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Lenses and Waves

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by

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Preface

"Le doute fait peine a l'esprit.
C'est pourquoy tout le monde
se range volontiers a l'opinion de ceux
qui pretendent avoir trouvè la certitude."[1]

This book evolved out of the dissertation that I defended on April 1, 1999 at the University of Twente. At the successive stages of its development critical readers have cast doubts on my argument. It has not troubled my mind; on the contrary, they enabled me to improve my argument in ways I could not have managed on my own. So, I want to thank Floris Cohen, Alan Shapiro, Jed Buchwald, Joella Yoder, and many others. Most of all, however, I have to thank Casper Hakfoort, who saw the final text of my dissertation but did not live to witness my defence and the further development of this study of optics in the seventeenth century.

This book would not have been possible without NWO (Netherlands Organisation for Scientific Research) and NACEE (Netherlands American Commission for Educational Exchange) who supplied me with a travel grant and a Fulbright grant, respectively, to work with Alan Shapiro in Minneapolis. The book would also not have been possible without the willingness of Kluwer Publishers and Jed Buchwald to include it in the Archimedes Series, and the unrelenting efforts of Charles Erkelens to see it through.

During the years this text accompanied my professional and personal doings, numerous people have helped me grow professionally and personally. I want to thank Peter-Paul Verbeek, John Heymans, Petra Bruulsema, Kai Barth, Albert van Helden, Rienk Vermij, Paul Lauxtermann, Lissa Roberts, and many, many others.

Still, the idea to study Huygens and his optics would not have even germinated – let alone that this book would have matured – without my life companion, Anne, with whom I now share a much more valuable creation. Thank you.

Fokko Jan Dijksterhuis
Calgary, June 2004

This book is dedicated to Casper Hakfoort
In memory of Lies Dijksterhuis

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[1] Undated note by Christiaan Huygens (probably 1686 or 1687), *OC21*, 342

Chapter 1
Introduction – 'the perfect Cartesian'
Christiaan Huygens, optics & the scientific revolution

"ΕΥΡΗΚΑ. The confirmation of my theory of light and refractions", proclaimed Christiaan Huygens on 6 August 1679. The line is accompanied by a small sketch, consisting of a parallelogram, an ellipse (though barely recognizable as such) and two pairs of perpendicular lines (Figure 1). The composition of geometrical figures does not immediately divulge its meaning. Yet, it conveys a pivotal event in the development of seventeenth-century science.

What is it? The parallelogram is a section – the principal section – of a piece of Iceland crystal, which is a transparent form of calcite with extraordinary optical properties. It refracts rays of light in a strange way that does not conform to the established laws of refraction. The ellipse represents the propagation of a wave of light in this crystal. It is not an ordinary, spherical wave, as waves of light are by nature, but that is precisely because the elliptical shape explains the strange refraction of light rays in Iceland crystal. The two pairs of lines denote the occasion for Huygens' joy. They are unnatural sections of the crystal, which he had managed to produce by cutting and polishing the crystal. They produced refractions exactly as his theory, by means of those elliptical waves, had predicted.

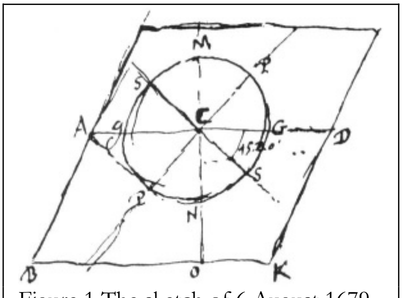

Figure 1 The sketch of 6 August 1679

The elliptical waves were derived from the wave theory he had developed two years earlier, with the formulation of a principle of wave propagation. Like ordinary spherical waves, these elliptical waves were hypothetical entities defining the mechanistic nature of light. Now, seventeenth-century science was full of hypotheses regarding the corpuscular nature of things. But Huygens' wave theory was not just another corpuscular theory. His principle defined the behavior of waves in a mathematical way, based on a theory describing the mechanics of light propagation in the form of collisions between ether particles. The mathematical character of Huygens' wave theory is historically significant. Huygens was the first in the seventeenth century to fully mathematize a mechanistic explanation of the properties of light. As contrasted to the qualitative pictures of his contemporaries, he could derive the exact properties of rays refracted by

Iceland crystal, including refractions that could only be observed by cutting the crystal along unnatural sections. The sketch records the experimental verification of Huygens' elliptical waves and, with it, the confirmation of his theory of light and refractions.

This brief synopsis explains what 'actually' happened on that 6th of August in 1679. The various terms and concepts will be explicated later on in this book. For this moment, it suffices to make clear the core of Huygens' wave theory and its historical significance. For Huygens the successful experiment meant the confirmation of his explanation of strange refraction and his wave theory in general. In the context of the history of seventeenth-century optics, and of the mathematical sciences in general, the importance of the event lies in the twofold particular nature of Huygens' theory: a mathematized model of the mechanistic nature of light considered as a hypothesis validated by experimental confirmation. With the mathematical form of his theory, Huygens can be said to have restored the problematic legacy of Descartes' natural philosophy, by defining mathematical principles for the mechanistic explanation of the physical nature of light. The hypothetical-deductive structure of his theory implied the abandonment of the quest for certainty of that same Cartesian legacy and of seventeenth-century science in general. Huygens presented waves of light, the inextricable core of his account of optical phenomena, explicitly as hypothetical entities whose certainty is inherently relative. In so doing, he set off from Descartes in a direction diametrically opposite to Newton, the principal other restorer of mechanistic science.

About a decade after the EUREKA of 6 August 1679, Huygens published his wave theory of light and his explanations of ordinary and strange refraction in *Traité de la Lumière* (1690). This book established his fame as a pioneer of mathematical physics as evidenced by the fact his principle of wave propagation is still known and used in various fields of modern physics under the name 'Huygens' principle'. Its historical importance is also generally acknowledged. According to Shapiro, Huygens stood out for his "…continual ability to rise above mechanism and to treat the continuum theory of light mathematically."[1] E.J. Dijksterhuis calls it the high point of mechanistic science and its creator the first 'perfect Cartesian': "In Huygens does Cartesian physics for the first time take the shape its creator had in mind."[2] How did this come about? How did Christiaan Huygens came to realize this historical landmark? Or more specifically, how did he arrive at his wave theory of light? That is the central question of this study.

A history of Traité de la Lumière

Unlike its eventual formulation in *Traité de la Lumière*, the development of Huygens' wave theory has hardly been subject to historical investigation. The

[1] Shapiro, "Kinematic optics", 244. (For referencing see page 267)

[2] Dijksterhuis, *Mechanization*, IV: 212 & 283 (references to this book will be made by section numbers). It should be noted that Dijksterhuis mainly focuses on the mathematical model of wave propagation.

first step for such a study is to go into the papers documenting Huygens' optics. The historian who does so on the basis of existing literature, awaits a surprise. There is much more to Huygens' optics than waves. He elaborated a comprehensive theory of the dioptrical properties of lenses and their configurations in telescopes, that goes by the title of *Dioptrica*. A second surprise is in store when one takes a closer looks to these papers on geometrical optics. The papers on dioptrics cover the Huygens' complete scientific career and form the exclusive content of the first two decades of his optical studies. The wave theory and related subjects are fully absent; not before 1672 do they turn up. In other words, the optics that brought Huygens future fame dates from a considerably late stage in the development of Huygens' optics.

