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ORTHOGONAL FREQUENCY  
DIVISION MULTIPLEXING  
FOR  
WIRELESS COMMUNICATIONS

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# ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING FOR WIRELESS COMMUNICATIONS

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# CONTENTS

<b>PREFACE</b>	<b>xi</b>
<b>1 INTRODUCTION</b>	<b>1</b>
<i>by Gordon Stüber</i>	
1.1 High Rate Wireless Applications	2
1.2 Wireless Channel	3
1.2.1 Path Loss and Shadowing	4
1.2.2 Multipath-Fading	5
1.3 Interference and Noise	6
1.4 Orthogonal frequency Division Multiplexing	8
1.4.1 OFDM Concept	8
1.4.2 Channel Capacity and OFDM	11
1.5 Synchronization and Channel Estimation	12
1.6 Peak-to-Average Power Ratio	13
1.7 MIMO OFDM	14
1.8 Outline of This Book	15
1.9 Summary and Further Reading	17
<b>2 BASIC CONCEPTS</b>	<b>19</b>
<i>by Ye (Geoffrey) Li</i>	
2.1 Basic OFDM	19
2.1.1 OFDM	19
2.1.2 FFT Implementation	21
2.1.3 Cyclic Extension, Power Spectrum, and Efficiency	21
2.1.4 Comparison with Single-Carrier	24
2.1.5 Design Example	25

2.1.6	Baseband versus Passband	25
2.2	Impairments of Wireless Channels to OFDM Signals	28
2.2.1	Time-Varying Impairments	28
2.2.2	Effect of Sampling Clock Offset	34
2.2.3	Effect of Timing Offset	35
2.2.4	Effect of Delay Spread	37
2.2.5	System Nonlinearity	39
2.3	Other Multicarrier Modulation	41
2.3.1	Orthogonal Approach	41
2.3.2	Filter Approach	43
2.3.3	General Multicarrier Modulation	45
<b>3</b>	<b>PERFORMANCE OPTIMIZATION</b>	<b>47</b>
	by <i>John M. Cioffi</i> and <i>Louise M. C. Hoo</i>	
3.1	History of OFDM Optimization	48
3.2	Channel Partitioning	49
3.2.1	Eigenfunction Transmission	51
3.2.2	Overlap, Excess Bandwidth, and Guard Period	54
3.2.3	Discrete-Time Channel Partitioning	54
3.2.4	Partitioning for OFDM	58
3.2.5	Stationary Equalization for Finite-length Partitioning	62
3.2.6	Finite-Length TEQ	63
3.3	Loading of Parallel Channels	66
3.3.1	Single-Channel Gap Analysis	66
3.3.2	A Single Performance Measure for Parallel Channels - Geometric SNR	70
3.3.3	Water-Filling Optimization	71
3.3.4	Margin Maximization	73
3.3.5	Loading Algorithm Classification	74
3.3.6	Computing Water Filling for RA Loading	75
3.3.7	Computing Water-Filling for MA Loading	77
3.3.8	Loading with Discrete Information Units	79
3.3.9	Sorting and Run-time Issues	91
3.3.10	Dynamic Loading	92
3.3.11	Multiuser Loading	94
3.4	Optimization through Coding	104
3.4.1	COFDM	105
3.4.2	Coded DMT	110

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3.4.3	Turbo DMT	111
3.5	Conclusion and Projections	112
<b>4</b>	<b>SYNCHRONIZATION</b>	<b>113</b>
	by <i>Sarah K. Wilson</i>	
4.1	Overview of Synchronization Schemes	114
4.1.1	Timing Offset Estimation	115
4.1.2	Frequency Offset Estimation	116
4.1.3	Acquisition Versus Tracking	117
4.2	Timing Offset Estimation	118
4.2.1	Pilot-based Methods	119
4.2.2	Non-Pilot-based Methods	126
4.3	Frequency Offset Estimation	131
4.3.1	Pilot-based Methods	133
4.3.2	Non-Pilot-based Methods	139
4.4	Joint Time- and Frequency Offset Estimation	139
4.4.1	Pilot-based	140
4.5	Sampling Clock Offset Estimation and Correction	142
4.6	Summary and Further Reading	143
<b>5</b>	<b>CHANNEL ESTIMATION</b>	<b>145</b>
	by <i>Ye (Geoffrey) Li</i>	
5.1	Introduction	145
5.1.1	Differential and Coherent Detection	145
5.1.2	OFDM Systems with Channel Estimator	147
5.1.3	Organization of the Chapter	149
5.2	Decision-Directed Channel Estimation	149
5.2.1	MMSE Estimation using Frequency Domain Correlation	150
5.2.2	MMSE Estimation using both Time- and Frequency Domain Correlations	153
5.2.3	Robust Estimation	155
5.2.4	Performance Evaluation	160
5.3	Pilot-Symbol-Aided Estimation	163
5.3.1	Grid Design	164
5.3.2	Direct Interpolation	165
5.3.3	Filtering Approaches	170
5.3.4	Performance Evaluation	172
5.4	MIMO Channel Estimation	176

5.4.1	Basic Channel Estimation	178
5.4.2	Optimum Training Sequences for Channel Estimation	180
5.4.3	Simplified Channel Estimation	183
5.4.4	Enhanced Channel Estimation	183
5.5	Summary and Further Reading	185
Appendix 5A:	Derivation of MMSE Channel Estimator	187
Appendix 5B:	MSE of MMSE Channel Estimator	189
Appendix 5C:	Mismatch Analysis	190
Appendix 5D:	Derivation of MMSE Transform based Approach	193
<b>6</b>	<b>PEAK POWER REDUCTION TECHNIQUES</b>	<b>199</b>
	by <i>Chintha Tellambura</i> and <i>Mathias Friese</i>	
6.1	Introduction	199
6.2	PAPR-Properties of OFDM Signals	200
6.2.1	Maximum PAPR of an $N$ Subcarrier OFDM Signal	204
6.2.2	Estimating True PAPR from Discrete Time Signals	205
6.3	PAPR-Reduction with Signal Distortion	208
6.3.1	Peak-Clipping Effect on System Performance	208
6.3.2	PAPR-Reduction by Clipping and Filtering	212
6.4	Limits for Distortionless PAPR-Reduction	215
6.5	Techniques for Distortionless PAPR-Reduction	217
6.5.1	Selective Mapping	217
6.5.2	Optimization Techniques	221
6.5.3	Modified Signal Constellation	231
6.5.4	PAPR-Reduction Effect on the System Performance	232
6.5.5	Algebraic Coding	234
6.6	PAPR Reduction for Multicarrier CDMA - (MC-CDMA)	242
6.7	PAPR Reduction for Multicode CDMA	242
6.8	Concluding Remarks	243
<b>7</b>	<b>SYNCHRONIZATION FOR MIMO OFDM</b>	<b>245</b>
	by <i>Gordon Stüber</i> and <i>Apurva Mody</i>	
7.1	Introduction	245
7.2	MIMO System Model	246
7.3	Preamble and Pilot Structures	250
7.3.1	Preamble	250
7.3.2	Pilots	251

