
ENERGY AND ENVIRONMENT

GERAD 25th Anniversary Series

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- **Energy and Environment**
Richard Loulou, Jean-Philippe Waaub, and Georges Zaccour, editors

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Edited by

RICHARD LOULOU
GERAD and McGill University

JEAN-PHILIPPE WAAUB
GERAD and Université du Québec à Montréal

GEORGES ZACCOUR
GERAD and HEC Montréal

Richard Loulou
GERAD & McGill Univ.
Montréal, Canada

Jean-Philippe Waub
GERAD & Univ. du Québec à Montréal
Montréal, Canada

Georges Zaccour
GERAD & HEC Montréal
Montréal, Canada

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Foreword

GERAD celebrates this year its 25th anniversary. The Center was created in 1980 by a small group of professors and researchers of HEC Montréal, McGill University and of the École Polytechnique de Montréal. GERAD's activities achieved sufficient scope to justify its conversion in June 1988 into a Joint Research Centre of HEC Montréal, the École Polytechnique de Montréal and McGill University. In 1996, the Université du Québec à Montréal joined these three institutions. GERAD has fifty members (professors), more than twenty research associates and post doctoral students and more than two hundreds master and Ph.D. students.

GERAD is a multi-university center and a vital forum for the development of operations research. Its mission is defined around the following four complementarily objectives:

- The original and expert contribution to all research fields in GERAD's area of expertise;
- The dissemination of research results in the best scientific outlets as well as in the society in general;
- The training of graduate students and post doctoral researchers;
- The contribution to the economic community by solving important problems and providing transferable tools.

GERAD's research thrusts and fields of expertise are as follows:

- Development of mathematical analysis tools and techniques to solve the complex problems that arise in management sciences and engineering;
- Development of algorithms to resolve such problems efficiently;
- Application of these techniques and tools to problems posed in related disciplines, such as statistics, financial engineering, game theory and artificial intelligence;
- Application of advanced tools to optimization and planning of large technical and economic systems, such as energy systems, transportation/communication networks, and production systems;
- Integration of scientific findings into software, expert systems and decision-support systems that can be used by industry.

One of the marking events of the celebrations of the 25th anniversary of GERAD is the publication of ten volumes covering most of the Center's research areas of expertise. The list follows: **Essays and Surveys in Global Optimization**, edited by C. Audet, P. Hansen and G. Savard; **Graph Theory and Combinatorial Optimization**,

edited by D. Avis, A. Hertz and O. Marcotte; **Numerical Methods in Finance**, edited by H. Ben-Ameur and M. Breton; **Analysis, Control and Optimization of Complex Dynamic Systems**, edited by E.K. Boukas and R. Malhamé; **Column Generation**, edited by G. Desaulniers, J. Desrosiers and M.M. Solomon; **Statistical Modeling and Analysis for Complex Data Problems**, edited by P. Duchesne and B. Rémillard; **Performance Evaluation and Planning Methods for the Next Generation Internet**, edited by A. Girard, B. Sansò and F. Vázquez-Abad; **Dynamic Games: Theory and Applications**, edited by A. Haurie and G. Zaccour; **Logistics Systems: Design and Optimization**, edited by A. Langevin and D. Riopel; **Energy and Environment**, edited by R. Loulou, J.-P. Waaub and G. Zaccour.

I would like to express my gratitude to the Editors of the ten volumes, to the authors who accepted with great enthusiasm to submit their work and to the reviewers for their benevolent work and timely response. I would also like to thank Mrs. Nicole Paradis, Francine Benoît and Louise Letendre and Mr. André Montpetit for their excellent editing work.

The GERAD group has earned its reputation as a worldwide leader in its field. This is certainly due to the enthusiasm and motivation of GERAD's researchers and students, but also to the funding and the infrastructures available. I would like to seize the opportunity to thank the organizations that, from the beginning, believed in the potential and the value of GERAD and have supported it over the years. These are HEC Montréal, École Polytechnique de Montréal, McGill University, Université du Québec à Montréal and, of course, the Natural Sciences and Engineering Research Council of Canada (NSERC) and the Fonds québécois de la recherche sur la nature et les technologies (FQRNT).

Georges Zaccour
Director of GERAD

Avant-propos

Le Groupe d'études et de recherche en analyse des décisions (GERAD) fête cette année son vingt-cinquième anniversaire. Fondé en 1980 par une poignée de professeurs et chercheurs de HEC Montréal engagés dans des recherches en équipe avec des collègues de l'Université McGill et de l'École Polytechnique de Montréal, le Centre comporte maintenant une cinquantaine de membres, plus d'une vingtaine de professionnels de recherche et stagiaires post-doctoraux et plus de 200 étudiants des cycles supérieurs. Les activités du GERAD ont pris suffisamment d'ampleur pour justifier en juin 1988 sa transformation en un Centre de recherche conjoint de HEC Montréal, de l'École Polytechnique de Montréal et de l'Université McGill. En 1996, l'Université du Québec à Montréal s'est jointe à ces institutions pour parrainer le GERAD.

Le GERAD est un regroupement de chercheurs autour de la discipline de la recherche opérationnelle. Sa mission s'articule autour des objectifs complémentaires suivants :

- la contribution originale et experte dans tous les axes de recherche de ses champs de compétence ;
- la diffusion des résultats dans les plus grandes revues du domaine ainsi qu'auprès des différents publics qui forment l'environnement du Centre ;
- la formation d'étudiants des cycles supérieurs et de stagiaires post-doctoraux ;
- la contribution à la communauté économique à travers la résolution de problèmes et le développement de coffres d'outils transférables.

Les principaux axes de recherche du GERAD, en allant du plus théorique au plus appliqué, sont les suivants :

- le développement d'outils et de techniques d'analyse mathématiques de la recherche opérationnelle pour la résolution de problèmes complexes qui se posent dans les sciences de la gestion et du génie ;
- la confection d'algorithmes permettant la résolution efficace de ces problèmes ;
- l'application de ces outils à des problèmes posés dans des disciplines connexes à la recherche opérationnelle telles que la statistique, l'ingénierie financière, la théorie des jeux et l'intelligence artificielle ;
- l'application de ces outils à l'optimisation et à la planification de grands systèmes technico-économiques comme les systèmes énergétiques, les réseaux de télécommunication et de transport, la logistique et la distributive dans les industries manufacturières et de service ;

- l'intégration des résultats scientifiques dans des logiciels, des systèmes experts et dans des systèmes d'aide à la décision transférables à l'industrie.

