]
$\sigma_s$	normal stress of solids (granular pressure), see Section 5.2 [ML <sup>-1</sup> T <sup>-2</sup> ] <i>or</i> component of $\sigma_{10}$ associated with particle grading, see Eq. 6.6 [-]
$\sigma_{10}$	standard deviation of log ( $V_m/V_{50}$ ), see Eq. 6.6 [-]
$\tau$	shear stress [ML <sup>-1</sup> T <sup>-2</sup> ]

$\tau_o$	shear stress at flow boundary [ $ML^{-1}T^{-2}$ ]
$\tau_s$	intergranular component of shear stress [ $ML^{-1}T^{-2}$ ]
$\tau_y$	yield-point shear stress [ $ML^{-1}T^{-2}$ ]
$\tau_{12}$	shear stress at stratified-flow interface [ $ML^{-1}T^{-2}$ ]
$\phi$	angle of internal friction of particles (static) [-]
$\phi'$	angle of internal friction of particles (dynamic) [-]
$\Phi$	Durand variable, see Eq. 4.4 [-]
$\psi$	Head coefficient for a pump [-]
$\Psi$	Durand variable, see Eq. 4.5 [-]
$\omega$	angular velocity in radians per second [ $T^{-1}$ ]

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## **Chapter 2**

# **REVIEW OF FLUID AND PARTICLE MECHANICS**

### **2.1 Introduction**

Before considering the flow of mixtures of liquids and solid particles, we must first treat the flow of single-phase liquids and the motion of particles in liquids. These topics also provide an introduction to concepts, terminology and notation used in later chapters. The treatment in this chapter is at the level of a review, intended primarily to reinforce knowledge which readers will have encountered in their undergraduate engineering curriculum, but may have not utilized in the interim. As in other parts of this book, the level of presentation is directed to practical engineering application. This chapter is not intended as an introduction to fluid mechanics in general, to turbulence, or to the micromechanics of particle-fluid systems. All these subjects have significance for fundamental research, but they are not required for an engineering treatment of slurry flow.

To characterise a simple fluid such as water only two material properties are required: density and viscosity. The density, denoted by  $\rho$ , represents the mass of fluid per unit volume. The viscosity is a measure of the resistance of the fluid to deformation by shearing, and is best illustrated by reference to a conceptual experiment illustrated in Fig. 2.1. The fluid fills a gap of

thickness  $y$  between two flat parallel plates. The plate forming the  $y = 0$  plane is kept stationary, while the other plate (at  $y = Y$ ) is moved parallel to

the first at a steady velocity  $U$ . The fluid immediately in contact with each solid plate keeps the same velocity as the plate; this is known as the 'non-slip boundary condition'. Thus the velocity of the fluid at  $y = 0$  is zero and at  $y = Y$  is  $U$ . The fluid velocity,  $u$ , at any intermediate position is  $yU/Y$ . The quantity  $U/Y$  is the velocity gradient in the fluid, denoted by  $du/dy$ . Its significance is that it represents the rate of shear deformation of the fluid; it is therefore known as the 'rate of shear strain' or, more simply, the 'shear rate'.

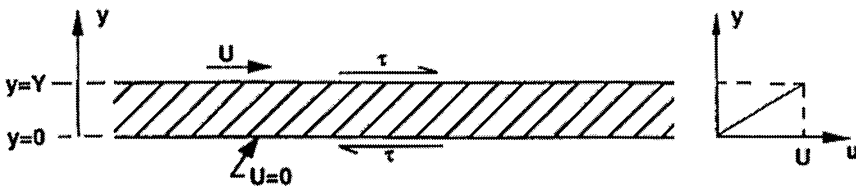


Figure 2.1. Shear deformation of a fluid (schematic)

To maintain the steady motion in Fig. 2.1, it is necessary to apply a force to the moving plate and an equal and opposite restraining force to the stationary plate. These forces are parallel to the plates, and in the direction of their relative motion. The force on one plate per unit area is known as the 'shear stress', denoted by  $\tau$ . As shown in Fig. 2.1, it represents the shear stress exerted by each plate on the fluid in the gap. Simple Newtonian mechanics dictates that the fluid exerts an equal and opposite shear stress on the plate, and that the shear stress within the fluid at any plane parallel to the plates is also  $\tau$ .

