Humans-with-Media and the Reorganization of Mathematical Thinking

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Humans-with-Media and the Reorganization of Mathematical Thinking

Information and Communication Technologies, Modeling, Visualization and Experimentation

Preface by Ubiratan D'Ambrosio
Afterword by Ole Skovsmose

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Dedication

This book is dedicated to

Anne Kepple

Contents

Contributing Authors

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Mónica E. Villarreal is a calculus professor at the Faculty of Agronomy of the University of Córdoba. She concluded her doctorate in mathematics education at UNESP, Rio Claro. She has supervised Masters students and has directed various research projects in Argentina. She is well published in Spanish, Portuguese and English. She is a consultant of BOLEMA, one of the most important mathematics education journals in Brazil, and of Revista de Educación Matemática, a journal in Argentina.

Preface

As we enter into the 21st Century, the presence of technology, particularly techno-science, in everyday life is overwhelming. Institutions in the modern world are affected by this presence.

Like Janus, the ancient Roman god whose double-faced head signified his knowledge of the present and the future, education has always been a two-faced enterprise. The past establishes goals and methods of Education, and the other face tries to capture the future and suggests and proposes new directions of thought and new styles of behavior for the generation which, in a few years, will take over both routines and societal innovation. History tells us that this face of Education has always been sensitive to emerging technologies.

Technologies of communication and information have been particularly influential in new directions of society, in particular of education. The transition from orality to writing marked a new role for the teacher. From the sole repository of accumulated knowledge, the teacher became a guide and interpreter of registered knowledge. The emergence of hardware, in the form of documents and books, initiated a companionship between teacher and hardware. It is also remarkable how the emergence of writing strengthened individual memory, contrary to the concerns of Thamus when Theuth explained to him the discovery of writing. The conservative king was afraid that the new invention would implant forgetfulness in the souls of men. Something similar occurred in Europe with the introduction of the technology of calculation of Indian and Arabic origins, which strengthened the analytic instruments of the philosophers of the late European Middle Age, thus paving the way for the Renaissance and Modern Age. We are now

living new possibilities in our communicative and analytic capabilities, thanks to the powerful new technology of communication and information.

Marcelo Carvalho Borba and Mónica Villarreal embraced the Janus metaphor when they decided to write this book. They are able to review, critically, the most relevant current educational practices, which largely reflect our past, and to venture into the future, proposing new directions for education. The same care of critically regarding the past is present in their views of the future. Thus, this book does not get trapped by the marvels suggested by the new, amazing, technologies.

It is a fact that billions are spent in education worldwide. But they risk being lost if we insist on declining educational models and practices. This big loss is unbearable for most countries, where human resources, so necessary for their future, receive an obsolete, and in most cases, useless, education. Even the more prosperous economies are very much concerned with the downgrading of their education, in spite of enormous resources available. We all agree that technology, by itself, is not the guarantee of a good education. But it is undeniable that lack of technology may hinder progress in education. Borba and Villarreal point to the key issues related to this paradoxical situation, avoiding sameness.

The book has an exemplary organization. In 10 chapters, the authors examine all the issues raised by the emerging technologies which are relevant for mathematics education. The challenges to the educator, from the cognitive dimensions to the political issues, are all dealt with by the authors. Although the book has originated in Brazil, the concerns are common to both the less and the more prosperous economies.

The authors claim a *de facto* evolution of the species towards higher levels of humanity, in the sense of a species impregnated with respect, solidarity and team spirit. This is particularly noticeable when they focus on the interaction of humans and technology. Refusing a common concern that technology leads to lack of humanity, the authors, drawing from many examples from the history of culture, claim the opposite. Indeed, there has been an interaction between humans and the technology they have created, and the evolution of the human species results from this interaction, to the point of a true merging of technologies in everyday life and, remarkably, in the way we think and act. The authors examine these facts, pointing out, very convincingly, that it is the responsibility of education to guide this merging to the ultimate goal of humanity. This is absolutely necessary for the survival, with dignity, of civilization.

