

# Image Correlation for Shape, Motion and Deformation Measurements

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# Image Correlation for Shape, Motion and Deformation Measurements

Basic Concepts, Theory and Applications

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# Preface

The impetus for the enclosed book is the nexus of accelerating interest and use of computer-vision-based measurement methods and the accumulation of sufficient theoretical, computational and experimental findings for development of a comprehensive treatment of the subject. With over five decades of active R&D in the field of computer vision for experimental measurements, either as originators, developers or practitioners of the method(s), the authors have both the commitment and the background necessary to provide the readership with a balanced, comprehensive treatment of the subject and its application to real-world problems.

As envisioned by the authors, the book is a reference book for engineers, scientists and educators seeking to employ advanced, image-based, non-contacting measurement methods to measure the shape and deformation of a material undergoing thermal, mechanical or variable environmental conditions. Since the methods have been shown to be broadly applicable in fields such as civil engineering, mechanical engineering, material science, electronic packaging, biomedical, manufacturing, joining, photogrammetry and others, the text should serve as a reference for all faculty, research scientists and students employing these methods in their investigations.

Encompassing both basic theoretical formulations in the areas of image correlation and computer vision and also recent developments and practical applications in two-dimensional, three-dimensional and volumetric image correlation methods in the fields of non-contacting measurements and experimental mechanics, the level of expertise is envisioned as an advanced supplement for an upper-level undergraduate class or as a companion text for a graduate-level class in measurements, experimental mechanics or non-contacting, vision-based image analysis methods with special emphasis in solids. Though the material contains a complete summary of concepts and also background material needed to develop a strong foundation in vision-based methods, the text does not include problem sets.

The scope of the work includes aspects of the broad area of non-contacting measurement of shape and deformation using images of an object, with special emphasis on computer vision and volumetric imaging. Specifics addressed in the various chapters include (a) ray optics, (b) single camera computer vision and calibration

concepts including distortion correction, (c) multi-camera, stereo-vision principles and calibration methods, (d) digital image correlation for image matching, including error assessment and (e) experimental details for single-camera, multi-camera and volumetric imaging applications.

Theoretical developments presented in the book are based on general concepts obtained from a wide range of sources, some of which are original to the authors. Basic optics concepts were abstracted from a combination of textbooks. Single camera and multi-camera calibration concepts were abstracted from archival research papers, research books and textbooks. Digital image correlation principles for image matching were developed using research papers, references, and text books, or are original with the authors and presented herein for the first time. Volumetric image correlation principles were primarily extracted from research articles. Error assessment developments use basic probabilistic concepts, but are otherwise original to the authors. Most experiments and associated developments for both two-dimensional and three-dimensional computer vision experiments are based on work performed by the authors and their students. All appendices were abstracted from a combination of text books, research articles and reference books.

Chapter 1 provides an introduction and in-depth literature survey. Chapter 2 covers basic optical developments applicable for modeling vision systems. Chapters 3 and 4 provide the theoretical foundation for calibration and measurements using single camera and stereo camera vision systems. Chapter 5 outlines the essential concepts underlying digital image correlation for motion measurements. Specific items discussed include (a) image matching methods, (b) subset shape functions, (c) intensity pattern metrics, (d) intensity pattern interpolation for discretely sampled patterns and (e) quantitative error estimates in 2D motion. In the latter section, the authors highlight original contributions that provide quantitative metrics for the errors that are expected during image correlation during simple translation experiments. Developments include probabilistic estimates for the mean and variance in the measurements due to the effects of intensity interpolation, intensity pattern noise and intensity pattern contrast, as well as appropriate interpretations of the results using signal-processing concepts. Chapter 6 presents details regarding the application of a single camera imaging system for in-plane measurement of deformations, including the potentially deleterious effects of out-of-plane motion. Applications include (a) successful measurement of the stress–strain response of an aluminum alloy using the surface strain field measured on a planar specimen undergoing uniaxial tension by 2D-DIC, (b) details of extensive experimental studies that clearly highlight the effects of out-of-plane motion on single-camera motion measurements, (c) use of a far-field microscope for micro-scale measurements of crack tip deformations with nanometer accuracy, (d) determination of material properties through an inverse methodology enabled by full-field 2D-DIC deformation measurements and (e) use of a scanning electron microscope for deformation measurements in a  $20 \times 20 \mu\text{m}$  field of view, including the development and application of nanoscale patterns. Chapter 7 describes in detail the application of stereo-vision camera imaging systems for general three-dimensional measurement of surface deformations. Applications include (a) a synchronized four-camera stereovision system for the