---

7.4	Time Synchronization and Sample/RF Frequency Offset Estimation	253
7.4.1	Sample/RF Frequency Offset Estimation	257
7.4.2	There is a Time for Channel Estimation	259
7.5	Simulation Results	260
7.6	Summary and Further Reading	262
Appendix 7A:	Gram-Schmidt Orthonormalization to Make $\mathbf{S}_k$ s Unitary	264
Appendix 7B:	Optimum Weight Calculation for Sampling Frequency Offset Estimates	266
Appendix 7C:	MSE Analysis for the Sampling Frequency/ Residual RF Oscillator Frequency Offset Estimator	267
Appendix 7D:	MSE Analysis of the Channel Estimator	269
<b>BIBLIOGRAPHY</b>		<b>271</b>
<b>SUBJECT INDEX</b>		<b>301</b>

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# PREFACE

*Orthogonal frequency division multiplexing* (OFDM) has been shown to be an effective technique to combat multipath fading in wireless channels. It has been and is going to be used in various wireless communication systems. This book gives a comprehensive introduction on the theory and practice of OFDM for wireless communications. It consists of seven chapters and each has been written by experts in the area. Chapter 1, by G. Stüber, briefly motivates OFDM and multicarrier modulation and introduces the basic concepts of OFDM, Chapter 2, by Y. (G.) Li, presents design of OFDM systems for wireless communications, various impairments caused by wireless channels, and some other types of OFDM related modulation. Chapters 3 to 6 address different techniques to mitigate the impairments and to improve the performance of OFDM systems. Chapter 3, by J. Cioffi and L. Hoo, focuses on system optimization techniques, including channel partitioning, loading of parallel channels, and optimization through coding. Chapter 4, by S. K. Wilson, addresses timing and frequency offset estimation in OFDM systems. It also briefly discusses sampling clock offset estimation and correction. Chapter 5, by Y. (G.) Li, deals with pilot aided and decision-directed channel estimation for OFDM systems. Chapter 6, by C. Tellambura and M. Friese, discusses various techniques to reduce the peak-to-average power ratio of OFDM signals. Chapter 7, by G. Stüber and A. Mody, presents recent results on synchronization for OFDM systems with multiple transmit and receive antennas for diversity and multiplexing. To facilitate the readers, about 300 subject indexes and 300 references are given at the end of the book.

This book is designed for engineers and researchers who are interested in learning and applying OFDM for wireless communications. The readers are expected to be familiar with technical concepts of communications theory, digital signal processing, linear algebra, probability and random processes.

It can be also used as a textbook for graduate courses in advanced digital communications. Nevertheless, to accommodate readers having a variety of technical backgrounds, most of the key concepts in our book are developed with detailed derivations and proofs.

Even though each chapter is written by different people, we have tried to make symbols, notations, writing styles in different chapters consistent.

The editors of the book would like to thank L. Cimini, Jr. for initiating the book project, discussing skeleton of the book, identifying potential chapter contributors, and providing insight comments on the first draft of almost every chapter. The editors are also deeply indebted to J. Cioffi, L. Hoo, S. Wilson, P. O. Börjesson, P. Ödling, J. J. van de Beek, C. Tellambura, M. Friese, and A. Mody, who not only have done important and crucial work in OFDM related research topics but also contributed chapters in this book.

In particular, Y. (G.) Li would like to thank Professor S.-X. Cheng of Southeast University, P. R. China, who first introduced the concept of parallel modem (an OFDM related modulation) to him about 20 years ago. He wishes to thank some of his previous colleagues at AT&T Labs - Research, including, L. Cimini, Jr., N. Sollenberger, J. Winters, and J. Chuang, for their technical advising and help in his OFDM related research. He also thanks his wife, Rena, for constant support and his sons, Frank and Micheal, for understanding while carrying out the book project.

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Atlanta, Georgia

# INTRODUCTION

Gordon Stüber

Digital bandpass modulation techniques can be broadly classified into two categories. The first is single-carrier modulation, where data is transmitted by using a single *radio frequency* (RF) carrier. The other is multi-carrier modulation, where data is transmitted by simultaneously modulating multiple RF carriers. This book is concerned with a particular type of multi-carrier modulation known as *orthogonal frequency division multiplexing* (OFDM). OFDM has gained popularity in a number of applications including digital subscriber loops, wireless local area networks. It is also a strong contender for fourth generation cellular land mobile radio systems.

OFDM transmits data in parallel by modulating a set of orthogonal sub-carriers. OFDM is attractive because it admits relatively easy solutions to some difficult challenges that are encountered when using single-carrier modulation schemes on wireless channels. Simplified frequency domain equalization is often touted as a primary advantage of OFDM over single-carrier modulation with conventional time-domain equalization. However, frequency domain equalization can be applied just as easily to single-carrier modulation techniques as it can to OFDM. Perhaps the greatest benefit of using OFDM is that the modulation of closely-spaced orthogonal sub-carriers partitions the available bandwidth into a collection of narrow sub-bands. Motivated by the water-pouring capacity of a frequency selective channel, adaptive transmission techniques can be readily used to increase the overall bandwidth efficiency. One such possibility is to use adaptive bit loading techniques, where the modulation alphabet size on each sub-carrier is adjusted according to channel conditions. A larger signal constellation is used on sub-carriers where the received signal-to-noise ratio is large, and vice versa. As will be shown later in this book, OFDM waveforms are resilient to timing errors, yet highly sensitive to frequency offsets and phase noise in the transmitter and receiver RF and sampling clock oscillators. These characteristics are op-

posite those of single-carrier modulation, which is more sensitive to timing errors and less sensitive to frequency offsets. This is a manifestation of the long OFDM modulated symbol duration and the closely-spaced orthogonal sub-carriers. Hence, OFDM has its own set of unique implementation challenges that are not present in single-carrier systems. This book provides a comprehensive treatment of these challenges and their solutions.

## 1.1 High Rate Wireless Applications

The demand for high speed wireless applications and limited RF signal bandwidth has spurred the development of power and bandwidth efficient air interface schemes. Cellular telephone systems have gone through such a growth process. After the introduction of the first analog cellular systems in the early 1980s, subscriber growth for basic cellular voice services increased dramatically. This led to the introduction of several second generation digital cellular standards in the early 1990s, such as the *Global System for Mobile communication* (GSM) and *Code Division Multiple Access* (CDMA), with the objective of providing greater system capacity so that the growing demand for voice services could be accommodated with scarce bandwidth resources.

