# The Essence of Dielectric Waveguides

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To
Our Families
Vivian, John, and Evelyn; Siblings-Dorothy, Richard, and Vicky
Karen, Susan and Lee

# PREFACE

*"It is our responsibility as scientists, knowing the great progress which comes from a satisfactory philosophy of ignorance, the great progress which is the fruit of freedom of thought, to proclaim the value of this freedom, to teach how doubt is not to be feared but welcomed and discussed; and to demand this freedom as our duty to all coming generations" —— Richard Feynman, 1955 ——*

First, as students from Cal Tech and MIT and then as researchers and teachers from other universities and industry, we are benefited greatly from the philosophy of learning practiced by these and other distinguished universities in the US, namely, learn and teach the fundamentals and not the fashions. Under this guiding light, this comprehensive book was formed, covering the most important modern topics on guided waves. As such, it may be used as a research reference book or as a textbook for senior undergraduate students or first-year graduate students. The lectures for an one-semester or one-quarter course on guided waves along surface wave structures can begin with a review of EM fundamentals (Chap. 2), and then move on to a discussion on the general important and relevant characteristics of these guided surface waves (Chap. 3). Then follows the rigorous analytic treatment for canonical structures (planar, circular, and elliptical) (Chaps. 4–8). By the end of these lectures, the students would have gained a very solid theoretical foundation on this subject. Then the fun part starts. The students can now learn how they may make use of their fundamental knowledge to treat the many modern up-to-date applications: linear and nonlinear wave propagation in fibers, solitons in fibers and WDM beams propagation in fibers (Chaps. 9 and 10), plasmon (sub-wavelength) waves (Chap. 12), waves in periodic structures (photonic structures) (Chap. 13), surface waves on metamaterial (artificial material) and other exotic (moving medium) structures (Chap. 14). Finally, the students can now be introduced to the many numerical approaches (Chap. 15) that can be used on the various guided wave structures, with the comforting knowledge that they possess the necessary theoretical foundation to correctly interpret the numerical data.

Substantial amount of the material of the text appears in book form for the first time. References are given to the original sources. However, unintentional oversight by us is unavoidable. For this the authors offer their apologies. It is curious to note that many popular references (with many citations in the literature) may not represent the papers published by the originators of the concepts. Special care has been taken by us not to follow this erroneous path. References are listed at the end of each chapter for clarity and ease of usage.

As far as nomenclatures and symbols are concerned, we have not been able to have a given symbol to represent a single unique entity throughout the whole book.

Instead, we only make sure that a given symbol clearly and uniquely represents a single entity in that chapter. Whenever possible, universally accepted nomenclatures are used to represent vector and scalar quantities.

It is with deep gratitude and great pleasure for us to acknowledge the significant guidance and encouragement given to us by Professors C. H. Papas, J. R. Whinnery, and R. W. Gould. We also wish to acknowledge with special thanks to Dr. Peter Siegel who introduced us to the field of terahertz research and who planted the seed for us to pursue the writing of this book. Throughout our professional careers, we benefited greatly from the many positive advice and encouragement from our colleagues. We express our deepest thanks and gratitude to them. Finally, we express our sincerest thanks to Marshall Kwong for his dedicated professional graphic arts work for this book, without which this book would be incomplete.

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*Los Angeles*

## CONTENTS

# 1

# INTRODUCTION

The increasing capabilities of digital computation have altered the way electromagnetic problems are being solved. It is no longer necessary that analytical solutions be obtained. Many practical problems with complicated geometries for which there are no closed form analytic solutions can now be solved numerically. Nevertheless, understanding the fundamental behavior (the essence) of the solutions must still be gained from analytic solutions of canonical problems. In other words, correct interpretation of the numerical results must depend on knowing the essence of guided waves on certain related canonical structures. Therefore, the primary goal of this book is to provide an insight into this essence.

Review of the wave guiding structures over the whole electromagnetic spectrum shows that, for frequencies below 30 GHz, mostly metal-based structures are used, and for frequencies above 30 GHz, increasing skin-depth losses in metal requires that low-loss structures be made without the use of any metallic material. Hence, the importance of pure dielectric waveguides for carrying large bandwidth signals is established. See Fig. 1.1 for a display of spectral regions in which certain guiding structures are useful. It is seen that the useful spectrum for dielectric waveguides can span more than seven decades, from $10^9$ to $10^{16}$ Hz.

