Concept Mapping in Mathematics

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Editor

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Research into Practice

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Foreword

It is a real pleasure to write a foreword to this first book that seeks to illustrate how concept mapping can be used to facilitate meaningful learning in mathematics. I believe the authors succeed in showing that mathematics can be more than the memorization of procedures to get answers to textbook types of problems. Through all of my elementary and high school studies of mathematics, I thought that what was required was to learn the procedures for getting answers, and I recall thinking that after doing 2 or 3 textbook problems of a given type, mathematics was rather tedious and relatively boring. By contrast, I saw the study of science as the search for understanding of fundamental concepts, such as the nature of matter, energy, and evolution. This I found to be exciting and I was always eager to seek deeper understanding of basic science concepts. It came as a somewhat shocking surprise to me when I was studying calculus at the University of Minnesota that there were fundamental mathematics concepts also, such as limit, slope, proportionality, etc. I recall feeling cheated that none of my teachers had helped me to gain a *conceptual* understanding of mathematics!

Another fact that I pondered during my youth was that there were child prodigies who could do unusually difficult mathematics at ages 10 or 12, but there were very few such prodigies in sciences, literature, or history. With the invention of the concept mapping tool, it became clear to me why the latter was the case in some disciplines. To achieve relative mastery in a field of science, there were many concepts that had to be learned and understood. In contrast, in music or mathematics, if one gains an early understanding of a few dozen fundamental concepts, such as those discussed in the chapters of this book, you can move on to understanding of major domains of mathematics, and perhaps even do some creative mathematics. History has shown this to be the case. Of course, we are all familiar with the musical early genius of Wolfgang Amadeus Mozart. My hypothesis is that if we can transform the teaching of mathematics to a field that is *conceptually transparent* to students from pre-school through high school, this might become one of the easiest fields of study instead of the opaque and often-dreaded study mathematics has been for so many students.

As a first pioneering effort, this book omits discussion of many issues that hinder the kind of school instruction in mathematics that would make the subject conceptually transparent and meaningful to all students. Nevertheless, the authors of the following chapters provide examples of how to teach mathematics better, and data to support the validity of the idea that teaching mathematics for understanding of mathematics concepts is better than what we are now doing in most school classrooms.

This book can also serve as a primer for mathematics teachers at all levels, who wish to make the study of mathematics a meaningful learning experience for all students. It can also provide guidance for future research using concept mapping tools that can expand our understanding of meaningful teaching and learning in mathematics. I see a bright future for the improvement of mathematics education.

Pensacola, FL Joseph D. Novak

Contents

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Introduction

Karoline Afamasaga-Fuata'i

This book is the first comprehensive book on concept mapping in mathematics. It provides the reader with an understanding of how the meta-cognitive tool, namely, hierarchical concept maps, and the process of concept mapping can be used innovatively and strategically to improve planning, teaching, learning, and assessment at different educational levels. The book consists of a collection of articles on research conducted critically to examine the usefulness of concept maps in the educational setting ranging from primary grade classrooms through secondary mathematics to preservice teacher education, undergraduate mathematics and post-graduate mathematics education. A second meta-cognitive tool, called vee diagrams, was also critically examined by some authors particularly its value in improving mathematical problem solving and mathematical modeling in physics.

Thematically, the book flows from a historical development overview of concept mapping in the sciences to applications of concept mapping in mathematics by teachers and preservice teachers as a means of analyzing mathematics topics, planning for instruction and designing assessment tasks including applications by school and university students as learning and review tools. The book provides case studies and resources that have been field tested with school and university students alike. The findings presented have implications for enriching mathematics learning and making problem solving more accessible and meaningful for students.

The theoretical underpinnings of concept mapping and of the studies in the book include Ausubel's cognitive theory of meaningful learning, constructivist and Vygotskian psychology to name a few. There is evidence particularly from international studies such as the Program for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) and mathematics education research, which suggest that students' mathematical literacy and problem solving skills should be enhanced through students collaborating and interacting as they work, discuss and communicate mathematically. This book proposes the meta-cognitive strategy of concept mapping as one viable means of promoting, communicating and explicating students' mathematical thinking and reasoning publicly

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in a social setting (e.g., mathematics classrooms) as they engage in mathematical dialogues and discussions.

Whilst a number of books have been published on concept mapping in the sciences and science education, none is dedicated to mathematics and mathematics education.

Shared commitments to develop and promote meaningful learning of mathematics and a more conceptual understanding of problem solving beyond simply knowing the formulas and procedures subsequently led the contributing authors to investigate the value of concepts maps as an educational tool in a variety of settings with different types of students from primary to university levels. The book, organised into four main parts, presents a diversity of applications as researched by the contributing authors.

Part I provides a historical overview of the development of concept mapping by Joseph Novak, the inventor of hierarchical concept maps. While Part II focuses on research conducted in primary mathematics with primary student teachers, teachers and students, Part III focuses on research conducted in secondary mathematics with secondary student teachers, teachers and students. Research conducted in university mathematics with university teachers and students are presented in Part IV whereas Part V poses questions about potential directions for future research in concept mapping in mathematics.

Individual chapters within each of Parts I–V are briefly summarised below:

Part I: A Historical Overview of Concept Mapping

In Chapter 1, Joseph Novak and Alberto Cañas describe how his research team, in response to a need for a tool to describe explicit changes to children's conceptual understanding, invented *concept mapping* in 1972. Underlying the research program and the development of the concept mapping tool was an explicit cognitive psychology of learning and an explicit constructivist epistemology. As well as describing the various applications of concept maps since its creation, leading up to the development of a concept mapping software at the Institute for Human and Machine Cognition to facilitate concept mapping, Novak and Cañas further propose *A New Model for Education*.

