Computer Engineering in Applied Electromagnetism

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This book contains the papers presented at the International Symposium on Electromagnetic Fields in Electrical Engineering ISEF'03 which was held in Maribor, Slovenia on September 18-20, 2003. ISEF conferences have been organized since 1985 as a common initiative of Polish and European researchers who deal with electromagnetic fields applied to electrical engineering. Until now the conferences have been held every two years, either in Poland or in one of the European academic centres known from its electromagnetic research. The city of Maribor is well-known in the world for its beauty and academic flavour as well as for its researchers’ achievements in the area of applied and computational electromagnetism.

Almost 200 papers were submitted as abstracts and after selection process 159 papers were accepted for the presentation at the Symposium, and almost all of them (ca. 90%) were presented both orally and in poster sessions. The papers published in this book were refereed by the sessions’ chairmen and the papers accepted for further publication were divided into two parts: these of more computational aspect and those of more applicable nature. The latters are published here, while the first part went to COMPEL journal.

It is the tradition of the ISEF meetings that they comprise a vast area of computational and applied electromagnetics. Moreover, the ISEF symposia aim at joining theory and practice, thus the majority of papers are deeply rooted in engineering problems, being simultaneously at a high theoretical level. Bearing this tradition, we hope to touch the heart of the matter in electromagnetism. The main topics of ISEF meetings are listed below:

- Computational Electromagnetics
- Electromagnetic Engineering
- Computational Techniques
- Coupled Field and Special Applications
- Bioelectromagnetics and Electromagnetic Hazards
- Magnetic Materials Modelling.

The papers in the book have been grouped in three sections which cover the above topics:

- Computational Techniques
- Electromagnetic Engineering
- Special Applications.

It makes some order in reading but also it somehow represents the main directions which are penetrated by researchers dealing with contemporary electromagnetics. Looking at the content of the book, one may also notice that more and more researchers go into the investigation of new applications of electromagnetics, especially those connected with medicine, biology and material sciences. The computational techniques which were under development during the last three decades and which are being still developed serve as good tools for discovering new electromagnetic phenomena. This conclusion is unnecessarily shared by all the readers but we try to show here the trend which can be clear from the book.
A more conventional approach is presented in the first paper of the volume which was the invited paper at the conference. The author is Professor Jan Sykulski from Southampton University, UK, who is well-known for being the key person of the International Compumag Society from its beginning. His writing shows contemporary computational techniques placed in the history of the subject, and the examples which are quoted in the paper are just confined to classical applications, as electrical machines and the like.

The three chapters are prefaced by short introductory remarks. They will help readers in looking for some particular topic in which they are interested.

We, the Editors of the book, would like to express our thanks to our colleagues who have contributed to the book by refereeing the papers at the conference as well as in the publishing process. We also convey our thanks to Kluver Academic Publishers for their effective collaboration in shaping this editorial enterprise. As ISEF conferences are organised biannually, we do hope to keep our strong links with Kluwer.

Mladen Trlep
Chairman of the Organising Committee

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Scientific Secretary

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Chairman of the ISEF Symposium
INVITED PAPER
COMPUTATIONAL ELECTROMAGNETICS: A TOOL, AN ART OR BLACK MAGIC?

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Abstract – The paper is an attempt to review the state of the art in Computational Electromagnetics (CEM) with a focus on applications related to electrical engineering. Design and optimisation, as well as development of new materials, are emphasised as of paramount importance in the real engineering world. Modern computational methods based on finite elements and related techniques have now become a mature design tool, but the complexity of the underlying mathematics and physics often hampers widespread use of these efficient techniques. Recent advances in general purpose software are encouraging but much remains to be done in improving the standards of education to remove the mist of mystery surrounding the subject.

Introduction

The research activity known as Computational Electromagnetics (CEM) has evolved alongside the modern developments in the digital computing hardware. Moreover, CEM is both a special case and part of the broader subject of computational mechanics. The speciality arises in many obvious ways, e.g. free space is an unbounded magnetic ‘material’, there is a vast range of physical dimensions encountered with critical feature sizes often varying over many orders of magnitude, the fundamental properties of Maxwell’s equations are different to equations governing other physical phenomena. There is also a very broad spectrum of frequencies encountered: from DC to daylight. Activities of the CEM community are well organised within the International Compumag Society [1,2], an independent international organisation in existence since 1993 with nearly 700 members from over 40 countries. The IEE Professional Network on Electromagnetics [3] is also gaining momentum and establishing itself as an international forum for discussion. The IEEE Magnetics Society [4] and ACES [5] manage the activities in North America. Journals such as IEE Proceedings [6], IEEE Transactions on Magnetics [7] and COMPEL [8] contain a significant number of papers showing fundamental advances and applications of CEM. There are many conferences reporting regularly on recent developments, including COMPUMAG [9,10], CEFC [11], CEM [12,13] and, of course, ISEF [14].