The trajectory to a species impregnated with respect, solidarity and team spirit meets with obstacles of a political nature. We may appeal, again, to the Janus metaphor when referring to the conservative opposition to the new, to a trend in Education to favor sameness. It is the strength of the ethos of a

society that supports this face. On the other side, acquiescence allows the absorption of the new. Caution, necessary in every step of human action, should not hinder venturing into the new. This is another merit of this book. The authors, well aware of the need of caution, implemented, with all the required instruments of monitoring and evaluation, many innovative projects. Most of the projects of technological innovation in Mathematics Education, internationally recognized, received attention of the authors and were the subject of careful research. The description of the projects, accompanied by the results of their research and by very important remarks, will be extremely valuable for those wishing to innovate. A rich bibliography helps the mathematics educators in the process of carrying on their projects inspired by this book.

Borba and Villarreal have written an excellent book. Combining high scholarship with sound and careful methodology, they give to the reader, not only mathematics educators, a support for being innovative in entering the future.

Writing this preface was a most pleasant experience. It is always an honor to be invited by colleagues and friends to write a preface. But when the authors are former students, as in this case, the honor is multiplied. I am reassured that the good moments we had together were enriching to all of us.

Ubiratan D'Ambrosio
São Paulo, August 2004

Foreword

The ideas that have matured and given fruit in this book have their roots in diverse places, and here we would like to tell part of the story of the path leading to the book's completion.

In 1988, Marcelo Borba began his doctoral studies at Cornell University, U.S.A, becoming a member of the Mathematics Education Research Group, led by Jere Confrey. Since then, he has reflected on computers, their presence in education, and related epistemological issues, until arriving at the notion of *humans-with-media*, the backbone of this book, which represents an attempt to break the dichotomy between humans and technology, with relevant consequences for the classroom.

He returned to his native Brazil in 1993, where he started working at the State University of São Paulo (UNESP), one of the most important centers of mathematics education in Brazil and Latin America, and where he had earned his masters degree in 1987 conducting research on ethnomathematics. Former concerns about culture, social justice, and political dimensions of education - although never abandoned by him while studying abroad - gained new impulse and began to interact with some of the epistemological issues related to computers. He began making the connections between ethnomathematics, phenomenology, epistemology, technology and mathematics, and a project for a book started to develop. An outline of the book was presented to Kluwer and approved in 1996. Personal problems and other professional assignments made it impossible to finish the book at that time.

These problems, however, seem to have had positive consequences. Some of the ideas, especially the notion of humans-with-media, matured and became intertwined with new data from ongoing research. Several articles

were published, mainly in Portuguese, and the project of the book was revised. Although Lévy continues to be the main reference for this work, with his notion of the thinking collective and his view of technology, the writings of other authors like Kerckhove and Castells helped to transform some of the main ideas. He has been bringing Levy's ideas to mathematics education for almost a decade, but in this book, he intertwines his ideas with examples from research in new ways.

Other changes took place, as well, especially when he invited Mónica Villarreal, from the University of Córdoba, Argentina, to help him with this endeavor. The original invitation was for her to help with the literature review and some specific chapters, however she quickly transformed into a co-author of the book, helping to clarify many of the ideas, and adding her touch to it. Villarreal came twice to Brazil to work on the book, in 2002 and 2003, sponsored by FAPESP, a funding agency of the State of São Paulo, Brazil. Prior to that, she obtained her doctoral degree in the Graduate Program of Mathematics Education from UNESP, Rio Claro, in 1999. It was during this period that she started to interact with Marcelo Borba as his advisee and member of their research group, GPIMEM. The constant dialogue, exchange, informal conversation, and the contact with new research perspectives and new authors, changed Mónica Villarreal's perspective regarding what it means to teach and learn mathematics and to conduct research in mathematics education. Upon her return to Argentina, she resumed her activities as researcher and professor at the University of Córdoba, initiating a new phase in her relationship with GPIMEM as an associated researcher.

Anne Kepple, a researcher in public health nutrition, made an enormous contribution to this book. Although her job was to edit the English written by a Brazilian and an Argentinean, she did much more than this. As someone who knows qualitative research very well, she helped us to balance the book by challenging claims and assertions made in earlier versions of it. She also suggested readings, paths to be followed in the investigations, and provided motivation when some obstacles seemed insurmountable. We would like to thank her in a very special way.

We also would like to thank Ole Skovsmose who motivated Marcelo Borba every chance he had to publish this book as he believed that the ideas regarding technology and mathematics education were original. We would also like to thank Ubiratan D'Ambrosio and Maria Bicudo who have strongly influenced us on issues regarding ethnomathematics and education in a dialogical perspective. The reader will see that parts of this book have been previously published in different forms and languages. The reader will always find complete references in the text to these publications.