Part II: Primary Mathematics Teaching and Learning

In Chapter 2, Karoline Afamasaga-Fuata'i presents the case study of a primary student teacher who, over one semester, applied concept maps and vee diagrams as tools to conceptually analyse the *Measurement* strand of a primary mathematics syllabus, to communicate her subsequent interpretations and understanding of syllabus outcomes, and to pedagogically plan learning activities to ensure the development of students' conceptual understanding of *length, volume, surface area,* and *capacity*.

In Chapter 3, Jean Schmittau and James Vagliardo use a case study to illustrate the power of concept mapping to reveal both the centrality of the *Positional* concept within elementary mathematics and the pedagogical content knowledge required to teach the concept of positional system and other mathematics concepts to which it is related.

Chapter 4 by Karoline Afamasaga-Fuata'i explores a post-graduate student's use of concept maps and vee diagrams as tools to analyse the *Fraction* strand of a primary mathematics curriculum and related problems and to record his developing pedagogical understanding over the semester as a consequence of social critiques and further revision.

Karoline Afamasaga-Fuata'i and Greg McPhan, in Chapter 5, presents a case study to highlight the kinds of concerns and issues that can impede the introduction of concept mapping to real classrooms and to demonstrate ways in which concept maps can be used to reinforce and review learning of mathematics and science topics in two primary classrooms. The ultimate highlight was the initiative by the two primary teachers and their students to come together for peer tutoring and peer collaborations as the older students mentored and assisted the younger ones in using a computer software *Inspiration*^{TM} to collaboratively construct concept maps.

Part III: Secondary Mathematics Teaching and Learning

In Chapter 6, Edurne Pozueta Mendia and Fermín González present a study to illustrate how concept maps can be used to monitor and identify the extent of secondary students' meaningful learning of *Proportionality* after teaching an innovative instructional module. Comparing individually-constructed concept maps to an expert map enabled them to distinguish students who had learnt proportionality more meaningfully from those who learnt by rote learning or had misconceptions.

In Chapter 7, Jean Schmittau examines how concept mapping can be used to assess whether secondary teachers possess the requisite knowledge to teach both concepts and procedures with understanding premised on the view that mathematical algorithms are not merely mechanical procedures to be learned by rote, but as fully conceptual cultural historical products.

Karoline Afamasaga-Fuata'i in Chapter 8 describes the case of a secondary student teacher, to demonstrate how concept maps can be used to provide a macro view of a two-year mathematics curriculum and to innovatively develop a teaching sequence and lesson plan on *Derivatives*.

James Vagliardo in Chapter 9 explores how concept maps may be used to highlight the importance of mediating a deeper meaning of *Logarithms* and its connections to other mathematical ideas by locating its conceptual essence from a cultural-historical context.

In Chapter 10, Maria Ramirez, Mario Aspee, Irma Sanabria & Neyra Tellez provide practical guidelines that are theoretically driven to assist mathematics students to construct concept maps and vee diagrams to illustrate their understanding of mathematical functions used to model physical phenomena.

William Caldwell explores in Chapter 11 the potential of concept mapping for increasing meaningful learning in mathematics at middle school level for both teachers and students and examines the results of mathematical professional development and student learning activities using concept mapping.

Part IV: University Mathematics Teaching and Learning

In Chapter 12, Karoline Afamasaga-Fuata'i presents a study that investigated the use of concept maps and vee diagrams to learn about new advanced mathematics topics students had not encountered before. Students displayed their developing understanding and knowledge on concept maps and vee diagrams for public scrutiny.

Rafael Pérez Flores in Chapter 13 deals with a particular way of using concept maps to contribute to engineering students' meaningful learning of mathematics by implementing a didactic strategy that is guided by the professor's concept maps, to facilitate the development of students' critical thinking and understanding and the application of these process in solving mathematics problems.

In Chapter 14, Karoline Afamasaga-Fuata'i presents the case of a student who used concept maps to illustrate and communicate his evolving understanding of *Differential Equations* over the semester as a result of his own research and revisions subsequent to social critiques during seminar presentations and individual consultations.

In Chapter 15, Karoline Afamasaga-Fuata'i explores a group of students' use of concept maps and vee diagrams as tools to visually display their developing and growing understanding of the conceptual structure of selected topics and the connections between this structure and procedures for solving problems.

Part V: Future Directions

In Chapter 16, Karoline Afamasaga-Fuata'i provides a synopsis of chapter findings and implications for incorporating concept mapping in real classrooms. Also included are suggestions of potential directions for future research in concept mapping in mathematics education.

Part I
A Historical Overview of Concept Mapping

Chapter 1
The Development and Evolution of the Concept Mapping Tool Leading to a New Model for Mathematics Education

Joseph D. Novak and Alberto J. Cañas

A research program at Cornell University that sought to study the ability of first and second grade children to acquire basic science concepts and the affect of this learning on later schooling led to the need for a new tool to describe explicit changes in children's conceptual understanding. *Concept mapping* was invented in 1972 to meet this need, and subsequently numerous other uses have been found for this tool. Underlying the research program and the development of the concept mapping tool was an explicit cognitive psychology of learning and an explicit constructivist epistemology, described briefly in this paper.

In 1987, collaboration began between Novak and Cañas and others at the Institute for Human and Machine Cognition, then part of the University of West Florida. This led to the development of software to facilitate concept mapping, evolving into the current version of CmapTools, now widely used in schools, universities, corporations, and governmental and non-governmental agencies.

CmapTools allows for selective use of Internet and other digital resources that can be attached to concept nodes and accessed via icons on a concept, providing a kind of knowledge portfolio or knowledge model. This capability permits a new kind of learning environment wherein learners build their own knowledge models, individually or collaboratively, and these can serve as a basis for life-long meaningful learning. Combined with other educational practices, use of CmapTools permits a New Model for Education, described briefly. Preliminary studies are underway to assess the possibilities of this New Model.