In this way new and more specific questions arise regarding the question 'how did Huygens arrive at his wave theory'? What exactly was his optics? How did he move from *Dioptrica* to *Traité de la Lumière*? And what does this teach us about the historical significance of his wave theory, Huygens' creation of a physical optics, and the character of his science? The point is that Huygens' dioptrics turns his seemingly self-explanatory wave theory into a historical problem. It did not develop from some innate cartesianism, for he was no born Cartesian, certainly not in optics at least. Fully absent from *Dioptrica* is the central question of *Traité de la Lumière*: what is the nature of light and how can it explain the laws of optics. Huygens first raised this question in 1672 – five years before he found his definite answer (which he confirmed another two years later in 1679). His previous twenty years of extensive dioptrical studies give scarcely any occasion to expect that this man was to give the mechanistic explanation of light and its properties a wholly new direction. In view of *Dioptrica*, the question is not only how Huygens came to treat the mechanistic nature of light in his particular way, but even how he came to consider the mechanistic nature of light in the first place.

In the literature on Huygens' optics, mechanistic philosophy has been customarily considered a natural part of his thinking. Only Bos, in his entry in the *Dictionary of Scientific Biography*, points at the relatively minor role mechanistic philosophizing played in his science before his move to Paris in the late 1660s.[3] The question of how the wave theory took shape in Huygens' mind thus becomes all the more intriguing. What caused Huygens to tackle this subject he had consistently ignored throughout his earlier work on optics? How do *Dioptrica* and *Traité de la Lumière* relate and what light does the former shed on the latter? Part of the answer is given by the fact that only at the very last moment, short before its publication, Huygens decided to change the title of his treatise on the wave theory from 'dioptrics' to 'treatise on light'. In his mind the two were closely connected, questions now

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[3] Bos, "Huygens", 609. Van Berkel further alludes to the influence of Parisian circles on the prominence of mechanistic philosophy in Huygens' oeuvre: Van Berkel, "Legacy", 55-59.

are: how exactly and what does this mean for our understanding of Huygens' optics?

In addition to the question of where in Huygens' oeuvre *Traité de la Lumière* properly belongs, a more general question may be asked: where in seventeenth-century science may this kind of science be taken to belong? Were the questions Huygens addressed in *Traité de la Lumière* part of any particular scientific discipline or coherent field of study? In the course of my investigation, it has became increasingly clear to me that the term 'optics' is rather problematic regarding the study of light in the seventeenth century, just like the term 'science' in general. Our modern understanding of 'optics' implies an investigation of phenomena of light much like *Traité de la Lumière*: a mathematically formulated theory of the physical nature of light explaining the mathematical regularities of those phenomena. Yet, optics in this sense was only just beginning to develop during the seventeenth century. The term 'optics' in the seventeenth century denoted the mathematical study of the behavior of light rays that we are used to identify with geometrical optics. This is what Huygens pursued in *Dioptrica*, prior to developing his wave theory. A general question regarding the history of optics raised by Huygens' *Traité de la Lumière* is how a new kind of optics, a physical optics, came into being in the seventeenth century and how this related to the older science of geometrical optics. This transformation of the mathematical science of optics is manifest in the title Huygens eventually chose for his treatise.

Huygens' optics

This book offers in the first place an account of the development of Huygens' optics, from the first steps of *Dioptrica* in 1652 to the eventual *Traité de la Lumière* of 1690. The following chapters take a chronological course through his engagements with the study of light, whereby the historical connection of its various parts sets the perspective. Terms like 'optics' and 'science' are problematic historically. Nevertheless, for sake of convenience, I will freely use them to denote the study of light in general and natural inquiry, except when this would give rise to (historical) misunderstandings. When discussing their historical character and development specifically, I will use appropriately historicizing phrases.

Chapter two discusses *Tractatus* – the unfinished treatise of 1653 on dioptrics that marks the beginning of Huygens' engagement with optics. *Tractatus* contained an comprehensive and rigorous theory of the telescope and I will argue that this makes it unique in seventeenth-century mathematical optics. Huygens was one of the few to raise theoretical questions regarding the properties and working of the telescope, and almost the only one to direct his mathematical proficiency towards the actual instruments used in astronomy. Kepler had preceded him, but he had not known the law of refraction and therefore could not derive but an approximate theory of lenses and their configurations. Some four decades afterwards, and two decades after the publication of the sine law, Huygens was the first to apply it to spherical lenses and remained so for almost two decades more. Chapter three discusses his practical pursuits in dioptrics leading into his subsequent treatise on dioptrics, *De Aberratione* of 1665. Huygens made a unique effort to employ dioptrical theory to improve the telescope. The contrast with Descartes is particularly conspicuous, for Huygens did not fit the telescope into the ideal mold prescribed by theory but directed his theory towards the instruments that were practically feasible. The effort was unsuccessful, for with his new theory of light and colors Newton made him realize the futility of his design. Taking into account Huygens' background in dioptrics sheds, I will argue, new light on the famous dispute with Newton in 1672.

These chapters are confined to what we would call geometrical optics and to its relationship to practical matters of telescopy.[4] I try to explain what this science was about and what was particular about the way Huygens pursued it. These chapters offer a fairly detailed account of *Dioptrica* within the context of seventeenth-century geometrical optics, and as such open fresh ground in the history of science. At the turn of the century, Kepler had laid a new foundation for geometrical optics. Image formation now became a matter of determining where and how a bundle of diverging rays from each point of the object is brought into focus again (or not) instead of tracing single rays from object point to image point. Of old, the ray was the bearer of the physical properties of light, but in the course of the seventeenth century this began to be qualified and questioned. In the wake of Kepler and Descartes the mathematical science of optics gradually transformed into new ways of doing optics. The traditional, geometrical way of doing optics did not vanish, though. It was ray optics in which the question of the nature of light need not penetrate further than determining the physical properties of rays in their interaction with opaque and transparent materials. This is the mathematical optics Huygens grew up with and that set the tone in his earliest dealings with the physics of light propagation. Only on second thought did he focus on the new question what is light and how can this explain its properties. This transformation is the subject of the next chapters.