measurement of large, out-of-plane deformations in polymer beam specimens, (b) dynamic deformation measurements using high speed stereo cameras and (c) development and use of 3D stereomicroscopy systems for microscale deformation and shape measurements. Chapter 8 presents both theoretical foundations and experimental results from a series of volumetric image correlation measurements in a polymeric foam undergoing compression loading. Chapter 9 provides a brief discussion of methods to estimate errors during stereovision measurements using the results obtained in Chapter 5 for image based motion bias and variability. Chapter 10 provides a summary of practical considerations regarding both 2D and 3D image correlation measurement methods. Included in this lengthy chapter are (a) an engineering approach for estimating the appropriate speckle size in a pattern along with examples using the approach, (b) methods for patterning a material and issues related to pattern adherence, (c) a simple approach to estimate the speckle size in an image, (d) an engineering approach for determining the depth of field and field of view along with an example, (e) a simple formula to estimate the effect of out of plane motion on image measurements, (f) estimation of errors in 2D image-plane matching and (g) measurement issues at both high and low magnification. Appendices A through I provide the background information that the authors believe is important to provide the foundation necessary for both essential concepts in computer vision and the practical application of the method for shape and deformation measurements. Areas covered include continuum mechanics; linear algebra; surface strain estimation; non-linear optimization; basic concepts in statistics and probability; introduction to projective geometry; rotation tensor formulations; spline functions and triangulation.

In closing, the authors have believed for a long time in the usefulness and broad applicability of vision-based methods. Hence, though the effort required has been significant, the authors are delighted that the book a reality. We have enjoyed the opportunity to make a contribution that should further expand opportunities for those with measurement needs that can be addressed by digital image correlation methods.

Columbia  
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# Overview

As envisioned by the authors, the book is a reference book encompassing the basic theoretical formulations in the areas of image correlation and computer vision, recent developments and practical applications of two-dimensional, three-dimensional and volumetric image correlation methods in the fields of non-contacting measurements and experimental mechanics. Since the methods have been shown to be broadly applicable in fields such as civil, mechanical, material science, electronic packaging, biomedical, manufacturing, joining, photogrammetry and others, the text should serve as a reference for all faculty, research scientists and students employing these methods in their investigations. Though the material contains a complete summary of concepts and also background material needed to develop a strong foundation in vision-based methods, the current version provides limited examples and does not contain problem sets. Therefore, the text should serve as a reference for the researcher, or provide the material needed for a first course in the area. Chapter 1 provides an introduction and in-depth literature survey. Chapter 2 covers basic optical developments applicable for modeling vision systems. Chapters 3 and 4 provide the theoretical foundation for calibration and measurements using single camera and stereo camera vision systems. Chapter 5 outlines the concepts underlying digital image correlation for motion measurements. Specific items discussed include (a) image matching methods, (b) subset shape functions, (c) intensity pattern metrics, (d) intensity pattern interpolation for discretely sampled patterns and (e) error estimates in 2D motion. In the latter section, the authors highlight original contributions that provide quantitative metrics for the errors that are expected during image correlation during simple translation experiments. Developments include probabilistic estimates for the mean and variance in the measurements due to the effects of intensity interpolation, intensity pattern noise and intensity pattern contrast, as well as appropriate interpretations of the results using signal-processing concepts. Chapter 6 presents details regarding the application of a single camera imaging system for in-plane measurement of deformations, including the potentially deleterious effects of out-of-plane motion. Applications include (a) successful measurement of the stress-strain response of an aluminum alloy using the surface strain field measured on a planar specimen undergoing uniaxial tension by 2D-DIC, (b) details of

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# Chapter 1

## Introduction

### 1.1 Literature Survey

As used in this article, the term “digital image correlation” refers to the class of non-contacting methods that acquire images of an object, store images in digital form and perform image analysis to extract full-field shape, deformation and/or motion measurements. Digital image registration (i.e. matching) has been performed with many types of object-based patterns, including lines, grids, dots and random arrays. One of the most commonly used approaches employs random patterns and compares sub-regions throughout the image to obtain a full-field of measurements. The patterns may occur on solid surfaces or may be a collection of particles in a fluid medium.

#### *1.1.1 Early History*

The early history of image-based measurements appears to reside in the field of photogrammetry, for which there is a wealth of literature. As noted by Doyle [65] and Gruner [96], the discussions of perspective and imagery date back to the writings of Leonardo da Vinci in 1480 and his related studies in 1492. Key developments over the next 3 centuries, including the work of Heinrich Lambert who developed the basic mathematics relating perspective and imaging (*The Free Perspective*, 1759), had their greatest impact after photography was invented. Attributed to Niepce (1765–1833), the first practical photographs were made by Daguerre in 1837.