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Introduction: The Invention of Concept Mapping

During the 1960s, Novak's research group first at Purdue University and then at Cornell University sought to develop a coherent theory of learning and theory of knowledge that would form a basis for more systematic research in education and a scientific basis for school curriculum design. We found that Ausubel's assimilation theory of learning presented in his *Psychology of Meaningful Verbal Learning* (1963) spoke to what we were most interested in, namely, how do learners grasp the meanings of concepts in a way that permits them to use these concepts to facilitate future learning and creative problem solving? Ausubel stressed the distinction between learning by rote and learning meaningfully. Rote learning or memorizing information may permit short-term recall of this information, but since it does not involve the learner in actively integrating new knowledge with concepts and propositions already known, it does not lead to an improved organization of knowledge in the learner's cognitive structure. In contrast, meaningful learning requires that the learner chooses actively to seek integration of new concept and propositional meanings into her/his cognitive structure, thereby enhancing and enriching her/his cognitive structure. Since the 1960s, many studies have shown that what distinguishes the naive or novice learner from the expert is the extent to which the person has a highly organized cognitive structure and metacognitive strategies to employ this knowledge in new learning or novel problems solving (Bransford, Brown, & Cocking, 1999). In short, one builds expertise in any discipline by building powerful knowledge structures that characterize the key intellectual achievements in that discipline, as well as strategies to use this knowledge.

Also occurring in the 1960s was a philosophical movement away from *positivism* where knowledge creation was seen as a search for "truths" unfettered by prior ideas or emotion. Kuhn's (1962) book, *The Structure of Scientific Revolutions* marked a turning point toward *constructivist* ideas that saw knowledge creation as a human endeavor that involved changing methodologies and paradigms and an evolving set of ideas and methodologies leading to useful but evolving paradigms and ideas. We saw that this constructivist epistemology and cognitive psychology was equally applicable to mathematics and mathematics teaching. The challenge was: "how do we get educators and the school contexts to change to enhance the utilization of these new insights" (Novak, 1986, p. 184). As we proceeded in our mathematics education studies, we found we could work with a theory of learning that explained how new concepts are acquired and used that complemented a theory of knowledge that focused on the evolving creation of new concepts and problem solving approaches.

Working with elementary school children, we sought to design new instruction in such a way that meaningful learning would be enhanced, and to demonstrate that such learning could facilitate future learning and problem solving. To do this we found that we needed an assessment method that could monitor the evolving knowledge frameworks of our learners. Moreover, we were interested in demonstrating that young children (ages 6–8) could acquire significant science concepts and that this learning would facilitate later learning. The advances in learning theory and

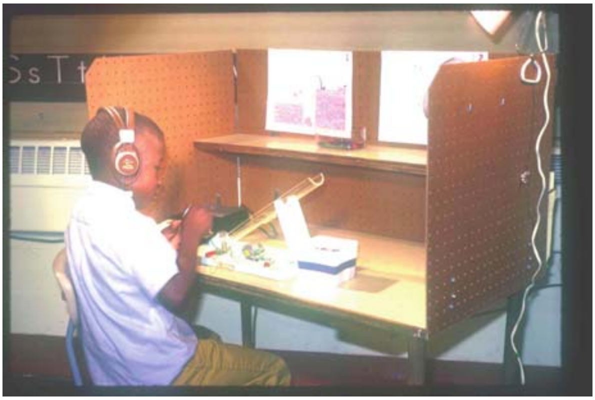

Fig. 1.1 An audio-tutorial carrel unit showing a 7 year old student learning about energy transformations

epistemology permitted Novak to construct a theory of education, first presented in 1977 (Novak, 1977) and modified and elaborated in 1998 (Novak, 1998). This theory has been guiding our work for some three decades.

Given our interest in teaching young children basic science concepts such as the nature of matter, energy, and energy transformations and related ideas, we were faced with the reality that most primary grade teachers did not posses the knowledge to teach these ideas. Therefore, we developed a series of audio-tutorial lessons where children were guided by audio instruction in the manipulation of pertinent materials and presented the vocabulary needed to code the concepts they were learning. Figure 1.1 shows an example of one of these audio-tutorial lessons and the carrel unit in which they were presented.

Lessons proceeded on a schedule of one new lesson every two weeks placed in classrooms in Ithaca Public schools. Earlier studies had shown that a variety of paper and pencil tests were inadequate for monitoring the growth in the children's understanding of concept meanings, so we used interviews to probe the children's knowledge. This, however created another problem in that it was difficult to see in the interview transcripts just how the children's cognitive structure was changing and how new concepts were being integrated into the child's cognitive structure. After struggling with the problem for some weeks, Novak's research group came up with the idea of transforming the interview transcripts into a hierarchically arranged picture showing the concepts and proposition revealed in the interview. We called the resulting drawing a *concept map*. We found that we could now see explicitly what concept and propositions were being integrated into a child's mind as they progressed through the audio-tutorial lessons, and also in later years as they encountered school science studies.

Figure 1.2 shows an example of a concept map drawn from an interview with one child at the end of grade two and another for the same child in grade 12.

