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Solar System Astrophysics

Background Science and the Inner Solar System

Springer

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Preface

This work is appearing in two parts because its mass is the result of combining detailed exposition and recent scholarship. Book I, dealing mainly with the inner solar system, and Book II, mainly on the outer solar system, represent the combined, annually updated, course notes of E. F. Milone and W. J. F. Wilson for the undergraduate course in solar system astrophysics that has been taught as part of the Astrophysics Program at the University of Calgary since the 1970s. The course, and so the book, assumes an initial course in astronomy and first-year courses in mathematics and physics. The relevant concepts of mathematics, geology, and chemistry that are required for the course are introduced within the text itself.

Solar System Astrophysics is intended for use by second- and third-year astrophysics majors, but other science students have also found the course notes rewarding. We therefore expect that students and instructors from other disciplines will also find the text a useful treatment. Finally, we think the work will be a suitable resource for amateurs with some background in science or mathematics. Most of the mathematical formulae presented in the text are derived in logical sequences. This makes for large numbers of equations, but it also makes for relatively clear derivations. The derivations are found mainly in Chapters 2–6 in the first volume, *Background Science and the Inner Solar System,* and in Chapters 10 and 11 in the second volume, *Planetary Atmospheres and the Outer Solar System.* Equations are found in the other chapters as well but these contain more expository material and recent scholarship than some of the earlier chapters. Thus, Chapters 8 and 9, and 12–16 contain some useful derivations, but also much imagery and results of modern studies.

The first volume starts with a description of historical perceptions of the solar system and universe, in narrowing perspective over the centuries, reflecting the history (until the present century, when extra-solar planets again have begun to broaden our focus). The second chapter treats the basic concepts in the geometry of the circle and of the sphere, reviewing and extending material from introductory astronomy courses, such as spherical coordinate transformations. The third chapter then reviews basic mechanics and two-body systems, orbital description, and the computations of ephemerides, then progresses to the restricted three-body and n-body cases, and concludes with a discussion of perturbations. The fourth chapter treats the core of the solar system, the Sun, and is not a bad introduction to solar or stellar astrophysics; the place of the Sun in the galaxy and in the context of other stars is described, and radiative transport, optical depth, and limb-darkening are introduced. In Chapter 5, the structure and composition of the Earth are discussed, the Adams–Williamson equation is derived, and its use for determining the march of pressure and density with radius described. In Chapter 6, the thermal structure and energy transport through the Earth are treated, and in this chapter the basic ideas of thermodynamics are put to use. Extending the discussion of the Earth's interior, Chapter 7 describes the rocks and minerals in the Earth and their crystalline structure. Chapter 8 treats the Moon, its structures, and its origins, making use of the developments of the preceding chapters. In Chapter 9, the surfaces of the other terrestrial planets are described, beginning with Mercury. In each of the three sections of this chapter, a brief historical discussion is followed by descriptions of modern ground-based and space mission results, with some of the spectacular imagery of Venus and Mars. The chapter concludes with a description of the evidence for water and surface modification on Mars. This concludes the discussion of the inner solar system.

The second volume begins in Chapter 10 with an extensive treatment of the physics and chemistry of the atmosphere and ionosphere of the Earth and an introduction to meteorology, and this discussion is extended to the atmospheres of Venus and Mars. Chapter 11 treats the magnetospheres of these planets, after a brief exposition of electromagnetic theory. In Chapter 12, we begin to treat the outer solar system, beginning with the gas giants. The structure, composition, and particle environments around these planets are discussed, and this is continued in Chapter 13, where the natural satellites and rings of these objects are treated in detail, with abundant use made of the missions to the outer planets. In Chapter 14, we discuss comets, beginning with an historical introduction that highlights the importance of comet studies to the development of modern astronomy. It summarizes the ground- and space-based imagery and discoveries, but makes use of earlier derivations to discuss cometary orbits. This chapter ends with the demise of comets and the physics of meteors. Chapter 15 treats the study of meteorites and the remaining small bodies of the solar system, the asteroids (*aka* minor planets, planetoids), and the outer solar system "Kuiper Belt" objects, and the closely related objects known as centaurs, plutinos, cubewanos, and others, all of which are numbered as asteroids. The chapter ends with discussions of the origin of the solar system and of debris disks around other stars, which point to widespread evidence of the birth of other planetary systems. Finally, in Chapter 16, we discuss the methods and results of extra-solar planet searches, the distinctions among stars, brown dwarfs, and planets, and we explore the origins of planetary systems in this wider context.

At the end of nearly every chapter we have a series of challenges. Instructors may use these as homework assignments, each due two weeks after the material from that chapter were discussed in class; *we* did! The general reader may find them helpful as focusing aids.

