FUZZY MULTI-CRITERIA DECISION MAKING

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FUZZY MULTI-CRITERIA DECISION MAKING

Theory and Applications with Recent Developments

Edited By

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PREFACE

Multiple criteria decision making (MCDM) is a modeling and methodological tool for dealing with complex engineering problems. Decision makers face many problems with incomplete and vague information in MCDM problems since the characteristics of these problems often require this kind of information. Fuzzy set approaches are suitable to use when the modeling of human knowledge is necessary and when human evaluations are needed. Fuzzy set theory is recognized as an important problem modeling and solution technique. Fuzzy set theory has been studied extensively over the past 40 years. Most of the early interest in fuzzy set theory pertained to representing uncertainty in human cognitive processes. Fuzzy set theory is now applied to problems in engineering, business, medical and related health sciences, and the natural sciences. Over the years there have been successful applications and implementations of fuzzy set theory in MCDM. MCDM is one of the branches in which fuzzy set theory found a wide application area. Many curriculums of undergraduate and graduate programs include many courses teaching how to use fuzzy sets when you face incomplete and vague information. One of these courses is fuzzy MCDM and its applications.

This book presents examples of applications of fuzzy sets in MCDM. It contains 22 original research and application chapters from different perspectives; and covers different areas of fuzzy MCDM. The book contains chapters on the two major areas of MCDM to which fuzzy set theory contributes. These areas are fuzzy multiple-attribute decision making (MADM) and fuzzy multiple-objective decision making (MODM). MADM approaches can be viewed as alternative methods for combining the information in a problem's decision matrix together with additional information from the decision maker to determine a final ranking, screening, or selection from among the alternatives. MODM is a powerful tool to assist in the process of searching for decisions that best satisfy a multitude of conflicting objectives.

The classification, review and analysis of fuzzy multi-criteria decision-making methods are summarized in the first two chapters. While the first chapter classifies the multi-criteria methods in a general sense, the second chapter focuses on intelligent fuzzy multi-criteria decision making.

The rest of the book is divided into two main parts. The first part includes chapters on frequently used MADM techniques under fuzziness, e.g., fuzzy Analytic Hierarchy Process (AHP), fuzzy TOPSIS, fuzzy outranking methods, fuzzy weighting methods, and a few application chapters of these techniques. The third chapter includes the most frequently used fuzzy AHP methods and their numerical and didactic examples. The fourth chapter shows how a fuzzy AHP method can be jointly used with another technique. The fifth chapter summarizes fuzzy outranking methods, which dichotomize preferred alternatives and nonpreferred ones by establishing outranking relationships. The sixth chapter presents another commonly used multi-attribute method, fuzzy TOPSIS and its application to selection among industrial robotic systems. The seventh chapter includes many fuzzy scoring methods and their applications. The rest of this part includes the other most frequently used fuzzy MADM techniques in the literature: fuzzy information axiom approach, intelligent fuzzy MADM approaches, gray-related analysis, and neuro-fuzzy approximation.

The second part of the book includes chapters on MODM techniques under fuzziness, e.g., fuzzy multi-objective linear programming, quasi-concave and non-concave fuzzy multi-objective programming, interactive fuzzy stochastic linear programming, fuzzy multi-objective integer goal programming, gray fuzzy multi-objective optimization, fuzzy multi-objective geometric programming and some applications of these techniques. These methods are the most frequently used MODM techniques in the fuzzy literature.

The presented methods in this book have been prepared by the authors who are the developers of these techniques. I hope that this book will provide a useful resource of ideas, techniques, and methods for additional research on the applications of fuzzy sets in MCDM. I am grateful to the referees whose valuable and highly appreciated works contributed to select the high quality of chapters published in this book. I am also grateful to my research assistant, Dr. Ihsan Kaya, for his invaluable effort to edit this book.