This child was obviously a meaningful learner and not only were some misconceptions remediated, but he developed an excellent knowledge structure for this area of science. The results from this 12-year longitudinal study demonstrated several

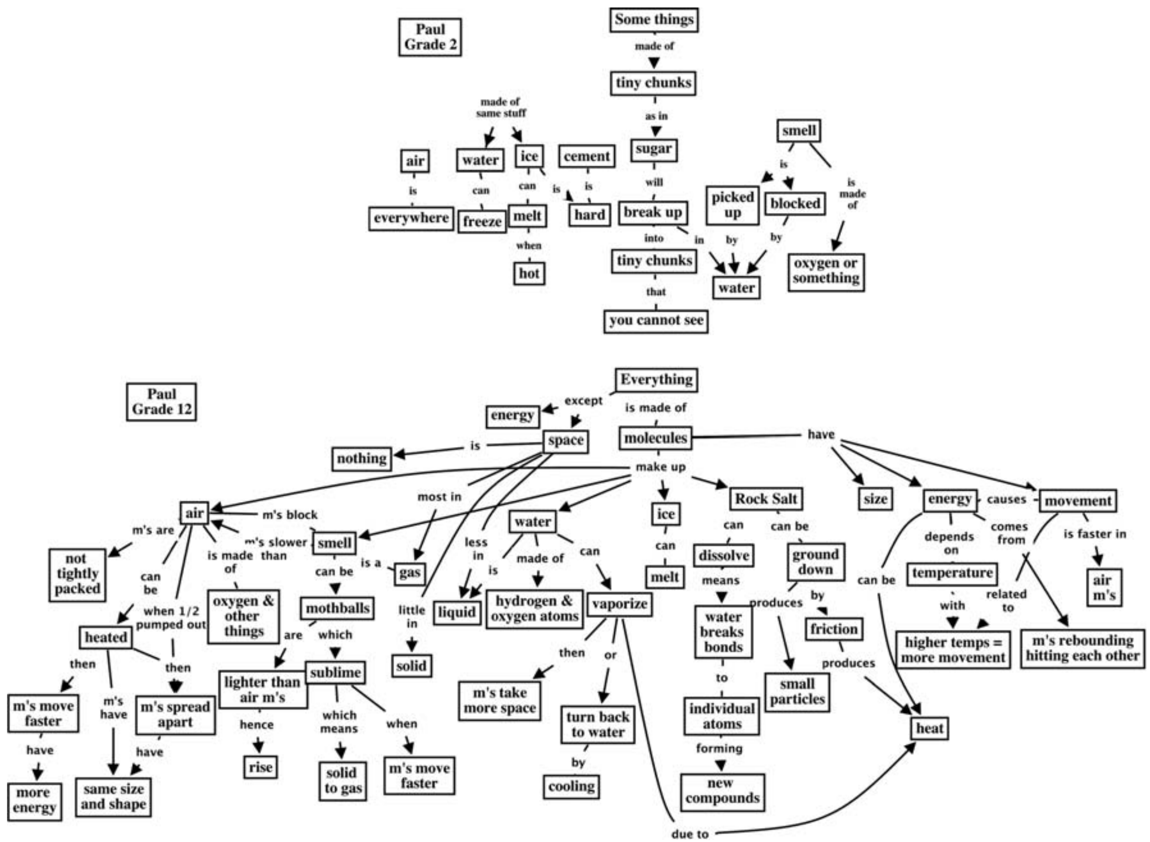

Fig. 1.2 Two concept maps drawn from interviews with children in grade 2, (*top*) and grade 12 (*bottom*). These show the enormous growth in conceptual understanding for this child over 10 years of schooling

things: concept maps could be a powerful knowledge representation and assessment tool; young children can acquire significant understanding of basic science concepts (widely disputed in the 1960s and 70s); technology can be used to deliver meaningful instruction to students; early meaningful learning of science concepts highly influenced learning in later science studies (Novak & Musonda, 1991).

Shortly after we developed the concept map tool for the assessing of changes in learner's cognitive structure, we found that our staff and others were reporting that concept maps were very helpful as a study tool for virtually any subject matter. This led Novak to develop a course called "Learning To Learn", and he taught this course at Cornell for some 20 years. One of the outcomes from the course was the book *Learning How to Learn* (Novak & Gowin, 1984). The book has been published in 9 languages and remains as popular as when it was first published. Concept maps are also powerful metacognitive tools helping students to understand the nature of knowledge and the nature of meaningful learning (Novak, 1990). More recently we have found concept maps to be an excellent tool for capturing expert knowledge for archiving and training in schools and corporations, and also for team problem solving. Beginning some 10 years ago, the Institute for Human and Machine Cognition has developed *CmapTools*, software that not only facilitates building concept maps but also offers new opportunities for learning, creating, and using knowledge, as will be discussed further.

The Use of Concept Maps in Mathematics

Our early work using concept maps in mathematics was focused on demonstrating how mathematical ideas could be represented in this form. Cardemone (1975) showed how the key ideas in a remedial college math course could be represented using concept maps. He found that the use of concept maps could help teachers design a better sequence of topics and helped students see relationships between topics. Minemier (1983) found that when students made concept maps for the topics they studied they not only performed better on problems solving tests but they also gained increased confidence in their ability to do mathematics. Fuata'i (1985, 1998) used concept maps along with vee diagrams with Form Five students in Western Samoa. She found that students became more autonomous learners and better at solving novel problems as compared with students not using these tools. Figure 1.3 shows an example of a concept map produced by one of her students.

CmapTools and the Internet

CmapTools goes beyond facilitating the construction of concept maps through an easy-to-use map editor, leveraging on the power of technology and particularly the Internet and WWW to enable students to collaborate locally or remotely in the construction of their maps, search for information that is relevant to their maps, link all types of resources to their maps, and publish their concept maps,