Conceptually, an experiment like that in Fig. 2.1 could be used to measure  $\tau$  as a function of  $du/dy$ . In practice, flat plates are inconvenient; practicable ways to measure viscosity are introduced in Chapter 3. For a simple fluid like water, the relationship between  $\tau$  and shear rate takes the form shown in Fig. 2.2, with  $\tau$  linearly proportional to  $du/dy$ . This type of relationship defines a 'Newtonian fluid', and the constant of proportionality is known as the 'shear viscosity' (or simply the 'viscosity') and will be denoted by  $\mu$ . Thus

$$\tau = \mu \frac{du}{dy} \quad (2.1)$$



Equation 2.1 is the 'constitutive equation' of a Newtonian fluid. The ratio  $\mu/\rho$  is known as the 'kinematic viscosity' of the fluid, denoted by  $\nu$ .

Values for the density and viscosity of water are given in Table 2.1. For practical purposes, over the range of conditions encountered in the industries using hydraulic transport, liquids are incompressible so that  $\rho$  and  $\mu$  can be taken as independent of pressure. However they both depend on temperature. Specifically, the viscosity of water decreases significantly with increasing temperature.

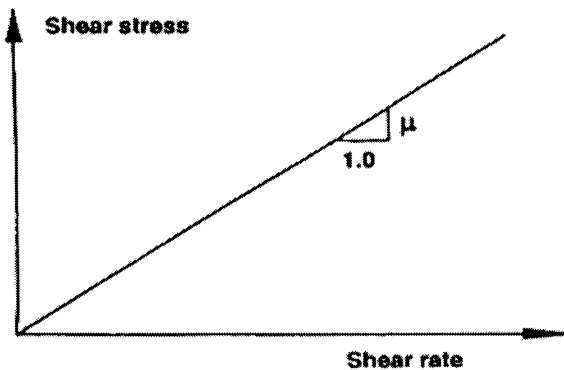


Figure 2.2. Characterisation of a Newtonian fluid (schematic)

Table 2.1. Properties of Water

Temperature (°C)	Density, $\rho$ (kg/m <sup>3</sup> )	Viscosity, $\mu \times 10^3$ (Pa.s)	Kinematic viscosity, $\nu \times 10^6$ (m <sup>2</sup> /s)	Vapour pressure, $p_v$ (kPa)
0	999.8	1.781	1.785	0.61
5	1000.0	1.518	1.519	0.87
10	999.7	1.307	1.306	1.23
15	999.1	1.139	1.139	1.70
20	998.2	1.002	1.003	2.34
25	997.0	0.890	0.893	3.17
30	995.7	0.798	0.800	4.24
40	992.2	0.653	0.658	7.38
50	988.0	0.547	0.553	12.33
60	983.2	0.466	0.474	19.92
70	977.8	0.404	0.413	31.16
80	971.8	0.354	0.364	47.34
90	965.3	0.315	0.326	70.10
100	958.4	0.282	0.294	101.33

## 2.2 Basic Relations for Flow of Simple Fluids

Much of this book is concerned with steady motion, in which the mean velocity at any point does not change with time (although it may vary with location). Whether the flow is steady or unsteady, analysis is based on three fundamental laws: the continuity balance (or conservation of matter); linear or angular momentum balances (which amount to the application of Newton's laws to fluids); and the mechanical energy balance (which is essentially the first law of thermodynamics applied to fluids). Some simple applications of these laws to solid-liquid mixtures are considered in Section 2.4. For steady flow of an incompressible fluid in a pipe or conduit, the continuity equation states simply that the volumetric flow is the same through each section across the pipe. Consider a pipe in which the diameter changes between sections A and B, as shown schematically in Fig. 2.3. If the total volumetric flowrate in the pipe is  $Q$ , and the pipe is taken to be 'running full', then the mean velocity at section A is

$$V_A = 4Q/\pi D_A^2 \quad (2.2)$$

where  $D_A$  is the internal pipe diameter at A, so that the pipe's cross-sectional area is  $\pi D_A^2/4$ . Similarly, at section B the mean velocity is

$$V_B = 4Q/\pi D_B^2 \quad (2.3)$$

The equation of continuity for this section of pipe then takes the form

$$Q = \pi D_A^2 V_A /4 = \pi D_B^2 V_B /4 \quad (2.4)$$

For cases in which it is necessary to consider the local velocity in the pipe at distance  $r$  from the axis, i.e.  $u(r)$ , then the total volumetric flow is evaluated from the integral

$$Q = \int_0^{D/2} u 2\pi r dr = 2\pi \int_0^{D/2} u r dr \quad (2.5)$$

in which the area of the element from  $r$  to  $(r+dr)$  is  $2\pi r dr$  and the velocity through it is  $u$ . Specific applications of Eq. 2.5 are considered below, and in Chapter 3.