November 1, 2007 *E. F. Milone* & *W. J. F. Wilson*

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These volumes owe their origin to more than 30 years of solar system classes in the Astrophysics Program at the University of Calgary, called, at various times, Geophysics 375, Astrophysics 301, 309, and 409. Therefore, we acknowledge, first, the students who took these courses and provided feedback. It is also a pleasure to thank the following people for their contributions:

David Mouritsen, formerly of Calgary and now Toronto, provided for Chapter 1 and our covers an image of his original work of art, an interpretation of Kepler's *Mysterium Cosmographicum*, in which the orbits of the planets are inscribed within solid geometric figures.

In Chapter 3, the Bradstreet and Steelman software package, *Binary Maker 3* was used to create an image to illustrate restricted three-body solutions.

University of Calgary Professor Emeritus Alan Clark gave us an image of an active region and detailed comments on the solar physics material of Chapter 4; Dr Rouppe van der Voort of the University of Oslo provided high-quality images of two other active region figures, for Chapter 4; the late Dr Richard Tousey of the US Naval Research Laboratory provided slides of some of the images, subsequently scanned for Chapter 4; limb-darkened spectral distribution plots were provided by Dr Robert L. Kurucz, of the Harvard-Smithsonian Center for Astrophysics; Dr. Charles Wolff, of Goddard Space Flight Center, NASA, reviewed the solar oscillations sections and provided helpful suggestions.

Dr. D. J. Stevenson provided helpful criticism of our lunar origins figures, and Dr. Robin Canup kindly prepared panels of her lunar simulations for our Fig. 8.10.

Dr Andrew Yau provided excellent notes as a guest lecturer in Asph 409 on the Martian atmosphere and its evolution, which contributed to our knowledge of the material presented in Chapters 9, 10, and 11; similarly, lectures by Professor J. S. Murphree of the University of Calgary illuminated the magnetospheric material described in Chapter 11.

NASA's online photo gallery provided many of the images in Chapters 8, 9, 12, 13, 14, and some of those in Chapter 15; additional images were provided by the Naval Research Laboratory (of both the Sun and the Moon). Some of these and other images involved work by other institutions, such as the U.S. Geological Survey, the Jet Propulsion Laboratory,

Arizona State Univ., Cornell, the European Space Agency, the Italian Space Agency (ASI), CICLOPS, CalTech, Univ. of Arizona, Space Science Institute, Boulder, the German Air and Space Center (DLR), Brown University, the Voyagers and the Cassini Imaging Teams, the Hubble Space Telescope, University of Maryland, the Minor Planet Center, Applied Physics Laboratory of the Johns Hopkins University, and the many individual sources, whether cited in captions or not, who contributed their talents to producing these images.

Dr. John Trauger provided a high resolution UV image of Saturn and its auroras for Chapter 12.

Dr William Reach, Caltech, provided an infrared mosaic image of Comet Schwassmann–Wachmann 3, and Mr John Mirtle of Calgary provided many of the comet images for Chapter 14, including those of Comets 109P/Swift–Tuttle, C/1995 O1 (Hale–Bopp), C/Hyakutake, Lee, C/Ikeya–Zhang, Brorsen–Metcalfe, and Machholz; Professor Michael F. A'Hearn of the University of Maryland for his critique of the comet content of Chapter 14.

Dr Allan Treiman, of the Lunar and Planetary Institute, Houston, was kind enough to provide background material on the debate over the ALH84001 organic life question, mentioned in Chapter 15. Mr Matthias Busch made available his *Easy-Sky* images of asteroid family distributions, and the Minor Planet Center provided the high-resolution figures of the distributions of the minor planets in the inner and outer solar system.

Dr Charles Lineweaver, University of New South Wales, provided a convincing illustration for the brown dwarf desert, illustrated in Chapter 16; University of Calgary graduate student Michael Williams provided several figures from his MSc thesis for Chapter 16.

Mr Alexander Jack assisted in updating and improving the readability of equations and text in some of the early chapters, and he and Ms Veronica Jack assisted in developing the tables of the extra-solar planets and their stars for Chapter 16.

In addition, we thank the many authors, journals, and publishers who have given us permission to use their figures and tabular material or adaptations thereof, freely. Finally, it is also a pleasure to thank Springer editors Dr Hans Koelsch, Dr Harry Blom, and their associate, Christopher Coughlin, for their support for this project.

Contents

Background Science and the Inner Solar System

1. Perceptions of the Solar System in History

The solar system has been around for a long time! Our perceptions of it, on the other hand, date back, arguably, only to the Upper Paleolithic (∼70,000 to ∼10,000 years ago).