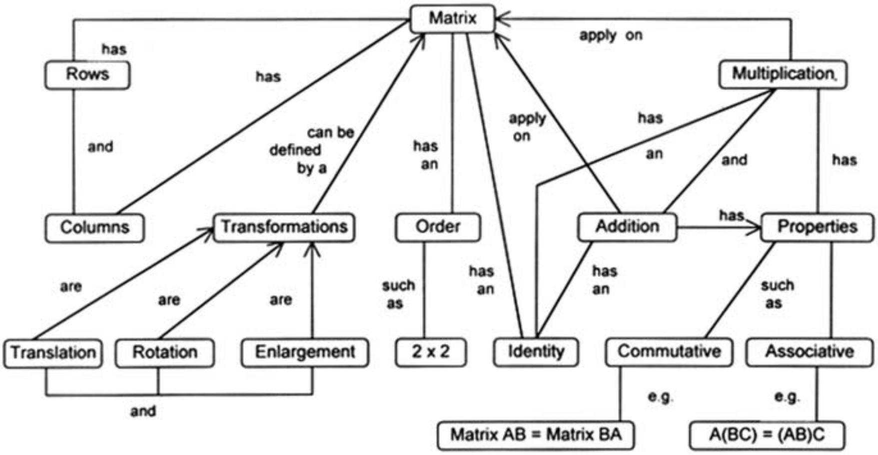

Fig. 1.3 A concept map made by a Western Samoan student in a study by Fuata'i (1998, p. 65)

(Cañas et al., 2004)[1]. CmapTools facilitates the linking of digital resources (e.g. images, videos, text, Web pages, or other digital concept maps) to further explain concepts through a simple drag-and-drop operation. The linked resources are depicted by an icon under the concept that represents the type of resource linked. By storing the concept maps and linked resources on a CmapServer (Cañas, Hill, Granados, C. Pérez, & J. D. Pérez, 2003), the concept maps are converted to Web pages and can be browsed through a Web browser. The search feature in CmapTools takes advantage of the context provided by a concept map to perform Web searches related to the map, producing more relevant results that may include Web pages and resources, or concept maps stored in the CmapTools network (Carvalho, Hewett, & Cañas, 2001). Thus, a small initial map can be used to search for relevant information which the student can investigate, which leads to an improved map, to another search, and so on. The student can link relevant resources found to the map, create other related maps, and organize these into what we call a *knowledge model* (Cañas, Hill, & Lott, 2003). Figure 1.4 illustrates a concept map made with CmapTools and the insets show some of the resources attached to this map that can be opened by clicking on the icon for the resource. Other examples will be given in subsequent chapters of this book.

A New Model for Education

Given the new technological capabilities available now, and combined with new ideas for applying the latest thinking about teaching and learning, it is possible to propose a New Model for Education (Novak & Cañas, 2004; Cañas & Novak, 2005). The New Model involves these activities that will be further elaborated:

[1] CmapTools can be downloaded and used at no cost from: http://cmap.ihmc.us

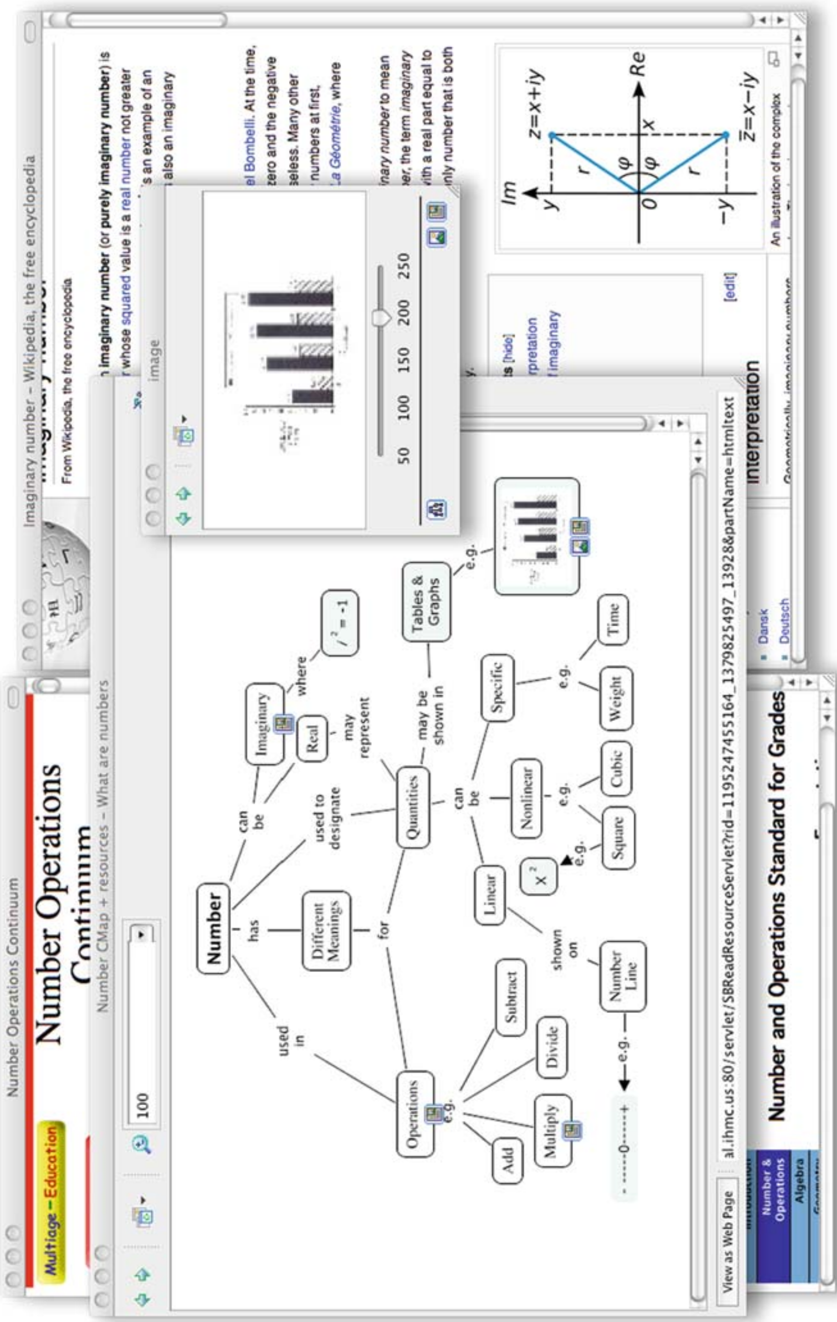

Fig. 1.4 A concept map about numbers showing some resources that have been opened by clicking on the various icons under the concepts

1. Use of “expert skeleton concept maps” to *scaffold* learning.
2. Use of CmapTools to build upon and expand the expert skeleton concept maps by drawing on resources available on the CmapServers, on the Web, plus texts, image, videos and other resources.
3. Collaboration among students to build “knowledge models”.
4. Explorations with real world problems providing data and other information to add to developing knowledge models.
5. Written, oral, and video reports and developing knowledge models.
6. Sharing and assessing team knowledge models.