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May 2008

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MULTI-CRITERIA DECISION MAKING METHODS AND FUZZY SETS

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Abstract: Multi-criteria decision making (MCDM) is one of the well-known topics of decision making. Fuzzy logic provides a useful way to approach a MCDM problem. Very often in MCDM problems, data are imprecise and fuzzy. In a real-world decision situation, the application of the classic MCDM method may face serious practical constraints, because of the criteria containing imprecision or vagueness inherent in the information. For these cases, fuzzy multi-attribute decision making (MADM) and fuzzy multi-objective decision making (MODM) methods have been developed. In this chapter, crisp MADM and MODM methods are first summarized briefly and then the diffusion of the fuzzy set theory into these methods is explained. Some examples of recently published papers on fuzzy MADM and MODM are given.

Key words: Multi-criteria, multi-attribute, multi-objective, decision making, fuzzy sets

1. INTRODUCTION

In the literature, there are two basic approaches to multiple criteria decision making (MCDM) problems: multiple attribute decision making (MADM) and multiple objective decision making (MODM). MADM problems are distinguished from MODM problems, which involve the design of a "best" alternative by considering the tradeoffs within a set of interacting design constraints. MADM refers to making selections among some courses of action in the presence of multiple, usually conflicting, attributes. In MODM problems, the number of alternatives is effectively

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infinite, and the tradeoffs among design criteria are typically described by continuous functions.

MADM is the most well-known branch of decision making. It is a branch of a general class of operations research models that deal with decision problems under the presence of a number of decision criteria. The MADM approach requires that the choice (selection) be made among decision alternatives described by their attributes. MADM problems are assumed to have a predetermined, limited number of decision alternatives. Solving a MADM problem involves sorting and ranking.

MADM approaches can be viewed as alternative methods for combining the information in a problem's decision matrix together with additional information from the decision maker to determine a final ranking, screening, or selection from among the alternatives. Besides the information contained in the decision matrix, all but the simplest MADM techniques require additional information from the decision maker to arrive at a final ranking, screening, or selection.

In the MODM approach, contrary to the MADM approach, the decision alternatives are not given. Instead, MODM provides a mathematical framework for designing a set of decision alternatives. Each alternative, once identified, is judged by how close it satisfies an objective or multiple objectives. In the MODM approach, the number of potential decision alternatives may be large. Solving a MODM problem involves selection.

It has been widely recognized that most decisions made in the real world take place in an environment in which the goals and constraints, because of their complexity, are not known precisely, and thus, the problem cannot be exactly defined or precisely represented in a crisp value (Bellman and Zadeh, 1970). To deal with the kind of qualitative, imprecise information or even ill-structured decision problems, Zadeh (1965) suggested employing the fuzzy set theory as a modeling tool for complex systems that can be controlled by humans but are hard to define exactly.

Fuzzy logic is a branch of mathematics that allows a computer to model the real world the same way that people do. It provides a simple way to reason with vague, ambiguous, and imprecise input or knowledge. In Boolean logic, every statement is true or false; i.e., it has a truth value 1 or 0. Boolean sets impose rigid membership requirements. In contrast, fuzzy sets have more flexible membership requirements that allow for partial membership in a set. Everything is a matter of degree, and exact reasoning is viewed as a limiting case of approximate reasoning. Hence, Boolean logic is a subset of Fuzzy logic. Human beings are involved in the decision analysis since decision making should take into account human subjectivity, rather than employing only objective probability measures. This makes fuzzy decision making necessary.

This chapter aims at classifying MADM and MODM methods and at explaining how the fuzzy sets have diffused into the MCDM methods.

2. MULTI-ATTRIBUTE DECISION MAKING: A CLASSIFICATION OF METHODS

MADM methods can be classified as to whether if they are non-compensatory or compensatory. The decision maker may be of the view that high performance relative to one attribute can at least partially compensate for low performance relative to another attribute, particularly if an initial screening analysis has eliminated alternatives that fail to meet any minimum performance requirements. Methods that incorporate tradeoffs between high and low performance into the analysis are termed "compensatory." Those methods that do not are termed "noncompensatory."

In their book, Hwang and Yoon (1981) give 14 MADM methods. These methods are explained briefly below. Additionally five more methods are listed below.