Le fait marquant des célébrations du 25^e du GERAD est la publication de dix volumes couvrant les champs d'expertise du Centre. La liste suit : **Essays and Surveys in Global Optimization**, édité par C. Audet, P. Hansen et G. Savard ; **Graph Theory and Combinatorial Optimization**, édité par D. Avis, A. Hertz et O. Marcotte ; **Numerical Methods in Finance**, édité par H. Ben-Ameur et M. Breton ; **Analysis, Control and Optimization of Complex Dynamic Systems**, édité par E.K. Boukas et R. Malhamé ; **Column Generation**, édité par G. Desaulniers, J. Desrosiers et M.M. Solomon ; **Statistical Modeling and Analysis for Complex Data Problems**, édité par P. Duchesne et B. Rémillard ; **Performance Evaluation and Planning Methods for the Next Generation Internet**, édité par A. Girard, B. Sansò et F. Vázquez-Abad ; **Dynamic Games : Theory and Applications**, édité par A. Haurie et G. Zaccour ; **Logistics Systems : Design and Optimization**, édité par A. Langevin et D. Riopel ; **Energy and Environment**, édité par R. Loulou, J.-P. Waub et G. Zaccour.

Je voudrais remercier très sincèrement les éditeurs de ces volumes, les nombreux auteurs qui ont très volontiers répondu à l'invitation des éditeurs à soumettre leurs travaux, et les évaluateurs pour leur bénévolat et ponctualité. Je voudrais aussi remercier Mmes Nicole Paradis, Francine Benoît et Louise Letendre ainsi que M. André Montpetit pour leur travail expert d'édition.

La place de premier plan qu'occupe le GERAD sur l'échiquier mondial est certes due à la passion qui anime ses chercheurs et ses étudiants, mais aussi au financement et à l'infrastructure disponibles. Je voudrais profiter de cette occasion pour remercier les organisations qui ont cru dès le départ au potentiel et la valeur du GERAD et nous ont soutenus durant ces années. Il s'agit de HEC Montréal, l'École Polytechnique de Montréal, l'Université McGill, l'Université du Québec à Montréal et, bien sûr, le Conseil de recherche en sciences naturelles et en génie du Canada (CRSNG) et le Fonds québécois de la recherche sur la nature et les technologies (FQRNT).

Georges Zaccour
Directeur du GERAD

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Contributing Authors

MUSTAPHA BABIKER
Arab Planning Institute, Kuwait
babiker@mit.edu

VALENTIN BERTSCH
University of Karlsruhe, Germany
valentin.bertsch@wiwi.uni-
karlsruhe.de

FRANCISCO CABO
Universidad de Valladolid, Spain
pcabo@eco.uva.es

DANAI DIAKOULAKI
National Technical University of Athens,
Greece
diak@chemeng.ntua.gr

LAURENT DROUET
HEC Geneva, Switzerland
laurent.drouet@hec.unige.ch

JUTTA GELDERMANN
University of Karlsruhe, Germany
jutta.geldermann@wiwi.uni-
karlsruhe.de

ALAIN HAURIE
HEC Geneva, Switzerland
alain.haurie@hec.unige.ch

MARK JACCARD
Simon Fraser University, Canada
jaccard@sfu.ca

HENRY D. JACOBY
Massachusetts Institute of Technology,
USA
hjacob@mit.edu

AMIT KANUDIA
McGill University and GERAD, Canada
amit.kanudia@gerad.ca

PIERRE L. KUNSCH
Vrije Universiteit Brussel, Belgium
pkunsch@vub.ac.be

MARYSE LABRIET
Université du Québec à Montréal and
GERAD, Canada
maryse.labriet@gerad.ca

RICHARD LOULOU
McGill University and GERAD, Canada
richard@haloa.ca

ALAN S. MANNE
Stanford University, USA
asmanne@attglobal.net

GUIOMAR MARTÍN-HERRÁN
Universidad de Valladolid, Spain
guiomar@eco.uva.es

MARÍA PILAR MARTÍNEZ-GARCÍA
Universidad de Murcia, Spain
pilarmg@um.es

GIORGOS MAVROTAS
National Technical University of Athens,
Greece
mavrotas@chemeng.ntua.gr

SERGEY PALTSEV
Massachusetts Institute of Technology,
USA
paltsev@mit.edu

PIERRE-OLIVIER PINEAU
University of Victoria, Canada
ppineau@uvic.ca

JOHN M. REILLY
Massachusetts Institute of Technology,
USA
jreilly@MIT.EDU

OTTO RENTZ
University of Karlsruhe, Germany
otto.rentz@wiwi.uni-karlsruhe.de

RICHARD G. RICHEL
Electricity Producers Research Institute
(EPRI), USA
richchels@epri.com

STEPHAN SCHOTT
Carleton University, Canada
sschott@ccs.carleton.ca

JOHAN SPRINGAEL
University of Antwerp, Belgium
johan.springael@ua.ac.be

PHILIPPE THALMANN
EPFL, Switzerland
philippe.thalman@epfl.ch

MARTIN TREITZ
University of Karlsruhe, Germany
martin.treitz@wiwi.uni-karlsruhe.de

KATHLEEN VAILLANCOURT
McGill University and GERAD, Canada
kvailancourt@hotmail.com

DENISE VAN REGEMORTER
Katholieke Universiteit Leuven,
The Netherlands
denise.vanregemorter@econ.
kuleuven.ac.be

MARC VIELLE
Université Toulouse 1 Sciences Sociales,
France
marc.vielle@univ-tlse1.fr

LAURENT VIGUIER
EPFL, Switzerland
laurent.viguiier@epfl.ch

JEAN-PHILIPPE WAAUB
Université du Québec à Montréal and
GERAD, Canada
jean-philippe.waaub@gerad.ca

Preface

This volume on energy and environmental modeling describes a broad variety of modeling methodologies, embodied in models of varying scopes and philosophies, ranging from top-down integrated assessment models to bottom-up partial equilibrium models, to hybrid models. Other articles call upon multicriteria and differential games methodologies.