Expert Skeleton Concept Maps

The idea behind the use of expert skeleton concept maps is that for most students (and many teachers) it is difficult to begin with a “blank sheet” and begin to build a concept map for some topic of interest. By providing a concept map prepared by an expert with 10–15 concepts on a given topic, this “skeleton” concept map can help the learner get started by providing a “scaffold” for building a more elaborate concept map. Vygotsky (1934), Berk and Winsler (1995) and others point out that the apprentice learner is often very insecure in their knowledge and needs both cognitive and affective encouragement. While the teacher can best provide the latter, the skeleton concept map can provide the cognitive encouragement to get on with the learning task. Moreover, students (and often teachers) may have misconceptions or faulty ideas about a topic that would impede their learning if they were to begin with a “blank sheet”. The scaffolding provided by the expert map can get the learner off to a good start, and as they begin to research relevant resources and to add concepts and resources to their map, there is a good chance that their misconceptions will also be remediated (Novak, 2002). Figure 1.5 shows an example of an expert skeleton concept map.

Adding Concepts and Resources Using CmapTools

It is well known today that meaningful learning requires that the learner chooses to interact with the learning materials in an active way and that he/she seeks to integrate new knowledge into her/his existing knowledge frameworks (Novak, 1998; Bransford et al., 1999). Through the drag-and-drop feature of CmapTools that allows for linking supplemental resources to a concept map by simply dragging and dropping the icon for an URL, an image, video, another concept map or any other digital resource on a concept, a learner can build an increasingly complex knowledge model for any domain of knowledge, which can serve as a starting point for later related learning. Moreover, CmapTools provide for easy collaboration between learners either locally or remotely, and either synchronously or asynchronously. When the recorder option of CmapTools is turned on, it will record step-by-step the *history* of the creation of a concept map, indicating the sequence of building steps and who did what at each step.

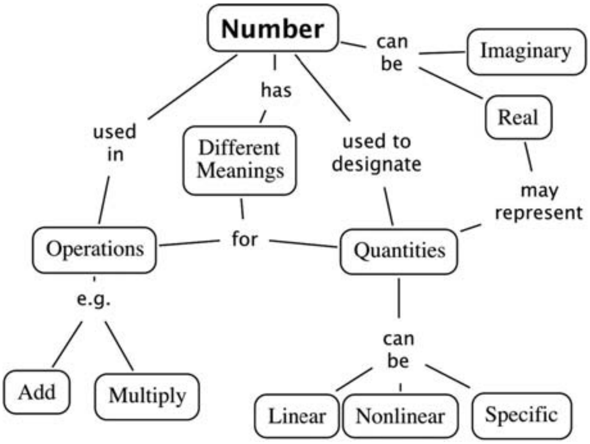

Fig. 1.5 An example of an "expert skeleton concept map" that can serve as a starting point for building a knowledge portfolio about number ideas. Figure 1.4 above shows an example of how this skeleton map could be elaborated using CmapTools and resources drawn from the Web. In general, the number of concepts expected to be added by the student is proportional to (e.g. two or three times) the number of concepts originally in the skeleton map

Obviously this also provides a new tool for cognitive learning studies, but such work is just beginning. For example, Miller, Cañas, & Novak (2008) using the Recorder tool has shown that in the process of learning how to construct concept maps, patterns in map changes for teachers in Proyecto Conéctate al Conocimiento in Panama (Tarté, 2006) were similar for teachers who had no previous experience using computers when compared with those who reported previous experience.

A major problem in mathematics studies is that there is rarely clear focus on the concepts underlying the mathematical operations learners are asked to do. It is even more rare to have an explicit record of the *conceptual thinking* of the learners as they progress in their studies, and a record the learner can turn to when related materials are studied. Another important advantage is that concept maps can be easily related to one another, for example as sub-concept maps in a more general, more encompassing concept map. Examples of this are shown in other chapters.

Collaboration Among Students

With the rediscovery of the studies of Vygotsky (1934) in the past 20 years, educators are increasingly recognizing the importance of social exchange in the building of cognitive structure, as well as for motivation for meaningful learning. Although the work of the Johnson brothers (1988) and others have shown some of the merits of "cooperative learning", most of these studies could not take advantage of the facilitation offered by CmapTools for cooperative learning. Often the advantages of cooperative learning were found to be small at best. We need new research studies showing the effect of collaboration on learning using CmapTools.

Exploration with Real World Problems

One of the important conclusions from many recent studies on "situated cognition" is the importance of placing learning into a meaningful, real world context. We recognize this value in our New Model and urge that whenever possible, new ideas in mathematics should be introduced within the framework of some real world problem. While math teachers have been using for decades the idea that ratio and proportion problems, for example, should be introduced with tangible activities, such as using objects of different densities for comparison, most of these activities have not made explicit the *mathematics concepts* involved in the problems given, and the focus has been mostly on procedures to get the "correct" answer. Of course, this has also carried over into physics teaching and teaching in other subjects.