#### *1.1.2 Photogrammetry, 1850–Present*

With the invention and refinement of photographic methods, developments in the field of photogrammetry have often been separated into four relatively distinct

phases (Konecny [144]). These phases are (1) plane photogrammetry (1850–1900), (2) analog photogrammetry (1900–1950), (3) analytical photogrammetry (1950–1985) and (4) digital photogrammetry (1985–present). From these four phases, the most enduring contributions are in the mathematical developments.

Specifically, the relationship between projective geometry and perspective imaging developed by Sturms and Haick (1883), the fundamental geometry of photogrammetry described by Sebastian Finsterwald (1899), the projective equations and their differentials for stereo-imaging which are fundamental to analytical photogrammetry developed by Otto von Gruber (1924), analytical solutions to the equations of photogrammetry in terms of direction cosines given by Earl Church (1945) and development of the principles of modern multi-station analytical photogrammetry using matrix notation by Dr. Hellmut Schmid (1953).

The work of Schmid is particularly relevant in that he not only developed the equations but also performed a “rigorously correct least squares solution” using any number of perspective views, along with a detailed study of error propagation. It is important to note that the early focus of photogrammetry was to extract 3D shape of objects through multi-view comparison of photographic records.

### ***1.1.3 Digital Image Correlation – Background and Related Activities***

Some of the first work in the area of image correlation was performed in the early 1950s by Gilbert Hobrough (1919–2002), who compared analog representations of photographs to register features from various views [17]. In 1961, Hobrough designed and built an instrument to “correlate high-resolution reconnaissance photography with high precision survey photography in order to enable more precise measurement of changeable ground conditions” [17], thereby being one of the first investigators to attempt a form of digital image correlation to extract positional information from the image correlation/matching process.

As digitized images became available in the 1960s and 1970s, researchers in artificial intelligence and robotics began to develop vision-based algorithms and stereo-vision methodologies in parallel with photogrammetry applications for aerial photographs. Rosenfeld [243] provides an extensive bibliography and a summary of the developments from 1955–1979. As noted by Rosenfeld, the areas of primary emphasis in this research community during the early years of digital image processing were (a) character recognition, (b) microscopy, (c) medicine and radiology and (d) photogrammetry/aerial photography, with engineering applications for shape and deformation measurements either non-existent or rare.

While digital image analysis methods were undergoing explosive growth, much of the field of experimental solid mechanics was focused on applying recently developed laser technology. Holography [82, 100, 117, 154], laser speckle [57], laser speckle photography [18, 19, 76, 175, 319], laser speckle interferometry [180, 267], speckle shearing interferometry [125, 153], holographic interferometry [327], moiré

interferometry [224] and ultra high density moiré interferometry [225] are typical examples of the type of measurement techniques developed for use with coherent light sources. A particularly interesting example using laser speckle was developed by Yamaguchi in 1986 [343], and employed a linear sensor to determine in-plane translation. In most cases, the measurement data (surface slopes, displacements, displacement gradient combinations) is embedded in the photographic medium, typically in the form of a fringe pattern. Since the recording process is generally nonlinear, resulting in difficulties in extracting partial fringe positions with high accuracy, the most common process employed by experimental mechanicians was a laborious determination of estimates for fringe center locations at a few points.

### ***1.1.4 Digital Image Correlation***

Given the difficulties encountered by experimental mechanicians during the post-processing of photographically recorded measurement data, and the burgeoning growth of image processing methods in the vision community, it was natural for researchers to employ the recent progress in digital imaging technology and develop (a) methods for digitally recording images containing measurement data, (b) algorithms to analyze the digital images and extract the measurement data and (c) approaches for automating the entire process. In many cases, the characteristic pattern used to compare subsets and extract full-field information was obtained by either coherent light illumination or through application of a high contrast pattern with incoherent illumination, resulting in a full-field, random pattern, or white light speckle pattern.

#### **1.1.4.1 Two-Dimensional Measurements, 1982–1999**

One of the earliest papers to propose the use of computer-based image acquisition and deformation measurements in material systems was written by Peters and Ranson in 1982 [215]. Interestingly, the original application envisioned developing and digitally recording a full-field pattern by subjecting an object to ultrasonic waves both before loading (the reference image) and during the loading process (the deformed image). By recording the resulting reflected wave pattern prior to and after applying load, the authors proposed a method for analyzing the resulting full-field recorded digital “images”. The method suggested a comparison of the digital images for various small regions (known as subsets) throughout the images before and after deformation, locating the positions of each of these subsets after deformation through digital image analysis. As part of the envisioned approach, the authors suggested using fundamental continuum mechanics concepts governing the deformation of small areas as part of the “matching process”.