In Chapter 1, F. Cabo, G. Martín-Herrán and M.P. Martínez-García analyze the existence of a sustained growth in two regions connected by trade. This trade flows from a resource-based economy, the South, to a developed country, the North. The natural resource intensive good produced in the South is used as a input in the North which produces a final consumption good. Investment in this latter is devoted either to increase the capital stock in the final output sector or to improve the productivity of the resource-based input in an environmental R&D sector. A differential game between these two regions allows us to determine endogenously the price of the traded good. The balanced growth paths that ensure a permanent growth of consumption in both regions not exhausting natural resources are characterized. The transition period towards these balanced paths is also presented.

In Chapter 2, L. Drouet, A. Haurie, M. Labriet, P. Thalmann, M. Vielle, and L. Viguier report on the coordinated development of a regional module within a world computable general equilibrium model (CGEM) and of a bottom-up energy-technology-environment model (ETEM) describing long term economic and technology choices for Switzerland to mitigate GHG emissions in accordance with Kyoto and post-Kyoto possible targets. The chapter discusses different possible approaches for coupling the two types of models, and describes a scenario built from a combined model where the residential sector is described by the bottom-up model and the rest of the economy by the CGEM. Results are presented and commented.

In Chapter 3, J. Geldermann, M. Treitz, V. Bertsch and O. Rentz present the contribution to the RODOS system of a multicriteria and multi-stakeholder decision analysis model based on the tool Web-HIPRE in ensuring the transparency of decision processes within off-site emergency management. The real-time on-line RODOS decision support system helps manage conflicting objectives related to a nuclear or radiological accident in Europe. Special attention is paid to the evaluation of

long term countermeasures. The RODOS modular structure and client server functionality are well suited for integrating this alternative evaluation model involving judgments and preferences. Web-HIPRE offers both MAVT and AHP elicitation methods for decision support with multiple stakeholders. A case study consisting of a hypothetical accident scenario illustrates the benefits from the model. Sensitivity analyses are also performed.

In Chapter 4, by M. Jaccard, the premise is that energy-economy models are especially useful to policy makers if they indicate the effect of energy and environment policies on the technology choices of businesses and consumers - what is called endogenous modeling of technological change. The CIMS hybrid model described in this chapter is technologically explicit, like a bottom-up engineering model, but also behaviorally realistic, like a top-down macro-economic model. With this combination, it can simulate packages of policies that include economy-wide emissions charges and technology-specific regulations and subsidies. Recent improvement to the model involves estimation of its behavioral parameters from discrete choice surveys of business and consumer technology preferences.

In Chapter 5, A. Kanudia, M. Labriet, R. Loulou, K. Vaillancourt, and J.-Ph. Waaub present the new multiregional global MARKAL-TIMES model and several recent applications to global energy-environment issues. The development of the model was motivated by the need to analyze international energy and environmental issues such as climate change, using a detailed, technology rich modeling framework. Three applications are described. First, the model is applied to conduct the cost-effectiveness analysis of Greenhouse Gas (GHG) emission abatement, whereby constraints on CO₂ emissions are added to the base case formulation. The model then computes the cost-efficient response of the energy system to these emission targets. Another application addresses the issue of "who pays" for emission reductions (whereas the cost-effectiveness analysis addressed the "who acts" issue). More precisely, the model is used to devise and evaluate certain allocation rules for attributing initial emission rights to regions in a cap-and-trade system. The third application uses World MARKAL in a cost-benefit mode, i.e. the model is augmented with damage costs resulting from climate change, and the composite model is run without any pre-set targets on emissions or concentration using different scenarios depending on whether the regions cooperate or not when confronted to the threat of damages. This last application makes systematic use of game theoretic concepts.

In Chapter 6, P.L. Kunsh and J. Springael present a simulation technique (system dynamics, VENSIM code) that provides insights to public policy makers regarding the complex issues related to a market of tradable CO_2 permits. A dynamic model is designed to overcome limitations due to static and deterministic pollution-control models. The model aims at answering questions related to (1) the number of permits that should be fed into the market each year, and (2) the priorities to be given to abatement technologies. A deterministic dynamic permit model is first proposed. Then, the model addresses the uncertainties related to abatement technologies marginal cost curves as a function of the abatement levels. Fuzzy reasoning techniques rather than probability based risk approaches are used to reconcile (aggregate) the diverging expert opinions (credibility scoring levels) and to take into account uncertainties on marginal abatement costs. This model runs on a rather short horizon (five years) assuming regular updating of data.

In Chapter 7, A. Manne and R. Richels present the most recent incarnation of the MERGE general equilibrium model. MERGE is a model for estimating the regional and global effects of greenhouse gas reductions. It quantifies alternative ways of thinking about climate change. The model contains submodels governing: the domestic and international economy; energy-related emissions of greenhouse gases; non-energy emissions of GHGs; and global climate change - market and non-market damages. These submodels are fully integrated in a series of regional general equilibrium models, each consisting of a constrained non-linear convex optimization program. A global equilibrium is computed via an iterative sequence of optimizations of the regional models.

In Chapter 8, G. Mavrotas and D. Diakoulaki present the application of Mixed Integer Multiple Objective Linear Programming (MIMOLP) to the power generation expansion problem of Crete for the period 2005-2020. This modelling approach was motivated by the need to effectively incorporate environmental (CO_2 emissions) and social considerations in energy decisions. The model with integer variables is solved using the Multi-Criteria Branch and Bound (MCBB) method which generates all the efficient points for problems with combinatorial features. The Crete case study is described and the model is used to compute a cost-efficient configuration of the energy systems (capacity expansion) that complies with the objective of minimizing CO_2 emissions, and exploits the flexibility offered by the forthcoming emission trading mechanism. The results are synthesized by a trade-off curve between CO_2 emissions and costs over the time horizon. It shows that considerable emission reductions can be achieved at relatively low costs. Each solution corresponds to an

investment plan provided by the model as other useful information for decision makers.