2.1 Dominance

An alternative is "dominated" if another alternative outperforms it with respect to at least one attribute and performs equally with respect to the remainder of attributes. With the dominance method, alternatives are screened such that all dominated alternatives are discarded. The screening power of this method tends to decrease as the number of independent attributes becomes larger.

2.2 Maximin

The principle underlying the maximin method is that "a chain is only as strong as its weakest link." Effectively, the method gives each alternative a score equal to the strength of its weakest link, where the "links" are the attributes. Thus, it requires that performance with respect to all attributes be measured in commensurate units (very rare for MADM problems) or else be normalized prior to performing the method.

2.3 Maximax

The viewpoint underlying the maximax method is one that assigns total importance to the attribute with respect to which each alternative performs best. Extending the “chain” analogy used in describing the maximin method, maximax performs as if one was comparing alternative chains in search of the best link. The score of each chain (alternative) is equal to the performance of its strongest link (attribute). Like the maximin method, maximax requires that all attributes be commensurate or else pre-normalized.

2.4 Conjunctive (Satisficing)

The conjunctive method is purely a screening method. The requirement embodied by the conjunctive screening approach is that to be acceptable, an alternative must exceed given performance thresholds for all attributes. The attributes (and thus the thresholds) need not be measured in commensurate units.

2.5 Disjunctive

The disjunctive method is also purely a screening method. It is the complement of the conjunctive method, substituting “or” in place of “and.” That is, to pass the disjunctive screening test, an alternative must exceed the given performance threshold for at least one attribute. Like the conjunctive method, the disjunctive method does not require attributes to be measured in commensurate units.

2.6 Lexicographic

The best-known application of the lexicographic method is, as its name implies, alphabetical ordering such as is found in dictionaries. Using this method, attributes are rank-ordered in terms of importance. The alternative with the best performance on the most important attribute is chosen. If there are ties with respect to this attribute, the next most important attribute is considered, and so on. Note two important ways in which MADM problems typically differ from alphabetizing dictionary words. First, there are many fewer alternatives in a MADM problem than words in the dictionary. Second, when the decision matrix contains quantitative attribute values, there are effectively an infinite number [rather than 26 (i.e., A-Z)] of possible scores with a correspondingly lower probability of ties.

2.7 Lexicographic Semi-Order

This is a slight variation on the lexicographic method, where "near-ties" are allowed to count as ties without any penalty to the alternative, which scores slightly lower within the tolerance ("tie") window. Counting near-ties as ties makes the lexicographic method less of a "knife-edged" ranking method and more appropriate for MADM problems with quantitative data in the decision matrix. However, the method can lead to intransitive results, wherein A is preferred to B, B is preferred to C, but C is preferred to A.

2.8 Elimination by Aspects

This method is a formalization of the well-known heuristic, "process of elimination." Like the lexicographic method, this evaluation proceeds one attribute at a time, starting with attributes determined to be most important. Then, like the conjunctive method, alternatives not exceeding minimum performance requirements—with respect to the single attribute of interest, in this case—are eliminated. The process generally proceeds until one alternative remains, although adjustment of the performance threshold may be required in some cases to achieve a unique solution.

2.9 Linear Assignment Method

This method requires, in addition to the decision matrix data, cardinal importance weights for each attribute and rankings of the alternatives with respect to each attribute. These information requirements are intermediate between those of the eight methods described previously, and the five methods that follow, in that they require ordinal (but not cardinal) preference rankings of the alternatives with respect to each attribute. The primary use of the additional information is to enable compensatory rather than noncompensatory analysis, that is, allowing good performance on one attribute to compensate for low performance on another.