The 1990s also seen a tremendous growth of Internet related services and applications, mostly using a wired Internet Protocol (IP) infrastructure. With the growing demand for wireless data and multimedia applications, cellular telephony and the Internet have become convergent technologies. This has led to the development of third generation cellular standards, such as *Wideband CDMA* (WCDMA) and *cdma2000*, that support wireless voice, data, and multimedia applications. With the pervasiveness of the Internet, the cellular telephone network is evolving from a circuit switched to a packet switched IP-based core network. Such an infrastructure can support not only delay insensitive applications such as mobile data, but delay sensitive applications such as *voice over IP* (VoIP) as well.

The growth of the Internet also led to the development of various *wireless local area network* (WLAN) standards, such as those developed under IEEE802.11, to permit mobile connectivity to the Internet. Such services typically operate in unlicensed bands. With a surging demand for wireless Internet connectivity, new WLAN standards have been developed including IEEE802.11b, popularly known as Wi-Fi that provides up to 11 Mb/s raw data rate, and more recently IEEE802.11a/g that provides wireless connectivity with speeds up to 54 Mb/s. IEEE802.11b uses a signaling technique

Data Rate	6, 9, 12, 18, 24, 36, 48, 54 Mb/s
Modulation	BPSK, QPSK, 16-QAM, 64-QAM
Coding	1/2, 2/3, 3/4 CC
Number of subchannels	52
Number of pilots	4
OFDM symbol duration	4 $\mu$ s
Guard interval	800 ns
Subcarrier spacing	312.5 kHz
3 dB bandwidth	16.56 MHz
Channel spacing	20 MHz

**Table 1.1.** Key parameters of the IEEE 802.11a OFDM standard, from [1].

based on *complementary code keying* (CCK), while IEEE802.11a uses OFDM which is the subject of this book. The main physical layer parameters of the IEEE 802.11a OFDM standard are summarized in Table 1.1. Dual mode radio access devices have been developed allowing access both public cellular networks and private WLANs, to provide a more ubiquitous and cost efficient connectivity.

More recent developments such as IEEE802.16 *wireless metropolitan area network* (WMAN) standard address *broadband fixed wireless access* (BFWA), that provides a last mile solution to compete with wireline technologies such as *Asymmetric Digital Subscriber Loop* (ADSL), coaxial cable, and satellite. Similar to IEEE802.11a, IEEE802.16 uses OFDM. The emerging *Mobile Broadband Wireless Access* (MBWA) IEEE802.20 standard extends IEEE802.16 to mobile environments. Once again, IEEE802.20 is based on OFDM. OFDM is also being considered in the IEEE802.11n standard that considers *Multiple-Input Multiple-Output* (MIMO) systems, where multiple antennas are used at the transmitter for the purpose of spatial multiplexing or to provide increased spatial diversity. Finally, OFDM has also found application in *Digital Audio Broadcast* (DAB) and *Digital Terrestrial Video Broadcast* (DVB-T) standards in Europe and Japan.

## 1.2 Wireless Channel

To comprehend the benefits and drawbacks of OFDM, we must first understand the basic characteristics of the radio propagation environment. Radio signals generally propagate according to three mechanisms; reflection,

diffraction, and scattering. The appropriate model for radio propagation depends largely on the intended application, and different models are used for the different applications such as cellular land mobile radio, WMANs, and indoor WLANs. In general, however, radio propagation can be roughly characterized by three nearly independent phenomenon; path loss attenuation with distance, shadowing, and multipath-fading. Each of these phenomenon is caused by a different underlying physical principle and each must be accounted for when designing, evaluating, and deploying any wireless system to ensure adequate coverage and quality of service.

### 1.2.1 Path Loss and Shadowing

It is well known that the intensity of an electromagnetic wave in free space decays with the square of the radio path length,  $d$ , such that the received power at distance  $d$  is

$$\Omega_p(d) = \Omega_t k \left( \frac{\lambda_c}{4\pi d} \right)^2 \quad (1.2.1)$$

where  $\Omega_t$  is the transmitted power,  $\lambda_c$  is the wavelength, and  $k$  is a constant of proportionality. Although it may seem counter-intuitive, path loss is essential in high capacity frequency reuse systems, the reason being that a rapid attenuation of signal strength with distance permits the bandwidth to be reused within a close physical proximity without excessive interference. Such principles form the basis for cellular mobile radio systems.

Free space propagation does not apply in a typical wireless operating environment, and the propagation path loss depends not only on the distance and wavelength, but also on the antenna types and heights and the local topography. The site specific nature of radio propagation makes theoretical prediction of path loss difficult, except for simple cases such as propagation over a flat, smooth, reflecting surface. A simple path loss model assumes that the received power is

$$\Omega_p \text{ (dBm)}(d) = \mu_{\Omega_p \text{ (dBm)}}(d_o) - 10\beta \log_{10}(d/d_o) + \epsilon_{\text{(dB)}} \text{ (dBm)} \quad (1.2.2)$$

where  $\mu_{\Omega_p \text{ (dBm)}}(d_o) = E[\Omega_p \text{ (dBm)}(d_o)]$  is the average received signal power (in dBm) at a known reference distance. The value of  $\mu_{\Omega_p \text{ (dBm)}}(d_o)$  depends on the transmit power, frequency, antenna heights and gains, and other factors. The parameter  $\beta$  is called the path loss exponent and is a key parameter that affects the coverage of a wireless system. The path loss exponent lies in the range  $3 \leq \beta \leq 4$  for a typical cellular land mobile

radio environment. Usually, the path loss exponents are determined from empirical measurement campaigns.

The parameter  $\epsilon_{(\text{dB})}$  in (1.2.2) represents the error between the actual and estimated path loss. It is usually modelled as a zero-mean Gaussian random variable (in decibel units). This error is caused by large terrain features such as buildings and hills, and is sometimes known as shadowing or shadow fading. Shadows are generally modelled as being log-normally distributed, meaning that the probability density function of received power in decibel units,  $\Omega_{(\text{dBm})}(d)$ , is

$$p_{\Omega_p(\text{dBm})}(d)(x) = \frac{1}{\sqrt{2\pi}\sigma_\Omega} \exp\left\{-\frac{(x - \mu_{\Omega_p(\text{dBm})}(d))^2}{2\sigma_\Omega^2}\right\} \quad (1.2.3)$$

where

$$\mu_{\Omega_p(\text{dBm})}(d) = \mu_{\Omega_p(\text{dBm})}(d_o) - 10\beta \log_{10}(d/d_o) \quad (\text{dBm}) . \quad (1.2.4)$$

The parameter  $\sigma_\Omega$  is the shadow standard deviation, and usually ranges from 5 to 12 dB, with  $\sigma_\Omega = 8$  dB being a typical value for cellular land mobile radio applications. Shadows are spatially correlated, and sometimes modelled as having an exponential decorrelation with distance.