Using this approach, in 1983 Sutton et al. [288] developed numerical algorithms and performed preliminary experiments using optically recorded images to show

that the approach, known today as 2D Digital Image Correlation (2D-DIC), was feasible when using optically recorded images. Anderson et al. [13] performed rigid body motion measurements using the algorithms, demonstrating that both planar translations and rotations can be reliably estimated through 2D-DIC image matching. Chu et al. in 1985 [48] performed a series of experiments to demonstrate that the method can be applied to quantify rotations and deformations in solids. To improve the speed of the analysis process, in 1986 Sutton et al. [280] demonstrated the use of gradient search methods so that subset-matching could be performed with sub-pixel accuracy throughout the image to obtain a dense set of full-field, two-dimensional displacement measurements. Tian and Huhns [305] discussed the use of several search methods capable of sub-pixel accuracy, showing that both coarse-fine and gradient search approaches are viable. Over the next decade, these procedures were validated, modified, improved and numerical algorithms refined [37, 294]. In the late 1980s, Sutton et al. [287] performed one-dimensional numerical simulations to provide initial estimates for the accuracy of deformation measurements in image correlation. In an effort to quantify internal deformations in a composite sheet, in 1989 Russell et al. [245] performed X-ray radiography before and after deformation. After digitizing the radiographs, 2D-DIC was used to determine the average through-thickness strain fields.

In the decade of the 1990s, limited studies were performed to assess the accuracy of the method. For example, Sjö Dahl [262–264] discussed the accuracy of measurements in “electronic speckle photography”. However, in most cases, investigators began applying the method to measure surface deformations in planar components. Research using 2D-DIC in fracture mechanics began in the early 1980s and continues to this day; summaries are provided in [291, 292]. Early work by McNeill et al. [189] demonstrated the use of 2D-DIC measurements for stress intensity factor estimation. Sutton et al. used local crack tip plastic zone measurements [295] to estimate the zone affected by “three-dimensional effects”. Dawicke and Sutton [62] measured crack opening displacement with 2D-DIC. Using work performed by Lyons that extended 2D-DIC for strain measurements at high temperature [176], Liu et al. [161] made full-field creep measurements at 700°C in alloy 718 over several hundred hours. Amstutz et al. [11, 12] measured tensile and shear deformations in Arcan specimens undergoing mixed mode I/II loading. Han et al. developed a high magnification optical system to measure deformations around stationary [101] and growing [102] crack tips under nominally Mode I loading.

In addition to the work in fracture mechanics, investigators used 2D-DIC to understand material deformation behavior including metals [56, 307, 308], plastics [168, 314], wood [355], ceramics [49] and tensile loading of paper [40, 279]. In the late 1990s and 2000, investigators applied 2D-DIC to study damage in composites [85, 93, 99] and concrete [46, 47].

Prior to recent emphasis on nano-science and nanotechnology, efforts of investigators such as Davidson and Lankford [60, 61] in the 1980s extended 2D-DIC concepts to the microscale for large deformation measurements near fatigue flaws. Additional high magnification studies [241, 274, 278, 290] were performed using 2D-DIC. Of particular note is the effort to extend the work of Davidson and

Lankford, as described in the work of Sutton et al. [290]. The authors developed a fully automated image acquisition and analysis system using a Questar far-field microscope to image a  $700 \times 700 \mu\text{m}$  crack tip region. Riddell et al. [241] employed the system to measure crack tip deformations as part of a study focused on crack closure assessment.<sup>1</sup> Details regarding this micro-scale application are given in Section 6.4.

Though much of the development and application in 2D digital image correlation remained focused at the University of South Carolina through the mid-1990s, a unique extension of 2D-DIC for scanning tunneling microscopy (STM) was completed by Prof Knauss and his students. In 1998, Vendroux and Knauss [321–323] reported their STM work. By acquiring STM images and using a modified gradient search approach to perform digital image correlation, the authors evaluated sequential STM images to extract deformation measurements at the sub-micron scale. In the same time frame, Doumalin et al. [64] discussed the use of image correlation using images from a scanning electron microscope for deformation measurements at higher magnification.

While novel methods were being developed for quantitative measurements at structural and micro length scales in solid mechanics, in 1984 Peters et al. [216] applied 2D-DIC to measure the velocity field in a seeded, two-dimensional flow field, demonstrating that the approach can be used in fluid systems. In the field of fluid mechanics, investigators used a variety of illumination sources and various high speed imaging concepts such as rotating drum and rotating mirror (frame/streak) camera systems to extend the method into a wide range of areas. Today, particle image velocimetry (PIV) [6] and digital PIV are accepted methods for extracting 2D motion measurements from images of fluid particles, providing local velocity and motion measurements in fluid systems.