Note at this point that quantitative attribute values (data in the decision matrix) do not constitute cardinal preference rankings. Attribute values are generally noncommensurate across attributes, preference is not necessarily linearly increasing with attribute values, and preference for attribute values of zero is not generally zero. However, as long as the decision maker can specify an ordinal correspondence between attribute values and preference, such as "more is better" or "less is better" for each attribute, then the ordinal alternative rankings with respect to each attribute that are needed by the linear assignment method are specified uniquely. Thus, the evaluation/performance rankings required by the linear assignment method are easier to derive than the evaluation/performance ratings required by the five methods that follow. The cost of using ordinal rankings rather than cardinal ratings is that the method is only "semi-compensatory," in that incremental changes in the performance of an alternative will not enter into the analysis unless the changes are large enough to alter the rank order of the alternatives.

2.10 Additive Weighting

The score of an alternative is equal to the weighted sum of its cardinal evaluation/preference ratings, where the weights are the importance weights associated with each attribute. The resulting cardinal scores for each alternative can be used to rank, screen, or choose an alternative. The analytical hierarchy process (AHP) is a particular approach to the additive weighting method.

2.11 Weighted Product

The weighted product is similar to the additive weighting method. However, instead of calculating "sub-scores" by multiplying performance scores times attribute importances, performance scores are raised to the power of the attribute importance weight. Then, rather than summing the resulting subscores across attributes to yield the total score for the alternative, the product of the scores yields the final alternative scores. The weighted product method tends to penalize poor performance on one attribute more heavily than does the additive weighting method.

2.12 Nontraditional Capital Investment Criteria

This method entails pairwise comparisons of the performance gains (over a baseline alternative) among attributes, for a given alternative. One attribute must be measured in monetary units. These comparisons are combined to estimate the (monetary) value attributed to each performance gain, and these values are summed to yield the overall implied value of each alternative. These implied values can be used to select an alternative, to rank alternatives, or presumably to screen alternatives as well.

2.13 TOPSIS (Technique for Order Preference by Similarity to Ideal Solution)

The principle behind TOPSIS is simple: The chosen alternative should be as close to the ideal solution as possible and as far from the negative-ideal solution as possible. The ideal solution is formed as a composite of the best performance values exhibited (in the decision matrix) by any alternative for each attribute. The negative-ideal solution is the composite of the worst performance values. Proximity to each of these performance poles is measured in the Euclidean sense (e.g., square root of the sum of the squared distances along each axis in the “attribute space”), with optional weighting of each attribute.

2.14 Distance from Target

This method and its results are also straightforward to describe graphically. First, target values for each attribute are chosen, which need not be exhibited by any available alternative. Then, the alternative with the shortest distance (again in the Euclidean sense) to this target point in “attribute space” is selected. Again, weighting of attributes is possible. Distance scores can be used to screen, rank, or select a preferred alternative.

2.15 Analytic Hierarchy Process (AHP)

The analytical hierarchy process was developed primarily by Saaty (1980). AHP is a type of additive weighting method. It has been widely reviewed and applied in the literature, and its use is supported by several commercially available, user-friendly software packages. Decision makers often find it difficult to accurately determine cardinal importance weights for a set of attributes simultaneously. As the number of attributes increases, better results are obtained when the problem is converted to one of making a series of pairwise comparisons. AHP formalizes the conversion of the attribute weighting problem into the more tractable problem of making a series of pairwise comparisons among competing attributes. AHP summarizes the results of pairwise comparisons in a “matrix of pairwise comparisons.” For each pair of attributes, the decision maker specifies a judgment about "how much more important one attribute is than the other."

Each pairwise comparison requires the decision maker to provide an answer to the question: "Attribute *A* is how much more important than Attribute *B*, relative to the overall objective?"

2.16 Outranking Methods (ELECTRE, PROMETHEE, ORESTE)

The basic concept of the ELECTRE (ELimination Et Choix Traduisant la Réalité or Elimination and Choice Translating Reality) method is how to deal with outranking relation by using pairwise comparisons among alternatives under each criteria separately. The outranking relationship of two alternatives, denoted as $A_i \rightarrow A_j$, describes that even though two alternatives *i* and *j* do not dominate each other mathematically, the decision maker accepts the risk of regarding A_i as almost surely better than A_j. An alternative is dominated if another alternative outranks it at least in one criterion and equals it in the remaining criteria. The ELECTRE method consists of a pairwise comparison of alternatives based on the degree to which evaluation of the alternatives and preference weight confirms or contradicts the pairwise dominance relationship between the alternatives. The decision maker may declare that she/he has a strong, weak, or indifferent preference or may even be unable to express his or her preference between two compared alternatives. The other two members of outranking methods are PROMETHEE and ORESTE.

