Exploring Probability in School

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Graham A. Jones
(Editor)

Exploring Probability in School

Challenges for Teaching and Learning

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GRAHAM A. JONES

PREFACE

During 1973 when I was undertaking my doctoral dissertation on young children's probabilistic thinking, I wrote to the late Efraim Fischbein to ask him some questions about my own study and to clarify some of his own work on probabilistic intuitions. In those days before the e-mail and fax technology, I was delighted to receive a very warm letter from him in less than a month. In addition to congratulating me on investigating probabilistic thinking and responding to my questions, he said in a gracious yet forthright manner that he wished that my research had involved some teaching. He went on to say that teaching probability involved special challenges because of the connection that needed to be made between theoretical and experimental notions of probability. He thought that making this connection with young children would be especially exciting and he urged me to take up the challenge in my later research.

In a very real sense, I have harbored Fischbein's challenge that has been repeated many times through his prodigious and thoughtful writing. For example, in talking about the need for an instructional program that involved both theoretical and experimental activities Fischbein and Gazit wrote, "It is that reciprocal dynamic of theoretically computed probabilities and observed relative frequencies that may best contribute to the development of efficient probabilistic intuition" (1984, p. 3).

As well as emphasizing the importance of the connection between theoretical and experimental probability, Fischbein also raised other issues about the challenges of teaching probability such as dealing with the primary intuitions that students brought to the classroom and providing appropriate representations to make probability concepts more accessible.

Accordingly this book is a response to Fischbein's enduring research and devotion to the challenges of teaching and learning probability. In initiating and editing *Exploring Probability in School: Challenges for Teaching and Learning*, I have tried to capture not only the spirit of Fischbein's original letter to me but also to gather together the writings of an international group of researchers who have made special contributions to the teaching and

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learning of probability at all levels of the school curriculum. Even though this book is a delayed response to Fischbein's challenge, the advent of probability and statistics in the mathematics curriculum at all levels makes the publication of this book on the teaching and learning of probability most timely.

I believe that the book will have special interest to a widespread audience. The book's journey through the landscape of extensive research on the learning and teaching of probability will appeal to the growing group of researchers and graduate students in fields like mathematics education, mathematics, and psychology. Curriculum developers, teacher educators and teachers will also find much to capture their interest in the learning activities and teaching implications presented at different levels of schooling. I also believe that writings on probability and people's responses to random phenomena possess a distinctive charm that hopefully will appeal to readers outside of the spheres of interest that I have foreshadowed above.

I wish to express my warmest thanks to all of the authors for their many contributions to this book. In addition to their special expertise that made each of the chapters a reality, they provided help in numerous ways to me and other authors in the team. I also want to pay special tribute to Shayne Mahon who took such wonderful pride in preparing and formatting our manuscripts through two phases of preparation. To my wife, Marion, I thank her for encouraging support over more than 40 years and for the expertise she provided in reading the chapters of this book and guiding it to a more coherent form of written communication.

I wish to add my special appreciation to Alan Bishop and members of the editorial board of the Mathematics Education Library series for their encouragement and enduring support for this project. I also thank the external reviewers who provided much valuable feedback and insight both in the proposal and review stages of this project. My sincere thanks to the editorial team at Kluwer Academic Publishers: In particular, I wish to thank Marie Sheldon, editor responsible for the Mathematics Education Library series and her assistant, Mary Panarelli, for responding to our many questions at various stages of the process and for presenting the book in such an attractive and professional manner. Finally, to Dr. John Le Blanc, Indiana University, thank you for getting me started on this creative endeavour.

1. REFERENCE

Fischbein, E., & Gazit, A. (1984). Does the teaching of probability improve probabilistic intuitions? *Educational Studies in Mathematics, 15*, 1-24.