In Chapter 9, S. Paltsev, H. Jacoby, J. Reilly, L. Viguier, and M. Babiker study the role played by existing fuel taxes in determining the welfare effects of exempting the transportation sector from measures to control greenhouse gases. To evaluate this role, the MIT Emissions Prediction and Policy Analysis (EPPA) model was modified to disaggregate the household transportation sector. This improvement requires an extension of the GTAP data set that underlies the model. The revised and extended facility is then used to compare economic costs of cap-and-trade systems differentiated by sector, focusing on two regions: the USA where the fuel taxes are low, and Europe where the fuel taxes are high. The authors find that the interplay between carbon policies and pre-existing taxes leads to different results in these regions: in the USA exemption of transport from such a system would increase the welfare cost of achieving a national emissions target, while in Europe such exemptions will correct pre-existing distortions and reduce the cost.

In Chapter 10, P.-O. Pineau and S. Schott study how electricity pricing and technology choices can affect greenhouse gases (GHG) emissions and capacity requirements. They use an innovative approach to model the electricity market under a time of use tariff, where they account for the cross-price elasticity of demand between peak and off-peak periods. Applying their model to the Canadian province of Ontario, they show that the combined effect of time of use prices and an "allowance price" for GHG emissions could cut capacity requirements by 20% to 30%, while price increases would be moderate if nuclear technology is chosen. If natural gas or coal technologies were chosen, off-peak price would increase by at least 13% while peak price would increase by a minimum of 20%. In terms of absolute GHG emissions, a reduction compared to the 2003 situation is only possible if coal is phased out and replaced with either nuclear or natural gas power plants. The contribution of this chapter is in its integration of two critical elements required to analyze the electricity sector: (1) how tariff structures and technologies have significant impacts on demand and capacity requirements, and (2) how price and cross-price elasticities are important demand management tools.

In Chapter 11, D. Van Regemorter describes an approach to integrate the interactions between environmental targets in an energy system optimization model, MARKAL/TIMES, so as to allow for an integrated policy evaluation. The environmental problems considered are global warming and local air pollution, both linked to energy production and

consumption, and their abatement possibilities are interrelated. This explains the choice of a partial equilibrium model for the energy market to study these policy questions. With the damage generated by emissions integrated in its objective function, the model allows to optimally compute trade-offs between environment protection and economic costs. The MARKAL/TIMES model and the integration of the externalities are described. The data used for the quantification and the valuation of the externalities linked to the supply and use of energy rely heavily on the ExternE EU project dedicated to the evaluation of the external cost of energy. An application with the Belgian MARKAL/TIMES model is presented.

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The Editors would like to express their gratitude to the authors for their contributions and timely responses to our comments and suggestions. We wish also to thank Francine Benoît and André Montpetit for their expert editing of the volume.

RICHARD LOULOU
JEAN-PHILIPPE WAAUB
GEORGES ZACCOUR

Chapter 1

NORTH–SOUTH TRADE AND THE SUSTAINABILITY OF ECONOMIC GROWTH: A MODEL WITH ENVIRONMENTAL CONSTRAINTS

Francisco Cabo
Guiomar Martín-Herrán
María Pilar Martínez-García

Abstract We present a model of trade between two different regions, North and South. The South specializes in a natural resource intensive good which is sold to and used as an input in the North. Assuming an environmental R&D sector in the North, which increases the efficiency of the traded good, the North–South trade and the natural resource management are modeled in a dynamic way. The existence of a sustained growth in the North, which allows a permanent growth of consumption in the South without exhausting natural resource, is proved. Transitional dynamics is also studied.

1. Introduction

Through the ages, countries have reached beyond their own borders to obtain raw and essential materials. Today's surer communications and increased trade have greatly enlarged this process and endowed it with far-reaching ecological implications. Thus, the pursuit of sustainability needs to take international economic relations into account. Moreover, the conservation of ecosystems on which the global economy depends must be guaranteed. International economic exchanges must also be made beneficial for all countries involved, ensuring the improvement of living conditions in poorer countries. Today, for many developing countries, neither of these conditions is met.

In this paper we present a model of trade between two different regions, North and South, and we studIIIn our model, international trade

is the channel through which part of the economic growth in the North is transmitted to the South. We shall prove that income can grow constantly in both countries while at the same time guaranteeing environmental conservation.

The sustainability of economic growth has been a subject of great attention in recent years. In the literature on endogenous growth several authors include environmental variables within their models in order to study economic policies that guarantee sustainable growth (Gradus and Smulders, 1993; Huang and Cai, 1994; Ligthart and van del Ploeg, 1994; Verdier, 1995; Bovenberg and Smulders, 1995, 1996; Musu, 1996; Bovenberg and Mooij, 1997, among others). However, all these models focus on an isolated country and do not consider trade relations with other countries. In fact, most of the papers on endogenous growth regarding environmental problems do not consider more than one region. Even those that consider several countries assume that all countries are identical, see, for example, Hettich (2000).

Unlike the previous works, and following Cabo, Escudero and Martín-Herrán (2002) we present a model with two different regions, North and South, that trade with each other. Within this framework of North–South trade and following the static approach laid down, for example, by Chichilnisky (1994); Panayotou (1994); Copeland and Taylor (1994, 1995), and Cabo (1999), we assume that the South specializes in a natural resource intensive good which is used as a productive input in the North. The North–South trade and the management of the environment is modeled in a dynamic way so that sustainable economic growth can be analyzed. A unique renewable resource is harvested to produce a resource intensive good. The North buys this good in the international market and uses it as an input to produce final output. Hereinafter, we will refer to this intermediate good, traded from South to North, as the resource-based input.

More ecological production technologies must be developed to sustain the current standard of living without depleting natural resources. One of the main achievements of the endogenous growth theory is to explain technological change endogenously. Romer (1990) incorporates R&D activities in an endogenous growth model. Grossman and Helpman (1991) (Chapters 3 and 4), Aghion and Howitt (1992) and Barro and Sala-i-Martin (1995) (Chapters 6, 7 and 8) are also classical references. In these models technological progress is attained by devoting resources to research activities. Technological knowledge either augments labor productivity in the final output production function, expands product variety or improves product quality. Thus, if human inventiveness has no limit the economy will grow indefinitely. In our paper, technolog-

ical knowledge enhances the productivity of the resource-based input. In the literature on environment and economic growth, Bovenberg and Smulders (1995, 1996) and Musu (1996) consider a technological knowledge of this kind. Although Musu (1996) assumes that the technological progress is a by-product of capital accumulation, we, following Bovenberg and Smulders (1995, 1996), incorporate an environmental R&D sector which is devoted to increasing the efficiency of the resource-based input in the North's output production.