In chapters four and five my focus shifts, along with Huygens', to the mechanistic nature of light. In 1672 a particular problem drew his attention to the question what light is and how its properties can be explained: the strange refraction in Iceland Crystal which created a puzzle regarding the physics of refraction that Huygens wanted to solve. These two chapters discuss the three stages of his investigation, his first analysis of the mathematics of strange refraction in 1672 and his eventual solution by means of elliptical waves in 1677 and its confirmation in 1679. Huygens' first attack on the problem of strange refraction is historically significant because he approached it along traditional lines of geometrical optics. Only in second

[4] Preliminary results are published in: Dijksterhuis, "Huygens' *Dioptrica*" and Dijksterhuis, "Huygens's efforts".

instance did he turn to the actual question underlying the problem: how exactly do waves of light propagate. And only in third instance, and forced by critical reactions, did Huygens seek for experimental validation of the theory he initially had developed primarily rationally. These chapters offer a new, detailed reconstruction of the origin of the wave theory on the basis of manuscript material that has not been taken into account earlier. It is also a reconstruction of how Huygens got from *Dioptrica* to *Traité de la Lumière*, in which I compare his approach to questions pertaining to the physical nature of light in the mathematical science of optics to his predecessors and contemporaries. Central themes are the way the nature of light was accounted for in the mathematical study of light in the seventeenth century and to relationships between explanatory theories of light and the laws of optics. I will argue that Huygens was the first to successfully mathematize a mechanistic conception of light. He was rivaled only by Newton, but for epistemological reasons he kept his hypotheses private.

Chapter six reviews the development of *Traité de la Lumière*, its significance for the history of seventeenth-century optics, and for our understanding of Huygens' science. After discussing the publication history of *Traité de la Lumière*, which reveals that Huygens disconnected it from *Dioptrica* only at the very last moment, I sketch some lines for a new perspective of the history of seventeenth-century optics in which traditional geometrical optics is taken into account as an important root. The mathematico-physical consideration of light of *Traité de la Lumière* was a particular answer to a new kind of question. A kind of question also addressed by such diverse scholars as Kepler, Descartes and Newton. In this sense, my study of the development of *Traité de la Lumière*, in particular in relationship with *Dioptrica*, is also a study of the origins of a new science of optics, nowadays denoted by the term 'physical optics'. Some instances of physical optics developed in the seventeenth-century, most notably by Huygens and Newton. But the primacy of the question 'what is the physical nature of light and how may this explain its properties? first had to be discovered and this only gradually came about in the pursuit of the mathematical science of optics. In the case of Huygens this emergence was particularly quiet. While solving the intriguing puzzle of strange refraction, he developed a new way of doing mathematical optics but he seems to have been hardly aware of the new ground he was breaking. At the close of this chapter, I discuss his alleged Cartesianism and I will argue that Huygens stumbled into becoming a 'perfect Cartesian' rather than determinedly and systematically create it.

This study is based on the optical papers in the *Oeuvres Complètes* and additional manuscript material. A large part of these have as yet not been studied. The *Oeuvres Complètes* split up Huygens' optics in two parts – volume 13 for *Dioptrica* and volume 19 for *Traité de la Lumière*. This subdivision along modern disciplinary lines resounds in the historical literature. E.J. Dijksterhuis, for example, separates explicitly 'geometrical optics' – where he merely mentions Huygens – and physical theories of optics.[5] No doubt all this has contributed its share to the fact that the relationship between *Traité de la Lumière* and *Dioptrica* – historical, conceptual as well as epistemical – has gone unexamined so far.[6] As for *Traité de la Lumière*, most historical interpretations are based on and confined to the published text. The additional manuscript material published in *OC19* has hardly been taken into account and no-one to my knowledge has used the original manuscript material in the *Codices Hugeniorum* in the Leiden university library.[7] Huygens' wave theory has been the subject of several historical studies. Each in their own way has been valuable for this study. E.J. Dijksterhuis gives an illuminating analysis of the merits of *Traité de la Lumière* as a pioneering instance of mathematical physics considered in the light of Descartes' mechanistic program. Sabra includes an account of Huygens' wave theory in his study of the historical development of the interplay of theory and observation in seventeenth-century optics. Shapiro offers a searching analysis of the historical development of the physical concepts underlying Huygens' wave theory. I intend to add to our growing historical understanding of *Traité de la Lumière* by reconstructing its origin and development in the context of his optical studies as a whole and of that of seventeenth-century optics in general.

Huygens' lifelong engagement with dioptrics as such has hardly been studied.[8] Even Harting, the microscopist who by mid-nineteenth century gives Huygens' telescopic work a central place in his biographical sketch, mentions dioptrical theory only in passing.[9] The editorial remarks in the 'Avertissement' of *OC13* form the main exception and are one of the few sources of information on the history of seventeenth-century geometrical optics in general. Some topics pertaining to seventeenth-century geometrical optics have been studied in considerable detail, but for the most part in the context of the seventeenth-century development of physical science. These are Kepler's theory of image formation, the discovery of the sine law and Newton's mathematical theory of colors and they are integrated in my

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[5] Dijksterhuis, *Mechanisering*, IV: 168-171, 284-287.

[6] Hashimoto hardly goes beyond noting that "… two works were closely related in Huygens's mind.": Hashimoto, "Huygens", 87-88.

[7] Dijksterhuis, *Mechanization*, IV: 284-287 and Sabra, *Theories*, 159-230 are confined to *Traité de la Lumière*. Shapiro uses some of the manuscripts published in *Oeuvres Complètes*. Ziggelaar, "How", draws mainly on *OC*19. Yoder has pointed out that the wave theory is no exception to the rule that in general, studies of Huygens' work tend to focus on his published works.

[8] Hashimoto has published a not too satisfactory article in which he discusses Huygens' dioptrics in general terms. Apart from some substantial flaws in his analyses and argument, Hashimoto fails to substantiate some of his main claims regarding Huygens' 'Baconianism'. Hashimoto, "Huygens", 75-76; 86-87; 89-90. For example, he reads back into *Tractatus* the utilitarian goal of *De aberratione* (60, compare my section 3.3.2), he thinks Huygens determined the configuration of his eyepiece theoretically (75, compare my section 3.1.2), maintains that *Systema saturnium* grew out of his study of dioptrics (89, compare my section 3.1.2) and that Huygens 'went into the speculation about the cause of colors' after his study of spherical aberration (89, compare my section 3.2.3)

[9] Harting, *Christiaan Huygens*, 13-14. Harting based himself on manuscript material disclosed in Uylenbroek's oration on the dioptrical work by the brothers Huygens: Uylenbroek, *Oratio*.

accounts of, respectively, seventeenth-century dioptrics in chapter 2, the epistemic role and status of explanations in optics in chapter 4, and Huygens' own dealings with colors in chapter 3 as well as his specific approach to mechanistic reasoning in chapter 5. Little literature on the history of the field of geometrical optics and its context exist.[10]

A substantial part of my argument is based upon comparisons with the pursuits of other seventeenth-century students of optics. In order to come to a historically sound understanding of what Huygens was doing, I find it necessary to find out how his optics relates to the pursuits of his predecessors and contemporaries. What questions did they ask (and what not) and how did they answer them? Why did they ask these questions and what answers did they find satisfactory? For example, in chapter 2 the earliest part of *Dioptrica* is compared with, among other works, Kepler's *Dioptrice* and Descartes' *La Dioptrique*. All bear the same title, yet the differences are considerable. Descartes discussed ideal lenses and did so in general terms only, rather than explaining their focusing and magnifying properties as Kepler had done. Huygens, in his habitual search for practical application, expressly focused on analyzing the dioptrical properties of real, spherical lenses and their configurations, thus developing a rigorous and general mathematical theory of the telescope. By means of such comparisons it is possible to determine in what way Huygens marked himself off as a seventeenth-century student of optics, or did not. These comparisons are focused on Huygens' optics, so I confine my discussions of seventeenth-century of optics to the mathematical aspects of dioptrics and physical optics. Other themes like practical dioptrics and natural philosophy in general will be treated only in relation to Huygens.