GRAHAM A. JONES AND THE AUTHORS

INTRODUCTION

In producing this book we are aware that we have stood on the shoulders of others who have had a profound influence on the destiny of probability teaching and learning. All of us cut our teeth on the monumental works of Jean Piaget and Barbel Inhelder's (1951/1975) *The Origin of the Idea of Chance in Children,* and Efraim Fischbein's (1975) *The Intuitive Sources of Probabilistic Thinking in Children.* More recently, Kapadia and Borovcnik's (1991) book on *Chance Encounters: Probability in Education,* and insightful reviews by Shaugnessy (1992) and Borovcnik and Peard (1996) have brought research on probability teaching and learning even closer to the classroom.

Building on the outstanding contributions of these trail blazers is a two-fold blessing; a primary blessing because they have all contributed so much to our own thinking and research; a secondary blessing because we had to take a perspective on probability teaching and learning that truly advances the field they have seeded. In addition, we have endeavoured to tap an audience that will influence the way the next generation of children think about probability.

We believe that the special mandate for this book lies in the fact that it has been spawned in the wake of probability becoming a mainstream area in the school curriculum (e.g., Australian Education Council [AEC], 1991, 1994; Department of Education and Science and the Welsh Office [DES], 1989; National Council of Teachers of Mathematics [NCTM], 1989, 2000). Not only has probability emerged as a mainstream area in the school mathematics curriculum, the study of probability is pervasive in that it commences in the primary grades and continues through into the college years. This pervasive injection of probability into the school curriculum is especially opportune because children of present and future eras will increasingly meet chance variation and random phenomena not only in mathematics but in the media, in meteorological, economic, and financial forecasting, and in social activities such as games, sports, and gambling. The introduction of probability in schools brings into sharp relief the need for a book that considers the teaching and learning of probability from the

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perspective of the salient research in the field and draws implications from the research for the classroom and other environments where chance and probability impact children's lives. More specifically, there is a need for a book that presents a coherent body of research-based knowledge on probability teaching and learning; one that can be used to inform those who will play key roles in guiding probability education: researchers, curriculum developers, educational policy makers, teacher educators, and teachers. We have endeavored to address all of these exigencies in *Exploring Probability in School: Challenges for Teaching and Learning.*

1. CHALLENGES IN TEACHING AND LEARNING PROBABILITY

The study of probability has distinctive characteristics that are not encountered by teachers and students in other areas of mathematics and, except for science, in other areas of the school curriculum. Although these distinctive features certainly broaden and strengthen the mathematics curriculum, they also create special challenges for teaching and learning. We have briefly alluded to some of these challenges in the Preface when reflecting on Fischbein's caveat about the pedagogical difficulties of dealing with the multifaceted perspectives of probability. In what follows, these teaching and learning challenges will be identified more precisely and expanded in setting the scene for the chapters of this book:

1. The cognitive demands of dealing with randomness in contrast with the deterministic thinking associated with most school mathematics.
2. The challenge of working with multiple conceptions of probability: (a) subjective, (b) classical or theoretical, and (c) frequentist or experimental (Fischbein, 1975).
3. The task of identifying probability knowledge and dispositions that will be helpful to students at various levels (Gal, 2002 & this volume).
4. The charge to interpret the research literature on probability teaching and learning in a way that provides insights for the teacher and his/her development of rich instructional programs.

The notion of randomness is problematic for both teachers and for students of all ages (e.g., Batanero, Henry & Parzysz, this volume; Fischbein & Schnarch, 1997; Shaughnessy, 1977). Mathematics curricula throughout the world abound in examples and tasks that require deterministic thinking and this penchant for deterministic approaches goes beyond the classroom into the scientific and quantitative experiences that are an integral part of our social and cultural lives. We meet random events everyday but we tend to evaluate them in the myopic mindset of deterministic lenses.

As we will see in several chapters of this book (e.g., Batanero et al.; Batanero & Sanchez; Langrall & Mooney; Watson), children cannot fully conceive probability until they have a multifaceted conception of it that, at very least includes an understanding of the classical and frequentist approaches and their relationship. Exploring these approaches brings teachers and children face to face with different conceptions of probability that range from intuitive beliefs, observation and experimental probability, and \theoretical calculations based on the sample space. In addition, as Borovcnik and Peard (1996) observe, "Cultural factors have made the development and treatment of the subject [probability] on the European continent different from Anglo-American approaches" (p. 239).