Most, if not all, of the studies described above employ direct image correlation principles for matching subsets and extracting full-field displacements. A parallel approach for performing the matching process that employed Fast Fourier Transforms (FFTs) was developed by Cheng et al. in 1993 [43]. Using well-known Fourier Transform properties relating frequency content to local displacement, the peak location in the frequency domain of an FFT was shown to be a viable alternative for estimating local displacement components for those applications where in-plane strains and rigid body rotations are small.

#### 1.1.4.2 Two-Dimensional Measurements, 2000–Present

In the past several years, 2D-DIC has undergone explosive growth worldwide; the authors identified 400+ archival articles<sup>2</sup> using the method since 2000.

Schreier et al. [252] demonstrated the importance of the image reconstruction process when attempting to improve the accuracy of the matching process. His

<sup>1</sup> The system remains in active use in the laboratory of R.S. Piascik and S.W. Smith at NASA Langley Research Center, as of the publication of this book.

<sup>2</sup> Science Citation Index literature search identifying use of 2D-DIC as part of measurement process, 2000–2007.

work showed that higher order spline interpolation functions can be used effectively to reconstruct the image intensity pattern and minimize measurement bias during the matching process, resulting in image position accuracy of 0.01 pixels or better in both the x and y directions when distortions are removed from the images. In a companion work, extending work by Lu and Cary [167], Schreier and Sutton [253] confirmed that quadratic shape functions provide some advantages when performing the matching process, especially for non-uniform strain fields, without appreciable increase in computational time. A summary of many of the key concepts noted above is given in two recent publications [285, 286]. Chapter 5 presents recent results for both bias and variability in 2D-DIC image displacement measurements.

Other researchers have presented modifications to various aspects of the 2D-DIC measurement process, including the search procedure [106, 352], correlation approach [123, 134, 210, 221, 304] and registration method [271, 273, 350]. With regard to registration methods, of note is the work of Cheng et al. [44] which proposed full-field deformation through pixel-by-pixel mapping using a B-spline functional form. Since that time, investigators have developed modified full-field DIC methods that combine analytical models with 2D-DIC measurements. For example, in a series of articles, Réthoré et al. [235–238] defined an “extended 2D-DIC” method using finite element concepts, with initial emphasis on crack specimen studies. Additional methods include a range of approaches [16, 135].

Inverse methods have been actively pursued in recent years for mechanical property estimation, including elastic properties [90, 91, 116, 150], properties in heterogeneous materials [163, 164], hyperelastic properties [92], micromechanics [113] and composites [179].

Even as additional studies have been performed to understand the theoretical fundamentals of the method, researchers and scientists have applied 2D-DIC in a breathtaking array of areas. For example, Chasiotis [41, 42], Jin [132], Li [159, 160, 341, 342] and others [270] have extended 2D-DIC for use in AFM systems, determining deformations and strains with spatial resolution on the order of 50 nm. As with the work of Davidson et al., the approach is most effective when deformations are large ( $\epsilon > 0.003$ ).

When relatively large deformations are expected, investigators have used a scanning electron microscope (SEM) with 2D-DIC [138, 146]. Recently, Sutton et al. [158, 282–284] have demonstrated that SEM images can be acquired with a high contrast random pattern [50, 255] under mechanical or thermal loading, corrected for spatial [130] and temporal distortions and used with 2D-DIC to extract elastic properties for fields of view on the order of  $10 \times 10 \mu\text{m}$ . In this work, the variation is small so that local strains on the order of 0.0005 can be reliably determined. Details for correcting SEM images and performing 2D-DIC for high magnification measurements with optimal accuracy are presented in Section 6.6.

In a recent study, Berfield et al. [28] performed experiments to acquire deformations on an internal plane containing a high contrast random pattern, extending 2D-DIC in a manner similar to seeding of fluids [6, 216].

Given the sheer volume of applications, it is only possible to present representative examples for how the method is being studied, modified and applied. Fracture



studies in recent years have included measurements in functionally graded materials [1–4], composites and concrete [8, 269, 297, 330], composites [9, 10, 103, 192, 202], asphalt [32], metals [211], polymers [131], rubber [71], shape memory alloys [58], electronic components and joints [258, 259], brittle materials [77, 190, 191], rock [177], wood [312] and general fracture parameter estimation methods [234, 239, 344–346].

Material characterization studies have included 2D-DIC measurements in thin materials and films [127, 128, 230, 246], polymers [141, 169, 182, 213, 223], metals [184, 220], heterogeneous materials [261, 299, 309], wood [129, 198, 199, 313], bio-materials [23, 45, 120, 157, 240, 331, 332, 347, 348], shape memory alloys [38, 193, 196, 197, 248], composites [78, 79, 214, 226, 244, 303, 315], asphalt [256], ceramics [126], glass wool [29], mineral wool [115], rock [31], glass [121], foams [133, 152, 227, 333, 334, 353, 354], clay [142], sands/soils [72, 98, 162, 231, 232, 339], concrete [222], paint [275, 276] and electronic components and joints [55, 66, 139, 185, 212, 272, 340, 349]. An interesting application was performed by Louis et al. [166]. In a manner analogous to the work of Russell et al. [245] for composites, the authors used X-Ray radiography with 2D-DIC to estimate the average strains in sandstone [166].