The Paleolithic evidence for interest in the Moon, for example, is in the form of possible tallies of days in such features as the 17-in. high sculpture called the *Venus of Laussel* in a rock shelter dated from 20,000 to 18,000 y BP in the Dordogne region of France (see Campbell 1988, pp. 65–66 for a discussion of its symbolic significance and Marshack 1972, p. 335 for arguments for its use as a tally) and the Blanchard bone (among other artifacts) with a complicated chain of crescent incisions, also investigated by Marshack. The amply endowed *Venus* holds in her right hand an upturned horn on which are incised 13 grooves (defining 14 non-groove areas), a number said to represent an approximation to the number of waxing crescents in a year ($12\frac{1}{3}$) and the number of days between new and full moon ($\sim 14\frac{3}{4}$) (Marshack 1972).

In the Neolithic (or "New Stone Age," roughly from 6500 to 1500 BC), the evidence for the importance of the Sun and the Moon, at least, is overwhelming. It can be found in the many ancient alignment sites in the British Isles and is echoed in Stone Age cultures around the world, at least according to some interpretations. See Thom (1972) for a flavor of that evidence or, for example, Kelley and Milone (2005, Chapter 6) for a more recent summary.

Aside from practical astronomy, with calendrical usefulness for both agriculture and religions, astrology plays an increasing role in late antiquity (first several centuries AD). For the past four millennia, and especially during Hellenic and Hellenistic times (prior to and after Alexander the Great, respectively), we do know what people thought regarding the nature and origin of the solar system. Without attempting detailed examination of each one, we can characterize the principal theories about the solar system (in earlier times, the entire cosmos) in as shown in Table 1.1.

Most of the many notable figures in Tables 1.1 and 1.2 are discussed in Kelley and Milone (2005). Here, we single out only three for further discussion.

Martianus Capella was a poet and summarizer, who probably wrote his allegorical poem *Satyricon* after the sack of Rome by the Huns in 410 AD, which he mentions, and before 429 AD, when Carthage was overrun by Vandals, which he does not. His description of a quasi-heliocentric system

Table 1.1. Ancient theories of the solar system

Pythagoras (*∼6th* BC): Earth at centre of planetary spheres (included Sun, Moon and the sphere of the fixed stars)

Anaxagoras of Clazomenae (c. 500–428 BC*)*: The Sun is a hot iron mass (based on meteorite evidence), bigger than the Peloponnesus; the Moon is a stone; the Earth, the centre of a cosmic vortex

Philolaus (*∼5th c.* BC*):* The Earth moves, while the stellar sphere is immobile. He felt that there should be ten planets (fixed star sphere included), so Philolaus invented an anti-Earth, perpetually located between the Earth and a central fire about which all the planets, including the sun, moved; orbits were circular, but not coplanar. This is the earliest recorded theory to consider the earth as a moving object—but it was to explain the daily western movement of the "fixed" stars and other objects, not the annual motions of the Sun or the planets

Eudoxus of Cnidus (∼408–355 BC*):* The fixed stars and each planet are carried on separate, concentric, rotating spheres, on various axes, centered on the Earth

Aristarchus of Samos (∼250 BC*)*: The Sun is at the center; the Earth both revolves around the Sun and rotates on its own axis; the Moon revolves around the Earth

Apollonius of Perga (∼220 BC*)*: Combinations of motion in circular orbits

Hipparchus (2nd c. BC*)*: The Earth is at the center; planets (including the Sun) revolve around the Earth; orbits are circular but non-concentric

Claudius Ptolemy (2nd c. AD*)*: The Earth is at the center; planets (including the Sun) revolve around the Earth; the orbits are combinations of circular motions, characterized by deferent orbits and epicyclic gyrations

Origen (3rd c. AD*)*: There exist a multiplicity of worlds, with the creation, fall, and redemption occurring on each

Martianus Capella ($\lesssim 5^{\text{th}}$ *c. ?)*: His *Satyricon* refers to a Sun-centered solar system; this popular work kept the notion alive in the West to Copernicus' time

Aryabhata (b. 476 AD*)*: He allowed the possibility of a heliocentric universe (but his work was not known in the West until after Copernicus)

(like Tycho's model, it had Mercury and Venus orbiting the Sun), kept alive this idea. Copernicus explicitly mentions Capella's (and not Aristarchus') discussion of the heliocentric system.

Much later, following the Renaissance and Reformation, Tycho Brahe and Johannes Kepler were important transition figures.

Brahe himself contributed to a break in the classical paradigm by demonstrating, with observational data, that comets moved among the orbits of planets, thus shattering once and for all the notion that rotating crystalline spheres bore the planets. His discovery of a supernova and his determination that it was a very distant object demolished the idea of the immutability of the heavens. Moreover, this and his cometary discoveries refuted the ideas of Aristotle, for centuries, the highest authority on scientific questions.