## 1.2.2 Multipath-Fading

A typical radio propagation environment exhibits multipath, where the plane waves incident on the receiver antenna arrive from many different directions with random amplitudes, frequencies and phases. Since the wavelength is relatively short (approximately 30 cm at 1 GHz), small changes in the location of the transmitter, receiver and/or scattering objects in the environment will cause large changes in the phases of the incident plane wave components. The constructive and destructive addition of plane waves combined with motion results in envelope fading, where the received envelope can vary by as much as 30 to 40 dB over a spatial distance equal to a fraction of a wavelength. Multipath-fading results in a doubly dispersive channel that exhibits dispersion in both the time and frequency domains. Time dispersion arises because the multipath components propagate over transmission paths having different lengths and, hence, they reach the receiver antenna with different time delays. Time dispersion causes intersymbol interference (ISI) that can be mitigated by using a time- or frequency domain equalizer in single-carrier systems, a RAKE receiver in CDMA systems, or frequency domain equalization in OFDM systems. Channel time variations due to mobility are

characterized by Doppler spreading in the frequency domain. Such time-variant channels require an adaptive receiver to estimate and track channel the channel impulse response or parameters such as the signal-to-noise ratio that are related to the channel impulse response.

A multipath-fading channel can be modelled as a linear time-variant filter having the complex low-pass impulse response

$$h(t, \tau) = \sum_{n=1}^N C_n e^{j\phi_n(t)} \delta(\tau - \tau_n) \quad (1.2.5)$$

where  $g(\tau, t)$  is the channel response at time  $t$  due to an impulse applied at time  $t - \tau$ , and  $\delta(\cdot)$  is the dirac delta function. In (1.2.5),  $C_n$ ,  $\phi_n$ , and  $\tau_n$  are the random amplitude, phase, and time delay, respectively, associated with the  $n$ th propagation path, and  $N$  is the total number of arriving multipath components. The time-variant phases  $\phi_n(t)$  are given by [2]

$$\phi_n(t) = 2\pi(f_{D,n}t + \phi_n) \quad (1.2.6)$$

where  $\phi_n$  is an arbitrary random phase uniformly distributed on the interval  $[-\pi, \pi]$  and

$$f_{D,n} = f_m \cos \theta_n \quad (1.2.7)$$

is the Doppler frequency associated with the  $n$ th propagation path, where  $f_d = v/\lambda_c$ ,  $\lambda_c$  is the carrier wavelength, and  $f_d$  is the maximum Doppler frequency occurring when the angle of arrival  $\theta_n = 0$ .

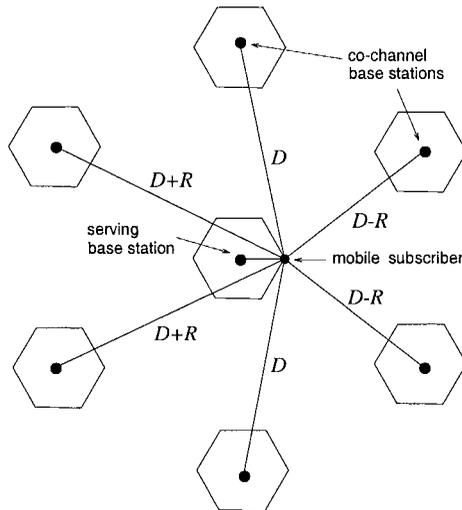
### 1.3 Interference and Noise

All communication systems are affected by thermal noise or *additive white Gaussian noise* (AWGN). However, wireless systems that employ frequency reuse are also affected by the more dominant *co-channel interference* (CCI). Co-channel interference arises when the carrier frequencies are spatially reused. In this case, the power density spectra of the co-channel signals overlap causing mutual interference. CCI places a limit on the minimum spatial separation that is required such that the carrier frequencies can be reused. CCI is the primary additive impairment in high capacity frequency reuse systems, such as cellular land mobile radio systems. Fig. 1.1 depicts the worst case forward channel co-channel interference situation in a cellular radio environment, which occurs when the mobile station is located at the corner of a cell at the maximum possible distance from its serving base

station. With omni-directional antennas, there are six primary co-channel interferers; two at distance  $D - R$ , two at distance  $D$ , and two at distance of  $D + R$ , where  $R$  is the cell radius. Using the simple path loss model in (1.2.4) and neglecting shadowing, the worst case *carrier-to-interference ratio* is

$$\begin{aligned} \frac{C}{I} &= \frac{1}{2} \frac{R^{-\beta}}{(D - R)^{-\beta} + D^{-\beta} + (D + R)^{-\beta}} \\ &= \frac{1}{2} \frac{1}{\left(\frac{D}{R} - 1\right)^{-\beta} + \left(\frac{D}{R}\right)^{-\beta} + \left(\frac{D}{R} + 1\right)^{-\beta}} \end{aligned} \quad (1.3.1)$$

where  $\beta$  is the propagation path loss exponent. The parameter  $D/R$  is called the co-channel reuse factor. In a hexagonal cell deployment  $D/R$  is related to the reuse cluster size,  $N$ , by  $D/R = \sqrt{3N}$ . Clearly, the  $C/I$  increases with the cluster size, thereby providing better link quality. However, at the same time the available bandwidth (and number of channels) per cell decreases, thereby increasing the new call and handoff call blocking probabilities.



**Figure 1.1.** Worst case co-channel interference on the forward channel.

Frequency reuse also introduces adjacent channel interference (ACI). This type of interference arises when adjacent cells use channels that are spectrally adjacent to each other. In this case, the power density spectrum of the desired and interfering signals partially overlap. Although ACI de-

grades link quality it is less severe than CCI, since the interfering signals do not completely overlap in frequency.

## 1.4 Orthogonal frequency Division Multiplexing

OFDM is a multi-carrier modulation technique where data symbols modulate a parallel collection of regularly spaced sub-carriers. The sub-carriers have the minimum frequency separation required to maintain orthogonality of their corresponding time domain waveforms, yet the signal spectra corresponding to the different sub-carriers overlap in frequency. The spectral overlap results in a waveform that uses the available bandwidth with a very high bandwidth efficiency. OFDM is simple to use on channels that exhibit time delay spread or, equivalently, frequency selectivity. Frequency selective channels are characterized by either their delay spread or their channel coherence bandwidth which measures the channel decorrelation in frequency. The coherence bandwidth is inversely proportional to the *root-mean-square* (rms) delay spread. By choosing the sub-carrier spacing properly in relation to the channel coherence bandwidth, OFDM can be used to convert a frequency selective channel into a parallel collection of frequency flat sub-channels. Techniques that are appropriate for flat fading channels can then be applied in a straight forward fashion.