This is an intellectual history of Huygens' optics and of seventeenth-century optics in general. The nature of the available sources – as well that of the man – are not suitable for some kind of social or cultural history. He operated rather autonomously, mainly because he was in the position to do so, and he was no gatherer of allies and did not try very hard to propagate his ideas about science and gain a following.

New light on Huygens

This study offers, in the first place, a concise history of Huygens' optics. Yet it is not a mere discussion of Huygens' contributions to various parts of optics. I also intend to shed more light on the character of Huygens' scientific personality. The issue of getting a clear picture of his scientific activity and its defining features is an acknowledged problem. In summing up a 1979 symposium on the life and work of Huygens, Rupert Hall

[10] For example, the precise application of the sine law to dioptrical problems, for example, has hardly been studied. Shapiro, "The Optical Lectures" is a valuable exception, discussing Barrow's lectures and their historical context. The relationship between the development of the telescope and of dioptrical theory – essential to my account of *Dioptrica* – has never been investigated in any detail. Van Helden has pointed out the weak connection between both in general terms: Van Helden, "The telescope in the 17th century", 45-49; Van Helden, "Birth", 63-68.

concluded that "… it isn't at all easy to understand how all the multifarious activities of this man's life fit together."[11] Huygens has been called the true heir of Galileo, the perfect Cartesian, and also a man deftly steering a middle course between Baconian empiricism and Cartesian rationalism.[12] Huygens himself has not been much of a help in this. He always was particularly reticent about his own motives. He was an intermediate figure between Galileo and Descartes on the one hand and Newton and Leibniz on the other but, lacking as he did a pronounced conception of the aims and methods of his science, he is difficult to situate among the protagonists of the scientific revolution.

In 1979, the most apt characterization of Huygens seemed to be that of an eclectic, who took up loose issues and solved them with the means he considered appropriate without some sort of central direction becoming apparent.[13] The original idea behind this study was that in *Traité de la Lumière* this eclecticism grew into a fruitful synthesis of mathematical, mechanistic and experimental approaches. This idea originates from the work my advisor, the Casper Hakfoort. In his study of eighteenth-century optics he formulated the idea when he pointed out the significance of natural philosophy for the development of optics, which in his view was hithertho neglected.[14] I consider it an honor to have been able to pursue this idea and to have had it bear unanticipated fruit. By trying to understand how the multifarious aspects of his optics fit together, I hope to be able to shed light on the character of his science in general and on his place in seventeenth-century science. *Dioptrica*, while adding to the standing impression of the great versatility of Huygens' oeuvre, has not changed my expectation that an understanding of the way the possible coherence of these aspects evolved may contribute to a better characterization of Huygens' science and of his place in seventeenth-century science as a whole.

In the final chapter of this book, I review my account of the development of Huygens' optics to see what light this may shed on his scientific personality. By way of conclusion it offers a sketch of his science, based on the previous chapters that go into the details – often technical – of his optics and its development. This chapter can be read independently as an essay on Huygens.

Huygens was a puzzle solver indeed, an avid seeker of rigorous, exact solutions to intricate mathematical puzzles. But these puzzles do have coherence, they all concerned questions regarding concrete, almost tangible subjects in the various fields of seventeenth-century mathematics. He was an eclectic, but only in comparison with the chief protagonists of the scientific

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[11] Hall, "Summary", 311.

[12] Westfall, *Construction*, 132-154; Dijksterhuis, *Mechanization*, 212; Elzinga, *Research program* and Westman, "Problem", 100-101.

[13] Hall, "Summary", 305-306. As regards his studies of motion, Yoder has further specified this characterization; Yoder, *Unrollling time*, 169-179.

[14] Hakfoort, *Optics in the age of Euler*, 183-184.

revolution, that set up schemes to lay new foundations for natural inquiry. Huygens did not have such a program and, as a result, his science seems to lack coherence, unless a coherence is looked for on a different level of seventeenth-century science. The essay therefore first forgets about Huygens' alleged Cartesianism to sketch the mathematician and his idiosyncratic focus on instruments. He was not a half-baked philosopher but a typical mathematician. A new Archimedes, as Mersenne foretold in 1647. The incomparable Huygens, as Leibniz said in 1695 upon the news of his death.[15] Then I ask anew how his Cartesianism fits into the picture. We may have trouble getting a balanced idea of what he was doing, but it appears that he was hardly aware of the size of the new ground he had been breaking. He had, in fact, developed a new mathematical science of optics.

[15] *OC1*, 47 and *OC10*, 721.

Chapter 2
1653 - 'Tractatus'
The mathematical understanding of telescopes

"Now, however, I am completely into dioptrics", Huygens wrote on 29 October 1652 in a letter to his former teacher in mathematics, Frans van Schooten, Jr.[1] His enthusiasm had been induced by a discovery in dioptrical theory he had recently made. It was an addition to Descartes' account of the refracting properties of curves in *La Géométrie*, that promised a useful extension of the plan for telescopes with perfect focusing properties Descartes had set out in *La Dioptrique*. During his study at Leiden University in 1645-6, Huygens had studied Descartes' mathematical works, *La Géométrie* in particular, intensively with Van Schooten. Although Van Schooten was professor of 'Duytsche Mathematique' at the Engineering school, appointed to teach practical mathematics in the vernacular to surveyors and the like. Huygens was not the only patrician son he introduced to the new mathematics: the future Pensionary Johan de Witt and the future Amsterdam mayor Johannes Hudde.

From 1647 Huygens and Van Schooten had to resort to corresponding over mathematics, when Huygens had to go to Breda to the newly established 'Collegium Auriacum', the college of the Oranges to which the Huygens family was closely connected politically. In 1649 Huygens had returned home to The Hague and now, in 1652, he was 'private citizen'. He did not feel like pursuing the career in diplomacy his father had planned for him and, with the Oranges out of power since 1650, not many duties were left to call on him. Huygens could, in other words, freely pursue his one interest, the study of the mathematical sciences. An appointment at a university was out of order for someone of his standing and, as Holland lacked a centrally organized church and a grand court, interesting options for patronage were not directly available.[2] So, with a room in his parental home at the 'Plein' in The Hague and a modest allowance from his father, he could live the honorable life of an 'amateur des sciences'. He enjoyed to company of his older brother Constantijn, who joined him in his work in practical dioptrics (see next chapter), and dedicated himself to mathematics. Geometry and mechanics were his main focus in these years, with his theories of impact and of pendulum motion and his invention of the

[1] *OC 1*, 215. "Nunc autem in dioptricis totus sum ..."