2.17 Multiple Attribute Utility Models

Utility theory describes the selection of a satisfactory solution as the maximization of satisfaction derived from its selection. The best alternative is the one that maximizes utility for the decision maker's stated preference structure. Utility models are of two types additive and multiplicative utility models. The main steps in using a multi-attribute utility model can be counted as 1) determination of utility functions for individual attributes, 2) determination of weighting or scaling factors, 3) determination of the type of utility model, 4) the measurement of the utility values for each alternative with respect to the considered attributes, and 5) the selection of the best alternative.

2.18 Analytic Network Process

In some practical decision problems, it seems to be the case where the local weights of criteria are different for each alternative. AHP has a difficulty in treating in such a case since AHP uses the same local weights of criteria for each alternative. To overcome this difficulty, Saaty (1996) proposed the analytic network process (ANP). ANP permits the use of different weights of criteria for alternatives.

2.19 Data Envelopment Analysis

Data envelopment analysis (DEA) is a nonparametric method of measuring the efficiency of a decision making unit such as a firm or a public-sector agency, which was first introduced into the operations research literature by Charnes et al. (1978). DEA is a relative, technical efficiency measurement tool, which uses operations research techniques to automatically calculate the weights assigned to the inputs and outputs of the production units being assessed. The actual input/output data values are then multiplied with the calculated weights to determine the efficiency scores. DEA is a nonparametric multiple criteria method; no production, cost, or profit function is estimated from the data.

2.20 Multi-Attribute Fuzzy Integrals

When mutual preferential independence among criteria can be assumed, consider that the utility function is additive and takes the form of a weighted sum. The assumption of mutual preferential independence among criteria is, however, rarely verified in practice. To be able to take interaction phenomena among criteria into account, it has been proposed to substitute a monotone set function on attributes set N called the fuzzy measure to the weight vector involved in the calculation of weighted sums. Such an approach can be regarded as taking into account not only the importance of each criterion but also the importance of each subset of criteria. Choquet integral is a natural extension of the weighted arithmetic mean (Grabisch, 1992; Sugeno, 1974).

3. MULTI-OBJECTIVE DECISION MAKING: A CLASSIFICATION OF METHODS

In multiple objective decision making, application functions are established to measure the degree of fulfillment of the decision maker's requirements (achievement of goals, nearness to an ideal point, satisfaction, etc.) on the objective functions and are extensively used in the process of finding "good compromise" solutions. MODM methodologies can be categorized in a variety of ways, such as the form of the model (e.g., linear, nonlinear, or stochastic), characteristic of the decision space (e.g., finite or infinite), or solution process (e.g., prior specification of preferences or interactive). Among MODM methods, we can count multi-objective linear programming (MOLP) and its variants such as multi-objective stochastic integer linear programming, interactive MOLP, and mixed 0-1 MOLP; multi-objective goal programming (MOGoP); multi-objective geometric programming (MOGeP); multi-objective nonlinear fractional programming; multi-objective dynamic programming; and multi-objective genetic programming. The formulations of these programming techniques under fuzziness will not be given here since most of them will be explained in detail in the subsequent chapters of this book with numerical examples. The intelligent fuzzy multi-criteria decision making methods will be explained by Waiel F. Abd El-Wahed in Chapter 2.

When a MODM problem is being formulated, the parameters of objective functions and constraints are normally assigned by experts. In most real situations, the possible values of these parameters are imprecisely or ambiguously known to the experts. Therefore, it would be more appropriate for these parameters to be represented as fuzzy numerical data that can be represented by fuzzy numbers.