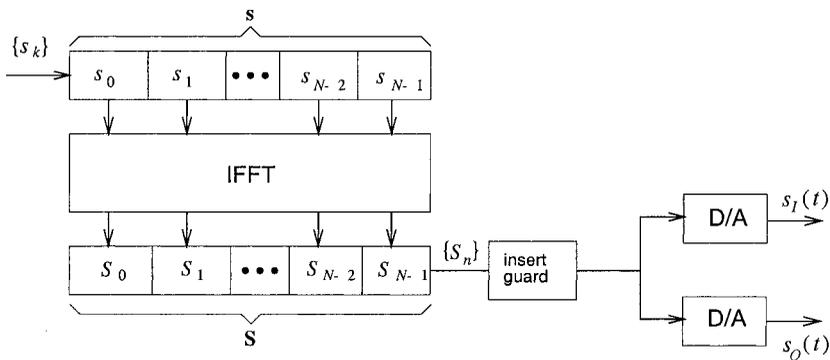
### 1.4.1 OFDM Concept

An OFDM modulator can be implemented as an  $N$ -point *inverse discrete Fourier transform* (IDFT) on a block of  $N$  information symbols followed by *digital-to-analog converter* (DAC) on the IDFT samples [3]. In practice, the IDFT can be implemented with the computationally efficient *inverse fast Fourier transform* (IFFT) as shown in Fig. 1.2. Let  $\{s_k, k = 1, \dots, N\}$  represent a block of  $N$  complex data symbols chosen from an appropriate signal constellation such as *quadrature amplitude modulation* (QAM) or *phase shift keying* (PSK). The IDFT of the data block is

$$S_n = \sum_{k=0}^{N-1} s_k \exp \left\{ \frac{j2\pi nk}{N} \right\}, \quad n = 0, 1, \dots, N-1, \quad (1.4.1)$$

yielding the time-domain sequence  $\{S_n, n = 1, \dots, N\}$ . To mitigate the effects of ISI caused by channel delay spread, each block of  $N$  IFFT coefficients is typically preceded by a *cyclic prefix* (CP) or a guard interval consisting of  $N_g$  samples, such that the length of the CP is at least equal to the channel

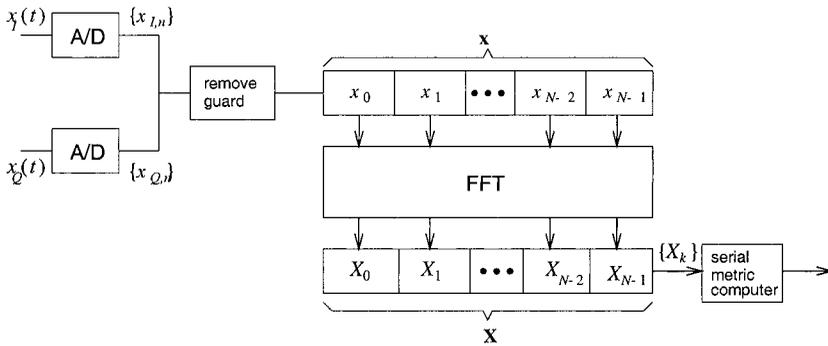
length  $N_h$  in samples, where  $\mu = \frac{T_h}{T_s} N$ ,  $T_h$  is the length of (continuous) channel, and  $T_s$  is the duration of a OFDM block or symbol. The cyclic prefix is simply a repetition of the last  $N_g$  IFFT coefficients. Alternatively, a cyclic suffix can be appended to the end of a block of  $N$  IFFT coefficients, that is a repetition of the first  $N_g$  IFFT coefficients. The guard interval of length  $N_g$  is an overhead that results in a power and bandwidth penalty, since it consists of redundant symbols. However, the guard interval is useful for implementing time and frequency synchronization functions in the receiver, since the guard interval contains repeated symbols at a known sample spacing. The time duration of an OFDM symbol is  $N + N_g$  times larger than the modulated symbol in a single-carrier system.



**Figure 1.2.** Block diagram of basic OFDM transmitter, from [2].

At the receiver, the received complex baseband signal is sampled with an *analog-to-digital converter* (ADC), usually with a sampling interval,  $\Delta_T = \frac{T_s}{N}$ . Sometimes fractional sampling is used, where the sample period is  $\frac{1}{M} \Delta_T$ , where  $M$  is an integer greater than one. For simplicity, assume here that  $M = 1$ . Then the combination of the DAC in the transmitter, the waveform channel, and the ADC in the receiver creates an overall discrete-time channel with tap spacing  $\Delta_T$ . After ADC, the  $N_g$  samples received during the guard interval of each OFDM symbol are discarded in the case of a cyclic prefix; in case of a cyclic suffix the  $N_g$  received samples at the beginning of an OFDM symbol are replaced with the  $\mu$  received samples at the end of the OFDM symbol. Under the condition that  $N_g \geq N_h$ , the *linear* convolution of the transmitted sequence of IFFT coefficients with the discrete-time channel is converted into a *circular* convolution. As a result, the effects of the ISI are completely and easily removed. After removal of the guard interval,

each block of  $N$  received samples is converted back to the frequency domain using an FFT as shown in Fig. 1.3. The FFT operation performs baseband demodulation. The  $N$  frequency domain samples are each processed with a simple one-tap Frequency Domain Equalizer (FDE) and applied to a decision device to recover the data symbols or to a metric computer if error correction coding is used. The one-tap FDE simply multiplies each FFT coefficient by a complex scalar.



**Figure 1.3.** Block diagram of basic OFDM receiver, from [2].

Ease of equalization is often touted as the primary advantage of OFDM. However, as mentioned earlier, similar equalization techniques can be applied to single-carrier systems as well. Such a technique is called *single-carrier frequency domain equalization* (SC-FDE). Similar to OFDM, SC-FDE systems transmit data in blocks of  $N$  symbols at a time. Each block of  $N$  data symbols is preceded by a cyclic prefix of length  $N_g$  that is simply a repetition of the last  $N_g$  samples in each length- $N$  block. Alternatively, a length- $N_g$  cyclic suffix can be appended to each length- $N$  block that is a repetition of the first  $N_g$  samples in the block. The length- $N + N_g$  block is then applied to a DAC, upconverted to RF, and transmitted over the waveform channel. The received waveform is downconverted to complex baseband and applied to an ADC. The receiver then removes the guard interval in exactly same fashion as an OFDM receiver. Afterwards, the length- $N$  time-domain sample sequence is converted to the frequency domain using an  $N$ -point FFT. Frequency domain equalization (FDE) is then applied to the  $N$  FFT coefficients. Similar to OFDM, the FDE simply multiplies each FFT coefficient by a complex scalar to perform zero-forcing or minimum mean square error equalization. Afterwards, the equalized samples are converted back to

the time-domain using an  $N$ -point IFFT and applied to a decision device or metric computer. The overall system complexity of SC-FDE is comparable to OFDM. The main difference is that OFDM uses an IFFT in the transmitter and an FFT in the receiver, while SC-FDE does not perform any transformation in the transmitter but employs an FFT/IFFT pair in the receiver.