## 1.1 Brief Historical Background

The concept of guiding electromagnetic waves either along a single conducting wire with finite surface impedance or along a dielectric rod/slab has been known for a long time. As early as 1899, Sommerfeld [1] conceived the idea of guiding a circularly symmetric TM[1] wave along a conducting wire with small surface resistivity. However, because of the large field extent outside the wire, this "open-wire" line remained a novelty with limited practical applications. In 1909, Sommerfeld treated the problem of an oscillating dipole above a finitely conducting plane [2]. He found theoretically that there existed not only a radiated wave due

[1] This notation and classification will be discussed in detail later.

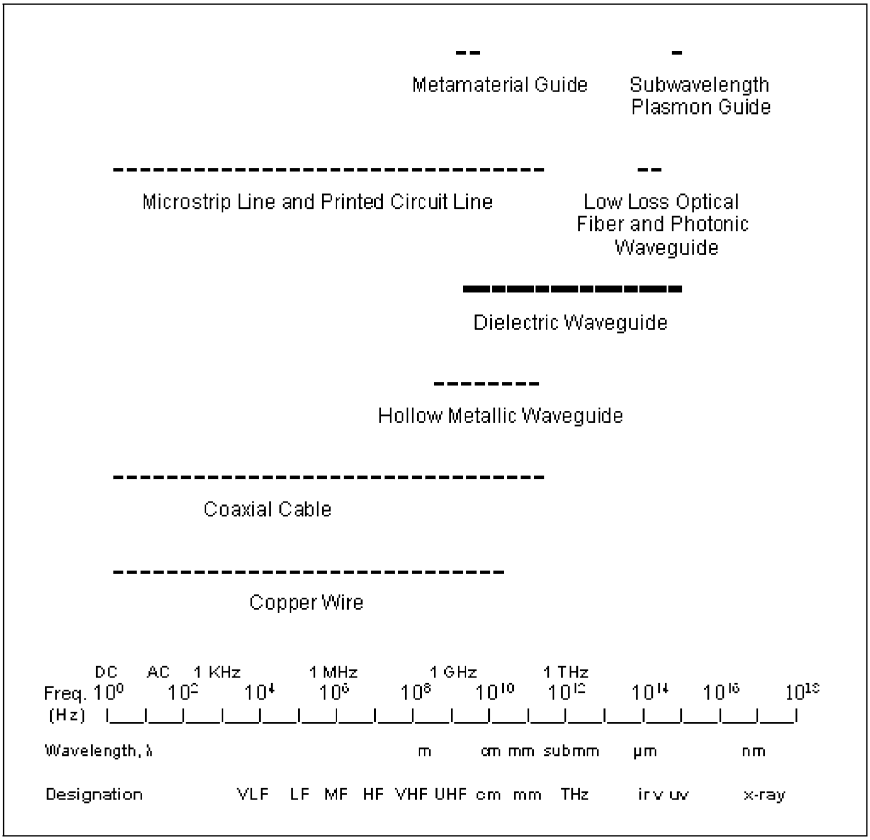


Figure 1.1. Spectral regions for various waveguides

to the oscillating dipole, but also a surface wave that traveled along the lossy surface [3]. This is the well-known "Sommerfeld Problem". In 1910, Hondros and Debye [4] demonstrated analytically that it was possible to propagate a TM wave along a lossless dielectric cylinder. Zahn [5] in 1915 and his two students, Ruter and Schriever [6], confirmed the existence of such a TM wave experimentally. Not until 1936 were the propagation properties of asymmetric waves on a round dielectric rod obtained by Carson et al. [7], who provided a complete mathematical analysis of this problem. It was noted in their paper that in order to satisfy the boundary conditions for the general case, a hybrid wave (i.e., the coexistence of longitudinal electric and magnetic fields) must be assumed. In other words, asymmetric TE and TM modes were inextricably coupled to each other along a circular dielectric rod. They also showed that (1) pure TE and TM waves could only exist in the circularly symmetric case and (2) there existed one and only one mode, namely the lowest order hybrid wave called the $HE_{11}$ mode, which possessed no cutoff frequency[2] and could propagate at all frequencies. All other circularly symmetric or nonsymmetric modes had cutoff frequencies. The dispersion relations of these modes were also obtained in their paper, but no numerical results were given. In their paper they also mentioned that Southworth in 1920 accidentally observed a guided TM wave in a trough of water. Later, in 1936, Southworth [8] described more detailed experimental measurements on the phase velocity and attenuation of the circularly symmetric TM wave on a circular dielectric guide. Soon afterwards, in 1938, Schelkunoff [9] wrote a paper on the coupled transmission line representation of the waves and the impedance concept, which became the foundation for the development of microwave circuits.