4. DIFFUSION OF FUZZY SETS INTO MULTI-CRITERIA DECISION MAKING

The classic MADM methods generally assume that all criteria and their respective weights are expressed in crisp values and, thus, that the rating and the ranking of the alternatives can be carried out without any problem. In a real-world decision situation, the application of the classic MADM method may face serious practical constraints from the criteria perhaps containing imprecision or vagueness inherent in the information. In many cases, performance of the criteria can only be expressed qualitatively or by using linguistic terms, which certainly demands a more appropriate method.

The most preferable situation for a MADM problem is when all ratings of the criteria and their degree of importance are known precisely, which makes it possible to arrange them in a crisp ranking. However, many of the decision making problems in the real world take place in an environment in which the goals, the constraints, and the consequences of possible actions are not known precisely (Bellman and Zadeh, 1970). These situations imply that a real decision problem is very complicated and thus often seems to be little suited to mathematical modeling because there is no crisp definition (Zimmermann and Zysno, 1985). Consequently, the ideal condition for a classic MADM problem may not be satisfied, in particular when the decision situation involves both fuzzy and crisp data. In general, the term "fuzzy" commonly refers to a situation in which the attribute or goal cannot be defined crisply, because of the absence of well-defined boundaries of the set of observation to which the description applies.

A similar situation is when the available information is not enough to judge or when the crisp value is inadequate to model real situations. Unfortunately, the classic MADM methods cannot handle such problems effectively, because they are only suitable for dealing with problems in which all performances of the criteria are assumed to be known and, thus, can be represented by crisp numbers. The application of the fuzzy set theory in the field of MADM is justified when the intended goals or their attainment cannot be defined or judged crisply but only as fuzzy sets (Zimmermann, 1987). The presence of fuzziness or imprecision in a MADM problem will obviously increase the complexity of the decision situation in many ways. Fuzzy or qualitative data are operationally more difficult to manipulate than crisp data, and they certainly increase the computational requirements in particular during the process of ranking when searching for the preferred alternatives (Chen and Hwang, 1992).

Bellman and Zadeh (1970) and Zimmermann (1978) introduced fuzzy sets into the MCDM field. They cleared the way for a new family of methods to deal with problems that had been inaccessible to and unsolvable with standard MCDM techniques. Bellman and Zadeh (1970) introduced the first approach regarding decision making in a fuzzy environment. They suggested that fuzzy goals and fuzzy constraints could be defined symmetrically as fuzzy sets in the space of alternatives, in which the decision was defined as the confluence between the constraints to be met and the goals to be satisfied. A maximizing decision was then defined as a point in the space of alternatives at which the membership function of a fuzzy decision attained its maximum value.

Baas and Kwakernaak's (1977) approach was widely regarded as the most classic work on the fuzzy MADM method and was often used as a benchmark for other similar fuzzy decision models. Their approach consisted of both phases of MADM, the rating of criteria and the ranking of multiple aspect alternatives using fuzzy sets.

Yager (1978) defined the fuzzy set of a decision as the intersection (conjunction) of all fuzzy goals. The best alternative should possess the highest membership values with respect to all criteria, but unfortunately, such a situation rarely occurs in the case of a multiple attribute decision-making problem. To arrive at the best acceptable alternative, he suggested a compromise solution by proposing the combination of max and min operators. For the determination of the relative importance of each attribute, he suggested the use of the Saaty method through pairwise comparison based on the reciprocal matrix.

Kickert (1978) summarized the fuzzy set theory applications in MADM problems. Zimmermann's (1985, 1987) two books include MADM applications. There are a number of very good surveys of fuzzy MCDM (Chen and Hwang, 1992; Fodor and Roubens, 1994; Luhandjula, 1989; Sakawa, 1993).

Dubois and Prade (1980), Zimmermann (1987), Chen and Hwang (1992), and Ribeiro (1996) differentiated the family of fuzzy MADM methods into two main phases. The first phase is generally known as the rating process, dealing with the measurement of performance ratings or the degree of satisfaction with respect to all attributes of each alternative. The aggregate rating, indicating the global performance of each alternative, can be obtained through the accomplishment of suitable aggregation operations of all criteria involved in the decision. The second phase, the ranking of alternatives, is carried out by ordering the existing alternatives according to the resulted aggregated performance ratings obtained from the first phase.