The task of identifying probability knowledge and dispositions that will be helpful to students at various ages is in its infancy because probability is new to the school mathematics curriculum especially at the elementary school level. Although the first curriculum documents incorporating probability have been around for at least 15 years (e.g., DES, 1989; NCTM, 1989), there is an urgent need to determine what constitutes "probability literacy" (Gal, this volume). Faced with a plethora of statistical and probability messages in the media and other sources, children are required to interpret the language of chance, to be familiar with the notion of randomness, to understand that events may vary in their degree of likelihood and independence, and to adopt a disposition that enables them to critically evaluate the probabilistic statements they read and hear (Gal, 2002). Questions such as "When do we introduce these notions of probability literacy?" and "How do we weave them into a coherent learning tapestry?" are all part of the challenge that we will try to address.

Finally, there is a need to interpret the extensive and diverse research literature on the learning and teaching of probability. On the one hand, there is evidence that people of all ages exhibit broad levels of probability thinking as well as misconceptions and biases that cannot be completely accounted for by age (e.g., Fischbein & Schnarch, 1997; Jones, Langrall, Thornton, & Mogill, 1997; Shaughnessy, 1977, 1992, Tversky & Kahneman, 1974; Watson, Collis, Moritz, 1997). On the other hand, there is a myriad of pedagogical and social issues that arise for teachers in designing probability instruction: building on students' conceptions and misconceptions, making connections between the classical and frequentist approaches to probability, utilizing emerging technologies, dealing with social and cultural issues that arise in the context of teaching probability, and enriching their own knowledge and beliefs about probability (Greer, 2001; Jones, Langrall, Thornton, & Mogill, 1999; Konold, Pollatsek, Well, Lohmeier, & Lipson, 1993; Pratt, 2000; Watson & Moritz, 1998). Consequently, there was a need for us as authors of this book to build a cognitive and pedagogical landscape for probability that will be capable of informing instruction. To this end, we will provide insights into the kind of probabilistic thinking that students bring to the classroom at various ages (see Batanero & Sanchez; Langrall & Mooney; Pfannkuch; Polaki; Pratt; Watson). We will also attempt to build an understanding of the pedagogical mechanisms and social culture that operate in classroom environments where probability is taught, learned, and assessed (see Greer & Mukhopadhyay; Jolliffe; Stohl).

These challenges and the research relating to them provide a fertile body of knowledge to support the intentions of this book. As we move to look more closely at the purpose and organization of the book, it will be evident that many of these challenges will be revisited in different themes and at different grade levels. In dealing with these different themes and grade levels our aim is to synthesize the key research, illuminate the important theoretical ideas and draw implications for the classroom.

2. PURPOSE OF THE BOOK

As mentioned earlier there has been a number of important reviews and expositions on the teaching of probability in recent times (e.g., Borovncnik & Peard, 1996; Kapadia & Borovcnik, 1991; Shaughnessy, 1992). All of these publications have contributed to the literature in teaching and learning probability and ipso facto the background that we bring to the writing of this book. Notwithstanding the importance of these contributions, we have the good fortune to be writing this book within an educational culture where, for the first time, probability has been taught across the school grade levels for an extended period of time. In essence we are writing in a context where researchers of the last decade have had the opportunity to draw on teachers' experiences in teaching probability and where classroom observations of even young children learning probability have been much more accessible.

The wider introduction of probability into the school curriculum has stimulated new research. This contemporary research, together with extant research in the field, needs to be documented and interpreted in such a way that it will bring fresh insights into the kind of issues educators face in supporting the learning of probability. More specifically the purpose of this book is to review and analyze the research literature with a view to addressing issues such as the following:

- the distinctive nature of chance, randomness and probability;
- the issue of probability literacy for our citizenry and the continuing need for probability at all levels of the school curriculum;

- the identification and examination of powerful ideas in probability that are accessible to students in the elementary, middle and high school years;
- the development of pedagogical knowledge that will enhance the learning of probability in the elementary, middle, and high school grades;
- the support and continuing education of teachers as they face the social and pedagogical challenges that are unique to the teaching and assessment of probability.