In 1943, Mallach [10] published his results on the use of the dielectric rod as a directive radiator. He showed experimentally that the radiation pattern obtained by the use of the asymmetric $HE_{11}$ mode produced only one lobe in the principal direction of radiation. Soon after Mallach's paper, Wegener [11] presented a dissertation in which the asymmetric $HE_{11}$ mode, together with the lowest order circularly symmetric TE and TM modes, were analyzed in detail. Not only were the numerical results of the propagation constants for these waves obtained, but also their attenuation characteristics. He also obtained a few experimental points substantiating his theoretical results. Apparently, he was not aware of the earlier Carson, Mead, and Schelkunoff's work. Elsasser in 1949 [12], independent of Wegener's work, published his computation on the attenuation properties of these three lowest-order modes. In a companion paper, Chandler [13] verified experimentally Elsasser's results on the dominant $HE_{11}$ mode. He found that the guiding effect was retained even when the rod was only a fraction of a wavelength in diameter. For this case, since the greater part of the guided energy was outside the dielectric rod, very little loss was observed. This was also the first time the cavity resonator technique for open dielectric structure was introduced to measure the attenuation constant of the $HE_{11}$ mode. It should be noted that the formula used by Chandler to obtain the attenuation constant, $\alpha$, from the measured $Q$ value was approximately correct, since it was derived assuming that the propagating mode was a TEM mode. The correct formula relating $\alpha$ and $Q$ for the hybrid $HE_{11}$ mode was given by Yeh in 1962 [14].

In the mid-1940s, Brillouin summarized his research on wave propagation in periodic structures in a book (1946) [15]. In 1951, Sensiper [16] wrote his thesis on wave propagation on a helical wire waveguide, a periodic structure waveguide. In 1954, Pierce [17] also provided the results on the interaction of electron beam

[2]The cutoff frequency does not have the conventional definition as that for the metal waveguides. Here it is defined that at the cutoff frequency the open dielectric waveguide structure ceases to act as a binding medium for the guided surface wave, and the wave can no longer be guided by the structure.

with slow waves guided by a periodic structure. The fundamental theory on wave propagation in periodic media is well established by these publications. At about the same time, the increasing demand for higher bandwidth low-loss transmission lines for transcontinental television and long-distance phone transmission provided the incentive to find new ways to transmit microwaves efficiently. King and Schlesinger [18] studied the dielectric image line (1954), while Goubau experimented with a conducting wire coated with a thin sheath of dielectric material, a modified version of the Sommerfeld line (1950) [19]. High loss or instability of the guided field due to the large field extent hampered further development of these approaches. During the 1950s, significant amount of research on the excitation of surface wave problem was carried out (See Collin's book [20]). These investigations together with Sommerfeld's research provided the basic understanding of the problem of wave excitation on a dielectric structure. Another notable effort was the concentrated research undertaken by the Bell Laboratory investigators on the transmission of millimeter wave in a oversized circular conducting tube supporting the low-loss circularly symmetric TM wave. (This approach turned out to be not very fruitful due to high loss caused by the modal instability of the low-loss circularly symmetric TM mode in an oversized waveguide.) At that time, the Bell Laboratory group chose not to investigate dielectric fiber as a viable optical waveguide due to its high dielectric losses. History tells us that this was an unfortunate decision.

Observation of waveguide modes in optical fibers was first reported by Snitzer and Hicks in 1959, then later in 1961 by Snitzer and Osterberg, and by Kapany and Burke in 1961 [21]. In 1961, Snitzer restudied the problem of wave guidance along an optical fiber (a circular dielectric cylinder). He provided detailed numerical computations on several lower-order modes and obtained field configurations for these modes [22]. In 1962, Yeh [23] solved the unique-canonical problem of surface wave propagation on an elliptical dielectric waveguide. Unlike the circular cylinder case where each mode can be described by a single order of Bessel function, each surface wave mode for an elliptical dielectric cylinder would require infinite sums of all orders of Mathieu functions. In other words, the dispersion relation of each mode on an elliptical dielectric cylinder must be represented by an infinite determinant of all orders of Mathieu functions. In this case no pure TE or TM mode can exist on an elliptical dielectric cylinder; all modes must be of a hybrid type, that is, the HE type. Yeh not only provided the complete analytical solution to this problem, he also obtained numerical solutions on the propagation constants as well as the attenuation constants for the dominant modes. Experimental verifications were also obtained by him. Independently, at about the same time, Lynkimov et al. [24] gave an analytic solution to this problem, but no experimental or detailed numerical results were given. One notes that the use of elliptical fiber is one way of making a polarization-preserving fiber [25]. In 1965, Bloembergen [26] wrote a book summarizing his research on wave propagation in nonlinear dielectrics. His work on nonlinear dielectric became the backbone of the later discovery of solitons in optical fibers.