### 1.4.2 Channel Capacity and OFDM

Consider a time-invariant frequency selective channel with transfer function  $H(f)$ , such that the amplitude response  $|H(f)|$  varies across the channel bandwidth  $W$ . The power spectral density of the additive Gaussian noise is  $S_{nn}(f)$  and may not be flat either. Shannon [4] proved that the capacity of such a non-ideal additive Gaussian noise channel is achieved when the transmitted power  $\Omega_t(f)$  is adjusted across the bandwidth  $W$  according to

$$\Omega_t(f) = \begin{cases} K - S_{nn}(f)/|H(f)|^2, & f \in W \\ 0, & f \notin W \end{cases} \quad (1.4.2)$$

where  $K$  is a constant chosen to satisfy the constraint

$$\int_W \Omega_t(f)df \leq \Omega_{av} \quad (1.4.3)$$

with  $\Omega_{av}$  being the average available power to the transmitter. One method for approaching the channel capacity is to divide the bandwidth  $W$  into  $N$  sub-bands of width  $W/\Delta f$ , where  $\Delta f = 1/T_s$  is chosen small enough so that  $|H(f)|^2/S_{nn}(f)$  is essentially constant within each sub-band. The signals in each sub-band may then be transmitted with the optimum power allocation  $\Omega_t(f)$ , while being individually coded to achieve capacity. From (1.4.2), the transmitter power should be high when  $|H(f)|^2/S_{nn}(f)$  is large and small when  $H(f)/S_{nn}(f)$  is small. In a practical system with a target bit error rate, this implies the use of a larger signal constellation and/or higher rate error correction code in sub-bands where  $|H(f)|^2/S_{nn}(f)$  is larger. The technique of using adaptive modulation and coding on the different OFDM sub-carriers requires knowledge of the channel at the transmitter. Such channel knowledge at the transmitter is readily available in OFDM systems that employ *time-division duplexing* (TDD), where the same set of sub-carrier frequencies are alternately used for transmission and reception in each direction of a full-duplex link. Reciprocity ensures that the channel in each direction has the same impulse response provided that the channel time-variations

are slow enough. Adaptive bit loading is more complicated in OFDM systems that employ *frequency division duplexing* (FDD) since the reciprocity principle does not apply due to the significant frequency decorrelation of the forward and reverse channels. With FDD, the channel must be estimated and, afterwards, full or partial information of the channel is relayed back to the transmitter for adaptation purposes.

## 1.5 Synchronization and Channel Estimation

An OFDM receiver operating in the acquisition mode must perform time synchronization, RF and sample clock frequency offset estimation and correction, and initial channel estimation. For systems that transmit information in a packetized or burst mode, these synchronization processes are usually aided by a synchronization preamble consisting of a training sequence or the concatenation of several training sequences. Training sequences in the synchronization preamble have length  $N_P = N/I$ , where  $N$  is the OFDM block length and  $I$  is integer. Each training sequence must have an appropriate cyclic guard interval. The synchronization preamble is periodically inserted into the stream of OFDM symbols containing the transmitted data. Synchronization algorithms can also exploit the cyclic guard interval of the OFDM symbol, since the guard interval consists of repeated symbols separated in time. The guard interval can be used to estimate the change in phase due to the channel and oscillator frequency offsets.

After acquisition has been achieved, the receiver enters the data mode and tracks the drift in the RF and sample clock oscillators, and variations in the channel. For applications characterized large Doppler spreads, such as land mobile radio, the channel coefficients are usually tracked by inserting additional OFDM pilot symbols or by using pilot sub-carriers followed by time and frequency interpolation. In WLAN and WMAN applications, the channel is relatively static since the user terminals are usually stationary or slowly moving. However, even in this case channel time variations are expected due to the presence of frequency offsets between transmitter and the receiver local RF and sample clock oscillators. Generally, the components used in the customer premises equipment are low cost and have low tolerances. A typical drift of 10-20 *parts per million* (ppm) is expected in the oscillators. Therefore, an OFDM signal with a bandwidth of 4 MHz, for example, may produce a sampling offset of 80 samples for every one second of transmission. Such RF and sampling frequency offsets cause phase rotation, amplitude distortion and may result in a complete loss of synchronization.

## 1.6 Peak-to-Average Power Ratio

Consider again the time-domain IFFT coefficients in (1.4.1). For purpose of illustration, suppose data symbols are chosen from *binary phase shift keying* (BPSK), such that  $s_k \in \{-1, +1\}$ . In this case (1.4.1) can be rewritten as

$$\begin{aligned} S_n &= S_n^{(R)} + jS_n^{(I)} \\ &= \sum_{k=0}^{N-1} s_k \cos \frac{2\pi nk}{N} + j \sum_{k=0}^{N-1} s_k \sin \frac{2\pi nk}{N}, \quad n = 0, 1, \dots, N-1. \end{aligned} \quad (1.6.1)$$

When  $N$  is large, the central limit theorem can be invoked such that  $S_n^{(R)}$  and  $S_n^{(I)}$  can be treated as independent zero-mean Gaussian random variables with variance  $\sigma^2 = N/2$ . Under this assumption, the  $|S_n|^2, n = 0, \dots, N-1$  are exponentially distributed random variables and  $E[|S_n|^2] = 2\sigma^2 = N$ . Treating the  $|S_n|^2$  as independent exponential random variables and applying order statistics, the peak value of  $|S_n|^2$ , denoted as  $S_{\max}^2 = \max_{0 \leq n \leq N-1} |S_n|^2$ , is a random variable with cumulative distribution function  $F_{S_{\max}^2}(y) = (1 - e^{-y/N})^N$ . The peak-to-average power ratio (PAPR) can be defined as

$$\text{PAPR} = \frac{S_{\max}^2}{E[|S_n|^2]} = \frac{S_{\max}^2}{N} \quad (1.6.2)$$

Note that the PAPR is a random variable due to the random data  $\{s_k, k = 0, \dots, N-1\}$ , and the probability that the PAPR exceeds a specified level  $z$  is

$$\begin{aligned} \text{Prob}(\text{PAPR} > z) &= \text{Prob}(S_{\max}^2 > zN) \\ &= 1 - F_{S_{\max}^2}(zN) \\ &= 1 - (1 - e^{-z})^N. \end{aligned} \quad (1.6.3)$$

Observe that  $\text{Prob}(\text{PAPR} > z)$  increases with the number of sub-carriers,  $N$ , for any level  $z \geq 1$ . We can also compute the mean PAPR using the probability density function  $f_{S_{\max}^2}(y) = (1 - e^{-y/N})^{N-1} e^{-y/N}$ . The result is

$$\begin{aligned} \overline{\text{PAPR}} &= \int_0^{\infty} y f_{S_{\max}^2}(y) dy \\ &= N \sum_{k=0}^{N-1} \binom{N-1}{k} \frac{(-1)^k}{(k+1)^2}. \end{aligned} \quad (1.6.4)$$

Again, observe that the average PAPR increases with  $N$ . For  $N = 16$ ,  $\overline{\text{PAPR}} = 3.38$  and for  $N = 32$ ,  $\overline{\text{PAPR}} = 4.06$ . Note that the above analysis of PAPR is based on a simple central limit theorem approximation. A more accurate and detailed analysis of PAPR is considered in Chapter 6 of this book.