Table 1.2. Post-medieval, pre-nineteenth century theories of the solar system

Nicholas of Cusa (1401–1464): He is said to have championed a Sun-centered theory; no explicit writings

Nicholas Copernicus (1473–1543): Sun-centered solar system; planets moved (as classically) in circular orbits

Tycho Brahe (1546–1601): Sun-centred planetary scheme—but Sun and planets revolve about the Earth

Johannes Kepler (1571–1630): Elliptical orbits; this is the first explicit departure from circular orbits. Keplerian empirical "laws"

René Descartes (1596–1650): The solar system is a complex of vortices; moons and planets arise from vortices within vortices

Isaac Newton (1642–1727): Planetary orbital motion due to gravity. The solar system is far from the stars (considered distant because of lack of parallax and relative motions) which are themselves, therefore, suns

Georges-Louis Leclerc Buffon (1707–1788): Collisional origin for the solar system (Sun with comet)

Immanuel Kant (1724–1794) and Simon de Laplace (1749–1827): The solar system had a nebular origin; contraction and conservation of angular momentum caused disk formation

Ernst Florenz Friedrich Chladni (1756–1827): The early aggregation of dust became planetesimals, and some of these, planets (Chladni 1794)

Kepler's early notion of the heliocentric planetary orbits carried on (crystalline) spheres inscribing and inscribed by the five regular polyhedral solids (see Figure 1.1) as expressed in the first half of his *Mysterium Cosmographicum* (Kepler 1596), evolved over his lifetime into a realization that the orbits were ellipses produced by forces that depended on the distance from the Sun. His persistence in trying to make sense of Tycho Brahe's highly precise data led to his conclusion that planetary orbits could not be circular. The consequences of this profound discovery resulted in the "Breaking of the Circle," in many ways (Nicholson 1950).

Thus their pursuit of the highest quality observational data and unflinching belief in the meaningfulness of those data led both of them to renounce the geocentric universe, although Brahe's was a last effort to incorporate the idea of a stationary Earth into a defensible model.

There are many nineteenth and twentieth century theories. Most of these theories involve either collisions or accretions or both. Table 1.3 presents some examples. Several of the theories, including the most recent, are cited in the references list. Note the trend from collisional theories to accretion theories in this interval.

Any thorough study of the solar system draws from chemistry, geology, and even biology, and numerous insights from those sciences will be brought into and used in this book. But, it is still basically astronomy. Observational

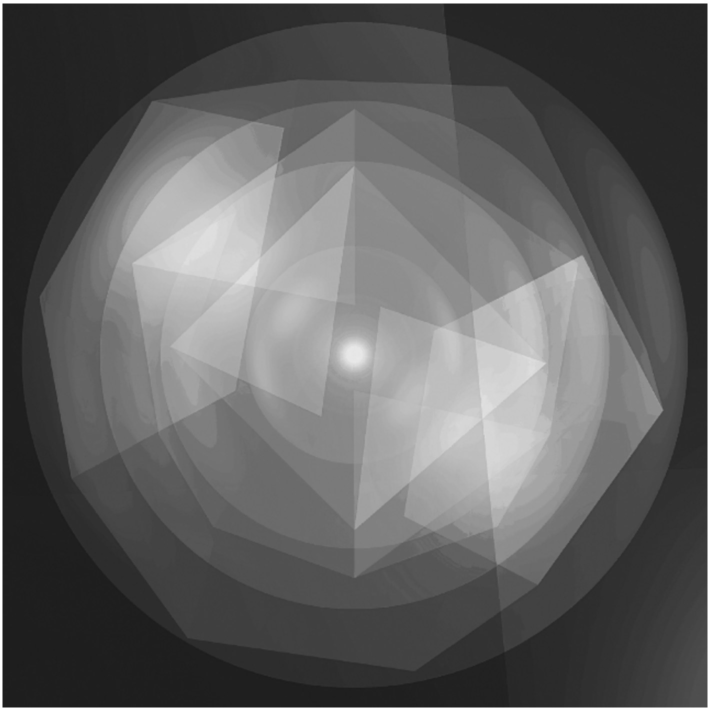

Fig. 1.1. An early Keplerian view of the solar system, inspired by Kepler's *Mysterium Cosmographicum* model of nested spheres and geometric solids. Original art by David Mouritsen (2005) and reproduced here with permission

astronomy has provided the basic data which are needed to understand the planets and other objects of the solar system, even if most of the new critical data now come from satellites and space probes, containing not only imaging cameras, but spectrographs (exploring the spectral energy distribution from radio and infrared to x-rays), magnetometers (to probe the structure and strength of magnetic fields), and particle detectors. But the latter does emphasize that remote sensing plays a vital role, and therefore our understanding of the solar system is more and more through "space science."

When we consider that the ultimate quest is to understand how the solar system came into being, how it evolves, and to what end, it is clear that a critical field of investigation has to be solar system dynamics. Therefore, this is an important area of study even for those who are not going to work for the Canadian, US, and European space agencies (CSA, NASA, and ESA, respectively) or for any of the space agencies developed in other countries around the world. We will take this subject up in a later chapter.