When real world activities are tied in with the use of CmapTools and the creation of knowledge models for the domains studied, research is beginning to show the resulting improvement in learning (Cañas, Novak, & González, 2004; Cañas & Novak, 2006; Cañas et al., 2008).

In the Proyecto Conéctate al Conocimiento (Tarté, 2006) effort in Panama, we are finding that the collaboration possibilities available with CmapTools are leading not only to sharing in knowledge building but also to a variety of social exchanges. During the training programs for teachers, teachers are invited to prepare a concept map in the form of biography referred to as "Who am I?". This has led to teachers and principals also building concept maps about their schools, communities and a variety of related exchanges (Sánchez et al., 2008). This personal engagement has had strong motivating effects for pursuing other collaborations and we expect this will increase over time as the social network grows. Figure 1.6 shows a sample montage of some of the work done by teachers and school principals in Panama. With thousands of teachers and students involved in this project, we are learning many new possibilities for ways to use concept maps to facilitate meaningful learning.

Written, Oral, and Video Reports and Developing Knowledge Models

While we will continue to see an increase in the use of electronic communications in the future, there will always be an important place for written and oral reports. Whether in schools or corporate settings, our students need to become effective written and oral communicators. In one of our early studies, we found that fifth grade children who prepared concept maps prior to attempting to write out their ideas not only wrote better stories but they were also better able to tell their stories (Ben-Amar, 1990). In fact, they wrote a play derived from their stories and it was so well received they were invited to present it at other elementary schools!

The full range of capabilities for organizing knowledge available using CmapTools is too recent to have an empirical research base to document the value of the possibilities created, including what we call A New Model for Education. Hopefully,

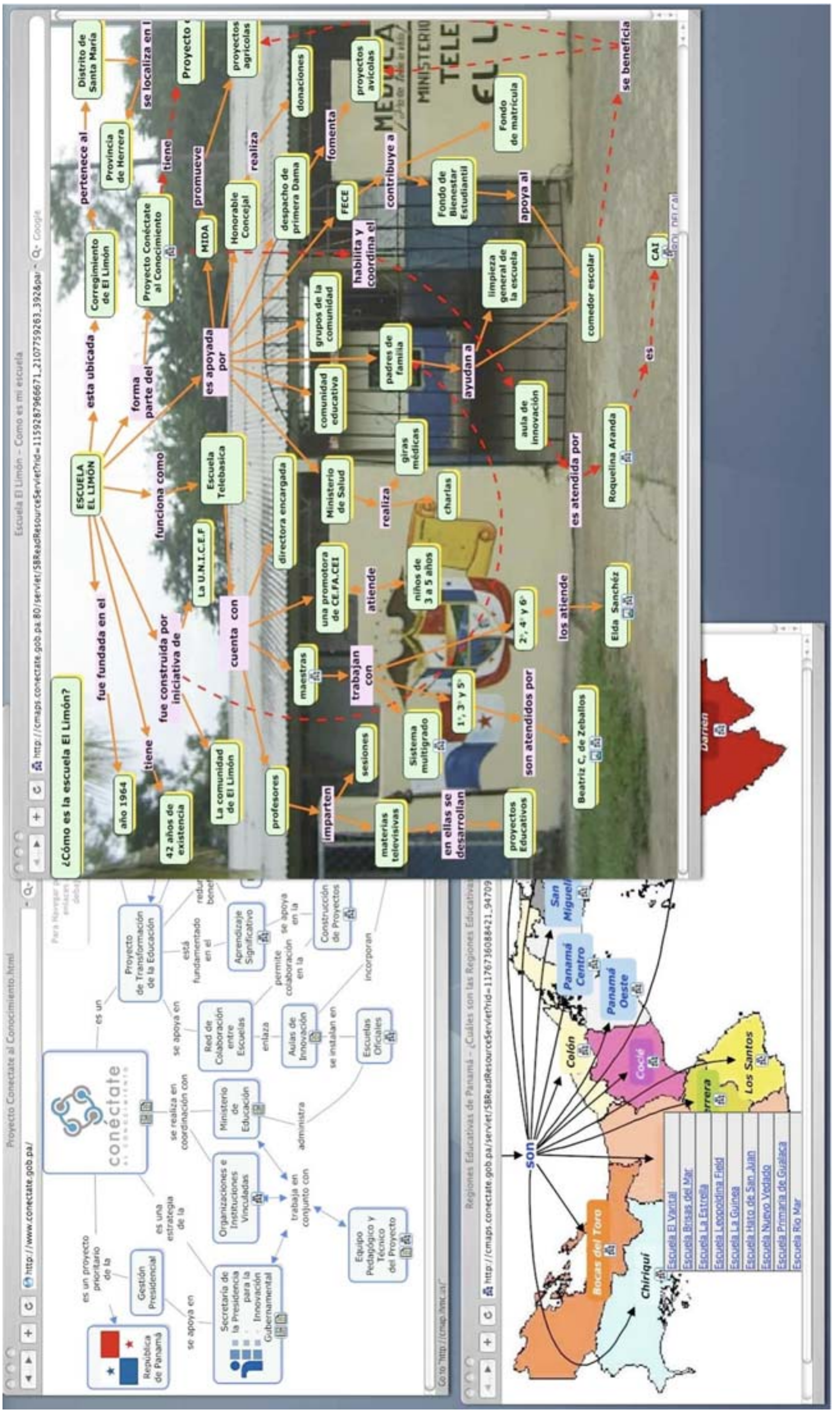

Fig. 1.6 Concept maps drawn by Panamanian teachers illustrating "Who am I?" concept maps posted on Web sites and serving to provide motivation and recognition, as well as a basis for collaborations

after the publication of this book, many empirical studies will be done to assess the value of CmapTools not only for improving instruction in mathematics, but also in improving students' ability to communicate their mathematics ideas and a new excitement for learning mathematics.