The high PAPR of OFDM signals is a fundamental drawback when compared to single-carrier modulation. Practical power amplifiers are linear only over a finite range of input amplitudes. In order to prevent saturation and clipping of the OFDM signal peaks, the amplifiers must be operated with sufficient "back-off" or head room. The required back-off increases with the PAPR and, hence, the number of sub-carriers  $N$ . However, increased back-off reduces the efficiency of the power amplifier. Generally, there are two solutions to the high PAPR problem of OFDM signals. The first is to reduce the PAPR of the transmitted signals through such methods as clipping and filtering, constrained coding, and selective mapping. The second is to use linearization techniques to increase the range of linearity of the power amplifier. Such PAPR reduction methods, however, reduce PAPR while sacrificing complexity and/or bandwidth efficiency.

## 1.7 MIMO OFDM

Wireless communications systems with multiple transmit and receiving antennas can exploit a dense scattering propagation environment to increase the channel capacity [5], [6]. Generally, there are two categories of MIMO techniques. One category improves the power efficiency by maximizing spatial diversity. Such techniques include delay diversity, and space-time block- and trellis-coding (STBC and STTC). The other category uses linear processing to increase data rate, typically under conditions where full spatial diversity would not be not achieved. Such techniques include Bell Labs Layered Space-Time (BLAST) [7].

The first transmit diversity approach for MIMO systems was delay diversity. Multiple transmit antennas send delayed copies of same signal, and maximum likelihood sequence estimation (MLSE) is used at the receiver to estimate the transmitted sequence. Decision feedback equalizers can also be used in such systems. The delay-diversity approach is simple and can be considered as a particular space-time code.

STBC and STTC can provide full spatial diversity for MIMO systems. STBC utilizes the orthogonality property of the code to achieve full diversity; however, it cannot achieve full-rate transmission when the number of

transmit antennas is greater than two. STTC uses enough trellis coding to guarantee full diversity; but the decoding complexity increases exponentially with the number of transmit antennas. Both STBC and STTC lack scalability with number of transmit antennas. As the number of transmit antennas is changed, different space-time codes are needed. STBC and STTC was developed for quasi-static flat fading channels. To apply STBC and STTC to frequency selective fading channels, they must be used in conjunction with other techniques such as equalization or orthogonal frequency division multiplexing (OFDM), that effectively generate one or more flat faded coding channels.

MIMO-OFDM arrangements have been suggested for frequency selective fading channels, where either STBC or STTC is used across the different antennas in conjunction with OFDM. Such approaches can provide very good performance on frequency selective fading channels. However, the complexity can be very high, especially for for a large number of transmit antennas. Another approach uses delay diversity together with OFDM on flat fading channels. For frequency selective fading channels, a cyclic delay diversity approach can be used with OFDM something we call multi-carrier delay diversity modulation (MDDM). Our work has shown that full spatial diversity can be achieved for MDDM on flat fading channels, provided that the minimum Hamming distance of the outer (pre-IFFT) error correcting code either equals or exceeds the number of transmit antennas, and to obtain good coding gain, a simple block interleaving will do [8]. Unlike STBC and STTC the scheme is scalable; the number of transmit antennas can be changed without changing the error correcting code. MDDM can easily handle frequency selective fading using MRC, or other type of combining depending on the environment.

## 1.8 Outline of This Book

Chapter 2 introduces the basic concepts of OFDM including FFT implementation, comparison with single-carrier modulation, and basic system design. Chapter 2 also analyzes the impact of the various impairments introduced by the wireless channel on OFDM performance. These impairments include frequency offsets in the RF and sample clock oscillators, channel time variations, sample timing offsets, multipath delay spread, and amplifier non-linearities. Finally, Chapter 2 briefly considers other approaches to multi-carrier modulation.

Chapter 3 is concerned with performance optimization of OFDM systems

with the objective of maximizing bandwidth efficiency. These optimization methods often rely on full or partial information of the channel transfer function at the transmitter. This information is used, for example, to adaptively load the OFDM sub-carriers, a technique commonly referred to as discrete multitone modulation (DMT). Chapter 3 discusses issues of partitioning the available bandwidth, (adaptive) loading of the parallel sub-channels, and optimization through error correction coding.

Chapter 4 addresses the synchronization of OFDM signals, including details of time and frequency synchronization. The time and frequency synchronization methods considered in Chapter 4 use either pilot-based methods or non-pilot-based methods that exploit the redundant symbols in the cyclic guard interval. Time and frequency synchronization processes can be implemented in a separate or joint fashion. Generally, the synchronization functionality has two modes, acquisition and tracking. Finally, the problem of sampling clock offset estimation and correction is considered.

Chapter 5, is concerned with channel estimation. With OFDM the channel transfer function must be estimated at each sub-carrier. There are three basic methods for channel estimation. The first is pilot-symbol aided estimation, where known pilot symbols are transmitted in the 2-D time-frequency grid. The second is decision directed estimation, where decisions on data symbols are used to update the channel estimates. The third category are blind channel estimation techniques that do not rely on the transmission of known data symbols.

Chapter 6 considers the important issue of OFDM peak-to-average power ratio (PAPR) reduction techniques. As mentioned earlier, multi-carrier modulation techniques such as OFDM exhibit a high PAPR. One method for reducing the PAPR is to intentionally limit or clip the OFDM waveform prior to amplification. Such methods distort the OFDM waveform that may result in a bit error rate degradation. Another class of methods uses distortionless methods such as constrained coding and selective mapping. Such methods do not distort the OFDM signal, but at the cost of bandwidth or computational complexity.

Finally, Chapter 7 considers MIMO-OFDM with an emphasis on synchronization. A complete suite of signal acquisition and tracking algorithms is presented for MIMO-OFDM systems. Our algorithms use a preamble to perform signal acquisition, which consists of time synchronization, RF and sampling frequency offset estimation, and channel estimation. This is followed by open loop tracking of the RF and sampling frequency offsets, and

the channel. We suggest a preamble structure and pilot matrix design for MIMO-OFDM systems with any number of transmit antennas that enable our algorithms to work efficiently. Simulation results are presented that account for all required signal acquisition and tracking functions in a system very similar to IEEE 802.16a.

## 1.9 Summary and Further Reading

This introductory chapter briefly introduced the main concepts of OFDM, while provided the motivation and outlining the scope of the remainder of the book.