Table 1.3. Nineteenth and twentieth century theories of the origin of the solar system

A. W. Bickerton (1842–1929): Star–Sun collision; explosive eruption forms planets

R. A. Proctor (1837–1888): Planetesimal aggregations

T. C. Chamberlain (1843–1928): Star–Sun collision; tidal eruption creates planets; (1904): Planetesimals

F. R. Moulton (1872–1952): Star–Sun collision; tidal eruption + planetary accretion

K. O. B. Birkeland (1867–1912): Ions in solar atmosphere form rings in solar magnetic field

S. A. Arrhenius (1859–1927): Direct Sun–star collision, leaving the Sun and long filament as remnants

H. Jeffreys (1891–1989): Grazing Sun–star collision, leaving long filament that fragmented

J. H. Jeans (1877–1946): Star–Sun collision producing tidal filament

H. P. Berlage (1856–1934): Solar particle emission lead to gaseous rings/disks

H. N. Russell (1877–1957): Binary star component disrupted, forming a filament

D. ter Haar (1938): Contracting, turbulent solar envelope developed into planets

H. O. G. Alfven (1908–1995): Sun collided with a gas cloud which became ionized, and formed rings in the Sun's magnetic field; electromagnetic braking and transfer of angular momentum

O. I. Schmidt (1891–1956): Sun collided with a swarm of interstellar bodies which became planets by accretion; refined by R. A. Lyttleton (1911–1995)

C. F. von Weizsäcker (1912–2007): Turbulent eddies in protosun formed planets and satellites

F. Hoyle (1915–2001): Sun's binary companion went supernova, producing gaseous shells; remnant star left the system

F. Whipple (1906–2004): Protosun captured dust cloud of large angular momentum

G. Kuiper (1905–1973): Gravitational instabilities in protosun's gaseous envelope became planets

V. S. Safronov (1917–1999): Aggregation of dust into planetesimals

A. G. W. Cameron (1924–2005): Gaseous protoplanet theory

C. Hayashi et al. (b. 1920): Aggregation into planetesimals ("Kyoto" school)

The investigation of the nature of the solar system points to several striking facts:

- The solar rotation and the revolution of all the planets are in the same sense: CCW as viewed from the north ecliptic pole (NEP).
- The orbits are very nearly coplanar (the biggest departures being for the innermost and the outermost (usually) planets—Mercury and Pluto); and,

Table 1.4. The Titius–Bode law

$r = 2^n \cdot 0.3 + 0.4$

Planet	n	Prediction	True a
Mercury	$-\infty$	0.4	0.39
Venus	0	0.7	0.72
Earth	1	1.0	1.00
Mars	2	1.6	1.52
Minor planets	3	2.8	$< 2.8 >$
Jupiter	4	5.2	5.20
Saturn	5	10.0	9.54
Uranus	6	19.6	19.18
Neptune	7	38.8	30.07
Pluto	8	77.2	39.46
Eris	9	154.0	67.78

again except for Mercury and dwarf and minor planets, very nearly circular; the spacing of the planets is not random, but is described by the Titius–Bode law.[1]

- Although the mass is strongly concentrated in the Sun, the angular momentum is not.
- The coplanar revolutions of the planets and the solar rotation (to be discussed later) already make a disk formation of the solar system more likely than a collisional origin.
- The low orbital eccentricities of the planets strengthen the case. The circularity of Neptune's orbit, the outermost and thus least strongly bound of all the major planets (Pluto, Eris, and other "dwarf planets" excepted from this category), is especially compelling.

These and other properties of the solar system will be reviewed at the beginning of Chapter 3 and again later, mainly in chapters 15 and 16 of Milone & Wilson (2008), when we consider the solar system's origins.

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[1] See Table 1.4. The Titius–Bode law (Bode 1772; Wurm 1787; Jaki 1972; Nieto 1972) can be expressed in the form:

$$r = (3 \times 2^n + 4)/10 \tag{1.1}$$

where $n = -\infty$, 0, 1, 2, 3, … 6 (7–9, Neptune–Eris, are not well represented). The relation can be better expressed in the more modern form,

$$r_n = r_0\, a^n \tag{1.2}$$

where r_n is the distance in AUs of the nth planet (Mercury is $n = 0$), in order of distance, from the Sun, and where $a \equiv 1.73$ (in the Blagg–Richardson formulation; see Nieto 1972). Note that one can either determine or assume the quantity r_0.