The engine of growth in this paper is not based uniquely on technological progress. A second source of growth stems from a learning-by-doing effect which indefinitely raises the productivity of the labor force. We follow the idea of Romer (1986) to eliminate diminishing returns to the factors, regarding knowledge as a by-product of capital accumulation. A firm that increases its physical capital learns at the same time how to produce more efficiently. Additionally, we assume that each firm's stock of knowledge is a public good which spills over instantly across all other firms and at no cost. This leads labor productivity to be dependent not on each firm's capital stock but on the whole economy's one. These two effects are taken into account in our paper to explain how labor affects the North's production function.

The model is stated as a differential game between North and South. The South possesses a natural resource and decides the extraction effort required to produce the resource-based input traded to the North. The South maximizes its discounted utility, which is an increasing function of its total income, given by the monetary value of the intermediate traded good. On the other hand, the North fixes the consumption, the demand for the resource-based input and the portion of the labor force devoted either to the final output sector or to the environmental R&D sector. This region equally maximizes its discounted utility, which is a function of its intertemporal consumption.

In this North–South trade model, we focus on the circumstances under which a balanced path follows from the optimal decisions of both players. We shall prove that along this balanced path Northern optimal choices regarding consumption, demand for the resource-based input and the labor share devoted either to the final output or to the R&D sector, guarantee the economic growth without being so demanding as to oblige the South to deplete its natural resource. Likewise, along this balanced path, the optimal extraction effort in the South is high enough to sustain economic growth but without endangering the conservation of the renewable natural resource. In consequence we refer to this balanced growth path as sustainable.

Most endogenous growth models which take into account environmental quality restrict the analysis to the steady state, which represents the sustainable growth path. In steady state, all variables grow at their long-run rates, and the use of natural resources is sustainable. However, in the real world, economies are not usually on sustainable growth paths, there might be imbalances between different sectors, or, as usually happens, the use of the natural resource might be above its sustainable level. Two aspects must be taken into account. Firstly, stability analysis will show whether there are transition paths to sustainability. Secondly, it is interesting to know how the economies behave during transitions. Martínez-García (2001, 2003) has proved that balanced paths in endogenous growth models are either unstable or possesses the saddle point property. In this paper we shall prove that this property is satisfied. Therefore, there exist transition economic policies that lead the economies to sustainability. In addition, we shall undertake a transitional dynamic analysis, which studies the dynamics of the model in the short and medium terms when it is not on the balanced path and shows losses and gains of either growth or welfare during the adaptation process.¹ This analysis has been largely ignored in the literature due to its complexity. Usually the study can be analytically carried out for the simplest models, although more complex specifications, such as our formulation, would require the implementation of numerical methods. In our paper we follow the numerical algorithm developed by Mulligan and Sala-i-Martin (1991, 1993), known as the time elimination method, which is well suited to characterizing the transitional dynamics of endogenous growth models.

The plan of this paper is as follows: in Section 2 we model the economies of two different regions and the trade relationships between them. Optimal paths in both regions as well as the equilibrium price of the traded good are studied in Section 3. We focus on the balanced path in Section 4, while transitional dynamics is analyzed in Section 5. Finally, we present our conclusions in Section 6.

2. The model

Two different regions that trade between themselves are modeled. The Northern region produces a unique final output, which can be used either to consume or to increase the stock of physical capital. The production process uses capital, labor and a resource-based input which is

¹For more information about the reasons to study the transitional dynamics see, for example, Hettich (2000) and Steger (2000).

produced in the South by harvesting from a renewable natural resource. We assume a learning-by-doing effect, which implies that the work force experience will have a positive effect on the labor productivity in the North. That is, when a firm invests in physical capital, at the same time its workers learn how to produce more efficiently.

In addition to the final output sector and following Bovenberg and Smulder (1995, 1996), we introduce an environmental R&D sector. This pure investment sector produces a technology that enhances the efficiency of the resource-based input in the production of final output. Thus the production function is given by:

$$Y(t) = K(t)^\alpha (h(t)R_N(t))^\beta (K(t)v(t)\bar{L})^{1-\alpha-\beta},$$

$$\alpha, \beta, \alpha + \beta \in (0, 1), \quad (1.1)$$

where $K(t)$ represents the capital stock, $R_N(t)$ the resource-based input, \bar{L} the constant labor and $h(t)$ the technological knowledge or efficiency of $R_N(t)$ in the production of $Y(t)$. All variables are evaluated at time t . From the total labor, the portion $v(t) \in (0, 1)$ is devoted to final output production. The remainder, $1 - v(t)$, goes to the environmental R&D sector increasing the productivity of the resource-based input according to the dynamic equation:

$$\dot{h}(t) = \eta(1 - v(t))h(t)\bar{L}, \quad \eta > 0. \quad (1.2)$$

The dynamics for the capital stock is given by the total production,² $Y(K, hR_N, Kv\bar{L})$, minus consumption, c , and the cost of the resource-based input,³ pR_N :

$$\dot{K} = Y(K, hR_N, Kv\bar{L}) - pR_N - c, \quad (1.3)$$

where p is the price of the resource-based input.

The North maximizes its intertemporal utility, discounted at the rate of time preference $\rho > 0$. It chooses consumption, demand for the resource-based input and the labor share devoted to the final output sector. The maximization problem is subject to the dynamics of the capital stock and the technological knowledge.

$$\max_{c, R_N, v} \int_0^\infty e^{-\rho t} \ln(c) dt,$$

²Henceforth, we omit time arguments when no confusion is caused by doing so.