Two events changed the tempo and direction of research on the optical fiber as a viable information transmission link: (1) Kao and Hockham [27] in 1964 recognized that if the impurities in optical fiber can be eliminated, the fiber may become a very low-loss transmission waveguide for optical signals; and (2) Kapron et al. [28] in 1970 minimized these impurities in fused silica, resulting in the successful making of optical fiber with optical transmission losses of approximately $20\,\mathrm{dB\,km}^{-1}$. These events awakened the researchers in the communication communities throughout the world. Major efforts were started in the U.S. (Bell Telephone Laboratories, Corning Glass Works, the Naval Electronics Laboratory Center in San Diego, and the Naval Research Laboratory), in the United Kingdom (Standard Telecommunications Laboratories and the British Post Office), in Japan (Nippon Electric Company and the Nippon Sheet Glass Company), and in Germany (AEG-Telefunken, Schott Glass Company, and the Siemens Company). Three major types of optical glass fibers were in contention: the solid core single-mode fiber, the liquid core fiber, and the solid core parabolic-index-variation multimode fiber. Because of the superior dispersion property (i.e., high bandwidth behavior) of the solid core single-mode fiber, it is now universally accepted as the long-distance fiber. At that time, the hope for an all-optical communication system also ignited a significant amount of research in integrated optical circuits, that is, planar imbedded optical dielectric waveguides [29]. Because the index of refraction of the core region and that of the cladding region of an optical fiber are quite close, Snyder in 1969 [30] and Gloge in 1971 [31] provided a new look on the modes that can exist in this so-called "weakly guiding" fiber. Since the 1980s, the emphasis of the research communities has been towards finding ways to increase the bandwidth capacity and to decrease the loss behavior of a single mode fiber. The use of WDM (Wavelength Division Multiplexed) scheme [32] and solitons [33,34] has provided the much sought-after improvement. From the 1990s until now, we find an explosion of novel dielectric waveguides due to the discovery of new materials.

The revolution in digital processing started in the 1950s finally took flight in the 1960s due to the rapid advances in the use of integrated circuits in digital computers. The impact has been incredible and far-reaching. Many heretofore unsolvable engineering or scientific problems could now be solved using a relatively straightforward numerical computational approach. Much advances in the application of numerical techniques to guided wave problems were therefore developed in the period from 1965 to 1980. For example, Yee in 1966 [35] developed the FDTD (Finite Difference Time Domain) algorithm to solve the Maxwell equations numerically; Mur in 1981 [36] developed an effective absorbing boundary condition for FDTD; Yeh and Wang in 1972 [37] made use of the two-point boundary value numerical approach to solve the graded-index fiber problem; Yeh and Lindgren [38] also found an efficient numerical way to solve the many layered guided wave structure problem; Yeh et al. in 1978 [39] and, few months afterwards, Feit and Fleck [40] applied the beam propagation method to treat the problem of wave propagation in single-mode or multimode fibers; Yeh et al. in 1975 [41] became the first group who successfully adopted the finite element technique to solve a large variety of single-mode optical waveguides; and Mariki and Yeh in 1985 [42] perfected the 3D TLM (Transmission-Line Matrix) technique based on the Schelkunoff's impedance concept to solve the arbitrarily shaped dielectric waveguide problem. Several numerical approaches (e.g., FDTD, Finite Element Method, Beam Propagation Method) have already been developed into commercial software packages where a given problem is viewed as a "blackbox" having input data (that specify the problem parameters) and output data (that provide numerical results). There is no need to understand the physics or engineering aspects of the problem. The increasing importance of these numerical approaches to treat guided waves in complex dielectric structures in such a mechanical manner is the reason why there is a need to write this book on the essence of dielectric waveguides. A more thorough discussion of these numerical techniques will be given in the chapter on numerical methods.

