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by

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To Marian
Also to Jennifer, Andrea, James and Julia

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Foreword

John Harris 1929–2003

In an engineering career which spanned almost fifty years, John Harris lost none of his enthusiasm for new fields of exploration. In his late sixties he began his research into fuzzy logic with his drive and sense of endeavour undimmed. At the time he was inaugural Chair of Industrial Engineering at the new National University of Science and Technology, Bulawayo, Zimbabwe, having dedicated almost his entire professional life to the development of educational opportunity in engineering science in sub Saharan Africa, a career which led him to Chair of the department of Mechanical Engineering at the University of Nairobi, Kenya, then subsequently to set up and chair the department of Mechanical Engineering in the University of Zimbabwe, Harare, thence to the Chair of the Mechanical Engineering department at the University of Malawi, returning for his last post to Zimbabwe, to the new university in Bulawayo.

The preface to this text, probably the last words my father ever wrote, reflects his belief in the relevance of fuzzy logic and its potential for applications at a functional level in society. He presents fuzzy logic as a theoretical perspective, a prism, through which he invites the viewer to share his understanding of industrial systems in our society.

The book was intended to be fourteen chapters. My father completed and edited the first nine chapters of this volume while fighting a titanic battle against disease. He chose to leave the expert care and comfort of St Michael's Hospice to return to die at home, where, against all odds and with the unfailingly patient and practical assistance of his son, James Harris, nine chapters were finalised. Chapters ten to fourteen inclusive existed in unedited form and some of this latter group of chapters include papers published previously in professional journals, which were intended to be reworked for inclusion. I am grateful to the Institution of Mechanical Engineers and to the Institution of Chemical Engineers for their permission to reproduce papers at chapters eleven, twelve and thirteen. The last five chapters therefore represent work in progress. When I came to work on the manuscript for chapter fourteen, the last of this volume, I found the pages suddenly empty of my father's pencilled edits.

Primarily, of course, this academic publication was created as an educational springboard text, to encourage others to take research forward and continue exploration of the fascinating interface between fuzzy logic and engineering.

I should like to thank all those who have helped and supported the efforts to bring this book to publication, including Derek Duffet for his kind assistance, Nathalie Jacobs and all at Springer Science and Business Media and my family, especially my mother, Marian Harris, to whom this book is principally dedicated.

Julia Harris
March 2005

Preface

Over the ages, there are many who have contemplated the perfect disc of the full moon in the night sky, the perfection of which we know to be an optical illusion made possible by the distance between the object and the observer. This is a factor which dissolves the imperfections that are apparent to the close observer. The perfect disc is an abstraction of the type which is of the essential nature of scientific and therefore engineering thinking. The abstractions enable precise relationships to be formulated without the hindrance of the complexity of imperfect detail. The human mind has a propensity to search for abstractions and also to classify and generalise them. The advance of civilisations, and more specifically here, science and engineering as we know it, would have been impossible without this phenomenon. It can, however, have penalties as in stereotyping, which have all too often in history lead to injustices.

Associated with the perfect abstractions of the mind is the concept of precision; that of flawless conformance to the ideal. This too has its roots in the philosophy of the Ancient Greeks and has a pervasive influence in many cultures, but especially in the western world where it has been a salient feature of scientific development. Another vital element of Ancient Greek philosophy which has governed thought not only in science but in many other fields such as law, medicine and theology, is that of classical; (Aristotelian) logic. This represents the strict classification of real or abstract objects into sets of which they are either perfect members or not.

Judging by the history of the past few centuries, science and engineering have been remarkably successful in the application of these principles. The fruits of success have been the results of the influence of the concepts of abstraction, precision and classical logic applied in a highly organised manner which the general public sees in the form of space flight, nuclear power and computers, for example. At the same time it will be recalled that there have also been periodic disasters that make national news, some natural others not such as aircraft and rail crashes, nuclear plant failures, submarine losses and many other types due to technical failures or human unreliability. Besides the catastrophic disasters that readily spring to mind, there are also cases of failure, not small in number, that could be traced to design failures, many of these do not make headline news. This does not include cases where a system simply does not meet the required performance specification and the loss is at the lower end of the criticality scale. Increasingly, system reliability is high on the public agenda.

Amongst the most valuable assets of any organisation is that of its professional knowledge base, part of which will be documented and explicit and part of which is dispersed and will reside in the minds of the organisation's staff. The latter is "free" in the sense that it has not been reduced to document form, it is volatile as staff transfer between different organisations. The total knowledge base governs the capacity and proficiency of an organisation to react to technical problems. In practice it would frequently be the case that the induction process for creating relationships would be accomplished by searching the free knowledge base to obtain consensus expert opinion, which may include contracted judgement. Fuzzy logic methods fit naturally and easily into the broader picture of knowledge engineering and more generally into asset management. These factors are to a large extent unrecognised in current engineering practice and offer the potential for a profound change in outlook at the strategic level. This text approaches the issue at a tactical level, but the full possibilities will be apparent.