Finally, it should be said that this book represents a consolidation of a way of conceptualizing research group, as Mónica Villarreal had been a regular member of the group, from 1995 through 1999, taking part in regular weekly meetings; and has since become an associated member of the group, which is a category created for members who have special projects, but do not participate regularly in the group, and are more likely to develop research within other research groups as well. Since 2000, we have worked together on-line and face-to-face, consolidating a fruitful thinking collective, and sharing research perspectives and academic discussions, intertwined with research conferences, dance sessions, and the very important, fun and indispensable conversations accompanied by good red wine.

Marcelo C. Borba and Mónica E. Villarreal

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All members and former members of our research group, GPIMEM, who influenced us directly in the development of this book - not only because of the mutual influence on our ideas, but because of their sharp criticism of earlier versions of some of the chapters. We would like to thank, in particular, Ana Paula Malheiros, Telma Gracias, Jonei Barbosa, Jussara Araújo, Nilce Scheffer, Francisco Benedetti, Rúbia Amaral, Audria Bovo, Fernanda Bonafini, Norma Alevatto, Miriam Penteado, Marcus Maltempi, Antonio Olímpio Junior, Ricardo Scucuglia, Ana Flávia Mussolini, Simone

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Marcelo C. Borba and Mónica E. Villarreal
August 2004

Chapter 1

WHY ANOTHER BOOK ABOUT TECHNOLOGY AND MATHEMATICS EDUCATION?

1. INTRODUCTION

Computers have been a theme of intense discussion within the mathematics education community for more than two decades. If the notion of computers is extended to include other devices, such as calculators, it can be said that the debate has been going on for over thirty years. This being the case, a reasonable question might be: Why write about computers? This book represents an attempt to respond to this interrogation from various perspectives. One response could be that technology has not been used intensively in education, despite the efforts of a substantial part of the mathematics education community and the presence of an ever-increasing number of studies about computers, calculators, graphing calculators, and mathematics. Therefore, this book could be seen as an attempt to explain such a discrepancy.

Another possible answer could be that, over the last 35 years, technology has changed so much that constant updating is necessary. In Brazil, some researchers in the field of technology and mathematics education argue that the expression 'new technology' (*novas tecnologias*) should no longer be used, as computers and calculators have been around long enough to not be considered new anymore. On many occasions, we have twisted this issue around and argued that, since technology is changing so fast, we can always use the adjective 'new', especially if we concentrate on the notion of interface, seen as a means of relating to information and communication technology. Interfaces such as the video monitor, the keyboard and the mouse, which became popular in the 80's, are examples of how computer

technology is renewed and transforms existing technology. Similarly, we can point to how flash technology and sensors, such as CBR[1] and CBL[2], began changing graphing calculators in the 90's; how newer versions of operational systems, such as Windows, became almost as friendly as the systems used for Macintosh since the 80's, resulting in the end of a 'direct contact' with the old-fashioned DOS system; how the Internet has changed computers, and how Java and www have changed the possibilities of the Internet. All of these changes have substantially transformed the way information technology can be used in education – transformations that we begin to examine in this book. Thus, this book could be seen as an attempt to examine how interfaces have changed the possible ways for one to learn in recent years.

Many authors, including us, have argued about the importance of coordinating multiple representations, now made possible with software that makes written, algebraic, tabular and graphical representations available. As technology has changed, and body movements can also be coordinated with standard representations in a more direct way, it has become possible to expand the notion of multiple representations. It can even be argued that the need exists to coordinate representations of the same type, for example, graphs produced by different media, such as computers and paper-and-pencil on the one hand, and body awareness of motion on the other. These different types of coordination have become part of knowing, and in this sense, it could be said that this book is about epistemological issues that arise when different technologies are associated with human beings.

Within the mathematics education community, one of the few issues on which there is consensus regarding the discussion about technology is that computers alone are not likely to bring any change, and that intense pedagogical discussion should be undertaken. In other words, if the decision is made to use technology in the classroom, the debate is still open regarding how to use them, from the perspective of the teacher and the students, as well as from the standpoint of other actors in the mathematics education landscape. We propose some pedagogical approaches in this book that we believe to be more resonant with these new technologies than others. In this regard, this book could also be seen as being about different pedagogies that could be used once it is decided that technology is relevant for education.