[2] Berkel, "Illusies", 83-84. In the 1660s Huygens would start to seek patronage abroad, first in Florence and then, successfully in Paris.

pendulum clock as the most renowned achievements. Yet, the discovery made late 1652 had sparked his interest in dioptrics, which largely dominated his scholarly activities the next two years. The letter to Van Schooten was the onset to a lifelong engagement with dioptrics, which nevertheless has little been studied historically.[3]

In the months following the letter to Van Schooten, Huygens elaborated a treatise that contained a mathematical theory of the dioptrical properties of lenses and telescopes. I will refer to this treatise as *Tractatus* and it is the subject of this chapter.[4] In *Tractatus* Huygens treated a specific set of dioptrical questions, directed at understanding the working of the telescope. In the first section of this chapter the content and character of the treatise are discussed. In the second section Huygens' approach to dioptrics is compared with that of contemporaries, by examining how other mathematicians dealt with the questions that stood central in *Tractatus*. In this discussion of seventeenth-century dioptrics the relationship between dioptrical theory and the development of the telescope is the central topic. I will argue that Huygens in his mathematical theory stood out for his focus on questions that were relevant to actual telescopes. In this he was the first to follow Kepler's lead; other theorists were absorbed by abstract questions emerging from mathematical theory for which men of practice, in their turn, did not care. In the next chapter Huygens' own telescopic practices are discussed. Now first the theoretical considerations of *Tractatus*.

2.1 The Tractatus of 1653

The background to Huygens' letter to Van Schooten was a problem with the lenses used in the telescopes of those days. Lenses were spherical, i.e. their cross section is circular. As a result they do not focus parallel rays perfectly. Rays from a distant point source that are refracted by a spherical surface do not intersect in a single point, rays close to the axis are refracted to a more distant point on the axis than rays farther from the axis (Figure 2). This is called spherical aberration and results in slightly blurred

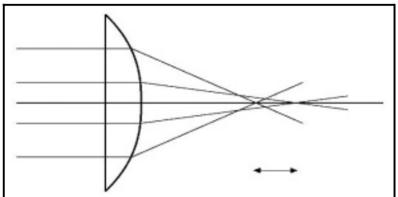
Figure 2 Spherical aberration

[3] The most thorough-going account still are the 'avertissements' by the editors of the *Oeuvres Complètes*. Southall, "Some of Huygens' contributions" reported on Huygens' dioptrics after the publication of volume 13. Harting, *Christiaan Huygens* had earlier discussed it briefly. In relationship with his astronomical work and his practical dioptrics, Albert van Helden, "Development" and Anne van Helden/Van Gent, *The Huygens collection* and "Lens production" discuss some topics. In the context of the history of seventeenth-century geometrical optics – which in its own right has little been studied – Shapiro, "'Optical Lectures'" mention Huygens' contributions. They are remarkably absent from the Malet, "Isaac Barrow" and "Kepler and the telescope". Hashimoto, "Huygens, dioptrics" is the only effort to discuss Huygens' dioptrics in the context of his broader oeuvre.

[4] *OC13*, 1-271. The editors of the *Oeuvres Complètes* have labeled it *Dioptrica, Pars I. Tractatus de refractione et telescopiis*. Its content stems from the 1650s. The original version of *Tractatus* does not exist anymore. A copy was made in Paris by Niquet – probably in 1666 or 1667, at the beginning of Huygens' stay in Paris – on which the text of the *Oeuvres Complètes* is based. The editors assume Niquet's copy of *Tractatus* is largely identical with the original 1653 manuscript; "Avertissement", XXX.

images. In *La Dioptrique* (1637), Descartes had explained that surfaces whose section is an ellipse or a hyperbola do not suffer this impediment. They are called aplanatic surfaces. Descartes could demonstrate this by means of the sine law, the exact law of refraction he had discovered some 10 years earlier. According to the sine law, the sines of incident and of refracted rays are in constant proportion. This ratio of sines is nowadays called index of refraction, it depends upon the refracting medium.

The discovery Huygens made in late 1652 sprang from Descartes' *La Géométrie*. Together with *La Dioptrique* and *Les Météores*, this essay was appended to *Discours de la methode* (1637). In *La Géométrie*, Descartes had introduced his new analytic geometry. In *La Géométrie* mathematical proof was given of the claim of *La Dioptrique* that the ellipse and hyperbola are aplanatic curves. In his letter to Van Schooten, Huygens wrote that he had discovered that under certain conditions circles also are aplanatic. This discovery implied that spherical lenses could focus perfectly in particular cases. Consequently, Huygens considered it of considerable importance for the improvement of telescopes.

Huygens' expectation that his discovery would be useful in practice, was fostered by the fact that Descartes' claims had turned out not to be practically feasable. Around 1650, no one had succeeded in actually grinding the lenses prescribed in *La Dioptrique*.[5] Apart from that, the treatise did not discuss the spherical lenses actually employed in telescopes. Descartes had applied his exact law only to theoretical lenses. When he made his discovery, Huygens must have realized that no-one had applied the sine law to spherical lenses yet. In the aftermath of his discovery, Huygens set out to correct this and develop a dioptrical theory of real lenses.

2.1.1 OVALS TO LENSES

In his letter to Van Schooten, Huygens did not explain the details of his discovery. He did so much later, in an appendix to a letter of 29 October 1654 that contained comments upon Van Schooten's first Latin edition of *La Géométrie*: *Geometria à Renato Des Cartes* (1649).[6] In book two, Descartes had introduced a range of special curves, ovals as he called them. This was not a mere abstract exercise, he said, for these curves were useful in optics:

> "For the rest, so that you know that the consideration of the curved lines here proposed is not without use, and that they have diverse properties that do not yield at all to those of conic sections, I here want to add further the explanation of certain ovals, that you will see to be very useful for the theory of catoptrics and of dioptrics."[7]

By means of the sine law, Descartes derived four classes of ovals that are aplanatic curves. If such a curve is the section of a refracting surface, rays

[5] With the possible exception of Descartes himself. See below, section 3.1

[6] *OC1*, 305-305.

[7] Descartes, *Geometrie*, 352 (*AT6*, 424). "Au reste affin que vous sçachiées que la consideration des lignes courbes icy proposée n'est pas sans usage, & qu'elles ont diverses proprietés, qui ne cedent en rien a celles des sections coniques, ie veux encore adiouster icy l'explication de certaines Ovales, que vous verrés estres tres utiles pour la Theorie de la Catoptrique, & de la Dioptrique."

coming from a single point are refracted towards another single point. In certain cases, the ovals reduce to the ellipses and hyperbolas of *La Dioptrique*. Huygens in his turn discovered that a particular class of these ovals may also reduce to a circle.

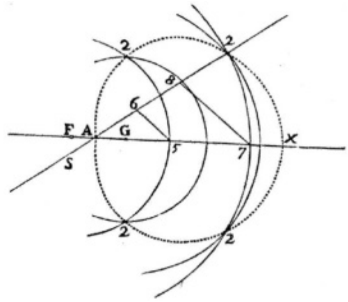

Figure 3 Cartesian oval.