There exist many books on fundamental aspects of field computation in applications relevant to electrical engineering (see for example [15-17]), as well as monographic publications devoted more specifically to finite-element aided modelling and simulation of electromechanical devices in electrical power engineering [18,19]. Design and optimisation feature prominently in literature; it is worth mentioning here some of the books devoted to optimal design in electricity and magnetism [20] and more general monographs on multi-objective optimisation [21,22]. Recent advances in material science and discovery of new types of materials (e.g. high temperature superconductivity) have an immense impact on the way in which modern designs of electromechanical devices are approached and present a continuing challenge to fundamental sciences in terms of development of new, cheaper, more efficient and accurate methods for electromagnetic field modelling and simulation. Coupled field systems are of particular importance as practical designs have to address simultaneously all aspects of performance of the device: electromagnetic, mechanical, thermal and economic. One of the reasons why progress is impeded is because the mathematics and physics behind the numerical formulations are often quite complex and thus few designers have sufficient skills to master available software. Nevertheless, recent progress with the general purpose software – such as OPERA [23], MAGNET [24], or ANSYS [25] – makes these programs extremely powerful tools in the hands of an expert.
Recent advances in CEM, supported by continuing increases of power and speed of computers, make finite element modelling an attractive alternative to well established semi-analytical and empirical methods, as well as to the still popular ‘trial and error’ approach. A typical system incorporating FE computation is demonstrated by Fig. 1 [15,16]. First, a designer has to build a computational model of his physical problem. This is a vital step, often underestimated, which may decide on a success or a failure of the whole process. Clearly, the model must be adequate for the results to be meaningful. This emphasises the significance of human input to the design and importance of an experiment and/or alternative models for solving the same problem, so that comparisons can be made. Once the model has been formulated the CAD system will facilitate finding a solution.

Parameterisation of the model is often desired, so that various parts – as well as the whole device – may be constrained in a convenient way, for example for optimisation or sensitivity analysis. A typical example of software development addressing this issue is the concept of Design Environment (DEM) [23,26,27] – see Fig. 2. A DEM facilitates the use of electromagnetic analysis software by providing an application specific shell to guide a non-specialist through the geometric design and physical property specification of a class of device. DEMs are created by experts and contain a parameterised model of a device with a set of decision making routines suggesting optimal representation of materials and boundary conditions, followed by automatic analysis of performance. The post-processor offers top level commands for specific tasks such as calculation of device parameters.

One of the more important issues associated generally with such CAD systems is the question of error estimation and the ability of the system to refine the model to improve the accuracy. Various adaptive schemes are available based on \( h, p \) or \( r \) mesh refinement [16] or dynamic bubbles shown in Fig. 3.

![Anisotropic mesh using dynamic bubbles](image1.png)

![3D model of an induction motor](image2.png)

**CAD of Electromechanical Devices**

Fig. 1. A typical CAD system for electromagnetics

Fig. 2. A general DEM and its relation to FE package

Fig. 3. Anisotropic mesh using dynamic bubbles [28]

Fig. 4. 3D model of an induction motor
The CEM community has gone a long way to address the needs of designers and contemporary commercial software is capable of solving static, quasi-static and full transient problems in 2D as well as in 3D. Nonlinearity of materials, permanent magnets, various shapes of excitation coils – these are just examples of what can easily be solved (see an example of Fig. 4). Finally, coupled problems can be handled involving interactions between electromagnetic field, motion and supplying circuit [29].

**Fundamental Formulations and Techniques**

There has been important progress in fundamental formulations providing more solid foundations for numerical field analysis. These have been reported at COMPUMAG [9], CEFC [11] and CEM [12]. Lack of space does not permit to elaborate on these developments here but some more exciting advances are mentioned. Equally, progress has been made in implementation of new techniques leading to more efficient, faster, more accurate and numerically stable algorithms. The following is a non-exhaustive list of such advances which have made the greatest impact on the CEM community.

- a new Finite Element Difference (FED) method,
- higher order Finite Difference Time Domain (FDTD) approach,
- further developments of the Transmission Line Matrix (TLM) method,
- advances of the Multiple Multipole Technique (MMT),
- the use of Finite Integration Technique (FIT),
- a new Subspace Projection Extrapolation (SPE) scheme,
- working field theory problems with Random Walks,
- formulations in terms of differential geometry,
- the usage of total/reduced magnetic vector potential and electric scalar potential,
- an introduction of Lie derivative as a tool for force computation,
- implementation of edge and facet elements,
- improved anisotropy models,
- efficient application of Continuum Design Sensitivity Analysis (CDSA).

**Modelling of New Types of Materials**

Discovery and/or development of new materials present a modelling challenge and often lead to reformulation of fundamental equations or methods of solution. We will focus here on recent advances in superconductivity. Ceramic superconducting materials were discovered in 1986 and their main advantage is that they can operate at liquid nitrogen temperature (78K) – hence the name High Temperature Superconductors (HTS) – and thus offer cheap and reliable technology (often compared to water cooling). With practical current densities 10 to 20 times larger than in conventional copper windings they have great potential in electric power applications (generators, motors, fault current limiters, transformers, flywheels, cables, etc.), as losses are significantly reduced. From the design point of view they offer a challenge because of very highly non-linear characteristics and anisotropic properties of materials, and due to unconventional design solutions. Fundamental characteristics and underlying physical processes are well described in literature. Some recent advances at Southampton University in the application of the HTS technology to electric power devices are described in [30-39].

There is continuing significant activity around the world in the development of HTS tapes and wires, applications of the technology to power devices and modelling of fundamental processes in the superconducting materials. From the practical point of view the ability to predict and reduce all ‘cold’ losses is of paramount importance to demonstrate economic advantages of HTS designs. The behaviour and characteristics of the highly non-linear and anisotropic HTS materials is markedly different to conventional conductors. A typical field distribution is depicted in Fig. 5, whereas the dependence of AC losses on field level is shown if Fig 6 [33]. It can be clearly seen that field direction is of paramount importance and thus steps need to be taken to ‘shape’ the leakage field in the device to avoid excessive losses and prevent the conductor from being exposed to fields higher than critical.
Low Temperature Superconductivity has not been very successful in electric power applications due to low reliability, high cost and difficult technology. HTS offer better thermal stability, cheaper cooling and improved reliability. Currently all conceptual HTS designs and small demonstrators use BSCCO tapes at temperatures between 20K and 30K because critical fields and currents are an order of magnitude better than at 78K and it is possible to have a core-less design. However, liquid neon or helium gas is needed leading to increased cost and complexity of refrigeration plant, reduced thermodynamic efficiency and worse reliability and higher maintenance requirements. All Southampton designs use cooling at 78/81/65/57 K (liquid nitrogen or air / sub-cooled nitrogen or air).