³We assume that no asset can be internationally traded. Furthermore, all firms maximize current profits, which proves that all firms behave in the same way with respect to the capital per labor unit and the resource-based input per labor unit. All these hypotheses, together with the perfect competition assumption, allow us to write expression (1.3).

$$\begin{aligned} \text{s.t. } \dot{K} &= K^\alpha (hR_N)^\beta (Kv\bar{L})^{1-\alpha-\beta} - pR_N - c, & K(0) &= K_0, \\ \dot{h} &= \eta(1-v)h\bar{L}, & h(0) &= h_0, \\ c, R_N &\geq 0, & v &\in (0, 1), \end{aligned}$$

where a logarithmic specification for the North's utility is assumed.

As far as the South is concerned, it harvests from a renewable natural resource. With the extracted amount it produces a resource-based input which is traded to the North and used to produce final output. The dynamics of the natural resource stock, s , is defined by a differential equation of the type described by Clark (1990). This equation equals the evolution of the resource to the natural resource growth, $F(s)$, minus the human depletion. The logistic growth function is considered to define $F(s)$. On the other hand, the rate of harvesting is proportional to the extraction effort and the stock of the natural resource:

$$\dot{s} = rs(1 - s/cc) - qEs, \quad (1.4)$$

$$0 \leq s \leq cc, \quad (1.5)$$

where r is the intrinsic growth rate, cc is the environmental carrying capacity or saturation level, q is a proportionality parameter and E is the extraction effort allocated to the natural resource.

The production of the resource-based input depends on the stock and the extraction effort of the natural resource:

$$R_S = \Phi(E, s) = Es^\theta, \quad (1.6)$$

where $\theta \in (0, 1)$ and $\Phi_E, \Phi_s > 0$. A higher stock of the natural resource leads to a higher productivity of the total effort and consequently to a greater production of the resource-based input. In this formulation θ represents the elasticity of the resource-based input with respect to the stock of the natural resource, $\varepsilon_s^{R_S}$.

The South has to decide the harvesting effort, E . This region maximizes its stream of utility discounted at rate $\rho' > 0$. No investment process occurs in the South, all income is consumed and the utility equals the logarithmic transformation of this consumption, $\ln(pR_S)$.

The maximization problem for this region can be expressed as:

$$\begin{aligned} \max_E \int_0^\infty e^{-\rho't} \ln(pEs^\theta) dt \\ \text{s.t. } \dot{s} &= rs(1 - s/cc) - qEs, & s(0) &= s_0, \\ E &\geq 0, & \theta &\in (0, 1). \end{aligned}$$

As well as in the North, the intertemporal elasticity of the South's utility is also constant and equal to one.

In the specified North–South trade model, each region maximizes utility and takes as given the world market price of the resource-based input. This fact suggests that either both regions are myopic (unaware of the effect of their decisions upon the price of the traded good), or that each of them represents a price-taker small open economy (the North a developed country and the South a developing one).

3. North and South’s optimal paths

Next we characterize optimal paths for both regions as well as the equilibrium price of the traded good. This price stems from equating South’s supply and North’s demand.

The current-value Hamiltonian for the North is given by,

$$H_N = \ln(c) + m_K [K^{1-\beta} (hR_N)^\beta (v\bar{L}^{1-\alpha-\beta} - pR_N - c] + m_h [\eta(1-v)h\bar{L}], \quad (1.7)$$

where m_K and m_h denote the shadow prices of the capital stock and the technological knowledge, respectively.

The first order conditions for an interior maximum are:

$$1/c = m_K, \quad (1.8a)$$

$$p = \beta h K^{1-\beta} (hR_N)^{\beta-1} (v\bar{L})^{1-\alpha-\beta}, \quad (1.8b)$$

$$m_h \eta h = m_K (1 - \alpha - \beta) K^{1-\beta} (hR_N)^\beta (v\bar{L})^{-\alpha-\beta}, \quad (1.8c)$$

$$\dot{m}_K = m_K [\rho - (1 - \beta) (hR_N/K)^\beta (v\bar{L})^{1-\alpha-\beta}], \quad (1.8d)$$

$$\dot{m}_h = m_h [\rho - \eta\beta/(1 - \alpha - \beta) v\bar{L} - \eta(1 - v)\bar{L}]. \quad (1.8e)$$

Equation (1.8a) says that the marginal utility of consumption should equal the shadow price of the capital stock (the marginal benefit of increasing the capital stock in one unit). The marginal productivity of the resource-based input equals its price in equation (1.8b). Condition (1.8c) states that the ratio between the marginal benefit of an additional unit of capital, m_K , and the marginal benefit of an additional unit of technological knowledge, m_h , is equal to the marginal effect on the technological growth of an extra-unit of labor in this sector divided by the marginal effect on the capital growth of an additional unit of labor in the final output sector. The necessary condition (1.8d) shows that the marginal productivity of the capital stock plus the rate of change of the marginal benefit of an additional unit of capital should equal the depreciation rate. At the same time the value of the marginal productivity of the technology plus this factor’s rate of growth plus the rate of change of the marginal benefit of an additional unit of technology should also be

equal to the depreciation rate. This condition corresponds to equation (1.8e).

From conditions (1.8b) and (1.8c) the optimal demand for the resource-based input and the optimal labor share in the final output sector can be written in terms of the price and the state and costate variables:

$$\begin{aligned} R_N &= \beta^{1/\alpha} [(1 - \alpha - \beta)/(\eta\beta)]^{(1-\alpha-\beta)/\alpha} [m_K/m_h]^{(1-\alpha-\beta)/\alpha} \\ &\quad \times [K/h]^{(1-\beta)/\alpha} [p/h]^{-1-\beta/\alpha}, \\ v\bar{L} &= \beta^{1/\alpha} [(1 - \alpha - \beta)/(\eta\beta)]^{(1-\beta)/\alpha} [m_K/m_h]^{(1-\beta)/\alpha} \\ &\quad \times [K/h]^{(1-\beta)/\alpha} [p/h]^{-\beta/\alpha}. \end{aligned} \quad (1.9)$$

The current-value Hamiltonian function for the South reads:

$$H_S = \ln(pEs^\theta) + m_s[rs(1 - s/cc) - qEs], \quad (1.10)$$

where m_s is the South's shadow price of the natural resource stock.

The first order conditions for an interior maximum in this region are:

$$E = 1/(qm_s s), \quad (1.11a)$$

$$\dot{m}_s = [\rho' - r(1 - 2s/cc) + qE]m_s - \theta/s. \quad (1.11b)$$

The optimal harvesting effort is negatively related to the current stock of the natural resource. However, at the same time, this effort is sensitive to the shadow price of the natural resource. Thus, the more highly the South values this stock, the lower the extraction effort.