Although by the mid-1960s, most of the fundamental concepts of guided wave propagation on linear dielectric structures have been uncovered and understood, it is the explosive revolutionary applications of these concepts in the modern world that establish the importance of understanding the essence of dielectric (surface) waveguides. Optical fibers, which are basically dielectric waveguides, are now routinely used as high-bandwidth communication links. Integrated optical circuits, also basically dielectric waveguides, are in the process of being used exclusively for super-speed computers. Recently, the pursuit of high data rate optical integrated circuits that are compatible with electronic integrated circuits has succeeded in the development of silicon based optical integrated circuits, sources, modulators, and detectors. The only remaining unexploited spectral region is the terahertz band. This is now being actively explored. It appears that, because of the high loss of metallic material in this spectrum, dielectric waveguides may be the only viable option for terahertz links. It should be pointed out that low-loss material in the terahertz region has yet to be found. Lack of suitable low-loss material in the terahertz spectrum means that the traditional optical fiber approach cannot be used to design a low-loss terahertz waveguide. Yeh and Shimabukuro in 2000 [43] found that the configuration of a high dielectric constant waveguide structure could affect greatly the loss behavior of the dominant TM-like mode. Hence, very low loss terahertz waveguide may be designed using this discovery. Other modern application areas for dielectric waveguides include the photonic crystal waveguide [44–47], basically an air or dielectric core surrounded by periodic dielectric structures; surface plasmon polaritons guides [48–50], basically a type of Sommerfeld guide; left-handed material (metamaterial) waveguide, that is a dielectric waveguide whose core region is made with artificial dielectrics with negative permittivity and negative permeability [51, 52]. The surface plasmon waveguide is of special interest in nanostructure research because of the subwavelength property of its guided wave. The peculiar behavior of waves guided by artificial metamaterial structure provides unique opportunity to invent new applications.

## 1.2 Scope of this Book

The plethora of dielectric waveguides and its vast modern applications mean that it is not possible to write an all-encompassing book on dielectric waveguides. Therefore, our goal is to write a "back to the basics" book that provides the foundation of dielectric waveguides that is useful, clear, and easy to understand.

Chapter 2 presents the fundamental electromagnetic equations with new insight in boundary conditions, classification of fields, the impedance concept, and the scalar-wave approach. Then, an over-all view of dielectric waveguides without delving directly into the specific solution of a given dielectric guided wave structure is presented in Chap. 3. The concepts given there are universally applicable to any dielectric waveguide. New and unique treatment on attenuation has been included.

Specific canonical dielectric guided wave structures will be treated in Chaps. 4–6. They are the planar, circular cylindrical, and elliptical cylindrical structures. Classical analytic modal solutions will be given and explained. The emphasis is to show how one may understand the wave guiding characteristics of a complex, perhaps more practical, dielectric structure from the knowledge of the fundamental solutions from these canonical structures. Approximate approaches for the rectangular dielectric waveguide structure and other structures with no known analytic solutions and inhomogeneous dielectric waveguides are considered in Chaps. 7 and 8.

Subsequent chapters (Chaps. 9–14) will deal with modern applications. Chapters 9 and 10 deal with linear or nonlinear optical fiber structures, where WDM propagation and WDM solitons will be emphasized. Chapter 11 deals with low-loss structures in the terahertz/millimeter wave region. Plasmon (subwavelength) waveguides are treated in Chap. 12. Chapter 13 deals with photonic crystal waveguides. Other uncommon structures, such as metamaterial structure, moving medium waveguide, and anisotropic material structures, are treated in Chap. 14.

Finally, a brief description of several important numerical techniques with examples will be given in Chap. 15.

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# 2

# FUNDAMENTAL ELECTROMAGNETIC FIELD EQUATIONS

All large-scale electromagnetic wave phenomena are governed by the Maxwell equations and the appropriate boundary conditions. In this chapter we shall discuss the fundamental equations and relations dealing with electromagnetic waves [1–3].

## 2.1 Maxwell Equations

On the basis of the established experimental laws, Maxwell postulated that the electromagnetic field vectors are subject to the following equations:

$$\boldsymbol{\nabla} \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t}, \tag{2.1}$$

$$\boldsymbol{\nabla} \times \mathbf{H}(\mathbf{r}, t) = \mathbf{J}(r, t) + \frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t}, \tag{2.2}$$

where

$\mathbf{E}(\mathbf{r}, t)$ = Electric field intensity ($\mathrm{V\,m^{-1}}$)

$\mathbf{H}(\mathbf{r}, t)$ = Magnetic field intensity ($\mathrm{A\,m^{-1}}$)

$\mathbf{D}(\mathbf{r}, t)$ = Electric displacement vector ($\mathrm{C\,m^{-2}}$)

$\mathbf{B}(\mathbf{r}, t)$ = Magnetic induction vector ($\mathrm{Wb\,m^{-2}}$)

$\mathbf{J}(\mathbf{r}, t)$ = Electric current density ($\mathrm{A\,m^{-1}}$)

These vectors are functions of space, $\mathbf{r}$ (in meters), and time, $t$ (in seconds). The mks or Giorgi system of units will be used throughout. On a macroscopic scale, the conservation of charge law can be expressed as follows:

$$\boldsymbol{\nabla} \cdot \mathbf{J}(\mathbf{r}, t) + \frac{\partial \rho}{\partial t} = 0, \tag{2.3}$$

here,

$\rho(r, t)$ = Electric charge density ($\mathrm{C\,m^{-3}}$).