The first device built and successfully tested in Southampton in 1997/98 was a small 10kVA demonstrator transformer (Fig. 7) [39]. A particularly satisfying result was the two-fold reduction of losses through introduction of magnetic flux diverters as demonstrated by Figs. 8.
A new 100 kVA, 2 pole HTS synchronous generator – nearing completion at Southampton – uses a magnetic core made of 9% cryogenic steel to lower the ampere-turns required by a factor of ten and to significantly reduce fields in the pancake coils made of BSCCO (Fig. 9). In terms of modelling the important issues are no-load tooth ripple losses due to the distortion of the fundamental flux density wave by the stator slotting, and full-load losses that include the effects of the MMF harmonics of the stator winding. Two models were used: full transient non-linear rotating (no-load, see Fig 10), and a combination of static and steady-state (full-load), respectively. Losses are released into liquid nitrogen and have to be removed using inefficient refrigeration system. Each 1W of loss requires between 15 – 25 W of installed refrigeration power at 78K (a similar figure at 4K would be about 1000 W).

**Optimisation**

Optimal design of electromechanical devices often necessitates repetitive usage of finite-element (FE) solvers, or other numerically intensive field computation. A direct way of incorporating field modelling into an optimisation loop is to call the FE package every time a function evaluation is needed. Although fairly straightforward in implementation, this on-line approach will normally lead to unacceptable computing times, as for each set of selected design parameters a full field analysis needs to be undertaken. The number of necessary calls to the FE software escalates as the number of design variables increases; moreover, additional calls are normally required to calculate each gradient of the objective function. Although theoretically this is of no consequence, in the design office environment such an approach becomes impractical.

The *Minimum Function Calls* (MFC) approach relies on evaluating the objective function a priori for a number of pre-determined cases and fitting an interpolating function through the data points [40]. The optimiser then uses the interpolating function rather than calling the FE directly. In this *Response Surface Methodology* (RSM) approach it is usual to use polynomial interpolating functions. Using the RSM reduces computing times dramatically, but care must be taken not to sacrifice accuracy. Extensive numerical experiments have shown that further significant improvements may be achieved by introducing *on-line learning with dynamic weighting* [40]. To illustrate the process a brushless permanent magnet (PM) motor has been optimised for efficiency (with minimum torque constraint) in terms of magnet height, tooth width and stack length. The convergence is illustrated in Fig. 11.

![Fig. 11. Convergence of efficiency and torque in a PM motor](image)

The deterministic approach, despite the addition of learning points, may not be able to avoid local minima traps. If this is identified as a potential problem stochastic techniques may offer a better choice. Most such techniques are very expensive in terms of number of necessary function evaluations and thus impractical. Some more recent methods, however, look more promising and one such technique introduced originally in [41] is reported here. It uses a combination of *Evolution Strategy*, *Differential Evolution* and *Multiquadrics Interpolation* (ES/DE/MQ) as shown in Fig. 12. This hybrid method has been shown to be able to avoid local minima traps for a number of test functions and achieves a significant reduction of the number of necessary function calls, making the approach...
suitable for computationally intensive FE design/optimisation problems. Moreover, the quality of the resultant optimum is comparable to, or better than, those obtained using other methods.

The **Neuro-Fuzzy Modelling (NFM)** [42] uses optimisation based on the Genetic Algorithm (GA) and the **Sequential Quadratic Programming (SQP)** method. In the NF/GA/SQP approach, an n-dimensional hyper-space is sampled initially using a grid structure or a suitable **Design of Experiment (DoE)** orthogonal array if the number of variables is high. The model data is subsequently employed to create a neuro-fuzzy model which provides an approximation of real function. The notion of **Membership Functions (MFs)** is introduced which can be described by Gaussian, generalised bell or other curves. During the supervised training process the parameters of each MF are modified using the back-propagation algorithm and the consequent parameters established using least squares, ultimately providing an approximation of the system under investigation. This empirical model effectively replaces the actual function generator – in this case the finite element solver – easing the computational cost when applying the optimisation routine. This comprises a GA to identify the locality of the global optimum followed by the SQP method to isolate it accurately. The latter is possible due to the extraction of derivative information from the neuro-fuzzy model.

There is growing interest in the ways in which the performance of a specific device could be modelled using a **neural network**. Such a network learns the shape of the hyper-surface and provides a fast evaluation of any point in it. Typically, the neural network is trained in a batch mode, prior to the optimisation process – essentially “off-line”. A recent attempt has been made to construct a system which can provide “on-line” training, i.e. a network which is capable of learning and modifying its behaviour as it is used [43]. Such a network has major benefits over a static system in that it can handle a large number of variations of a device and track developments in design related to material changes and manufacturing processes.

Design has to be put in the context of general trends in optimisation methods. The role of multi-objective tasks is increasing as practical designs often involve conflicting requirements. Such problems may be converted into single-objective tasks with a priori application of knowledge or imposition of a decision (e.g. weighting factors), but it is argued that information can easily be lost in the process. Instead the application of **Pareto Optimal Front (POF)** approximation is advocated. The mathematical theory of Pareto optimisation may be somewhat complicated [44], but some basic definitions and properties are easily explained using a special case of two objective functions being minimised as shown in Fig. 13.

Finally, often in practice, it is the improvement to the design, not necessarily a global optimum, which is of interest. Hence the sensitivity analysis is of great value as computing times are not affected by the number of design variables. The **Continuum Design Sensitivity Analysis (CDSA)** is particularly to be recommended as standard EM software may be used for extracting gradient information [45].