From equation (1.6) and the optimal effort in (1.11a), the optimal supply of the resource-based input also depends negatively on m_s and s :

$$R_S = 1/(qm_s s^1 - \theta). \quad (1.12)$$

The optimal supply of the resource-based input is independent of its price, p , or equivalently, the price elasticity of the supply for the resource-based input is zero. This supply determines the amount sold to and used in the North, and pR_S matches total income in the South.

By equating South's supply in (1.12) and North's demand in (1.9), the equilibrium price for the resource-based input can be written:

$$\begin{aligned} p &= \Psi [qm_s s^{1-\theta}]^{\alpha/(\alpha+\beta)} [m_K/m_h]^{(1-\alpha-\beta)/(\alpha+\beta)} \\ &\quad \times [K/h]^{(1-\beta)/(\alpha+\beta)} h, \end{aligned} \quad (1.13)$$

where constant $\Psi \equiv \beta[(1 - \alpha - \beta)/\eta]^{(1-\alpha-\beta)/(\alpha+\beta)} > 0$.

The first term in brackets in (1.13) represents the negative relationship between this price and the supply of the resource-based input. The remainder stems from the demand side. Note that the higher the relative value of the capital stock with respect to the value of the technological knowledge, m_K/m_h , the lower the labor share devoted to the environmental R&D sector, and consequently the lower the growth in the resource-based input productivity. Thus, the demand for this input increases and so does its price. This is what the second term in brackets highlights. On the other hand, higher physical capital enhances the demand for the resource-based input and, consequently, its price. Finally, the effect of a higher technological knowledge is twofold. It increases the efficiency of R_N , reducing the demand for this good and, at the same time, it speeds up the growth rate of the technology accumulation. This leads to a reduction in the labor share in the R&D sector and an increment in the final output sector. Again, a higher v increases the demand for the resource-based input. The former effect reduces the price while the latter raises it. If the output elasticity of the labor factor, $1 - \alpha - \beta$, is greater than the output elasticity of the resource-based factor, β , then the former effect is stronger and the price falls.

4. The balanced path

A balanced path is a trajectory where all variables grow at constant rates (which may in some cases be zero). The labor share devoted to the final output sector takes values between zero and one, while the natural resource stock has to be positive and lower than its carrying capacity. Thus, since v and s are upper and lower bounded, they cannot grow indefinitely at non-zero rate. These variables must be constant on a balanced path. On the other hand, since the production of the resource-based input depends on the extraction of the natural resource, which is bounded, this good also must remain constant on a balanced path. From now on we refer to this intermediate good as R , given that in equilibrium $R = R_S = R_N$.

First order condition (1.11b) for the South's maximization problem can be rewritten as:

$$\dot{m}_s/m_s = \rho' - \partial\dot{s}/\partial s - \theta/(sm_s).$$

Thus, from the optimal resource-based input in (1.12) and the dynamics of m_s , the growth rate of R can be deduced:

$$\begin{aligned} \dot{R}/R &= -\dot{m}_s/m_s - \dot{s}/s = -\rho' + \theta qE + \partial\dot{s}/\partial s - (1 - \theta)\dot{s}/s \\ &= -\rho' + \theta F(s)/s + \partial\dot{s}/\partial s - \dot{s}/s. \end{aligned} \quad (1.14)$$

Given the logistic growth function, $F(s)$, considered for the natural resource, the natural growth rate per unit of resource, $F(s)/s$, is greater than the marginal growth rate $F'(s)$ at any point. Thus, $\dot{s}/s - \partial\dot{s}/\partial s$ is positive, and from (1.14) a necessary condition for a non-zero constant resource-based input is $\theta F(s)/s > \rho'$. This inequality states that the output elasticity with respect to the stock of the natural resource in the Southern production process, $\varepsilon_s^{R_s} = \theta$, times the natural growth rate per unit of resource, surpasses the rate of time preference. A necessary condition for this inequality is:

$$\theta r - \rho' > 0. \quad (1.15)$$

From equation (1.14) and the dynamics of s , the growth rate of the resource-based input can be rewritten as a function of the resource stock:

$$\dot{R}/R = \theta r - \rho' - (1 + \theta)rs/cc.$$

From this equation, the resource-based input remains unchanged when the natural resource stock takes the constant value:

$$s^* = (\theta r - \rho')cc/[(1 + \theta)r], \quad (1.16)$$

which is feasible under condition (1.15). Conversely, when inequality (1.15) is not fulfilled the resource-based input falls indefinitely and no steady state is possible.

Next we turn our attention to the dynamics of the relevant variables in the North: consumption, capital stock, technological knowledge and the labor share devoted either to the R&D or the final output sector. The dynamics along the balanced path of the price of the resource-based input and the North and South's shadow prices are also studied.

By manipulating the North's first order conditions (1.8a) and (1.8d), the growth rate of consumption is,

$$\dot{c}/c = (1 - \beta)(hR/K)^\beta (v\bar{L})^{1-\alpha-\beta} - \rho.$$

As we have shown, v and R remain constant along a balanced path, thus the growth rate of consumption will also be constant if and only if h and K grow at the same rate.

Additionally, from (1.8c),

$$(1 - \alpha - \beta)(Rh/K)^\beta (v\bar{L})^{-\alpha-\beta}/\eta = (m_h/m_K)(h/K).$$

Therefore the shadow prices of the physical capital and the technological knowledge also grow identically along the balanced path.

>From equation (1.9), and taking into account that R , m_K/m_h and K/h do not change along the balanced path, then neither does the ratio

p/h . The price of the resource-based input grows at the same rate as h and hence at the same as K .

Moreover, since

$$\dot{K}/K = (hR/K)^\beta (v\bar{L})^{1-\alpha-\beta} - pR/K - c/K,$$

the capital growth rate will be constant if capital stock and consumption grow at the same rate.