This is the equation of continuity. Faraday's Law, Ampere's Law, Gauss' Law, and Coulomb's Law are included or can be derived from the Maxwell equations and the equation of continuity. For example, (2.1) is a statement of Faraday's Law, while (2.2), without the displacement current term, $\partial \mathbf{D}(\mathbf{r}, t)/\partial t$, is a statement of Ampere's Law. Maxwell postulated the existence of the displacement current term in (2.2) to express the wave nature of the electromagnetic fields. Since the divergence of the curl of any vector vanishes identically, taking the divergence of (2.1) yields

$$\boldsymbol{\nabla} \cdot \frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t} = \frac{\partial}{\partial t} \left( \boldsymbol{\nabla} \cdot \mathbf{B}(\mathbf{r}, t) \right) = 0 \tag{2.4}$$

or

$$\boldsymbol{\nabla} \cdot \mathbf{B}(\mathbf{r}, t) = 0. \tag{2.5}$$

This is Gauss' Law. From (2.5), the field of the magnetic induction vector $\mathbf{B}(\mathbf{r}, t)$ is solenoidal. The divergence of (2.2) gives

$$\boldsymbol{\nabla} \cdot \mathbf{J}(\mathbf{r}, t) + \nabla \cdot \frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t} = 0 \tag{2.6}$$

and, from (2.3), one obtains

$$\frac{\partial}{\partial t} \left[ \boldsymbol{\nabla} \cdot (\mathbf{D}(\mathbf{r}, t) - \rho(\mathbf{r}, t) \right] = 0 \tag{2.7}$$

or

$$\boldsymbol{\nabla} \cdot \mathbf{D}(\mathbf{r}, t) = \rho(\mathbf{r}, t). \tag{2.8}$$

This is Coulomb's Law. One notes that these divergence equations (2.5) and (2.8) are not independent relations of Maxwell equations (2.1) and (2.2) and the equation of continuity, (2.3). Limiting our investigation to linear phenomena, fields of arbitrary time variation can be constructed from harmonic solutions through the Fourier Transform Method, and there is no loss of generality with the assumption that the time-dependent variation of the fields may be factored out as follows:

$$\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{E}(\mathbf{r})\, e^{j\omega t}], \tag{2.9}$$

where $\omega$ is the harmonic frequency of the wave, Re means the real part of, and $\mathbf{E}(\mathbf{r})$, the electric field vector, is a spatially dependent, complex function. Similar time variations are assumed for the other field and source quantities, such as $\mathbf{D}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{D}(\mathbf{r})\, e^{j\omega t}]$, $\mathbf{H}(\mathbf{r}, t) = \mathrm{Re}[\mathbf{H}(\mathbf{r})\, e^{j\omega t}], \ldots,$ etc. The time-harmonic Maxwell equations and the continuity equation now take the forms

$$\boldsymbol{\nabla} \times \mathbf{E}(\mathbf{r}) = -j\omega \mathbf{B}(\mathbf{r}), \tag{2.10}$$

$$\boldsymbol{\nabla} \times \mathbf{H}(\mathbf{r}) = \mathbf{J}(\mathbf{r}) + j\omega \mathbf{D}(\mathbf{r}), \tag{2.11}$$

$$\boldsymbol{\nabla} \cdot \mathbf{J}(\mathbf{r}) = -j\omega \rho(\mathbf{r}). \tag{2.12}$$

The associated divergence equations are

$$\boldsymbol{\nabla} \cdot \mathbf{B}(\mathbf{r}) = \quad 0, \tag{2.13}$$

$$\boldsymbol{\nabla} \cdot \mathbf{D}(\mathbf{r}) = \quad \rho(\mathbf{r}). \tag{2.14}$$

The field vectors $\mathbf{E}(\mathbf{r})$, $\mathbf{D}(\mathbf{r})$, $\mathbf{H}(\mathbf{r})$, and $\mathbf{B}(\mathbf{r})$ are now spatially dependent complex functions. It is seen from (2.10) and (2.11) that given the source function $\mathbf{J}(\mathbf{r})$, there are four unknown quantities, $\mathbf{E},\ \mathbf{B},\ \mathbf{D}$, and $\mathbf{H}$, and two independent equations (2.10) and (2.11). Two additional independent equations relating the field quantities, $\mathbf{E},\ \mathbf{B},\ \mathbf{D}$, and $\mathbf{H}$ are needed in order that deterministic solutions for these quantities may be found. The needed equations are obtained from the constitutive relations.