Already it will be evident, that the book has a strong focus on the key ideas of probability and on the teaching and learning of probability in the various components or grade bands of the school curriculum: elementary, middle, and high school years. This is made more explicit in next section on structure and organization of the book.

3. ORGANIZATION OF THE BOOK AND CHAPTER OUTLINES

Following the Preface and the Introduction the first section provides a perspective on probability and aspects of probability education. The next three sections examine the teaching and learning of probability in the elementary, middle, and high school years. Our final section deals with important issues for teachers: contextual background within which probability is taught, assessment, and teacher education and development.

Section 1: Perspectives on Probability and Probability Education

In this section, we consider fundamental ideas like the nature of chance, randomness, and probability and what is meant by "probability literacy" for all citizens. There is also an overview of the theoretical notions of probability education that can be gleaned from more than 50 years of research on the teaching and learning of probability. In essence, this section endeavors to set in place the mathematical and educational building blocks of probability before we examine the more detailed aspects of teaching and learning probability in the various grade bands. On the one hand we need to consider the historical roots of chance and the various interpretations of probability. On the other hand, we need to know what kind of knowledge is associated with being literate in probability and also the kind of probability knowledge that students can be expected to bring to and learn in the classroom.

Chapter 1: The nature of chance and probability. The authors examine various interpretations of chance, randomness, and probability, and suggest implications for curricula and instruction. It is noted that, through the ages, people have been faced with uncertainty, largely through games of chance. However, it is only in the last four centuries that mathematicians have developed quantitative measures for randomness, that in turn gave birth to various conceptions of probability: the classical approach based on combinatorics, the experimental approach leading to the limit of stabilized frequencies, and finally to a conflict between objective and subjective points of view. In more recent times we see the emergence of modelling approaches that endeavor to distinguish between real random situations and their theoretical interpretations. By way of contrast to these mathematical developments, people of various ages bring erroneous intuitions about probability that certainly need to be addressed through instruction.

Chapter 2: Towards "probability literacy" for all citizens. This chapter focuses on "probability literacy", that is, the knowledge and dispositions that students need to develop to be considered literate in the real-world of probability. The chapter reviews various models of adult literacy, numeracy, and statistical literacy that define the terrain in which knowledge of probability is situated. It then sketches a model for probability literacy, that contains five basic elements of probability-related knowledge and some supporting dispositions. The author argues that both knowledge and dispositional elements are needed for adults to be able to effectively interpret and engage in probabilistic situations. Finally, the chapter examines implications of the proposed model for instructional practice and research.

Chapter 3: An overview of research into the teaching and learning of probability. The authors of this chapter present a historical review of research into the teaching and learning of probability during the last 50 years. The review reveals that the research during this time can be appropriately categorized into three major phases: the Piagetian period (1950-1969), the Post-Piagetian period (1970-1989) and the Contemporary period (1990-). The Piagetian period is dominated by Piaget and Inhelder's cognitive development research on children's probabilistic thinking and the research of psychologists who examined people's strategies when predicting in uncertain environments. The Post-Piagetian period is characterized by Fischbein's seminal research on intuitions, by continuing psychological research into decision-making under uncertainty, and by the emergence of research on students' probabilistic conceptions prior to and during instruction. The Contemporary period identifies research that occurred after probability materialized as a mainstream area in the school curriculum. The research of this latter period is the main focus of this book.

Section 2: Teaching and Learning Probability in the Elementary School

This section analyzes research on probability at the elementary school level (approximately ages 5 through 10 years). Because probability has only recently been introduced into the elementary school mathematics curriculum, we have attempted to synthesize from the research literature characteristics of young children's probabilistic thinking that are relevant for curriculum development and instruction. This synthesis will also incorporate the identification of powerful ideas in probability and combinatorics that are accessible and central for building early probability literacy.