**Industrial and Educational Requirements**

From the industrial perspective it is probably true to say that many managers still perceive computational electromagnetics as a kind of “black magic” – and yet these are the very people who would benefit most from using CEM techniques to reduce design times and costs. There is lack of appropriate skills to benefit fully from the enormous power and versatility of the available CEM tools. It may be argued that three categories of users are required in industry:

- those able to run EM software with basic understanding of field displays and ability to interpret the numerical results to incorporate them into design processes;
design experts who understand the language of electromagnetics and are capable of creating computational models using available software;

- EM software developers – the ultimate CEM experts producing computational tools.

Decision makers in industry – in their very best interest – need to support the universities and government in providing sufficient funding for both the development of CEM tools and for providing sufficient education at different levels. Academic institutions face a great opportunity of reversing the current trend of reducing the amount of EM teaching and making sure that relevant courses are available to undergraduates and engineers in industry. There is some cause for optimism as progress with CEM software development is accelerating and more programs find their way to design offices.

**Future Developments**

Further progress is required and a possible list of topics for research and development may include:

- adaptive meshing with particular emphasis on problems with strong skin effect,
- reliable error estimation (a posteriori and a priori),
- code development for high speed computing,
- efficient handling of non-linearity, hysteresis and anisotropy,
- modelling of new types of materials (e.g. composite, superconducting),
- incorporation of linear movement and rotation of some parts of the device,
- combined modelling of fields and circuits (e.g. supply electronic circuits),
- coupled problems (electromagnetic + stress + temperature, etc),
- optimisation (deterministic and stochastic, practical implications),
- integrated design systems (combined mechanical, electromagnetic, thermal, economic).

It can be argued, however, that CAD in Magnetics is already a mature practical tool for design and optimisation of a variety of electromechanical devices and the engineering community can benefit from tremendous advances that occurred in the field over the past many years.

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SECTION I
COMPUTATIONAL TECHNIQUES

Introductory Remarks

The first section is devoted to the problems connected with computational electromagnetics. The subject has been intensively developed in the last decades – one can place in time very roughly the starting date of the subject in the mid-seventies when the first COMPUMAG conference was held in Oxford, UK. Since that time many conferences have been held and the ISEF conference is one of them. Elderly participants can probably remember the first activities in this field which in the main were devoted to making numerical models in simple two-dimensional modes with stationary or quasi-stationary time variation. The content of the section shows that research on the problems is much more advanced: the 3-D numerical code regarding all the sophisticated material features, complicated geometry and arbitrary course in time is nowadays the tool for optimizing and designing processes. Very few authors consider the so-called direct problems, i.e. the problems of electromagnetic field computation. Just the opposite, most papers deal with the so-called inverse problems, i.e. the problems of shape or parameters designing. That is signum temporis (the sign of the times).

The papers which belong to this section can be divided into two subgroups, regarding the subject of their contents:

- improvement of computational techniques,
- optimization problems,

The first subgroup considers the problems which can be divided into two piles: one pile is represented by papers I-1, I-11, I-13 and I-14 and comprises some methodological considerations. They are very interesting, indeed, as they introduce analytical formulae to the algorithm or they analyze the features of the algorithm. Paper I-1 gives the solution of the problem which seems to be simple as its geometry is simple but, as a matter of fact, it is not a trivial problem. The authors give some analytical solutions of the problem and discuss them from the point of view of assumptions adopted. The authors of paper I-11 consider the problem of imaging two-dimensional current density distributions on the basis of the magnetic field that is generated by the currents. It is, in other words, the problem of regularization which appears when dealing with inverse problems. Paper I-13 deals with the calculation of magnetic fields created by permanent magnets by the use of the indirect boundary-integral model. One meets here a creative introduction of the boundary-integral method. Fictitious sources are introduced and considered as unique sources of the conservative magnetic field. The approach leads to advanced analytical expressions for usual magnetic quantities, by means of which various technical objects containing permanent magnets can be analyzed. Paper I-15 describes the application of the general load line method to a magnetic circuit problem with series and parallel branches of uniform cross sections consisting of the same homogeneous ferromagnetic core material. It also compares the results with those obtained by finite element analysis.

The second pile of papers considering computational techniques is more or less connected with computer software. Two of them (I-7, I-10) concern the problems of peripheries of numerical software: in (I-7) one can find the method of mesh generation, while in (I-10) the method of visualization is described. Paper (I-12) introduces some new technique of calculation, namely the coupling of field and circuit method.

The second group is connected with optimization and design techniques. As has already been mentioned, the main stream of activity in computational electromagnetics is directed at solving such problems. Indeed, one can find in the section many techniques of optimization as well as many objects

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to be optimized and designed. As to the techniques, three of them are mostly quoted: genetic and evolution strategies (I-2, I-18), simulated annealing (I-3) and methods based on sensitivity analysis (I-17).

The rest of the papers deal with other methods of optimization, such as experimental method (I-4, I-5, I-15), Taguchi method (I-8) and combined. One can observe even bigger variety when one looks at objects either designed or and optimized. We have here power transformers and shielding technique (I-5, I-6), small electric motors and devices (I-8, I-9, I-16), magnets (I-17), planar thick film filters (I-4), high voltage lines and insulators (I-2, I-15).

Of course, the techniques and the objects are crossed mutually and there is no rule which would prescribe one method to one object or there is neither this nor that method in its pure form.