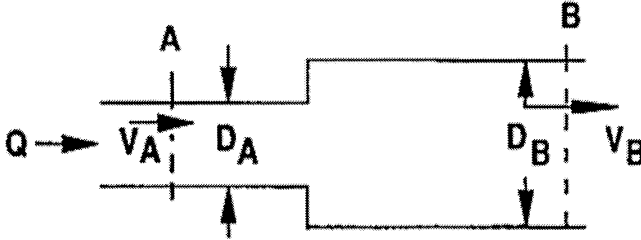


Figure 2.3. Flow in a pipe with change in diameter (schematic)

We next turn to the linear momentum equation, considering the simple case illustrated by Fig. 2.4: a fluid in steady motion through a straight horizontal pipe of constant diameter. Flow is ‘fully developed’, i.e. conditions do not vary between positions along the pipe. Therefore the shear stress exerted by the pipe walls on the fluid,  $\tau_o$ , is the same at all sections. Between two sections A and B, a distance  $L$  apart, the total area of pipe wall is  $\pi DL$  so that the total force exerted by the pipe walls on the fluid is  $\pi DL\tau_o$ . The linear momentum equation applied to this case of steady uniform fully-developed flow states that the total force on the fluid between sections A and B must be zero, (because the momentum flux across section A is equal to that across section B), giving

$$\frac{\pi D^2}{4} (p_A - p_B) + \pi DL \tau_o = 0 \tag{2.6}$$

where  $p_A$  and  $p_B$  are the (static) pressures in the fluid at the two sections. Rearranging Eq. 2.6

$$-\frac{dp}{dL} = -\frac{(p_A - p_B)}{L} = \frac{4\tau_o}{D} \tag{2.7}$$

or

$$\tau_o = -\frac{(p_A - p_B) D}{4L} \quad (2.8)$$

Equations 2.7 and 2.8 apply whether or not the fluid is Newtonian (i.e. obeys Eq. 2.1).

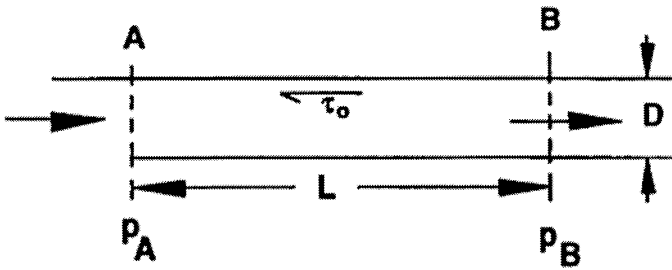


Figure 2.4. Flow in a straight horizontal pipe of constant diameter (schematic)

Another basic equation, for angular momentum, is needed for the analysis of centrifugal pumps. It will be presented in that context in Chapter 9. The energy equation is the next to be dealt with here. The mechanical energy balance for a flowing fluid is usually written in the form known as 'Bernoulli's equation'. It will be given here in terms of the 'head' of the fluid. 'Head' is a concept used extensively by Civil and Mining Engineers, and much of the literature on hydraulic conveying is written in terms of head and 'hydraulic gradient' (see below). Head is a measure of the mechanical energy of a flowing fluid per unit mass. It indicates the height by which the fluid would rise if the energy were converted to potential energy, and therefore has the units of length. The 'total dynamic head' of a fluid of density  $\rho$  flowing at velocity  $V$  in a pipe at elevation  $z$  above a reference level and at pressure  $p$  is

$$H = \frac{V^2}{2g} + \frac{p}{\rho g} + z \quad (2.9)$$

where the first term in this expression represents the kinetic energy of the fluid ('velocity head'), the second results from the static pressure in the fluid ('pressure head'), and the final term is the elevation ('static head').