### **1.1.4.3 Three-Dimensional Digital Image Correlation Measurements, Pre-2000**

Since two-dimensional digital image correlation requires predominantly in-plane displacements and strains, relatively small out-of-plane motion will change the magnification and introduce errors in the measured in-plane displacement. The effect was clearly highlighted in publications [68, 168] that attempted to estimate the effect of curvature on the sensor plane projections, with relatively large errors evident in the estimated deformations. Section 6.3 quantitatively demonstrates the deleterious effect of out-of-plane motion on 2D image motion measurements, while also confirming that stereo-vision measurements account for such motion by measuring all components simultaneously [289].

As early as the 1960s, photogrammetry principles developed for shape and motion measurements were used to estimate plate deflections [337]. From 1970–1990, the concept of digital correlation for use in photogrammetry was highlighted [5, 34, 108, 137], and photogrammetry used to estimate critical crack tip opening [143] and surface deformations through selective feature identification [329]. Morimoto and Fujigaki [194] discussed the use of multiple cameras with images of a deforming rectangular grid and FFT methods for image analysis and surface motion estimates. Combining stereo-vision principles with 2D-DIC concepts developed and used in single camera imaging, Chao et al. successfully developed, automated and applied a two-camera stereo vision system for the measurement of three-dimensional crack tip deformations [172, 173]. Cárdenas-García et al. [39] considered parallel and converging stereo configurations with DIC to identify features.



To overcome some of the key limitations of the method used in these studies (square subsets remained square in both cameras, mismatch in the triangulation of corresponding points and manual motions in a calibration process that was laborious and time consuming), the stereo-vision method was modified [109] to include (a) the effects of perspective on subset shape, (b) use of a grid with a range of calibration motions and (c) appropriate constraints on the analysis to include the presence of epipolar lines. As part of the on-going US aging aircraft research program, the modified system was used to obtain deformation measurements on a range of center crack panel sizes.<sup>3</sup> Results from the combined computational and experimental effort were presented by Helm et al. [110–112].

In the late 1990s, Synnergren et al. [300–302] and Lacey et al. [145] employed stereovision to make deformation measurements. Andresen [15] used photogrammetry principles and a stereo system to record and analyze images of a grating undergoing large deformation. Of particular note is the set of experiments by Synnergren et al. where the investigators obtained flash X-ray images of a specimen from two directions before and during high-rate loading. By comparing features in both sets of X-rays, the investigators extracted 3D estimates for specimen deformations. Lockwood and Reynolds [165] used SEM imaging to obtain stereo images via in-situ specimen tilting to obtain surface images of the specimen from two orientations. Without patterning the surface, the authors used image correlation and clearly discernible surface features to extract the matching image locations. This data was used with a simplified stereo imaging model to extract approximate 3D fracture surface shape information.

As investigators in the US were focused on developing robust measurement systems for a range of applications, European investigators working in the field of computer vision were also making significant progress. For example, Faugeras and Devernay [68] used a Taylor's series expansion of the unknown disparity function between two stereoscopic images to define a "subset shape function". For example, the first and second order expansions lead to the affine and quadratic shape functions used in many applications. Devy et al. [63] developed an improved methodology for 3D calibration. In their work, the authors defined the classic sum of squared reprojection errors (differences between the model-based image plane locations and the measured image plane locations) and optimized over the set of intrinsic and extrinsic model parameters; the model also included a radial distortion correction parameter. Lavest et al. [148] introduced the bundle adjustment technique (known for decades in the photogrammetry community) to the computer vision field and proposed an accurate, flexible calibration method based on this approach.

Garcia et al. [84] proposed an improved method for the calibration of a stereo-vision sensor and were the first to demonstrate that seven constraints are needed to reduce the family of calibration parameter solutions to a unique set when using bundle adjustment. As part of their work, the authors also extended the single camera calibration procedure commonly used in computer vision to the calibration of a stereo-camera system. Known as a stereo-rig system, the approach used by the

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<sup>3</sup> All experiments were performed in Bldg 1205 at NASA Langley Research Center in Hampton, VA.

authors included an efficient approach for calibration by bundle adjustment using a global minimization procedure, and demonstrated experimentally that the approach outperforms previous calibration approaches based on separate camera calibration.