The papers placed in the section show, more or less, the state of the art of computational techniques in applied electromagnetics. We devoted a little bit more space to papers dealing with some analytical solutions. It seems a little strange to work out such approaches, especially if one takes into account the common use of numerical modeling. It is believed, however, that such approaches can enrich pure numerical models providing them with a priori information on the problem considered. And, above all other aspects, the analytical solutions are pretty and they will never pass into oblivion.
I-1. STUDY OF THE ELECTROMAGNETIC FIELD INDUCED BY CURRENTS FLOWING IN THE CIRCULAR CONDUCTORS

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Abstract – Equations, describing the distribution of electromagnetic field induced by currents of (1.1) type flowing in the circular coil, including symmetrically infinitely long metal cylinder (fig.1), are presented in the paper. In spite of simply physical configuration, they are very complicated. Main research goals set by authors are presented below: a/ general analyze of electromagnetic field in the considered configuration, b/ examination of electromagnetic waves in the distant zone with respect to possible interferences, c/ examination of influence of Maxwell displacement currents on distribution of field in the environment of metal wall (circular cylinder) of curved surface. It is obvious, that in the case of surface, which is not curved, influence of Maxwell displacement currents on distribution of field in the near zone is negligible. Influence on the distribution of electrical field is essential and cannot be omitted even during the approximation [1]. However, in the case of cylindrical wall, Maxwell currents may influence significantly even on the distribution of magnetic field (in the case of exciting currents flowing in the linear conductors, which are parallel to the axis of cylinder) [2].

1. Electromagnetic field induced by current flowing in the circular coil without inside cylinder \((R_0 = 0)\)

We assume, that current flowing in the conductor has the following form:

\[ i(t, \theta) = i_0 \exp(j\omega + n\theta), \quad j = \sqrt{-1} \quad (1.1) \]

We will calculate spatial, complex amplitudes of electromagnetic field excited by the current of this type. Equation (1.1) can be treated as one of terms of complex Fourier series of angular function periodical with respect to the angle \(\theta\).

Retarded electromagnetic potential is a mathematical base for the calculations \(\tilde{A}\) [3]:

Fig. 1. Diagram of analyzed configuration
\[ \vec{A} = \frac{i_0 \mu_0}{4\pi} \int_{-\pi}^{\pi} e^{jn\theta} W_0(\rho) d\theta, \quad d\vec{\ell} = R(-\sin \theta, \cos \theta, 0) d\theta \] (1.2)

\[ W_0 = \frac{e^{-jk\rho}}{\rho}, \quad k = \frac{\omega}{c} = \omega \sqrt{\mu_0 \varepsilon_0}, \quad \rho^2 = R^2 + r^2 + z^2 - 2rR \cos(\theta - \varphi) \]

Differential \( d\vec{\ell} \) is written in the Cartesian coordinates \((x, y, z)\); all next calculations are carried out in the cylindrical coordinates \((r, \theta, z)\). Electric field \( \vec{E} \) and magnetic field \( \vec{H} \) can be expressed in the following form:

\[ \vec{H} = \frac{1}{\mu_0} \text{rot} \vec{A}, \quad \vec{E} = \frac{1}{j\omega \varepsilon_0} \text{rot} \vec{H} \] (1.3)

After some rather arduous mathematical calculations, on the basis of these equations we obtain [4]:

- (radial \( E_1 \), circumferential \( E_2 \) and axial \( E_3 \)) components of electric field:

\[ E_1 = \text{h}(\varphi) \int_0^\pi [W_3 - r(r - R \cos \tau)W_2] \sin n \tau \sin \tau \; d\tau, \quad E_2 = jh(\varphi) \int_0^\pi [W_3 \cos \tau - RrW_2 \sin^2 \tau] \cos n \tau \; d\tau \]

\[ E_3 = -h(\varphi) \int_0^\pi W_2 \sin n \tau \sin \tau \; d\tau, \quad h(\varphi) = \frac{i_0 R \; e^{jn\varphi}}{2\pi \varepsilon_0 \omega}, \quad W_1 = \frac{1}{\rho} \frac{dW_0}{d\rho} = -\left( \frac{jk}{\rho} + \frac{1}{\rho^2} \right) W_0, \] (1.4)

\[ W_2 = \frac{1}{\rho} \frac{dW_1}{d\rho} = \left( \frac{3}{\rho^4} + \frac{3jk}{\rho^3} - \frac{k^2}{\rho^2} \right) W_0, \quad W_3 = 2W_1 + \rho^2 W_2 = \left( \frac{1}{\rho^2} + \frac{jk}{\rho} - \frac{k^2}{\rho^2} \right) W_0 \]

- and adequately of magnetic field:

\[ H_1 = -zh(\varphi) \int_0^\pi [W_1(\rho) \cos n \tau \; d\tau, \quad H_2 = -jzh(\varphi) \int_0^\pi [W_1(\rho) \sin n \tau \sin \tau \; d\tau \]

\[ H_3 = h(\varphi) \int [r \cos \tau - R]W_1(\rho) \cos n \tau \; d\tau, \quad h(\varphi) = \frac{i_0 R \; e^{jn\varphi}}{2\pi} \] (1.5)

If the influence of Maxwell currents \((k = 0 \equiv c = \infty \text{ in equations (1.2)})\) is omitted and it is assumed, that \( i \) current does not change along the conductor \((n=0)\), we achieve the distribution of field in accordance with Biot-Savart formulas:

\[ E_1 = E_3 = 0, \quad E_2 = jh(\varphi) \int_0^\pi [W_3 \cos \tau - RrW_2 \sin^2 \tau] d\tau \] (1.6)
2. Description of electromagnetic field in the distant and near zone

On the basis of formulas (1.4), (1.5), we may present description of field in the distant zone. Consequently, in this zone $\rho^2 = r^2 + z^2$; functions $W_0, W_1, W_2$ do not depend on the integration parameter $\tau$. In this case, components of field different from zero will exist only if $n = 1$. Namely:

$$E_1 = \frac{\pi}{2} h_0 (W_3 - \rho^2 W_2),\quad E_2 = \frac{\pi}{2} jh_0 W_3,\quad E_3 = -\frac{\pi}{2} h_0 z r W_2$$

$$H_1 = 0,\quad H_2 = -\frac{\pi}{2} j h_0 W_1,\quad H_3 = \frac{\pi}{2} h_0 \rho W_1$$