In our final chapter, we take up the properties of extrasolar planets. The existence of disks around other stars and giant molecular clouds with which protostars are associated are further evidence for a disk origin of the solar system. Currently, planets are believed to arise from protostellar disks, but there are sharp disagreements over the separate roles of disk condensation and accretion of other condensates or larger—perhaps pre-existing clumps of matter. The importance of disks is empirically based not only on the observations of infrared tori seen around other stars (most famously, but far from exclusively, β Pictoris), but also on the basis of meteorite and theoretical studies. However, theoretical difficulties in explaining the formation of the lesser giants at their current locations in the solar system and the presence of "hot Jupiters" in other star systems strongly suggest dynamical migrations of planets from their points of origin, if the disk origin is to be sustained. Indeed, modern simulations provide mounting evidence that the lesser giants in the solar system, Uranus and Neptune, were formed closer to the Sun and were driven further out by dynamical interactions. In the last chapter of Milone & Wilson (2008) we will summarize what can be generalized about the origins of planetary systems and the prospects of finding terrestrial planets in other star systems.

Any study of the origin of the solar system must be a kind of mystery-solving expedition. So, we need to take note of the clues as we go along, as a police inspector in attempting to unscramble a forensic puzzle.

We start by providing some basic investigatory tools and then begin the search for clues in the dynamical and physical structure of the solar system.

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Challenges

[1.1] Try to categorize the theories of the origin of the solar system, listing the theories below each category. Is there any evidence for historical evolution or evidence of progress among the theories of a particular type?

[1.2] Compare the computed distances of the Titius–Bode and Blagg–Richardson laws to the mean distances of the planets from the Sun. A spreadsheet is the most convenient way of doing this. Can you formulate another relation that describes these distances precisely? (Hint: think non-linear. You can make use of a software package such as ***Tablecurve***[2] to find other relationships.)

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[2] Tablecurve 2D Automated Curve Fitting Software v2.0 1994 ed., copyrighted by AISN Software Incorporated.

2. Basic Tools and Concepts

In this chapter, from the Greeks (through much subsequent development), we derive the tools of spherical astronomy. We will describe the basic theorems of spherical trigonometry and emphasize the usefulness of the sine and cosine laws. We will also describe the ellipse and its properties, in preparation for a subsequent discussion of orbits.

2.1 Circular Arcs and Spherical Astronomy

All astronomical objects outside the solar system are sufficiently far away that their shifts in position due to parallax (caused by periodic motions of the Earth) and proper motion (caused mainly though not exclusively by the objects' own motion) are too small to be discerned—at least by the unaided eye. Historically, this suggested that these objects could be regarded as being fixed to a sphere, the *celestial sphere*, of some very large radius centered on the Earth. Objects within the solar system change position with time, e.g., a superior planet's orbital motion causes an eastward motion across our sky relative to the distant stars and the Earth's motion causes the planet to follow a retrograde loop. However, objects within the solar system can be referenced to the celestial sphere at any given instant of time.

We may wish to calculate the distance measured across the sky from one object to another knowing the distance of each of them from a third object, and also knowing an appropriate angle. ("Distance across the sky" is actually an arclength, measured in units of angle such as degrees or radians.) In doing this, we are in essence drawing arcs joining three objects to form a triangle on a spherical surface, so the mathematical relationships involved are those of spherical trigonometry. When we apply them to the sky we are practicing *spherical astronomy.* We note also that the objects do not need to be real; one or more of them can be a reference point, such as the north or south celestial pole.

A *spherical triangle* is a triangle on the surface of a sphere such that each side of the triangle is part of a *great circle*, which has as its center the center of the sphere (a *small circle* will have its center along a radius of the sphere). For example, the Earth's equator is a great circle (assuming a spherical Earth), and any line of latitude other than the equator is a small circle.

The procedure in spherical astronomy lies primarily in the calculation of one side or angle in a spherical triangle where three other appropriate quantities are known, e.g., we may wish to find the length (in degrees) of one side of a spherical triangle given two other sides and the angle formed between them or find one side given a second side and the angle opposite each side or find one angle given a second angle and the side opposite each angle.

First, we review the basics of spherical trigonometry and then derive the cosine and sine laws, analogous but not identical to the cosine and sine laws of plane geometry. Additional theorems relating the three angles and three sides (involving, for example, haversines[1]) can also be found, but will not be derived here; for such theorems, see Smart's (1977) or Green's (1985) spherical astronomy texts, for example.

Figure 2.1(a) shows an example of a spherical triangle. The three ellipses are great circles seen in projection, and the spherical triangle (marked by heavy lines) is formed by their intersections. In Figure 2.1(b), we label the three sides of the triangle a, b, and c, and the angles at the three corners A, B, and C. We also draw a radius from the center of the sphere to each corner of the triangle (dashed lines).

Figure 2.2(a) shows an enlarged view of the spherical triangle and the radii to the center of the sphere. A circle is coplanar with its radii, so the shaded cross-section in this figure is a plane triangle (i.e., with straight sides).