#### **1.1.4.4 Three-Dimensional Digital Image Correlation Measurements, 2000–Present**

As a result of the developments which have occurred in recent years, three-dimensional digital image correlation now is being used for a wide range of applications on both large and small structures (e.g. [111, 187]), as well as in the area of fluid measurements, where similar developments have led investigators to modify 2D PIV setups and employ multiple camera systems (i.e., stereoscopic PIV systems) to resolve all three components of velocity of imaged fluid particles [118, 119]. Of particular note is the use of a dual distribution of speckle sizes using complementary colors within a single pattern [187]. In this work, the authors used green or red filters to extract either the large or small pattern, depending upon whether full field information or local information is to be emphasized. Method development has included stereo optical microscopy with a novel distortion correction approach [130, 254]; the method has been used with existing surface features [147]. Section 7.4 describes the use of stereomicroscopy for deformation and shape measurements on specimens ranging from a few millimeters to 300  $\mu\text{m}$ . In recent years, high speed imaging systems have been developed for use in a variety of experimental studies. Basic studies of typical high speed imaging systems were performed by Tiwari et al. [306]. In this case, the authors have shown that results from their studies led the authors to conclude that ultra high speed cameras utilizing image intensifiers may introduce large errors in the image-based deformation measurements. Recently, Orteu et al. [206] have used stereovision images to obtain simultaneously both 3D motion measurements and temperature estimates through appropriate thermal calibration procedures.

Stereovision system applications have included measurements on flexible wings undergoing aerodynamic loading [7, 266], as well as measurement of shape and deformation on cylindrical surfaces [173, 174] and on balloon structures [328].

In addition to the early work noted previously, recent fracture studies have been performed under mixed mode I/II [174] and mixed mode I/III loading conditions [298], while other investigators measured side-necking in a fracture specimen [151]. Section 7.3 presents experimental details for application of high speed stereovision to the study of mixed mode I/III fracture in a ductile metallic specimen. For low ductility materials, such as concrete, Lecompte et al. considered the use of imaging for crack detection [149].

Material characterization has also been an active area including microscale studies in engineered materials [73, 74, 338] and bio-materials [201, 281, 311], foam [51, 97], ceramics [204, 242], composites [187, 195, 207, 208], polymers [136] and high rate events [250, 251, 260]. A recent area of focus has been the determination of residual stresses using surface deformation measurements [122, 186, 200].

Section 7.2 describes the use of four synchronized cameras in a multi-stereo-vision system application to quantify large deformations in an edge-cracked, highly ductile polymer undergoing out-of-plane bending.

#### 1.1.4.5 Volumetric Digital Image Correlation (VDIC)

Volumetric imaging and image analysis has been an active area of research and development within the medical community for decades, resulting in mature imaging capabilities. Micro and Macro Computer Aided Tomography (CT), Magnetic Resonance Imaging (MRI), Confocal Imaging Microscopy (CIM) and Positron Emission Tomography (PET) are examples of technology that are currently available for full-volume imaging of bio-material systems.

In addition to imaging technology, there is a wide range of software available to view volumetric images and analyze the images to extract information of interest to investigators. As a recent example, the work of Weiss, Veress, Phatak et al. [219, 324, 325] is noteworthy. In their studies, the authors developed methods for comparing images that have minimal feature content to extract full-field estimates for the deformations required to “warp” the image into the required shape.

Even as medical investigators developed software to analyze images for their applications, Bay et al. [26, 27, 188, 265] proposed that the 2D-DIC concepts used successfully for 2D and 3D computer vision be extended and used to match small sub-volumes before and after undergoing loading to obtain a full volumetric field of three-dimensional motions. The authors applied these principles to make quantitative deformation measurements in bone, demonstrating the efficacy of the proposed “volumetric digital image correlation” approach.

Applications of the approach include measurements on sandwich structures [247], bio-materials [326], steel powders [318], metallic alloys [171] and rock [156]. An interesting approach was developed by Germaneau et al. [86–89], using internally scattered light to obtain a volumetric pattern for imaging and analysis. The approach is qualitatively similar to the CIM approach used by Franck et al. [80], where volumetric images of a bio-material are acquired through point-by-point imaging of laser illuminated positions. Chapter 8 presents both theoretical background and experimental details for application of Volumetric DIC to study the behavior of foam under compression.

## 1.2 Discussion

Chapter 2 presents the essential elements of geometric optics that are employed in the theoretical developments. The section includes (a) thin and thick lens approximations, (b) depth of field and field of view formulae and (c) the general form of the pinhole model arising from geometrical optics approximations.

In Chapter 3, the theoretical foundation for single camera perspective image models is detailed, along with the relationship of various coordinate transformations to the camera calibration process. Additional topics introduced in this section include (a) distortion and various parametric and non-parametric distortion models and (b) modern camera calibration approaches using bundle adjustment, distortion and optimization algorithms.