In *La Géométrie* Descartes introduced the said class of ovals as follows (Figure 3). The dotted line is an oval of this class. If the right part 2X2 of the oval is the right boundary of a refracting medium, rays intersecting in point F are refracted to point G.[8] The oval is constructed as follows. Lines FA and AS intersect in A at an arbitrary angle, F is an arbitrary point on FA. Draw a circle with center F and radius F5. Line 56 is drawn, so that A5 is to A6 as the ratio of sines of the refracting medium. G is an arbitrary point between A and 5, S is on A6 with AS = AG. A circle with center G and radius S6 cuts the first circle in the points 2, 2. These are the first two points of the oval. This procedure is repeated with points 7 and 8, et cetera until the oval 22X22 is completed.[9] Huygens' discovered that the oval reduces to a circle when the ratio of AF to AG is equal to the ratio of A5 to A6, the ratio of sines.[10] This means that with respect to rays tending to F, a spherical surface 2X2 will focus them exactly in G. Van Schooten was a bit skeptical about Huygens' claim. Could such a simple fact have escaped Descartes? Nevertheless, he included it in the second edition of *Geometria à Renatio Des Cartes* (1659).[11]

Discovering that a spherical surface is aplanatic in certain cases is one thing, applying it to lenses in practice is another. It remained to be seen what shape the second surface should have and how it might be employed in telescopes. For one thing, it does not seem useful for objective lenses, the front and most important lens of a telescope that receives parallel rays. It appears the usefulness of the discovery was limited, for Huygens never returned to it in his dioptrical studies.[12]

The historical importance of the discovery lies in the fact that it aroused Huygens' interest in dioptrics. He did not exaggerate when he said he was engrossed in dioptrics. Not only its theory, practice too. Five days after his letter to Van Schooten, on 4 November, he wrote to Gerard Gutschoven, an acquainted mathematician in Antwerp.[13] After some introductory remarks,

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[8] Descartes, *Geometrie*, 358-359 (*AT6*, 430-431). The left part 2A2 is a mirror that reflects rays intersecting in G so that they (virtually) intersect in F, provided that it diminishes the 'tendency' of the rays to a given degree.

[9] Descartes, *Geometrie*, 353-354 (*AT6*, 424-426). The curve satisfies the equation $F2 - FA = n(G2 - GA)$.

[10] *OC1*, 305. See note 9: the equation becomes $AF = nAG$.

[11] Reproduced in *OC14*, 419.

[12] In *Tractatus*, he merely mentioned that a spherical surface is aplanatic for certain points: *OC13*, 64-67.

[13] *OC1*, 190-192.

Huygens launched a series of questions on the art of making lenses. What material are grinding moulds made of, how is the spherical figure of a lens checked, what glue is used to attach the lenses to a grip, et cetera. Only after these questions did he explain to Gutschoven that he wanted to know all these things because he had discovered something that would greatly improve telescopes.

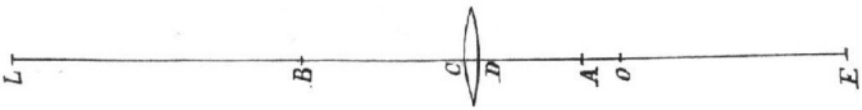

Figure 4 Focal distance of a bi-convex lens

The letter reveals that Huygens had already begun to investigate the dioptrical properties of spherical lenses. It contained a theorem on the focal distance of parallel rays refracted by a bi-convex lens CD (Figure 4).[14] AC and DB are the radii of the anterior and posterior side of the lens. L and E are determined by DL : LB =CE : EA = *n*, the index of refraction. O is found by EL : LB = ED : EO. Rays parallel to the axis EL come from the direction of L. Without proof Huygens said that O is the focus of the refracted rays. He said that he could prove this and that he had found many more theorems. A month later, in a letter of 10 December to André Tacquet, a Jesuit mathematician in Louvain, he added an important insight.[15] As a result of spherical aberration point O is not the exact focus. Nevertheless, it may be taken as the focus: "… since beyond point O no converging rays intersect with the axis."[16] In later letters to Tacquet and Gutschoven he called this point the 'punctum concursus'.[17] It is the where rays closest to the axis are refracted to. This definition would be fundamental to the theory of *Tractatus*, which apparently was well under way. Huygens told Tacquet that he had already written two books of a treatise on dioptrics: one on focal distances, another one on magnification. A third one on telescopes was in preparation.

Within a month or two after his letter to van Schooten, Huygens' understanding of dioptrics was rapidly developing. It was also developing in a particular direction. Huygens was studying the dioptrical properties of spherical lenses. He must have found out that little had been published on the subject. The only mathematical theory of spherical lenses was Kepler's *Dioptrice* (1611), but it lacked an exact law of refraction. Only Descartes had applied the sine law to lenses, but he had ignored spherical lenses. Huygens had begun to develop an exact theory of spherical lenses by himself. He combined this theoretical interest with an interest in practical matters of telescope making. He reported to have seen a telescope made by the famous craftsman Johann Wiesel of Augsburg. He was impressed and regretted that

[14] *OC1*, 192.
[15] *OC1*, 201-205.
[16] *OC1*, 204. "… adeo ut nullius radij concursus cum axe contingat ultra punctum O."
[17] *OC1*, 224-226.

Holland did not have such excellent craftsmen.[18] On 10 February, 1653, Gutschoven finally informed him on the art of lens making.[19] Huygens did not put the information to practice right-away, he first elaborated his dioptrical theory.

2.1.2 A THEORY OF THE TELESCOPE

Huygens had written to Tacquet that his treatise would consist of three parts: a theory of focal distances of lenses, a theory of the magnification produced by configurations of lenses, and an account of the dioptrical properties of telescopes based on the theory of the two preceding parts. The third part was not yet finished when Huygens wrote Tacquet, in fact he never elaborated in the form originally conceived. The third part of *Tractatus* as it is found in the *Oeuvres Complètes* is a collection of dispersed propositions collected by the editors. Only the first two seem to be from the 1650s.[20] In the arrangements of manuscripts Huygens made in the late 1680s, part one of *Tractatus* appears for the large part as it is found in the *Oeuvres Complètes*.[21] Judging from the various page numberings, Huygens has not edited it very much, except that he inserted – probably in the late 1660s – parts of his study of spherical aberration after the twentieth proposition. Part two of *Tractatus* has been reshuffled somewhat more, but the main line appears to be sufficiently original. In the following discussion of *Tractatus*, I follow the text of the *Oeuvres Complètes* in so far as it appears to reflect the original treatise.

Huygens coupled his orientation on the telescope with the mathematical rigor typical of him. Although he singled out dioptrical problems that were relevant to the telescope, he treated these with a generality and completeness that often exceeded the direct needs of explaining the working of the telescope. Huygens' rigorous approach is clear from the very start of *Tractatus*. Basic for his treatment of focal distances was the realization that spherical surfaces do not focus exactly. This had been noticed earlier and had been the rationale behind *La Dioptrique*. Nobody, however, had gone beyond the mere observation of spherical aberration. Huygens got a firmer mathematical grip on the imperfect focusing of lenses by defining which point on the axis may count as the focus. Although he only discussed focal points, Huygens took spherical aberration into account by consistently determining the focus as the 'punctum concursus'.