Now consider a liquid propelled by a centrifugal pump, as shown in Fig. 2.5. Upstream of the pump, at section A on the pump suction, the total dynamic head of the fluid is  $H_A$ . Similarly, at section B on the pump

discharge, the total dynamic head is  $H_B$ . The increase in head across the pump, which is a simple measure of the energy imparted to the fluid by the pump, is known as the 'total developed head', TDH; i.e.

$$TDH = H_B - H_A = \frac{V_B^2 - V_A^2}{2g} + \frac{p_B - p_A}{\rho g} + (z_B - z_A) \quad (2.10)$$

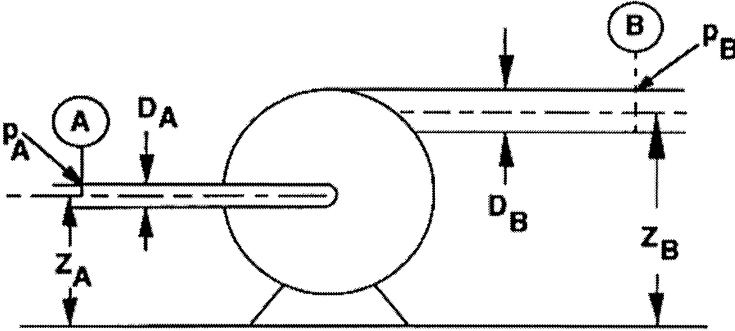


Figure 2.5. Fluid passing through a centrifugal pump (schematic)

In terms of the total volumetric flow rate,  $Q$ , and the pipe diameters at the two sections,  $D_A$  and  $D_B$ , (Eq. 2.2 and 2.3), the TDH equation becomes

$$TDH = \frac{8Q^2}{\pi^2 g} \left( \frac{1}{D_B^4} - \frac{1}{D_A^4} \right) + \frac{(p_B - p_A)}{\rho g} + (z_B - z_A) \quad (2.11)$$

The second term in Eq. 2.11, the 'pressure head', is normally by far the largest. The 'static head' term ( $z_B - z_A$ ) is usually relatively small, and depends on the pump geometry and dimensions. Centrifugal pumps are sometimes made with different suction and discharge diameters (see Chapter 8), so that the first or 'velocity head' term can become significant at large flowrates.

For a fluid flowing in a straight horizontal pipe of constant diameter, the only head term which varies along the pipe is that arising from changes in pressure. For the flow shown in Fig. 2.4, the reduction in head per unit length of pipe is known as the 'hydraulic gradient':

$$i = -\frac{(P_A - P_B)}{\rho g L} = \frac{4 \tau_o}{\rho g D} \quad (2.12)$$

Here  $i$  has been related to the wall shear stress,  $\tau_o$ , using Eq. 2.7. The units of the hydraulic gradient are (m head lost)/(m pipe run), or feet of head per foot of pipe. Thus the numerical value of  $i$  is independent of the system of units used. However,  $i$  is not strictly a dimensionless number: for example, if the flow took place on the moon or in any other environment of changed gravity, the value of  $i$  would be different even though  $\tau_o$  and the pressure gradient were unchanged.

A simple illustration of the significance of  $i$  is given by Fig. 2.6. If we imagine that 'sight glasses' are attached to the pipe - i.e. vertical open-ended transparent tubes - then the height to which the fluid rises in each sight glass shows the pressure head inside the pipe at that point. For steady fully-developed pipe flow, with constant hydraulic gradient and pressure gradient, the levels in sight glasses set along the pipe will lie on a straight line. This is known as the 'hydraulic grade line', and its inclination is the hydraulic gradient,  $i$ .

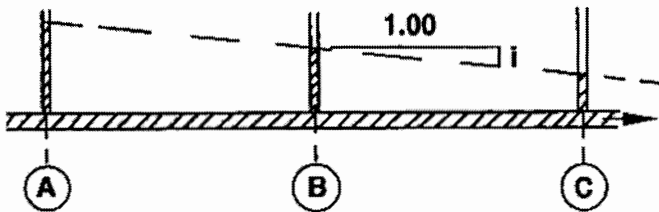


Figure 2.6. Significance of hydraulic gradient (schematic)

Consider now a pipe at an angle to the horizontal, as shown in Fig. 2.7. The pressure difference is measured between two sections, A and B, by connecting the pipe at these points to a manometer or a differential pressure sensor. Point B is  $\Delta z$  above A. The sensor is  $h$  above A, and therefore  $(\Delta z - h)$  below B, and the connections from the sensor to the pipe are filled with the same fluid as in the pipe, of density  $\rho$ . If the pressure in the pipe at A is  $p_A$ , then the pressure on the upstream side of the sensor is lower by the hydrostatic column  $h$ :

$$p_A' = p_A - \rho g h \quad (2.13)$$

Similarly, the pressure on the other side of the sensor is