It is important to distinguish between angles A, B, and C, which are the angles seen by an (two-dimensional) observer on the surface of the sphere and angles a, b, and c, which are the angles seen at the center of the sphere (Figure 2.2(b)). That is, angle A is the angle (measured in degrees or radians)

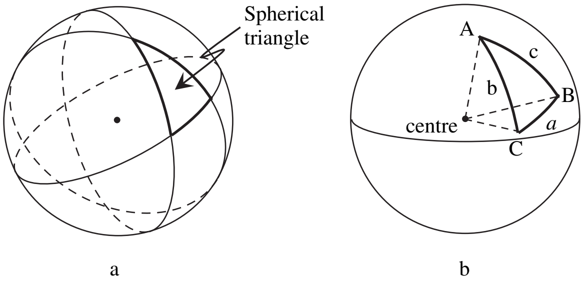

Fig. 2.1. The sphere with a spherical triangle on its surface

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[1] hav $\theta = 1/2\,[1 - \cos\theta]$.

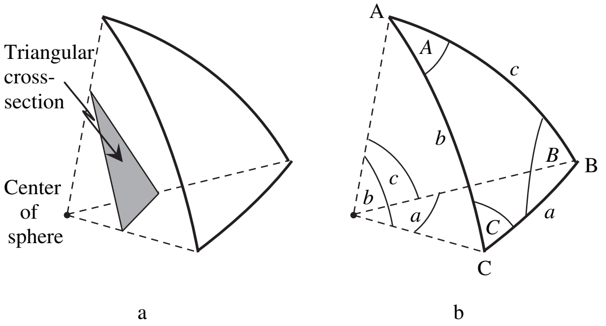

Fig. 2.2. Relating the sides of a spherical triangle to angles a, b, and c at the center of the sphere and angles A, B, and C on the surface of the sphere. The *heavy lines* are arcs of great circles; the *dashed lines* are radii of the sphere

at the intersection of sides b and c on the surface of the sphere, whereas angle a is the angular separation "across the sky" from point B to point C (also measured in degrees or radians) as viewed from the center of the sphere. The principal equations relating these quantities to each other are the cosine and sine laws of spherical astronomy, which we will now derive.

NB: Small circle arcs have a different relation to interior angles. The laws derived here, therefore, do not apply to them.

2.1.1 The Law of Cosines for a Spherical Triangle

In this section we demonstrate a proof of the cosine law:

$$\cos a = \cos b \cos c + \sin b \ \sin c \ \cos A$$

First, we derive some useful results for a right spherical triangle ($\angle C = 90°$ in Figure 2.3), which will also be useful in proving the sine law.

Take the radius of the sphere to be one unit of distance (OA = 1).

Note that any three points define a flat plane, so planes OAC, OAB, and OBC are each flat, and plane OAC is perpendicular to plane OBC. (To visualize this, it may help to think of arc BC as lying along the equator, in which case arc AC is part of a circle of longitude and arc AB is a "diagonal" great circle arc joining the two. Circles of longitude meet the equator at right angles, and longitudinal planes are perpendicular to the equatorial plane.)

From A, drop a line AD perpendicular to the plane OBC. Plane OAC ⊥ plane OBC, so point D is on the line OC. From D, draw a line DE ⊥ OB. (DE is, of course, not ⊥ OC.) Then:

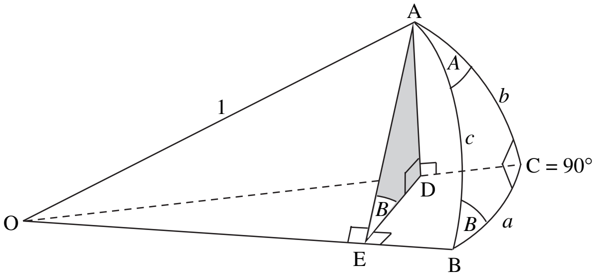

Fig. 2.3. Projection of a right spherical triangle onto a plane perpendicular to the base plane OBC. For convenience, we define points A and B to be located at the apices of angles *A* and *B*, respectively

Plane ADE ⊥ plane OBC because line AD ⊥ plane OBC
Plane ADE ⊥ line OB because DE ⊥ OB and AD ⊥ plane OBC
$\angle$ADE = 90° because plane OAC ⊥ plane OBC
$\angle$OED = 90° and $\angle$OEA = 90° because plane ADE ⊥ line OB

Now define a plane tangent to the sphere (i.e., ⊥ to line OB) at point B; then $\angle B$ lies in this tangent plane and, in fact, is equal to the angle of intersection of the two planes OAB and OBC. Plane ADE is parallel to the tangent plane, since both planes are ⊥ line OB, so it follows that:

$$\angle \mathrm{AED} = \angle B$$

Then

$$\sin c = \frac{\mathrm{AE}}{\mathrm{OA}} = \frac{\mathrm{AE}}{1} = \mathrm{AE}$$
$$\cos c = \frac{\mathrm{OE}}{\mathrm{OA}} = \mathrm{OE}$$
$$\sin b = \frac{\mathrm{AD}}{\mathrm{OA}} = \mathrm{AD}$$
$$\cos b = \frac{\mathrm{OD}}{\mathrm{OA}} = \mathrm{OD}$$