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[18] *OC1*, 215.

[19] *OC1*, 219-223.

[20] The decisions the editors made for the remaining propositions are sometimes somewhat mysterious. For example, the fourth proposition has been assembled of fragments from various folios. And from folio *Hug29*, 177 they put a diagram in part three of *Tractatus*, but they transferred the main contents to 'De telescopiis' (see section 6.1.2).

[21] On this arrangement see page 221. By the way, the two first propositions of part three are inserted after part one.

In part one of *Tractatus* he defined 'punctum concursus' as follows. In the third proposition, he defined the focus as the limit point of the intersections of refracted rays with the axis (Figure 5).[22] ABC is a plano-convex lens and parallel rays are incident from the direction of D. Consequently, they are only refracted by the spherical surface. Huygens showed that the closer rays are to the axis DE, the closer to E they reach it. Beyond E no refracted rays crosses the axis. This limit point E he defined as the 'punctum concursus' of the spherical surface ABC. If the surface is concave, rays do not intersect at all after refraction, they diverge. In this case, the 'punctum concursus' is the virtual focus, the limit point of the intersections with the axis of the backwards extended refracted rays.

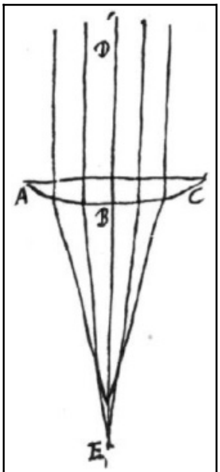

Figure 5 Punctum concursus

In the first part of *Tractatus*, Huygens derived focal distances of all types of spherical lenses by determining exactly the 'punctum concursus' in each case. Refraction of parallel rays by a lens consists in most cases of the two successive refractions by each side of the lens. Determining the focal distance thus consists of three problems. First, the refraction of parallel rays from air to glass by a spherical surface. Second, that of the refraction of converging or diverging rays from glass to air. Finally, combining both. Huygens built up his theory accordingly. He first derived theorems expressing focal distance of spherical surfaces for parallel rays in terms of their radii. Secondly, he derived theorems expressing the focal distance for non-parallel rays in terms of the radius and the focal distance for parallel rays. Finally, he expressed the focal distance of the various kinds of lenses in terms of the radii of their sides. In each case he took the thickness of the lens into account. Only afterwards did he derive simplified theorems for thin lenses, in which their thickness is ignored. I now sketch the typical case of a bi-convex lens, the theorem that Huygens included without proof in his letters to Gutschoven and Tacquet. The determination of the focal distances of other lenses – plano-convex, bi-concave, etc. – went along similar lines.

The focal distance of a bi-convex lens

The eighth proposition of *Tractatus* dealt with parallel rays refracted at the convex surface of a denser medium (Figure 6). AC is the radius of ABP; Q is a point on the axis AC so that AQ : QC = n, where n is the index of refraction. Huygens demonstrated that Q is the 'punctum concursus' of parallel rays OB, NP. A refracted ray BL intersects axis AC in a point L between A and Q. With the sine law BL : LC = AQ : QC = n. For any ray OB, BL is smaller than AL and AL is smaller than AQ. Therefore no refracted rays intersect the axis beyond

[22] *OC13*, 16-19

Q. In order to prove that Q is the ‘punctum concursus’ of ABP, consider ray NP and its refraction PK. PK is found with the sine law and KQ is therefore a given interval. On KQ choose L and draw T, close to A, so that LT : CL = AQ : CQ = *n*. Now PL : LC < PK : KC = *n*. PL is smaller than TL, which in its turn is smaller AL. A circle with center L and radius TL intersects the refracting surface ABP between A and P in a point B. Draw BL and BC and it follows that BL : LC = TL : LC = *n*. Therefore BO is refracted to L. So, the closer a paraxial ray is to the axis, the closer to Q the refracted ray will intersect with the axis. Q is the limit point of these intersections and therefore the ‘punctum concursus’. When the index of refraction AQ : QC = 3 : 2 – the approximate value for glass – AQ is exactly three times the radius AC.

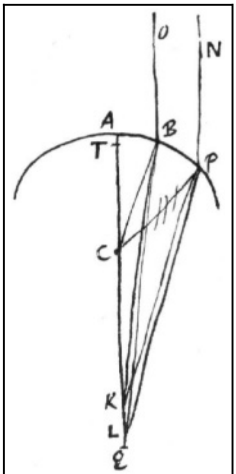

Figure 6 Refraction at the anterior side of a bi-convex lens

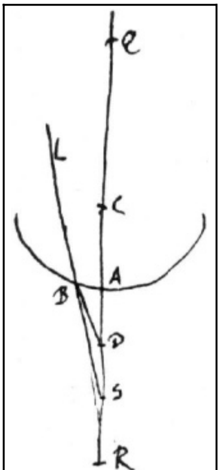

Figure 7 Refraction at the posterior side of a bi-convex lens.

The refraction at the posterior side of the lens is dealt with in the twelfth proposition. This case is more complex as the incident rays are converging due to the refraction at the anterior side. Huygens dealt with eight cases of non-parallel rays. [23] For all cases, he expressed the focal distance of the non-parallel rays in terms of the focal distance of the surface for paraxial rays. The case at hand is the fourth part of the proposition (Figure 7).[24] Rays converge towards a point S, outside the dense medium bounded by a spherical surface AB with radius AC. Q is the ‘punctum concursus’ of paraxial rays coming from R. With SQ : SA = SC : SD, the ‘punctum concursus’ D of the converging rays LB is found. Huygens’ proof consisted of a reversal of the first case treated in this proposition: rays diverging from D are refracted so that they (virtually) intersect in S.[25] This proof is similar to the one above.

Finally, in the sixteenth proposition of *Tractatus*, Huygens determined the focal distance of a convex lens by combining the preceding results. It was equal to the theorem he put forward in his letters to Tacquet and Gutschoven. CD is a bi-convex lens with radii of curvature AC and BD (Figure 8). The foci for paraxial rays are respectively E and L. According to the eighth proposition CE : EA = DL : LB = *n*. With the twelfth proposition,

[23] *OC13*, 40-79.
[24] *OC13*, 70-73.
[25] *OC13*, 42-47.

the 'punctum concursus' N for parallel rays from the direction of L is found with EL : ED = EB : EN. After the refraction at the surface C, the rays converge towards E; they are then refracted at the surface D towards N. In modern notation: $DN = \dfrac{\dfrac{nAC \cdot BD}{n-1} - BC \cdot CD}{n(AC+BD)-(n-1)CD}$, where DN is the focal distance measured from the anterior face of the lens. For rays coming from the other direction O is the 'punctum concursus'.