## 2.2 The Constitutive Relations

The constitutive relations are derived from the description of the macroscopic properties of the medium in the immediate neighborhood of the specified field point. In general, we shall assume that, at any given point in a given medium, the vector $\mathbf{D}$ and $\mathbf{H}$ may be represented as a function of $\mathbf{E}$ and $\mathbf{B}$.

$$\mathbf{D} = F_1(\mathbf{E}, \mathbf{B}), \tag{2.15}$$

$$\mathbf{H} = F_2(\mathbf{E}, \mathbf{B}). \tag{2.16}$$

The functional dependencies of these functions are obtained from the macroscopic physical properties of the medium [4]. The behavior of a material medium in an electromagnetic field can be described in terms of distributions of electric and magnetic dipoles. The medium can be characterized by two polarization density functions: $\mathbf{P}$, the electric dipole moment per unit volume, and $\mathbf{M}$, the magnetic dipole moment per unit volume. The polarization may be induced under the action of the field from other sources, or it may be virtually permanent and independent of external fields. The permanent polarizations will be designated by $\mathbf{P}_0$ and $\mathbf{M}_0$. A few examples are given below.

### *2.2.1 Simple Medium (Linear and Isotropic)*

A simple medium is taken to be (a) linear, where $\mathbf{D}$ is a linear function of $\mathbf{E}$ and $\mathbf{H}$ is a linear function of $\mathbf{B}$, and (b) isotropic, where $\mathbf{D}$ is parallel to $\mathbf{E}$ and $\mathbf{H}$ is parallel to $\mathbf{B}$. In this simple medium,

$$\mathbf{D} = \epsilon \mathbf{E}, \qquad \mathbf{H} = \frac{1}{\mu}\mathbf{B}. \tag{2.17}$$

The parameters $\epsilon$ and $\mu$, which represent the macroscopic electromagnetic properties, are, respectively, the permittivity and permeability of the medium. For isotropic inhomogeneous media, $\epsilon$ and $\mu$ may be functions of positions. For free-space,

$$\epsilon = \epsilon_0, \qquad \mu = \mu_0, \tag{2.18}$$

where $\epsilon_0 = 8.854 \times 10^{-12}$ $(\mathrm{F\,m^{-1}})$ and $\mu_0 = 4\pi \times 10^{-7}$ $(\mathrm{H\,m^{-1}})$ are, respectively, the free-space permittivity and free-space permeability. The relationships between the field vectors and the polarization vectors are defined as follows:

$$\mathbf{P} + \mathbf{P}_0 = \mathbf{D} - \epsilon_0 \mathbf{E} = (\epsilon - \epsilon_0)\mathbf{E} = \chi_{\mathrm{e}} \epsilon_0 \mathbf{E}, \tag{2.19}$$

$$\mathbf{M} + \mathbf{M}_0 = \frac{1}{\mu_0}\mathbf{B} - \mathbf{H} = \left(\frac{\mu}{\mu_0} - 1\right)\mathbf{H} = \chi_{\mathrm{m}} \mathbf{H}, \tag{2.20}$$

where $\chi_{\mathrm{e}}$ and $\chi_{\mathrm{m}}$ are called the electric and magnetic susceptibilities. The electric and magnetic polarization vectors are zero in free-space. Strictly speaking, the relations (2.19) and (2.20) are definable only for time-periodic phenomena, since in general $\epsilon$ and $\mu$ are functions of the frequency. The frequency dependence of the constitutive parameters is known as the *dispersive property* of the medium. Hence, these relations are applicable to other than time-periodic, time-varying fields only when, over the significant part of the frequency spectrum covered by the Fourier components of the time dependence, the constitutive parameters $\epsilon$ and $\mu$ are sensibly independent of frequency.