Additionally, by rejection of non significant values, we may assume:

$$W_1 = -\frac{j k}{\rho} W_0,\quad W_2 = -\frac{k^2}{\rho^2} W_0,\quad W_3 = -k^2 W_0$$

Similarly in the near zone (induction)

$$W_0 = \frac{1}{\rho^2},\quad W_1 = -\frac{1}{\rho^3},\quad W_2 = \frac{3}{\rho^5},\quad W_3 = \frac{1}{\rho^3}$$

3. Description of electromagnetic field, taking into account eddy currents in the metal cylinder

For the sake of range of paper, we limit the examination of influence of Maxwell displacement currents on the distribution of electromagnetic field on the surface of metal cylinder, to the case of axial symmetry only. With such an assumption, we may also take into account displacement current, though exciting current does not change along the circular conductor. There is another situation, when current flows in the infinite long straight conductor; In such a situation, it is possible to omit the displacement current only, if exciting current does not change along the conductor.

In the axially symmetrical case, electric field will have the circumferential component $E$ only and magnetic field will have the radial component $H_1$ and axial component $H_2$:

$$\vec{E} = (0, E, 0),\quad \vec{H} = (H_1, 0, H_2)$$

These components meet the equation:
\[ \Delta E + \left( k^2 - \frac{1}{r^2} \right) E = 0, \quad \Delta H_1 + \left( k^2 - \frac{1}{r^2} \right) H_1 = 0, \quad \Delta H_3 + k^2 H_3 = 0, \quad k = \frac{\omega}{c} \quad (3.2) \]

Additionally, we assume, that radius \( R_0 \) is great comparing to the depth of field penetration in the metal:
\[ R_0 \gg \sqrt{\frac{2}{\omega \mu \gamma}} \quad (3.3) \]

So, we may apply the impedance boundary conditions for \( r = R_0 \):
\[ E = -\frac{\alpha}{\gamma} H_3, \quad \alpha^2 = j \omega \mu \gamma \quad (3.4) \]

Solutions of equations (3.2) may be presented using Fourier integrals; If impedance condition (3.4) is met, we obtain:

a) if Maxwell displacement currents are omitted (\( k = 0 \equiv c = \infty \)):
\[ E = \int_0^\infty S(\tau) K_1(\tau \gamma) \cos \gamma \tau \, d\tau + E_+ (r, z), \quad E_\pm = \begin{cases} \int_0^\infty Q(\tau) K_1(\tau \gamma) \cos \gamma \tau \, d\tau, & r \geq R \\ \int_0^\infty Q_\pm(\tau) l_1(\tau \gamma) \cos \gamma \tau \, d\tau, & r \leq R \end{cases} \quad (3.5) \]

where:
\[ Q_\pm = \frac{K_1(\tau R)}{l_1(\tau R)} Q, \quad Q = \frac{\pi j \omega \mu_0 l_1(\tau R)}{4 [K_0(\tau R) l_1(\tau R) + K_1(\tau R) l_0(\tau R)]}, \]
\[ S = \frac{\alpha}{\gamma} [K_0(\tau R_0) l_1(\tau R_0) - j \omega \mu_0 l_1(\tau R_0)] Q_\pm \quad (3.6) \]
\[ H_1 = -\frac{1}{j \omega \mu_0} \int_0^\infty \tau \cdot S(\tau) K_1(\tau \gamma) \sin \gamma \tau \, d\tau + \frac{1}{j \omega \mu_0} \int_0^\infty \tau \cdot Q_\pm K_1(\tau \gamma) \sin \gamma \tau \, d\tau + H_{1\text{e}} \]
\[ H_{1\text{e}} = \begin{cases} -\frac{1}{j \omega \mu_0} \int_0^\infty \tau \cdot Q_\pm K_1(\tau \gamma) \sin \gamma \tau \, d\tau, & r > R \\ -\frac{1}{j \omega \mu_0} \int_0^\infty \tau \cdot Q_\pm l_1(\tau \gamma) \sin \gamma \tau \, d\tau, & r < R \end{cases} \quad (3.7) \]
We may determine the component \( H_3 \) from the formula:

\[
H_3 = \frac{1}{j \omega \mu_0} \int_0^\infty \tau \cdot SK_0(\pi) \cos \tau \, d\tau + H_{3*}, \quad H_{3*} = \begin{cases}
\frac{1}{j \omega \mu_0} \int_0^\infty \tau \cdot QK_0(\pi) \cos \tau \, d\tau, & r \geq R \\
-\frac{1}{j \omega \mu_0} \int_0^\tau \tau \cdot QI_0(\pi) \cos \tau \, d\tau, & r \leq R
\end{cases}
\] (3.8)

b/ taking into account Maxwell displacement currents \((k \neq 0)\):

\[
E = \int_0^{r_0} C_1 H_1^{(2)}(\delta r) \cos \tau \, d\tau + \int_{r_0}^\infty C_0 K_0(\delta r) \cos \tau \, d\tau + E_0
\]

\[
E_0 = \begin{cases}
\int_0^{r_0} C_1 J_1(\delta r) \cos \tau \, d\tau + \int_{r_0}^\infty C_3 I_1(\delta r) \cos \tau \, d\tau, & r \leq R \\
\int_0^{r_0} C_2 H_1^{(2)}(\delta r) \cos \tau \, d\tau + \int_{r_0}^\infty C_4 K_0(\delta r) \cos \tau \, d\tau, & r \geq R
\end{cases}
\] (3.9)