Some titles among recently published papers can show us the latest interest areas of MADM and MODM. Ravi and Reddy (1999) rank both coking and noncoking coals of India using fuzzy multi-attribute decision making. They use Saaty's AHP and Yager's (1978) fuzzy MADM approach to arrive at the coal field having the best quality coal for industrial use. Fan et al. (2002) propose a new approach to solve the MADM problem, where the decision maker gives his/her preference on alternatives in a fuzzy relation. To reflect the decision maker's preference information, an optimization model is constructed to assess the attribute weights and then to select the most desirable alternatives.

Wang and Parkan (2005) investigate a MADM problem with fuzzy preference information on alternatives and propose an eigenvector method to rank them. Three optimization models are introduced to assess the relative importance weights of attributes in a MADM problem, which integrate subjective fuzzy preference relations and objective information in different ways. Omero et al. (2005) deal with the problem of assessing the performance of a set of production units, simultaneously considering different kinds of information, yielded by data envelopment analysis, a qualitative data analysis, and an expert assessment. Hua et al. (2005) develop a fuzzy multiple attribute decision making (FMADM) method with a three-level hierarchical decision making model to evaluate the aggregate risk for green manufacturing projects.

Gu and Zhu (2006) construct a fuzzy symmetry matrix by referring to the covariance definition of random variables as attribute evaluation space based on a fuzzy decision making matrix. They propose a fuzzy AHP method by using the approximate fuzzy eigenvector of such a fuzzy symmetry matrix. This algorithm reflects the dispersed projection of decision information in general. Fan et al. (2004) investigate the multiple attribute decision making (MADM) problems with preference information on alternatives. A new method is proposed to solve the MADM problem, where the decision maker gives his/her preference on alternatives in a fuzzy relation. To reflect the decision maker's subjective preference information, a linear goal programming model is constructed to determine the weight vector of attributes and then to rank the alternatives.

Ling (2006) presents a fuzzy MADM method in which the attribute weights and decision matrix elements (attribute values) are fuzzy variables. Fuzzy arithmetic operations and the expected value operator of fuzzy variables are used to solve the FMADM problem. Xu and Chen (2007) develop an interactive method for multiple attribute group decision making in a fuzzy environment. The method can be used in situations where the information about attribute weights is partly known, the weights of decision makers are expressed in exact numerical values or triangular fuzzy numbers, and the attribute values are triangular fuzzy numbers. Chen and Larbani (2006) obtain the weights of a MADM problem with a fuzzy decision matrix by formulating it as a two-person, zero-sum game with an uncertain payoff matrix. Moreover, the equilibrium solution and the resolution method for the MADM game are developed. These results are validated by a product development example of nano-materials.

Some recently published papers on fuzzy MODM are given as follows: El-Wahed and Abo-Sinna (2001) introduce a solution method based on the theory of fuzzy sets and goal programming for MODM problems. The solution method, called hybrid fuzzy-goal programming (HFGP), combines and extends the attractive features of both fuzzy set theory and goal programming for MODM problems. The HFGP approach is introduced to determine weights to the objectives under the same priorities as using the concept of fuzzy membership functions along with the notion of degree of conflict among objectives. Also, HFGP converts a MODM problem into a lexicographic goal programming problem by fixing the priorities and aspiration levels appropriately. Rasmy et al. (2002) introduce an interactive approach for solving MODM problems based on linguistic preferences and architecture of a fuzzy expert system. They consider the decision maker's preferences in determining the priorities and aspiration levels, in addition to analysis of conflict among the goals. The main concept is to convert the MODM problem into its equivalent goal programming problem by appropriately setting the priority and aspiration level for each objective. The conversion approach is based on the fuzzy linguistic preferences of the decision maker. Borges and Antunes (2002) study the effects of uncertainty on multiple-objective linear programming models by using the concepts of fuzzy set theory. The proposed interactive decision support system is based on the interactive exploration of the weight space. The comparative analysis of indifference regions on the various weight spaces (which vary according to intervals of values of the satisfaction degree of objective functions and constraints) enables the study of the stability and evolution of the basis that corresponds to the calculated efficient solutions with changes of some model parameters. Luhandjula (1984) used a linguistic variable approach to present a procedure for solving the multiple objective linear fractional programming problem (MOLFPP). Dutta et al. (1992) modified the linguistic approach of Luhandjula such as to obtain an efficient solution to MOLFPP. Stancu-Minasian and Pop (2003) points out certain shortcomings in the work of Dutta et al. and gives the correct proof of theorem, which validates the obtaining of the efficient solutions. We notice that the method presented there as a general one does only work efficiently if certain hypotheses (restrictive enough and hardly verified) are satisfied.