These sides are shown in Figure 2.4. Then from ΔODE,

$$\cos a = \frac{\mathrm{OE}}{\mathrm{OD}} = \frac{\cos c}{\cos b}$$

or

$$\cos c = \cos a \ \cos b \tag{2.1}$$

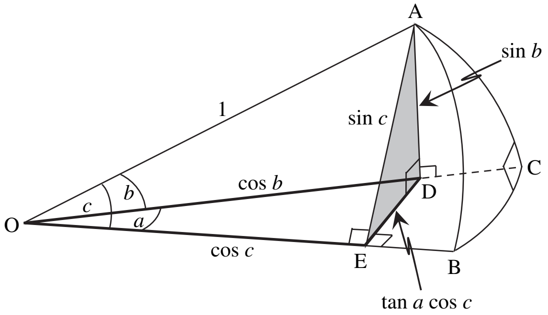

Fig. 2.4. Lengths of relevant sides, taking the radius of the sphere to be one unit of distance (OA = 1)

Also from ΔODE,

$$\tan\, a = \frac{\mathrm{DE}}{\mathrm{OE}} = \frac{\mathrm{DE}}{\cos\, c}$$

or

$$\mathrm{DE} = \tan\, a\ \cos\, c$$

Therefore, from ΔADE,

$$\cos\, B = \frac{\mathrm{DE}}{\mathrm{EA}} = \frac{\tan\, a\ \cos\, c}{\sin\, c} = \tan\, a\ \cot\, c$$

Now $\angle C$ has been defined to be a right angle, but there is nothing to distinguish $\angle A$ from $\angle B$. It follows that any rule derived for the left-hand triangle in Figure 2.5, below, has to be equally true for the right-hand triangle.

Thus, any rule derived for $\angle B$ is equally true for $\angle A$ with suitable relettering:

$$\cos\, A = \tan\, b\ \cot\, c \tag{2.2}$$

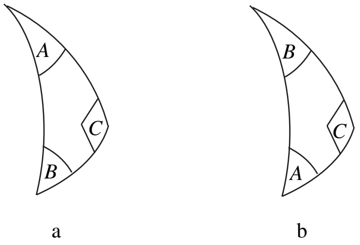

Fig. 2.5. Un-handedness of spherical triangles

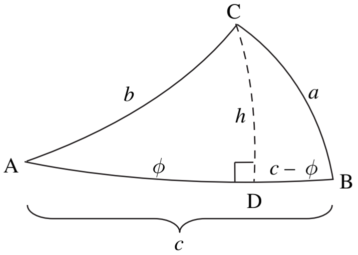

Fig. 2.6. The general spherical triangle as a combination of two right spherical triangles

(One could derive this directly by redrawing Figure 2.3 with the plane passing through point B perpendicular to the line OA instead of through point A perpendicular to the line OB.) Another equation can be obtained easily from ΔADE in Figure 2.4:

$$\sin\, B = \frac{\mathrm{AD}}{\mathrm{AE}} = \frac{\sin\, b}{\sin\, c}$$

$$\therefore\ \sin\, b = \sin\, c\ \sin\, B \tag{2.3}$$

With equations (2.1) to (2.3) in mind, we can now look at the general spherical triangle (no right angles), as shown in Figure 2.6.

Drop an arc $h \perp$ arc AB from point C to point D in Figure 2.6. This divides the triangle into two right spherical triangles.

Define arc AD to take up an angle ϕ as seen from the center of the sphere (point O in Figure 2.4). Then arc DB takes up angle $(c - \phi)$.

Apply equation (2.1) to each right spherical triangle in Figure 2.6; then,

ΔADC: side b is opposite to the right angle, so,

$$\cos\, b = \cos\, h\ \cos\, \phi \tag{2.4}$$

ΔBDC: side a is opposite to the right angle, so,

$$\cos\, a = \cos\, h\ \cos\, (c - \phi) \tag{2.5}$$

Divide equation (2.5) by equation (2.4) and use the standard trigonometric identity, $\cos(c - \phi) = \cos\, c\ \cos\, \phi + \sin\, c\ \sin\, \phi$, to get

$$\frac{\cos\, a}{\cos\, b} = \frac{\cos\, h\ \cos\, (c - \phi)}{\cos\, h\ \cos\, \phi} = \frac{\cos\, (c - \phi)}{\cos\, \phi}$$

$$= \frac{\cos\, c\ \cos\, \phi + \sin\, c\ \sin\, \phi}{\cos\, \phi} = \cos\, c + \sin\, c\ \tan\, \phi$$