\[
\delta^2 = k^2 - \tau^2, \quad \delta^* = \tau^2 - k^2, \quad \tau_0 = \frac{\omega}{c} = k
\]

\[
H_3 = -\frac{1}{j \omega \mu_0} \int_0^{r_0} C_5 \delta H_0^{(2)}(\delta r) \cos \tau \, d\tau - \int_{r_0}^\infty C_6 \delta K_0(\delta r) \cos \tau \, d\tau + H_{30}
\]

\[
H_{30} = -\frac{1}{j \omega \mu_0} \int_0^{r_0} C_1 \delta I_0(\delta r) \cos \tau \, d\tau + \int_{r_0}^\infty C_3 \delta I_0(\delta r) \cos \tau \, d\tau, \quad r \leq R
\] (3.10)

where:

\[
C_1 = -\frac{j \omega \mu_0}{\pi \delta} H_1^{(2)}(\delta R), \quad C_2 = -\frac{J_1(\delta R)}{\pi \delta} \frac{J_1}{M_1}, \quad C_3 = \frac{J_1(\delta R)}{\pi \delta} \frac{J_1}{M_1}, \quad C_4 = \frac{I_1(\delta R)}{\pi \delta} \frac{I_1}{M_1}, \quad C_5 = \frac{\delta H_0^{(2)}(\delta R)}{\pi \delta} \frac{\delta I_1}{M_1}
\]

\[
C_6 = \frac{\delta H_0^{(2)}(\delta R)}{\pi \delta} \frac{\delta I_1}{M_1} - \frac{\delta H_0^{(2)}(\delta R)}{\pi \delta} \frac{\delta I_1}{M_1} - \frac{\delta H_0^{(2)}(\delta R)}{\pi \delta} \frac{\delta I_1}{M_1}
\]

\[
\alpha^2 = j \omega \gamma
\]


\[ H_1 = \frac{1}{j\omega \mu_0} E_2 = -\frac{1}{j\omega \mu_0} \int_0^\infty \tau Q \cdot \sin \tau z \, d\tau \]  

(3.11)

**Conclusions**

On the basis of performed calculations, the following conclusions may be presented:

a/ In the distant zone, electromagnetic field excited by currents, flowing in the circular coil, appears only, if \( n=1 \) in the equation (1.2).

b/ If axial symmetry does not exist (\( n \neq 0 \)), all components of electromagnetic field occur in the cylindrical coordinates.

**References**


II-2. IMPROVEMENT OF EFFECTIVENESS OF HIGH-CURRENT LINES OPTIMIZATION BY MODIFICATION OF GENETIC ALGORITHMS AND PARALLELING OF THE COMPUTATION PROCESS

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Abstract – The work presents a mathematical model of electrodynamic parameters calculation, determining the optimization process of high-current lines. Special attention was paid to speeding up the optimization calculation with the use of genetic algorithms with different modifications and with the use of the computers including several parallel operating processors. The optimization calculation and the discussion of obtained results were presented.

Introduction

Optimization of engineering systems is often a highly complex process. Introduction of multi-criterial analysis of systems and more and more sophisticated mathematical methods designed for calculating of object parameters give rise to the problems of accuracy and duration of the optimization process. An important factor in such cases is appropriate choice of optimization method and the use of suitable modifications of the chosen method (according to considered technical problem). Additionally, parallel execution of the calculation may be considerably helpful in efficient realization of the task. The paper presents optimization problem of three-phase high-current lines. For the purpose of optimizing the genetic algorithms were applied, with their various modifications. For particular modification variants the effectiveness of optimization process was analyzed. The effect of parallel organization of the calculation process on the time of optimization search was examined.

Description of the system and electrodynamic computation methods

The object of the analyses of the present paper is a three-phase screened air-insulated high-current line (power busway). It consists of three phase conductors, made in the form of oval cross-section tubes, distributed symmetrically each 120°, inside a screen of circular cross-section.

Geometry of the system (Fig. 1) is characterized by the following values: thickness of conductor wall (g), major (a) and minor (b) axis of the oval cross-section, height of conductor suspension (h) and internal radius of the screen (R_s). Thickness of screen wall (t_s) is assumed to be constant and amounts to 3 mm.

External radius of the screen (R_{sz}) and height of insulator (h_{iz}) are determined by the above mentioned values. Hence, the analyzed system is characterized by five independent variables: a, b, g, h, R_s.

Fig 1. Geometrical parameters of the system
In order to determine selected electrodynamical parameters (reckoned among optimization elements) of shielded three-phase power busways (power loss, temperatures, electrodynamics forces, voltage gradients) the method of integral equations was used. Starting from the equations of magnetic vector potential $A$ written for the areas of the analyzed system respectively one may define the distribution of current density $J(r, \phi)$ of the phase conductor on the ground of an approximate solution of the system of integral equations obtained from the known relationship of electromagnetic field $E = -j\omega A$ and $J = \gamma E$

$$J(r, \phi) - J(r_0, \phi_0) + \frac{3}{4\pi} j\omega \mu_0 \gamma_c \int \int J(r', \phi') [K(r', \phi', r, \phi) - K(r', \phi', r_0, \phi_0)] dr' d\phi' = 0$$

(1)

$$\int J(r', \phi') r' dr' d\phi' = 1$$

(2)

where: $(r_0, \phi_0)$ is a reference point, $\gamma_c$ is conductivity of conductor material, $I$ - current intensity in the phase, $K(r', \phi', r, \phi)$ - a kernel of the integral equation:

$$K(r', \phi', r, \phi) = \sum_{i=1}^{\infty} \left[ a_i \sin(i(\phi - \phi')) + b_i \cos(i(\phi - \phi')) \right] \frac{x^i}{i} + \sum_{i=1}^{\infty} \left[ F_{1i} \sin(i(\phi - \phi')) + F_{2i} \cos(i(\phi - \phi')) \right] (r r')^i$$

(3)

Coefficients occurring in the equation were defined by the works [3, 4].