Chapter 4: Characteristics of elementary school students' probabilistic reasoning. This chapter presents a review and discussion of the literature pertaining to the probabilistic reasoning of elementary school children. The body of research that has emerged over the past 50 years is examined to highlight the current state of knowledge regarding the concepts, skills, and habits of mind necessary for elementary school children to develop an understanding of probability. The chapter is organized into two sections. The first section addresses the research pertaining to children's intuitions, understandings, and misconceptions in deterministic and chance situations. This section also examines children's conceptions of sample space, relative frequency and proportionality, and experimental and theoretical probabilities. The second section considers theoretical perspectives on instruction focusing on how children develop understandings necessary to reason probabilistically.

Chapter 5: Combinatorics and the development of children's combinatorial reasoning. Basic combinatorial ideas are an integral part of elementary probability and our teaching of probability needs to take account of this fact. Despite its importance in the mathematics curriculum, combinatorics continues to remain neglected, particularly at the elementary school level. The chapter addresses this concern by first exploring some elementary ideas of combinatorics and how they support children's early development in probability and problem-solving. Consideration is then given to various types of combinatorial problems and the relevant difficulties they present children. A review of research on children's combinatorial reasoning is presented in the second half of the chapter and the chapter concludes by looking at ways in which we might increase children's access to powerful ideas in combinatorics.

Section 3: Teaching and Learning Probability in the Middle School

This section provides an analysis of a number of key aspects of the teaching and learning of probability in the middle school (approximately ages 10 through 14 years). More specifically, this section examines research on middle school students' understanding of various concepts in probability including randomness, luck, experimental and theoretical probability, simple and compound events, conditional probability and independence. Implications are drawn from this research for curriculum and instruction in the middle school, including the use of technological learning environments.

Chapter 6: The probabilistic reasoning of middle school students. It is at the middle school level that the basic ideas about elementary probability are consolidated and more sophisticated concepts introduced for the first time. This chapter reviews the research dealing with a wide range of issues related to probabilistic reasoning. Although the calculation of simple probabilities and the use of proportional reasoning are addressed, other issues involve beliefs about luck, fairness and random behavior; the use of chance language and equiprobability bias; understanding of compound, conditional, and conjunctive events; appreciation of sampling and variation as part of probabilistic reasoning; and the application of probabilistic reasoning in context. In some instances reasoning of middle school students is more appropriate than that of older students and these distinctive characteristics suggest implications for the classroom.

Chapter 7: How do teachers foster students' understanding of probability? This chapter begins by arguing that, for middle school children, the notion of distribution and the law of large numbers are central constructs in the teaching and learning of probability. Yet the curriculum literature reveals that these key ideas are often avoided in the name of making probability more accessible. Considerable research has been generated in recent decades that catalogues what children do *not* know. Regrettably, we have only limited evidence of the intuitions and meanings for stochastic ideas that students have available as starting points for new pedagogical approaches. Through an illustrative case study, the author proposes that some technological approaches suggest how we might design tasks that not only generate purpose but also focus specifically on the use of distribution and the law of large numbers.

Chapter 8: Dealing with compound events. The ability to make valid probability predictions in the context of compound events (e.g. tossing a fair coin and a six-sided fair die) is a key learning goal for the middle school (e.g., NCTM, 2000). Using the notions of sample space and probability of an event as a context, this chapter analyses upper elementary and middle school students' ability to generate sets of outcomes associated with compound events and to make valid probability predictions in such situations. To reach this goal, the author examines two research orientations. The first pertains to research work that developed detailed accounts of students' probabilistic thinking in the form of cognitive frameworks. The second involves research studies that sought to document how students acquired more sophisticated ideas of dealing with compound events during instructional programs. Instructional and research implications arising from these examinations of the literature are discussed.