Several systems have previously used OFDM or other multicarrier techniques [3], [9], [10], [11], [12]. In particular, in the early 1960's, this multicarrier techniques were used in several high-frequency military systems, such as KINEPLEX [13], ANDEFT [14], KATHRYN [15], [16], where fast fading was not a problem. Similar modems have found applications in voice bandwidth data communications [17] to alleviate the degradations caused by an impulsive noise environment. More information on multicarrier related research in 1960's and 1970's can be found in [18], [19], [20] and the references therein. In 1985, Cimini first investigated OFDM for mobile wireless communications systems in [21]. In [22], Casas and Leung discussed the application of multicarrier techniques on mobile radio FM channels. Willink and Wittke [23] and Kalet [24] investigated the theoretical capacity of multicarrier systems. In 1990, Bingham [25] studied the performance and implementation complexity of OFDM and concluded that the time for OFDM had come. The application of original OFDM, clustered OFDM, and MC-CDMA in mobile wireless systems can be found in [26], [27], [28], [29]. For MIMO and BLAST, see the work of Foschini, [5], [7], [6]. Recently, several books [30], [31], [32] on OFDM have been published.

# BASIC CONCEPTS

Ye (Geoffrey) Li

In this chapter, we first introduce the basic concepts of *orthogonal frequency division multiplexing* (OFDM), discuss the advantages and disadvantages compared single-carrier modulation, and present an implementation example. We then address various impairments of wireless channels on OFDM systems. Finally, we briefly describe other forms of multicarrier modulation.

## 2.1 Basic OFDM

High data-rate is desired in many applications. However, as the symbol duration reduces with the increase of data-rate, the systems using single-carrier modulation suffer from more severe *intersymbol interference* (ISI) caused by the dispersive fading of wireless channels, thereby needing more complex equalization. OFDM modulation divides the entire frequency selective fading channel into many narrow band flat fading subchannels<sup>1</sup> in which high-bit-rate data are transmitted in parallel and do not undergo ISI due to the long symbol duration. Therefore, OFDM modulation has been chosen for many standards, including *Digital Audio Broadcasting* (DAB) and terrestrial TV in Europe, and *wireless local area network* (WLAN). Moreover, it is also an important technique for high data-rate transmission over mobile wireless channels. Here we introduce the basic concepts of OFDM.

### 2.1.1 OFDM

OFDM was first introduced in [3], which is the form used in all present standards. It can be regarded as a time-limited form of multicarrier modulation.

Let  $\{s_k\}_{k=0}^{N-1}$  be the complex symbols to be transmitted by OFDM mod-

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<sup>1</sup>Subchannel is sometimes also called subcarrier or tone.

ulation; the OFDM (modulated) signal can be expressed as

$$s(t) = \sum_{k=0}^{N-1} s_k e^{j2\pi f_k t} = \sum_{k=0}^{N-1} s_k \varphi_k(t), \quad \text{for } 0 \leq t \leq T_s, \quad (2.1.1)$$

where  $f_k = f_o + k\Delta f$  and

$$\varphi_k(t) = \begin{cases} e^{j2\pi f_k t} & \text{if } 0 \leq t \leq T_s, \\ 0 & \text{otherwise,} \end{cases} \quad (2.1.2)$$

for  $k = 0, 1, \dots, N-1$ .  $T_s$  and  $\Delta f$  are called the *symbol duration* and *subchannel space* of OFDM, respectively. In order for receiver to demodulate OFDM signal, the symbol duration must be long enough such that  $T_s \Delta f = 1$ , which is also called *orthogonality condition*.

Because of the orthogonality condition, we have

$$\begin{aligned} & \frac{1}{T_s} \int_0^{T_s} \varphi_k(t) \varphi_l^*(t) dt \\ &= \frac{1}{T_s} \int_0^{T_s} e^{j2\pi(f_k - f_l)t} dt \\ &= \frac{1}{T_s} \int_0^{T_s} e^{j2\pi(k-l)\Delta f t} dt \\ &= \delta[k - l], \end{aligned} \quad (2.1.3)$$

where  $\delta[k - l]$  is the delta function defined as

$$\delta[n] = \begin{cases} 1, & \text{if } n = 0, \\ 0, & \text{otherwise,} \end{cases}$$

Equation (2.1.3) shows that  $\{\varphi_k(t)\}_{k=0}^{N-1}$  is a set of orthogonal functions. Using this property, the OFDM signal can be demodulated by

$$\begin{aligned} & \frac{1}{T_s} \int_0^{T_s} s(t) e^{-j2\pi f_k t} dt \\ &= \frac{1}{T_s} \int_0^{T_s} \left( \sum_{l=0}^{N-1} s_l \varphi_l(t) \right) \varphi_k^*(t) dt \\ &= \sum_{l=0}^{N-1} s_l \delta[l - k] \\ &= s_k. \end{aligned} \quad (2.1.4)$$

### 2.1.2 FFT Implementation

From (2.1.4), an integral is used for demodulation of OFDM signals. Here we describe the relationship between OFDM and *discrete Fourier transform* (DFT), which can be implemented by low complexity *fast Fourier transform* (FFT), as briefly indicated in Section 1.4.1.

From the previous discussion, an OFDM signal can be expressed as

$$s(t) = \sum_{k=0}^{N-1} s_k e^{j2\pi f_k t}.$$

If  $s(t)$  is sampled at an interval of  $T_{sa} = \frac{T_s}{N}$ , then

$$S_n = s(n\Delta_s) = \sum_{k=0}^{N-1} s_k e^{j2\pi f_k \frac{nT_s}{N}}. \quad (2.1.5)$$

Without loss of generality, setting  $f_o = 0$ , then  $f_k T_s = k$  and (2.1.5) becomes

$$S_n = \sum_{k=0}^{N-1} s_k e^{j\frac{2\pi kn}{N}} = \text{IDFT} \{s_k\},$$

where IDFT denotes the *inverse discrete Fourier transform*. Therefore, the OFDM transmitter can be implemented using the IDFT. For the same reason, the receiver can be also implemented using DFT.

The FFT algorithm provides an efficient way to implement the DFT and the IDFT. It reduces the number of complex multiplications from  $N^2$  to  $\frac{N}{2} \log_2 N$  for an  $N$ -point DFT or IDFT. Hence, with the help of the FFT algorithm, the implementation of OFDM is very simple, as shown in Figures 1.3 and 1.4.

### 2.1.3 Cyclic Extension, Power Spectrum, and Efficiency

To deal with delay spread of wireless channels, a cyclic extension is usually used in OFDM systems. There are three different types of cyclic extensions, which are shown in Figure 2.1. Denote  $T_g$  the length of a cyclic extension that is inserted between OFDM blocks. From Fig. 2.1 (b), OFDM signal,  $s(t)$ , can be extended into  $\bar{s}(t)$  by

$$\bar{s}(t) = \begin{cases} s(t), & \text{if } 0 \leq t \leq T_s, \\ s(t - T_s), & \text{if } T_s < t \leq T_s + T_g (= T). \end{cases} \quad (2.1.6)$$