Sharing and Assessing Team Knowledge Models

Already indicated above are ways in which the sharing of individual and team knowledge models can be facilitated using the collaboration tools of CmapTools. However, many teachers want to know how they can *evaluate* knowledge models. In our own teaching, we have used a variety of strategies including having teams post their knowledge models anonymously and then asking students to rank the models from lowest to highest, including criteria for their rankings. Using digital knowledge models created with CmapTools, these can be posted on the class server and provide easy access for assessment. Students can be very insightful, and often brutally honest, in their assessments. Furthermore, serious assessment is an educational experience, and students learn how they can improve their own knowledge models.

In Conclusion

Our objective in this chapter was to provide a brief history of the development of the concept mapping tool, including the development of the computer software, CmapTools, designed to facilitate concept map making and to provide new opportunities for individual and collaborative learning. Although the research to date supports the value of concept mapping to facilitate meaningful learning (Coffey et al., 2003), very little research has been done in the field of mathematics education. It is our hope this book will encourage such research. We also hope to see studies in mathematics learning that will utilize what we call a New Model for Education, and that libraries of "expert skeleton concept maps" in mathematics will be posted on web sites. We observed an increase in the number of papers dealing with mathematics education presented at international conferences on concept mapping from 2004 to 2008 (Cañas, Novak, et al., 2004; Cañas & Novak, 2006; Cañas et al., 2008) and we are hopeful that even more and improved studies will be presented at following conferences (see http://cmc.ihmc.us).

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Part II
Primary Mathematics Teaching and Learning

Chapter 2
Analysing the "Measurement" Strand Using Concept Maps and Vee Diagrams

Karoline Afamasaga-Fuata'i

The chapter presents data from a case study, which investigated a primary student teacher's developing proficiency with concept maps and vee diagrams as tools to guide the analyses of syllabus outcomes of the "Measurement" strand of a primary mathematics syllabus and subsequently using the results to design learning activities that promote working and communicating mathematically. The student teacher's individually constructed concept maps of the sub-topics length, volume and capacity are presented here including some vee diagrams of related problems. Through concept mapping and vee diagramming, the student teacher's understanding of the mapped topics evolved and deepened, empowering her to confidently provide mathematical justifications for strategies and procedures used in solving problems which are appropriate to the primary level, effectively communicate her understanding publicly, and developmentally sequence learning activities to ensure future students' conceptual understanding of the sub-topics.

Introduction

Various *Professional Teaching Standards* point to the need for teachers of mathematics to have deep understanding of students' learning, pedagogical content knowledge of the relevant syllabus and the ability to plan learning activities that develop students' understanding, as essential to achieve excellence in teaching mathematics (AAMT, 2006). These Standards therefore imply that student teachers should develop deep knowledge and understanding of principles, concepts and methods they are expected to teach their future students. For example, the underlying theoretical principles of the New South Wales Board of Studies' *K-6 Mathematics Syllabus* (NSWBOS, 2002) encourage the development of students' conceptual understanding through an appropriate sequencing of learning activities and implementation of working and communicating mathematically strategies. To this end, this

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chapter proposes that the application of the metacognitive tools of hierarchical concept maps (maps) and vee diagrams (diagrams), and the innovative strategies of concept mapping and vee diagramming can influence (a) the development of students' meaningful learning and conceptual understanding and (b) the dynamics of working and communicating mathematically within a social setting. Therefore, the focus question for this chapter is: "*In what ways do hierarchical concept maps and vee diagrams facilitate the preparation of primary student teachers for teaching mathematics, in particular, the development of a deep understanding of the content of the relevant syllabus*?" This chapter presents the case study of a Bachelor of Education (Primary) student teacher (i.e., Susan) who concept mapped and vee diagrammed over a semester, in her third year mathematics education course in a regional Australian university.

Literature Review of Concept Mapping and Vee Diagrams

Ausubel's theory of meaningful learning, which defines meaningful learning as learning in which students actively make connections between what they already know and new knowledge, underpins concept mapping particularly its principle that learners' cognitive structures are hierarchically organized with more general, superordinate concepts subsuming less general and more specific concepts. Linking new concepts to existing cognitive structures may occur via *progressive differentiation* (reorganization of existing knowledge under more general ideas) and/or *integrative reconciliation* (synthesising many ideas into one or two when apparent contradictory ideas are reconciled) (Ausubel, 2000; Novak & Canãs, 2006). By constructing maps/diagrams, students illustrate publicly their interpretation and understanding of topics/problems. Hierarchical concept maps were first introduced by Novak as a research tool to illustrate the hierarchical interconnections between main concepts (nodes) in a knowledge domain with descriptions of the interrelationships (linking words) on the connecting lines. The basic semantic unit (*proposition*) describes a meaningful relationship as shown by the triad "valid node – valid linking words-> valid nodes" (Novak & Canãs, 2006; Novak & Gowin, 1984). Vee diagrams, in contrast, were introduced by Gowin as an epistemological tool, in the shape of a vee that is contextualised in the phenomenon to be analysed. The vee's left side depicts the philosophy and theoretical framework, which drive the analysis to answer the focus question. On the vee's right side are the records, methods of transforming the records to answer the focus question and value claims. The epistemological vee was later modified (Afamasaga-Fuata'i, 1998, 2005) to one that is focused on guiding the thinking and reasoning involved in solving a mathematics problem (examples are presented later).

Numerous studies examined the use of maps and/or diagrams as assessment tools of students' conceptual understanding over time in the sciences (Novak & Canãs, 2004; Brown, 2000; Mintzes, Wandersee, & Novak, 2000) and mathematics (Afamasaga-Fuata'i, 2004; Hannson, 2005; Liyanage & Thomas, 2002;