Li et al. (2006) improve the fuzzy compromise approach of Guu and Wu (1999) by automatically computing proper membership thresholds instead of choosing them. Indeed, in practice, choosing membership thresholds arbitrarily may result in an infeasible optimization problem. Although a minimum satisfaction degree is adjusted to get a fuzzy efficient solution, it sometimes makes the process of interaction more complicated. To overcome this drawback, a theoretically and practically more efficient two-phase max–min fuzzy compromise approach is proposed. Wu et al. (2006) develop a new approximate algorithm for solving fuzzy multiple objective linear programming (FMOLP) problems involving fuzzy parameters in any form of membership functions in both objective functions and constraints. A detailed description and analysis of the algorithm are supplied. Abo-Sinna and Abou-El-Enien (2006) extend the TOPSIS for solving large scale multiple objective programming problems involving fuzzy parameters. These fuzzy parameters are characterized as fuzzy numbers. For such problems, the α–Pareto optimality is introduced by extending the ordinary Pareto optimality on the basis of the α–level sets of fuzzy numbers. An interactive fuzzy decision-making algorithm for generating an α–Pareto optimal solution through the TOPSIS approach is provided where the decision maker is asked to specify the degree α and the relative importance of objectives.

5. CONCLUSIONS

The main difference between the MADM and MODM approaches is that MODM concentrates on continuous decision space aimed at the realization of the best solution, in which several objective functions are to be achieved simultaneously. The decision processes involve searching for the best solution, given a set a conflicting objectives, and thus, a MODM problem is associated with the problem of design for optimal solutions through mathematical programming. In finding the best feasible solution, various interactions within the design constraints that best satisfy the goals must be considered by way of attaining some acceptable levels of sets of some quantifiable objectives. Conversely, MADM refers to making decisions in the discrete decision spaces and focuses on how to select or to rank different predetermined alternatives. Accordingly, a MADM problem can be associated with a problem of choice or ranking of the existing alternatives (Zimmermann, 1985).

Having to use crisp values is one of the problematic points in the crisp evaluation process. As some criteria are difficult to measure by crisp values, they are usually neglected during the evaluation. Another reason is about mathematical models that are based on crisp values. These methods cannot deal with decision makers' ambiguities, uncertainties, and vagueness that cannot be handled by crisp values. The use of fuzzy set theory allows us to incorporate unquantifiable information, incomplete information, non obtainable information, and partially ignorant facts into the decision model. When decision data are precisely known, they should not be placed into a fuzzy format in the decision analysis. Applications of fuzzy sets within the field of decision making have, for the most part, consisted of extensions or "fuzzifications" of the classic theories of decision making. Decisions to be made in complex contexts, characterized by the presence of multiple evaluation aspects, are normally affected by uncertainty, which is essentially from the insufficient and/or imprecise nature of input data as well as the subjective and evaluative preferences of the decision maker. Fuzzy sets have powerful features to be incorporated into many optimization techniques. Multiple criteria decision making is one of these, and it is certain that more frequently you will see more fuzzy MCDM modeling and applications in the literature over the next few years.

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