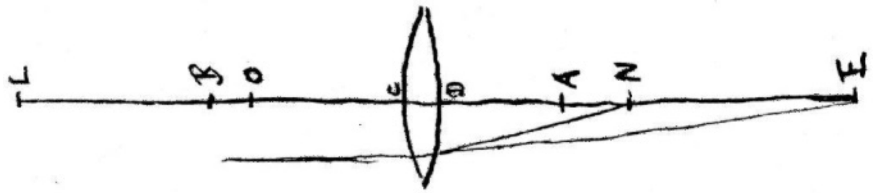

Figure 8 Focal distance DN of a bi-convex lens

The case of a bi-convex lens was only one out of many cases Huygens treated in the fourteenth to seventeenth proposition of *Tractatus*. Taking both spherical aberration and the thickness of the lens into account, he derived exact theorems for the focal distance of each type of lens. In each case, he also showed how to simplify the theorem when the thickness of the lens is not taken into account. In the case of a bi-convex lens, he started by comparing the focal distances CO and DN when the radii of both sides of the lens are not equal. Their difference vanishes when the thickness of the lens CD is ignored and both refractions are assumed to take place simultaneously. The focal distance N is then easily found by first determining point L with DL : LB = n and then AB : AD = DE : EA, or $DN = \dfrac{2AC \cdot BD}{AC+BD}$.[26] In the case of a glass lens (n = 3 : 2) LB is twice BD and (AC + BD) : AC = 2BD : DN. It follows directly that the focal distance is equal to the radius in the case of an equi-convex lens. In the twentieth proposition of *Tractatus*, Huygens extended the results for thin lenses to non-parallel rays. In this case rays diverge from a point on the axis relatively close to the lens and are refracted towards a point P found by DO · DP = DC2 (DO is the focal distance for parallel rays coming from the opposite direction). Huygens had to treat all cases of positive and negative lens sides separately, but the result comes down to the modern formula $\frac{1}{p}+\frac{1}{p'}=\frac{1}{f}$.[27]

In the remainder of the first book of *Tractutus*, Huygens completed his theory of focal distances by determining the image of an extended object, rounded off in the twenty-fourth proposition (Figure 9). The diameter of the image IG is to the diameter of the object KF as the distance HL of the image

[26] *OC13*, 88-89. Equivalent to the modern formula $\frac{1}{f} = (n-1)(\frac{1}{R_1}+\frac{1}{R_2})$.

[27] *OC13*, 98-109.

to the lens is to the distance EL of the object to the lens.[28] The point L has a special property that Huygens had established in the preceding proposition. An arbitrary ray that passes through this point leaves the lens parallel to the incident ray.[29] In the twenty-second proposition, Huygens had demonstrated that the focal distance LG of rays from a point K of the axis is more or less equal to that of a point E on the axis.[30] The triangles KLF and GLI are therefore similar, which proves the theorem.[31]

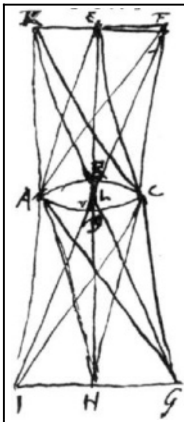

Figure 9 Extended image.

Images

The theory of focal distances formed the basis of Huygens' discussion of the properties of images formed by lenses and lens-systems in the second book of *Tractatus*. Huygens' theory of images is once again both rigorous and general. The central questions in book two were how to determine the orientation of the image and the degree of magnification. For the time being, Huygens ignored the question whether an image is in focus. In this way he could derive general theorems on the relationship between the shape of lenses and their magnifying properties. He then showed how these reduced to simpler theorems in particular cases, for example for a distant object. In the third book he showed what configurations produced focused images.

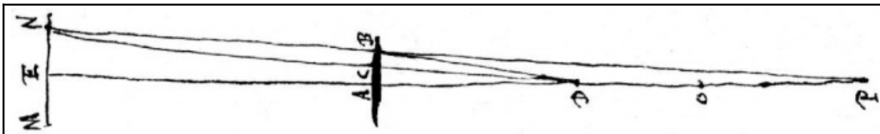

Figure 10 Magnification by a convex lens.

In the second and third propositions of book two, Huygens discussed a convex lens. His aim was to determine the magnification of the image for the various positions of eye D, lens ACB and object MEN (Figure 10). In order to distinguish between upright and reversed images, Huygens defined the 'punctum correspondens' (later called 'punctum dirigens').[32] This is the focus of rays emanating from the point where the eye is situated and is thus found by means of the theory of the first book. First, the eye D is between a convex

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[28] *OC13*, 122-125.

[29] *OC13*, 118-123. In modern terms, L is the optical center.

[30] *OC13*, 114-119.

[31] Huygens added that when the thickness of the less is taken into account, point V in the lens instead of L should be taken as the vertex of the triangle.

[32] *OC13*, 176n1.

lens ACB and its focus O. In this case, the object MEN is seen upright and magnified. The lens refracts a ray NBP to BD so that point N of the object is seen in B, whereas it would be seen in C without the lens. AB is larger than AC and on the same side of the axis. Huygens then showed that AB : AC = (AO : OD)·(ED : EP), which in the case of a distant object reduces to AB : AC = AO : OD.[33] If, on the other hand, the eye is placed in the focus (so that AD = AO) and NB is taken parallel to the axis, AB : AC = EO : AO which becomes infinitely large when the object is placed at large distance. In the next proposition, Huygens considered the cases where the eye is placed beyond the focus O. In this case the 'punctum correspondens' P is on the other side of the lens and the image will be reversed when the object is placed beyond it.

With the same degree of generality, in the fifth proposition Huygens discussed the images produced by a configuration of two lenses.[34] He figured (Figure 11, most left one) two lenses A and B with focal distances GA and HB, the eye C and the object DEF, all arbitrarily positioned on a common axis. He then constructed point K on the axis, the 'punctum correspondens' of the eye with respect to lens B, the ocular lens. Next, he constructed point L on the axis, the 'punctum correspondens' of point K with respect to lens A, the objective lens. In this way, a ray LD will be refracted by the two lenses to the eye via points M on lens A and N on lens B. Without lenses, the eye sees point F of the object – where DE = DF – along line COF. The degree of magnification is therefore determined by the proportion BN : BO. In this general case, the magnification follows from BN : BO = (HB : HC)·(AG : GK)·(EC : EL). Huygens derived this proportion for the case of a concave ocular and a convex objective, but the same applied to a system of two convex lenses.

In the adjoining drawings, Huygens sketched various positions of eye, lenses and object (Figure 11 gives four cases). These showed whether the image was upright or reversed. In addition, he showed how the general theorem reduced to a simpler one in particular cases. For example, when the 'punctum correspondens' of the ocular K and the focus of the objective G coincide, it reduces to (HB : HC)·(EC : AK). Likewise, the configurations used in practice were only a special case that Huygens discussed as he went along. If a concave ocular and a convex objective are positioned in such a way that BG = BH, where the ocular is between the objective and its focus, the magnification of a distant object is AG : BH. The same applies to two convex lenses that are positioned with their foci coinciding in between. In other words, the magnification is equal to the quotient of the focal distances of both lenses. In this roundabout way, Huygens proved what had been, and

[33] *OC13*, 174-179.
[34] *OC13*, 186-197.