Chapter 9: How can teachers build notions of conditional probability and independence? In this chapter the authors explore middle school students' probabilistic thinking in conditional probability and independence and argue that these concepts are both important and appropriate learning for the middle school curriculum. Following interpretations of conditional probability and independence, there is an examination of the emergence of these concepts in the school curriculum and and a presentation of arguments in favor of their inclusion. The chapter highlights research relating to middle school students' conceptions and misconceptions about conditional probability and independence and analyzes student strategies in dealing with these concepts prior to instruction. There is also a review of research that traces students' thinking in conditional probability and independence during instruction. Finally, implications are drawn for teaching and learning conditional probability and independence in the middle school.

Section 4: Teaching and Learning Probability in the High School

This section extends the two previous sections by considering the nature of high school students' (approximately ages 15 through 18 years) conceptions of probability in areas such as randomness, combinatorics, conditional probability, independence, and association. It documents what research tells us about high school students' understanding of probability distributions and the connections they make between probability and statistical inference.

Chapter 10: What is the nature of high school students' conceptions and misconceptions about probability? In this chapter the authors review research that focuses specifically on the probabilistic thinking of secondary school students and relate the students' thinking to core curriculum expectations for students of this age. In particular, the authors look at research associated with some key elements for the probability curriculum: combinatorial reasoning and problem solving, perception of randomness, probability misconceptions, conditional probability and independence, association in contingency tables, random variable and probability distribution, sampling and inference, and simulation. Examples of items taken from the research literature will be used to identify and describe the kinds of reasoning, errors and difficulties that high school students normally find in each of these topics. Finally, the implications of these research findings for teaching probability in the secondary school are examined.

Chapter 11: Probability and statistical inference: How do teachers enable learners to make the connection? This chapter examines research on informal inference and proposes possible teaching pathways towards formal inference. Reasoning with measures of center, distributional reasoning, sampling reasoning, and drawing a conclusion are considered as a means of laying down foundations for learners to interconnect probabilistic and statistical thinking. Both classical and empirical approaches to formal inference are discussed. Building on these approaches the chapter attempts to address the following key questions: What concepts of probability do we need to teach as a basis for inferential reasoning? What type of inferential statistics is appropriate for high school students? How can technology be used to support inferential reasoning?

Section 5: Teachers and Probability

In this section the authors deal with a number of important issues that relate to teachers and the teaching of probability at all levels. First, they examine the political, social and pedagogical challenges teachers face in dealing with probability in the classroom. Second, there is an analysis of issues related to assessment: both on going monitoring as well as more formative and summative assessments. Finally, there is a review of research that looks at questions relating to teacher education and teacher development and its impact on classroom instruction in probability.

Chapter 12: Teaching the mathematization of uncertainty: Historical, cultural, social, and political contexts. Beyond asking what probability should be taught in school, and how, lies the deeper question of why we should teach it. Addressing this question requires analysis of the historical, cultural, social, and political contexts. The emergence of mathematical probability from a complex of cultural practices, and the subsequent symbiotic development of theory and applications, throw light on a branch of mathematics that is closely and distinctively integrated with the phenomena it models. Challenges for the teaching of probability include the tendency to reduce it to its formal expression, and the weakness of probabilistic thinking in society at large. The authors argue for an emphasis on probability as a supremely multidisciplinary topic, highlighting the importance of modeling and the making of connections with people's lived experience.

Chapter 13: Assessing probabilistic thinking and reasoning. This chapter approaches the assessment of probabilistic thinking and reasoning from several different angles. The introduction sets the scene by covering the purposes of assessment and the definitions of probabilistic thinking and reasoning. Then, after a short section on types of assessment tasks, some specific examples of probability questions are given and considered in some depth. This is followed by a section on relevant research studies. The chapter concludes with a discussion of some assessment methods that might be used as alternatives to more traditional methods and an analysis of the role of the teacher in the assessment of probability.