Let us define new variables: $\tilde{c} = c/K$ and $\tilde{p} = p/K$. Note that a balanced path in the original variables corresponds to the steady state in variables \tilde{c} , v , \tilde{p} , R and s . Dynamic equations for these variables are⁴:

$$\dot{\tilde{c}} = \tilde{c}[\tilde{c} - \rho], \quad (1.17a)$$

$$\dot{v} = v[\beta\{\theta r - \rho' - (1 + \theta)rs/cc\} + \beta\eta + (\beta - 1)\tilde{p}R + (\beta - 1)\tilde{c}]/(\alpha + \beta) + \beta\eta v^2/(1 - \alpha - \beta), \quad (1.17b)$$

$$\dot{\tilde{p}} = \tilde{p}/(\alpha + \beta)[-\alpha\{\theta r - \rho' - (1 + \theta)rs/cc\} + \beta\eta + (\beta - 1)\tilde{p}R - (1 - \alpha - 2\beta)\tilde{c}], \quad (1.17c)$$

$$\dot{R} = R[\theta r - \rho' - (1 + \theta)rs/cc], \quad (1.17d)$$

$$\dot{s} = rs(1 - s/cc) - qRs^{1-\theta}. \quad (1.17e)$$

The equilibria for these five equations correspond to sustained growth paths. It is easy to show that there exists a unique balanced path with a constant and positive stock of the natural resource given by (1.16), as long as condition (1.15) is satisfied.

On the balanced path,⁵

$$\frac{\dot{m}_K^*}{m_K^*} + \frac{\dot{K}^*}{K^*} = \frac{\dot{m}_h^*}{m_h^*} + \frac{\dot{h}^*}{h^*} = \rho - \frac{\eta\beta}{1 - \alpha - \beta}v^* < \rho,$$

and therefore, transversality conditions, given by

$$\lim_{t \rightarrow +\infty} e^{-\rho t} m_K^*(t) K^*(t) = 0, \quad \lim_{t \rightarrow +\infty} e^{-\rho t} m_h^*(t) h^*(t) = 0, \quad (1.18)$$

are satisfied. On the other hand, since m_s and s remain constant on the balanced path, transversality condition

$$\lim_{t \rightarrow +\infty} e^{-\rho' t} m_s^*(t) s^*(t) = 0, \quad (1.19)$$

is also satisfied.

⁴For simplicity, from now on we assume \bar{L} equal to one.

⁵The star represents a variable on the balanced path.

Since the current-value Hamiltonians H_N and H_S , given by (1.7) and (1.10), are concave in state and control variables, necessary conditions for optimality, together with the transversality conditions (1.18) and (1.19), are also sufficient conditions for optimality. Moreover, given initial conditions for state variables, if we find a path converging toward the balanced path we have found an optimal solution.

4.1 Dynamics of the natural resource

As previously stated, a sustained growth path in the North involves the use of a constant amount of resource-based input. The same is true for the stock of the natural resource. Furthermore, by (1.12) the resource-based input and the natural resource stock remain motionless if and only if the shadow value of the natural resource is also constant.

First of all we analyze the dynamic system which displays the dynamics of the resource stock and its shadow value. The former is given by (1.4) and the latter by the first order conditions in (1.11b). From the optimal extraction effort in (1.11a), the system can be written as,

$$\dot{s} = rs(1 - s/cc) - 1/m_s, \quad (1.20a)$$

$$\dot{m}_s = m_s[\rho' - r(1 - 2s/cc)] + (1 - \theta)/s. \quad (1.20b)$$

The balanced path that guarantees a sustained economic growth is associated with constant values of s and hence m_s . Therefore, it is interesting to analyze the stability of the steady state for the system in (1.20a) and (1.20b). The steady state for the natural resource coincides with s^* in (1.16), which ensures a constant resource-based input. Additionally, the steady state for the shadow price takes the value,

$$m_s^* = (1 + \theta)^2 r / [(\theta r - \rho')(r + \rho')cc] = (1 + \theta) / [(r + \rho')s^*].$$

Under condition (1.15), which ensures a positive natural resource stock, the shadow price is also positive in steady state. Under this condition, the steady state shows a saddle point stability (property proved in the first Appendix).

At this point it is interesting to ascertain the dynamic relationship between the extraction effort and the resource stock. From (1.11a) and the differential equations (1.20a) and (1.20b) it is easy to derive the dynamics of the extraction effort,

$$\dot{E} = E[q\theta E - \rho' - rs/cc]. \quad (1.21)$$

The s - E phase plane in Figure 1.1, presents the unique interior steady state equilibrium, point A. If the initial condition is such that the resource stock is below its steady state value, s^* , then equilibrium A can

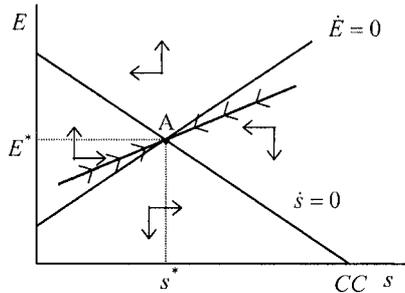


Figure 1.1. s - E phase plane

be reached as long as the South fixes a sufficiently low extraction effort, below the steady state equilibrium effort E^* . However, if this effort is too low, the system would diverge from the equilibrium to solutions with no extraction effort. Conversely, if the effort is too high, resource would fall while the effort would grow with no limit. Reverse reasoning applies when the natural resource stock is initially above s^* .

5. Transitional dynamics to sustainability

We would like to know whether a transition path to the sustainable growth solution exists. Moreover, along the transition period, growth rates of the relevant variables as well as the stock of the natural resource might not match their steady state values. Knowing how the model behaves in these transition periods is of great interest.

In our model, deviations of variable v from its steady state value represent imbalances between the final output and the innovation sectors, variable \tilde{p} measures imbalances in the price of the resource-based input, while variables R and s say if the exploitation of the natural resource is above or below its sustainable level. Initially, these variables might not be at their steady state values. The stability analysis in Section 5.1 studies the existence of transition paths converging on the steady state. In Section 5.2 we analyze transitional dynamics along these paths using numerical simulation and the time elimination method of Mulligan and Sala-i-Martin (1991, 1993).

5.1 Stability analysis

The five eigenvalues of the Jacobian matrix of system (1.17) evaluated on the unique balanced path with a constant stock of the natural