Chapter 4 presents two-dimensional and three-dimensional computer vision models with special emphasis on calibration procedures that are commonly employed.

Chapter 5 outlines fundamental concepts underlying digital image correlation for motion measurements. Topics include (a) image correspondence during matching, (b) matching methods including differential and template approaches, (c) optimization criteria with intensity pattern scaling and offset accommodation, (d) subset shape functions, (e) solution methods, (f) interpolation methods and (g) error estimates, including bias and variability estimation approaches.

Chapter 6 presents details regarding the application of a single camera imaging system for in-plane measurement of deformations, including the potentially deleterious effects of out-of-plane motion. Applications include (a) successful measurement of the stress-strain response of an aluminum alloy using the surface strain field measured on a planar specimen undergoing uniaxial tension by 2D-DIC, (b) details of extensive experimental studies that clearly highlight the effects of out-of-plane motion on single-camera motion measurements, (c) use of a far-field microscope for microscale measurements of crack tip deformations with nanometer accuracy, (d) determination of material properties through an inverse methodology enabled by full-field 2D-DIC deformation measurements and (e) use of a scanning electron microscope for deformation measurements in a  $20 \times 20 \mu\text{m}$  field of view, including the development and application of nanoscale patterns.

Chapter 7 describes in detail the application of stereo-vision camera imaging systems for general three-dimensional measurement of surface deformations. Applications include (a) a synchronized four-camera stereovision system for the measurement of large, out-of-plane deformations in polymer beam specimens, (b) dynamic deformation measurements using high speed stereo cameras and (c) development and use of 3D stereomicroscopy systems for microscale deformation and shape measurements.

Chapter 8 presents both theoretical foundations and experimental results from a series of volumetric image correlation measurements in a polymeric foam undergoing compression loading.

Chapter 9 provides a brief discussion of methods to estimate errors that are expected during stereovision measurements. Developments include approaches to obtain probabilistic estimates for the mean and variance in the measurements due to the effects of intensity interpolation, intensity pattern noise and intensity pattern contrast on image-plane matching locations for corresponding points.

Chapter 10 presents practical considerations for accurate measurements using digital image correlation methods. Topics include (a) general imaging issues including camera and lens selection, depth of field, field of view and image magnification,

(b) image artifacts, including image sensor defects, reflections and lens contamination, (c) speckle pattern application, (d) optimal speckle pattern structure, (e) optimal speckle size and engineering estimates with examples, (f) exposure time and image blurring, (g) out of plane motion and estimates for the effect on 2D measurements and (h) image matching errors and approaches for estimating errors in both 2D and stereo imaging.

Appendices A through I provide the background information that the authors believe is important to provide the foundation necessary for both essential concepts in computer vision and the practical application of the method for shape and deformation measurements. Areas covered include continuum mechanics; linear algebra; surface strain estimation; non-linear optimization; basic concepts in statistics and probability; introduction to projective geometry; rotation tensor formulations; optimal estimates for intensity scale and offset; spline functions and triangulation.

## Chapter 2

# Elements of Geometrical Optics

### 2.1 Optics of a Camera

Figure 2.1 shows a simple optical system that consists of a single, ideal lens, i.e. a thin lens. Such lenses are good approximations when (a) the angles and diameters of the focused, light rays are sufficiently small so that the Gauss approximation is appropriate and (b) geometrical aberrations and other optical defects can be neglected [107]. Next, we consider an optical system consisting of a single thick lens. Using ray optics to extract the basic formulae relating object positions to sensor locations, we will see that by neglecting the effect of blur due to defocus, these optical models can be combined into a single simple geometric model of perspective projection: the so-called pinhole model.<sup>1</sup>

#### 2.1.1 Thin Lens

A ray incident on a surface is described by its angle of incidence relative to the surface normal. In this context, a thin lens, or an association of lenses that are thin, should have a thickness (distance along the optical axis between the two outer surfaces of the lens) that is negligible compared to its focal length or to any of its dimensions. Lenses whose thickness is not negligible are called thick lenses.

The Gauss, or paraxial, approximation assumes that (a) the incident light rays make a small angle,  $\theta$ , relative to the optical axis of the lens and (b) the ray lies close to the optical axis as it traverses the imaging system. Under this condition, the following approximation is assumed to be valid:  $\sin \theta \simeq \tan \theta \simeq \theta$ . Using this approximation, ray tracing is relatively simple since (a) any ray going through the centre of the lens is not deviated, (b) all parallel rays pass through the focal plane of the lens and (c) rays passing through a thin lens are deviated at the lens' mid-plane.

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<sup>1</sup> In-depth developments in optics and applied optics are available in books such as Klein [140] and Born and Wolf [35].