Chapter 14: Probability in teacher education and development. The purpose of this chapter is to investigate the nature and development of teachers' probabilistic understanding. The chapter includes a discussion of how teachers' computational approaches to teaching mathematics can affect their capacity to have a deep understanding of the non-deterministic nature of probability and their ability to facilitate students' probabilistic understanding. The chapter discusses research on teachers' knowledge and beliefs about probability, their ability to teach probabilistic ideas, and their use of simulation tools. In addition, the author discusses lessons learned from teacher education projects aimed at developing teachers' knowledge about probability.

4. CONCLUDING COMMENT

As you continue your journey into *Exploring Probability in School: Challenges for Teaching and Learning,* we hope that you will capture some of the intriguing characteristics of chance and probability and the special role that it plays in our world and our children's world. It is also our desire that you will discover some of the fascination associated with the way that humans think about random phenomena and probability and ipso facto the challenge faced by researchers, educators, and teachers in trying to make powerful ideas in probability more accessible to students of all ages.

REFERENCES

Australian Education Council. (1991). *A national statement on mathematics for Australian Schools*. Carlton, VIC: Curriculum Corporation.

Australian Education Council. (1994). *Mathematics: A curriculum profile for Australian schools*. Carlton, VIC: Curriculum Corporation.

Borovcnik, M. & Peard, R. (1996). Probability. In A. Bishop, K. Clements, & C. Keitel, J. Kilpatrick, C. Laborde (Eds.), *International handbook of mathematics education* (Part 1, pp. 239-288). Dordrecht, The Netherlands, Kluwer.

Department of Education and Science and the Welsh Office. (1989). *National curriculum: Mathematics for ages 5 to 16*. York, UK: Central Office of Information.

Fischbein, E. (1975). *The intuitive sources of probabilistic thinking in children*. Dordrecht, The Netherlands: Reidel.

Fischbein, E., & Schnarch, D. (1997). The evolution with age of probabilistic, intuitively based misconceptions. *Journal for Research in Mathematics Education, 28*, 96-105.

Gal, I. (2002). Adult statistical literacy: Meanings, components, responsibilities. *International Statistical Review, 70*(1), 1-25.

Greer, B. (2001). Understanding probabilistic thinking: The legacy of Efraim Fischbein. *Educational Studies in Mathematics, 45*, 15-33.

Jones, G. A., Langrall, C. W., Thornton, C. A., & Mogill, A. T. (1997). A framework for assessing and nurturing young children's thinking in probability. *Educational Studies in Mathematics, 32*, 101-125.

Jones, G. A., Langrall, C. W., Thornton, C. A., & Mogill, A. T. (1999). Students' probabilistic thinking in instruction. *Journal for Research in Mathematics Education, 30*, 487-519.

Kapadia, R. & Borovcnik, M. (Eds.). (1991). *Chance encounters: Probability in education*. Dordrecht, The Netherlands: Kluwer.

Konold, C., Pollatsek, A., Well, A., Lohmeier, J., & Lipson, A. (1993). Inconsistencies in students' reasoning about probability. *Journal for Research in Mathematics Education, 24*, 392-414.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Piaget, J., & Inhelder, B. (1975). *The origin of the idea of chance in children* (L. Leake Jr., P. Burrell, & H. D. Fischbein, Trans.). New York: Norton (Original work published 1951).

Pratt, D. (2000). Making sense of the total of two dice. *Journal for Research in Mathematics Education, 31*, 602-625.

Shaughnessy, J. M. (1977). Misconceptions of probability: An experiment with a small-group, activity-based, model building approach to introductory probability at the college level. *Educational Studies in Mathematics, 8*, 285-316.

Shaughnessy, J. M. (1992). Research in probability and statistics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 465-494). New York: Macmillan.

Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science, 185*, 1124-1131.

Watson, J. D., Collis, K. F., & Moritz, J. B. (1997). The development of chance measurement. *Mathematics Education Research Journal, 9*, 60-82.

Watson, J. D., & Moritz, J. B. (1998). Longitudinal development of chance measurement. *Mathematics Education Research Journal, 10*, 103-127.