### *2.2.2 Anisotropic Medium [5–7]*

In an anisotropic material medium, the electromagnetic properties are functions of the field directions about a point. Thus, in general,

$$\mathbf{D} = \underline{\underline{\epsilon}} \cdot \mathbf{E} \qquad \underline{\underline{\epsilon}} = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix}, \tag{2.21}$$

$$\mathbf{B} = \underline{\underline{\mu}} \cdot \mathbf{H} \qquad \underline{\underline{\mu}} = \begin{bmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{bmatrix}. \tag{2.22}$$

Here, $\epsilon_{ij}$ and $\mu_{ij}$ are elements of the permittivity matrix and the permeability matrix describing the anisotropic characteristics of the medium. For inhomogeneous and anisotropic medium, $\epsilon_{ij}$ and $\mu_{ij}$ are functions of positions. For anisotropic and dispersive medium, $\epsilon_{ij}$ and $\mu_{ij}$ are functions of the frequency. The electromagnetic properties of a few common anisotropic material media are characterized as follows:

(a) Magnetized Ferrite Medium

$$\underline{\underline{\epsilon}} = \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_1 \end{bmatrix} = \text{a scalar} \qquad \underline{\underline{\mu}} = \begin{bmatrix} \mu_{11} & \mu_{12} & 0 \\ \mu_{21} & \mu_{22} & 0 \\ 0 & 0 & \mu_{33} \end{bmatrix} \tag{2.23}$$

with an impressed static magnetic field along the axial $z$-axis.

(b) Crystalline Medium

$$\underline{\underline{\epsilon}} = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{bmatrix} \qquad \underline{\underline{\mu}} = \begin{bmatrix} \mu_0 & 0 & 0 \\ 0 & \mu_0 & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} = \text{a scalar.} \tag{2.24}$$

(c) Uniaxial Medium

$$\underline{\underline{\epsilon}} = \begin{bmatrix} \epsilon_1 & 0 & 0 \\ 0 & \epsilon_2 & 0 \\ 0 & 0 & \epsilon_3 \end{bmatrix} \qquad \underline{\underline{\mu}} = \begin{bmatrix} \mu_0 & 0 & 0 \\ 0 & \mu_0 & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} = \text{a scalar.} \tag{2.25}$$

(d) Cold Plasma with Impressed Static Magnetic Field $B_0$

$$\underline{\underline{\epsilon}} = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & 0 \\ \epsilon_{21} & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \qquad \underline{\underline{\mu}} = \begin{bmatrix} \mu_0 & 0 & 0 \\ 0 & \mu_0 & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} = \text{a scalar.} \tag{2.26}$$

Here $\epsilon_{ij}$ is a function of frequency as follows:

$$\begin{aligned}
\epsilon_{11} &= \epsilon_0 \left[ 1 - \frac{\omega_{\mathrm{p}}^2 (\omega - j\nu)}{\omega \left[ (\omega - j\nu)^2 - \omega_{\mathrm{c}}^2 \right]} \right], \\
\epsilon_{22} &= \epsilon_{11}, \\
\epsilon_{12} &= j\epsilon_0 \left[ \frac{\omega_{\mathrm{p}}^2 \omega_{\mathrm{c}}}{\omega (\omega - j\nu + \omega_{\mathrm{c}}) (\omega - j\nu - \omega_{\mathrm{c}})} \right], \\
\epsilon_{21} &= -\epsilon_{12}, \\
\epsilon_{33} &= \epsilon_0 \left[ 1 - \frac{\omega_{\mathrm{p}}^2}{\omega (\omega - j\nu)} \right],
\end{aligned}$$

where $\omega_{\mathrm{p}} = \left( n_{\mathrm{e}} e^2 / m_{\mathrm{e}} \epsilon_0 \right)^{1/2}$ is the electron plasma frequency, $n_{\mathrm{e}}$ is the electron number density, $e$ is the electronic charge, $m_{\mathrm{e}}$ is the electron mass, and $\omega_{\mathrm{c}} = eB_0/m_{\mathrm{e}}$ is the electron cyclotron frequency, where $B_0$ is the impressed static magnetic induction along the $z$-axis. The term $\nu$ is the collision frequency of the electrons with the heavier particles.

### *2.2.3 Left-Handed Medium (Metamaterial) [8–10]*

A class of artificial media can be characterized as follows:

$$\mathbf{D} = -\epsilon \mathbf{E}, \qquad \mathbf{H} = -\frac{1}{\mu} \mathbf{B}. \tag{2.27}$$

A left-handed material medium (usually artificially made) is one with negative permittivity and negative permeability in the frequency range of interest. The index of refraction in such a metamaterial medium is also negative. The permittivity and the permeability of the medium are usually frequency dependent and lossy. In other words, they are complex quantities.

### *2.2.4 Conducting Medium*

According to Ohm's law

$$\mathbf{J} = \sigma \mathbf{E}, \tag{2.28}$$