In spite of the 'free will' of teachers and administrators, whether they do or do not want to use technology, it has been hard to avoid using it due to

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[1] CBR - Calculator Based Ranger - is a sonic movement detector that measures distances, velocity and acceleration.

[2] CBL is an interface that makes it possible for data, such as light intensity, temperature and electric tension, to be stored and transferred to the graph and table facilities of a graphing calculator.

social pressure from 'actors' such as politicians and business and school administrators. Such forces may use arguments such as "we must use computers because the labor force needs to be prepared for jobs in the future". In using computers not out of a conviction that they can be beneficial, but because it has been mandated, teachers may attempt to use them as little as possible, and when they do, make the minimum changes necessary in the structure of the curriculum and practices embedded in it. New technology in such an approach can be thought of as something that should not alter the *status quo* in school, nor 'touch' the way 'knowledge is transmitted in school'. New technology can therefore be 'domesticated'. Computers may be used as if they were 'electronic books', and graphing calculators as just a way of drawing graphs quickly. In this sense, this book represents an attempt to counter this way of conceptualizing technology in educational settings.

As suggested in the last paragraph, there is still an open question regarding the reasons why one should use technology. Twenty years ago, there was a debate between those who were opposed to using technology and those who were in favor of it because they believed it would improve teaching and learning (of mathematics). Although in most places technology is used to some degree in education, it could be the case that old questions regarding the use of technology are still open to debate, that issues regarding why one should use technology have gone unanswered. This book could then be seen as an attempt to propose another answer to why technology should be used in education.

This book is written for the mathematics education community, which could be loosely defined as being the set of researchers, professors and teachers who are interested in reflecting on the teaching and learning of mathematics, socio-cultural and political aspects of mathematics in school and society, and philosophical issues regarding the role of mathematics in education. Research in mathematics education has been gradually gaining identity in the last forty years or so. Most of members of this community like to debate theories about how one thinks and how one learns and teaches. In this book, we do not present a new theory regarding how to think about computers and education. We do, however, propose ideas, expressed in the form of theoretical constructs, about how we can overcome the dichotomy between humans and technology that underlies many of the difficulties that we, as a community, have experienced in implementing the use of technology in schools in ways that are not domesticated.

In addition to developing a theoretical discussion about the relationship between humans and computers, and what we call reorganization of thinking, we present several examples from research developed inside and outside the classroom. Examples help to shed light on the theoretical

discussion. Local experience often has the power of becoming convincingly acceptable for different contexts. That is why when examples are presented, many of us say, "now I know what you mean!" or "This makes sense", meaning that we related to the example. As examples shed light on the theory, many may find reason to disagree with the ideas presented in this book, and to therefore help keep the debate alive in our field. Examples may, in many instances, be the bridge between those who like theoretical discussion and those who do not. After presenting examples, we will return to the theoretical discussion presented and introduce some new features into the debate.

The examples are, for the most part, from research conducted by a group led by the first author of this book. This research group, called GPIMEM[3] (Grupo de Pesquisa em Informática, Outras Mídias e Educação Matemática[4]), based at the State University of São Paulo (UNESP), Rio Claro Campus, São Paulo, Brazil, is one of the few in the so-called Third World that has been developing long term research about the use of technology in mathematics education, a theme which is often, due to prejudice, restricted to the so-called First World countries. This research group is composed of almost 30 members who are professors, associated researchers, doctoral and masters students, technicians, and undergraduate students. The undergraduates, who are engaged in a genuinely Brazilian program that is called Scientific Initiation, develop research at their level, under the supervision of professors, and receive, in exchange, a small scholarship. Their experience becomes a powerful item in their Curriculum Vitae, if they decide to engage in research after graduation. In our research group, all members meet periodically in different subgroups to solve particular aspects of different projects, or to think about the overall goals of the group. This book could, therefore, be considered to be about the activities of this group.

In GPIMEM, we use various research procedures and views of knowledge that are integrated. We can therefore say that we develop different research methodologies for different kinds of research, although all of them would fit within the so-called qualitative research paradigm. We develop teaching experiments as a means of documenting closely the way students, teachers and even workers deal with technology as they learn mathematics. Besides being a source for epistemological debate, the analysis of the data that come from these experiments has enabled us, as a group, to develop curriculum as we listen closely to students. Such experiments are integrated with studies developed in the classroom and in other landscapes

[3] http://www.rc.unesp.br/igce/pgem/gpimem.html

[4] Technology, other media and Mathematics Education Research Group.

where other educational actors are present. We do develop extension work, which has also become a setting for research. A major struggle of GPIMEM has been to articulate basic research, both in the lab and in the classroom, with implementation and research that can be applied more rapidly. Doing this without being trapped by pressures from the market, and being able to think about social transformation, has been a challenge. This book could, then, be considered to be about the interface between research methodology and implementation of research results with social concerns in mind.