SECTION I

PERSPECTIVES ON PROBABILITY AND PROBABILITY EDUCATION

CARMEN BATANERO, MICHEL HENRY, BERNARD PARZYSZ

THE NATURE OF CHANCE AND PROBABILITY

Chapter 1

The theory of probabilities is at bottom nothing but common sense reduced to calculus; it enables us to appreciate with exactness that which accurate minds feel with a sort of instinct for which ofttimes they are unable to account... It teaches us to avoid the illusions which often mislead us; ... there is no science more worthy of our contemplations nor a more useful one for admission to our system of public education. (Laplace, 1986/1825, pp. 206-207)

1. INTRODUCTION

Epistemological problems play a fundamental role for mathematics educators, because analyzing the obstacles that have historically emerged in the formation of concepts can help us understand students' difficulties in learning mathematics. This is particularly important in the field of probability, where, in addition to the difficulty of understanding scientific knowledge as a theoretical interpretation of real phenomena, one has to deal with typical misconceptions and beliefs, and knowledge about future events that is often based on divinatory predictions that have arisen from a magical ancestral way of thinking. For centuries all speculation about future events was inconceivable, since the future only belonged to the omniscient and omnipotent glory of the supreme Creator as noted by Jacques Bernoulli (1713/1987) in introducing the fourth part of *Ars Conjectandi.* Mind you, this divine association was not an obstacle for players betting on games of chance; however, the quantitative control of these bets remained in the field of intuition.

Gerolamo Cardano, who connected betting to the enumeration of winning combinations, was the first to make progress in probabilistic thinking in the 16th century. However, the decisive step in probability thought was achieved by Blaise Pascal and Pierre de Fermat in their correspondence (Pascal 1654/1963a), and was exposed by Pascal in his *Traité du Triangle Arithmétique* (Pascal 1654/1963b, Edwards 1987). Ignoring metaphysics,

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Pascal and de Fermat quantified the winning chances for the players in the case when a game actually stops before one of them wins the prize and where equal probabilities were not appropriate. The assumption of equiprobability of the elementary outcomes in a fair game was the first criterion to estimate the probability of a compound event made up of these outcomes.

Since then, the concept of probability has received different interpretations according to the metaphysical component of people's relationships with reality (Hacking, 1975) and thus probability is a young area where formal development has been linked to a large number of paradoxes that show the disparity between intuition and conceptual development in this field (Borovcnik, Bentz, & Kapadia, 1991). For example, many students think that the events "obtaining 5 and 6" and "6 is obtained twice" are equally likely when throwing two dice. Other examples are given in Székely (1986) and through this chapter (e.g. the first historical probability problem posed to Galileo by the Grand Duke of Tuscany).

Even today, and in spite of having a satisfactory axiomatic system, there are still controversies over the interpretation of basic concepts and about their impact on the practice of statistics. Moreover, Borovcnik and Peard (1996, p. 249) remark that probabilistic reasoning is different from logical or causal reasoning and thus counterintuitive results in probability are found even at very elementary levels. This is in contrast with other branches of mathematics where counterintuitive results are encountered only when working at a high degree of abstraction. This fact explains the existence of erroneous intuitions and learning difficulties that still persist at the high school level (Batanero, Serrano & Green, 1998; Batanero & Sanchez, this volume; Fischbein, Nello, Marino, 1991; Jones & Thornton, this volume; Langrall & Mooney; this volume Shaughnessy, 1983, 1992; Watson, this volume). A well-known example is the following: when successive players try to pick at random the shortest stick among a set of sticks it is argued that the first player has the greatest probability to get the shortest stick, because successive players might be unable to get it. The fact that the probabilities are equal for all players in this example is contrary to naive probabilistic intuition.

In this chapter we will examine different interpretations of the nature of chance, randomness, and probability and will highlight how these multiple conceptions are complementary and can influence curriculum goals. Finally we include some implications for the teaching and learning of probability in schools.