It results from symmetry of the system that distribution of current density of two remaining phase conductors (S and T) is the same as in R but shifted by +120° and -120°, respectively.

Similar relationship remains in force for the shield, with an additional equation:

$$\int J_s(r', \phi') r' dr' d\phi' = 0$$

(4)

The presented system of integral equations may be solved in approximate manner using a moment method, being a variation of Ritz method. Methods of solving the above questions and determining the coefficients occurring in equations are discussed in detail in the works [3, 4].

Knowledge of approximate distribution of current density vector enables to determine the value of active power loss in the conductor and its shield (based on Joule's law). Knowledge of the active power losses and the distribution of power density emitted in the conductors and in the screen is necessary for determining thermal conditions of the system [3, 4]. Due to voltage and current values the optimization results depend rather on temperature distribution than on electrodynamic forces and voltage gradients. However, these parameters are also determined during the optimization process.

**Optimization criterion**

The optimization is aimed at determining such dimensions of the system for which a minimal amount of conductor and screen material shall be used, for assumed values of rated voltage, rated current, and the parameters characterizing the conductors and the screen. Therefore, the optimization consists in minimization of a function $S(u)$ determining the area of the busway cross-section, written in a general form [2, 3]:

$$S(u) = f(u_1, u_2, u_3, ..., u_r)$$

(5)

where: $u_1, u_2, u_3, ..., u_r$ - decisive variables ($r=5$), namely the values of $a, b, g, h, $ and $R_s$.

Constraints of the optimization process are admissible electrodynamical and thermal parameters (temperatures of working lines $T_{C_{max}}$ and of the screen $T_{S_{max}}$, electrical strength of the system $E_{max}$, the forces acting in steady conditions and in short-circuit state $F_{max}$), as well as admissible ranges of variation of the geometrical dimensions.
A set of the above mentioned constraints $Z_i$ in a general form is given by the function:

$$Z(u) = [Z_1(u), Z_2(u), ..., Z_k(u)]$$  \hspace{1cm} (6)

In order to convert the presented problem to a constraint-free optimization problem the criterion function $S_Z(u)$ has been modified:

$$S_Z(u) = S(u) + \sum_{i=1}^{k} P_i (Z_i)^2 N[Z_i]$$  \hspace{1cm} (7)

where:

$$N[Z_i]{\begin{cases}1 & \text{for } Z_i > 0 \text{ infringement of the constraints} \\0 & \text{for } Z_i \leq 0 \text{ fulfilment of the constraints} \end{cases}}$$

The formula (7) includes $P_i$ - a so-called penalty factor, of high value, being positive for the case of minimization $S(u)$ (and negative for maximization).

Due to the dependence of criterion function included extremized variables on delimiting parameters (the values of which are to be determined in result of complex numerical calculation) and due to the constraints included in the objective function, the modified criterion function can not be written in an explicit form. This imposes an additional problem that complicates the optimization calculations and considerably extends the time consumed for getting the results.

The optimizing method is based on genetic algorithms. Therefore, the criterial function was converted into an adaptation function (bringing the problem to a maximization task) [2].

**Genetic algorithm and effectiveness of its modification**

The genetic algorithm in its elementary version includes the following operations: selection according to the roulette rule, simple crossing with random linking, and simple mutation.

Optimization methods in their classical forms often do not take into account the nature of considered problem and, therefore, are less effective. However, their effectiveness may be improved in result of some modifications.

The selection according to the roulette principle is a process characterized by considerable variation, i.e. providing important spread of actual number of the copies around its expected value. The paper presents and implements modification of the selection operation based on the model of De Jong's expected values and Brindle's random selection using the remainders without repetitions.

In order to avoid the loss of the best existing individual two variants of a method of keeping the best of existing solutions were proposed: such a solution is kept 1) always, or 2) provided that the adaptation factor of the best individual of the $P(t+1)$ population is lower than for the best individual of the $P(t)$ population (exclusivity model).

Another modification method consists in scaling of the adaptation. In the presented algorithm linear scaling was applied. Adjustment scaling is aimed at preventing domination of the above average individuals at the beginning of the computation process, that could result, on the one hand, in stagnation of the procedure in a local maximum, and, on the other hand, in increasing the differences in accommodation factors of the individuals in final stage of the computation, when the average accommodation factor only slightly departs from its maximal value [2, 3].

During the calculation the course of the genetic algorithm was monitored for 100 generations. Probabilities of crossing and mutation were constant and amounted to 0.8 and 0.005, respectively. For computation purpose binary coding was used. Every individual (chromosome) included 20 genes (bits). Constant population size was maintained, including 20 individuals.

Examples of optimization search are shown in Figs. 2 - 9. In order to depict better the existing relationships the charts of absolute values of the adaptation coefficients have been used (referred to optimal values for each of the solutions).
Fig. 2. Optimization course in case of selection according to the roulette principle

Fig. 3. Optimization course in case of selection with regard to remainders without repetitions

Fig. 4. Optimization course in case of selection according to the roulette principle, scaling of accommodation factor

Fig. 5. Optimization course in case of selection with regard to remainders without repetitions, scaling of accommodation factor

Fig. 6. Optimization course in case of selection according to the roulette principle, scaling of accommodation factor, "unconditional" transferring of the best individual

Fig. 7. Optimization course in case of selection with regard to remainders without repetitions, scaling of accommodation factor, "unconditional" transferring of the best individual