In the above paragraphs, we have shown different reasons for a book about technology. A combination of the above topics is what the book is about. We present a discussion regarding theoretical issues; discuss examples, for the most part, from our research developed in Brazil; and raise questions about the problems of articulating research and implementing results in educational systems. We present a theoretical discussion that may help to convince some that pedagogy and curriculum should be changed substantially when qualitatively different media, such as information and computer technologies, are introduced in education.

In Chapter 2 we introduce the notion of reorganization of thinking and the idea that knowledge is always produced by collectives of humans-with-media. We present the notion that human thinking is reorganized by different media, such as computers and their evolving interfaces. By reorganization we mean that computers do not substitute humans, nor are they juxtaposed to them. They interact and are actors in knowing. They form part of a collective that thinks, and are not simply tools which are neutral or have some peripheral role in the production of knowledge.

In Chapter 3 we introduce our perspective of modeling as a pedagogical approach that has synergy with the use of information and communication technology. We do so contrasting it with the literature about problem posing and problem solving. The roots of modeling in Brazil are described and compared with the Danish project work. Different perspectives relating technology and modeling are presented. Finally, the limits of modeling are pointed out.

In Chapter 4, we present the experimental-with-technology approach. We discuss the meaning of experimentation in mathematics and mathematics education. We propose that modeling be used in conjunction with the experimental approach, an environment in which students raise conjectures, argue and 'prove'. The teacher has the role of coordinating the experiences of the students with what is traditionally accepted in academia. We also introduce a discussion about multiple representations and their relationship with media.

Within computer technology, visualization has taken on an important role, and this is why we devote Chapter 5 to this theme. We present an in-

depth discussion of the role of visualization in mathematics and mathematics education, stressing the value of this process in educational settings. We show how our perspective of humans-with-media as the basic unit that produces knowledge gives a new twist to the long time discussion about visualization in our community.

In Chapter 6, we present several examples from our research about modeling, showing how this pedagogical approach influences the use of technology by students, and how different technological actors, such as a function software or the Internet, and even paper and pencil, play a major role in the investigations developed by students in formal school settings. Almost all the examples come from classroom experiences in a mathematics course for biology majors where modeling was implemented as the pedagogical approach.

In Chapter 7, we return to the discussion of the experimental-with-technology approach and the process of visualization in educational settings, grounding it in examples from research. We suggest that experimentation and visualization are major attributes to be explored when computer technology is used. Examples in this chapter, as well as in the one preceding it, involve contents such as functions, derivative, integrals and associated topics from high school and early university-level mathematics curricula.

Chapter 8 is dedicated entirely to discussion about the Internet. Epistemological issues regarding the transformation of the notions of space and time that come with the presence of the Internet are addressed; as well as the nature of interaction in on-line education settings. Continuing education for mathematics teachers is emphasized in this chapter, as data from courses in which they participated are presented. Social issues, which permeate some of the examples of modeling, gain an important place in this chapter as we discuss how courses like this can be a path for giving access for teachers all over the country to a recognized mathematics education center in Brazil, like UNESP, the State University of São Paulo.

In Chapter 9, we apply to our own research group the notion of collective intelligence as we discuss how we integrate research, and the impossibility of an hierarchy in which the professor knows more, doctoral students know a little bit less, master students much less, and so on. We present a map of the research we develop, including the studies that are not discussed in detail in this book. Research methodology, procedures and epistemology are intertwined in this chapter.

Finally, in Chapter 10, we discuss the political dimensions of our research, as well as philosophical issues regarding the tension between psychological time, the pace of production of new technology, and elaboration of research about technology and mathematics education. We use elements from the history of education, regarding the introduction of the

notebook in schools, to discuss the political role of having access to information and computer technology. We locate the work we develop in Brazil in the international scene, discussing our concern about democracy, the right of access to information and communication technology, and the importance of collaborative work with researchers from different parts of the world.