Fuzzy logic has a much wider and deeper foundation than is implied in this practical text. To learn a new language one can study the grammar or learn by examples. Professional engineers and students with time constraints would usually opt for the latter method and this is the aim in both this and the companion text, *An Introduction to Fuzzy Logic Applications*. The benefits of the new language are; more flexibility and generality in the formulation and solution of problems; nonlinear problems are easily encompassed. Also input information is often more fully represented than in conventional treatments and furthermore carried through to the conclusions which consequently have a higher information content. This is a key factor, enabling more reliable decisions to be made by professional engineers and designers.

This work extends and complements that the companion text. It opens up new avenues of applications and new aspects of fuzzy logic. There is a stronger emphasis on the interpretation of the conclusions.

This radical new approach to problems in engineering science and also to professional engineering procedures avoids over representing the precision of information and knowledge and the approximate use of classical logic which is implicit in current practice. Applications in the research literature are fairly sparse and in the professional literature they are practically non-existent. It is anticipated that the absorption of fuzzy logic methods into engineering practice will take time, but it is certain that the advantages and the need to reappraise the current processing of information and formulation of knowledge will prevail and that the methods will become part of the engineer's intellectual tool kit. But in the meantime there is the need for a substantial educational drive to propagate the awareness of the potential of fuzzy logic methods. The way in which this will happen will be through further seminars, short courses and undergraduate or postgraduate courses, all of which require the support of suitable texts. At present there is a very limited selection of texts to serve this purpose. This text is addressed to engineering lecturers, researchers extending the frontiers of knowledge, professional engineers and designers and also students. A hallmark of fuzzy logic methods is that the cultural gap between researchers and practitioners is not apparent, the linguistic formulation of problems and conclusions is equally coherent to both.

I would like to acknowledge the invaluable help given by Derek Duffett and James Harris in the final stages of compiling the text. Any flaws that remain are mine. Comments on any aspects of the text would be welcome.

John Harris
October 2003

Chapter 1
Comments and Definitions

Most people during the course of their education acquire a knowledge of classical logic, even if it is only of a hazy nature. The hallmark is an acceptance that statements are either “true” or “false”: the car will or will not start, the tennis match has or has not been postponed. Such is the basis of classical (Aristotelian) logic (CL). Later, one appreciates that some propositions have an expectation element when they are concerned with future events: the car probably will not start tomorrow morning (based upon experience), the tennis match will probably be postponed if the weather is bad. Such uncertainty is sometimes treated by statistical methods, based upon historical data, though at a less sophisticated level, more often on simple guesswork.

Although CL is a special case of fuzzy logic (FL), its greater familiarity makes it a useful point of departure in this text.

(A guide to fuzzy logic is provided in the companion text, *An Introduction to Fuzzy Logic Applications*. See also Further Reading.)

1.1. Classical Logic Basics

1.1.1. DEFINITIONS

Sets. The basic concept in CL, as in FL, is that of a set. This is conceived as a collection of objects, which may be tangible or intangible, having some common attribute or feature, such as shape, colour, type or use (for example). The common denominator is the categorical characteristic which is the defining feature of the set. A set may comprise, for example, the whole numbers between 0 and 10, another example could be the members of staff of a particular hospital. Sets are denoted in this text by capital letters, thus X.

Subsets. These are sets within sets and have two attributes, one defining the set and one the subset. Subsets are also denoted by capital letters, thus $Y \subset X$, where $\subset$ means subset Y is contained within set X.

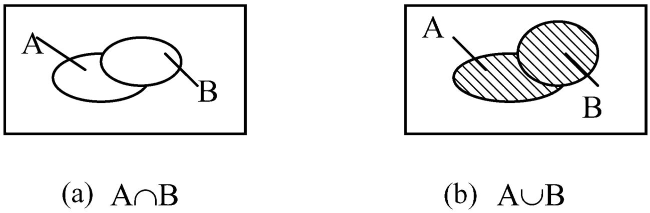

Figure 1.1. Venn diagrams of classical logic operations. (a) Intersection. (b) Union.

Elements. The individual members of a set are called its elements, and are denoted by lower case letters, thus p. Also, $p \in P$ means that element p is a member of the set P. Elements may be one or more discrete features, or point values of a continuum. The set containing only one element is called the unit set.

If the set is to be specified by listing all the elements, it is written thus; $\{a, b, c\}$, in which the order of the elements within the brackets is immaterial. As an example, $S = \{0, 1\}$ represents the set of electric light switch positions. The switch is either "on" or "off".

Special sets.

(i) Universal set. This comprises all the elements in the population. It is denoted by 1.
(ii) Null set or empty set. This contains no elements. It is denoted by Ø.

Complimentary set. The compliment of set X is set X', where $X + X' = 1$, which is the universal set. It follows that the complement of the universal set is the null set.

Logic Operations. Several CL operations on sets are defined. The intersection AND is denoted by ∩. The union OR is denoted by ∪. The two operations are conveniently illustrated by means of the Venn diagrams, as shown in Figures 1.1(a) and 1.1(b).

In the Venn diagrams in Figure 1.1, the surrounding rectangle represents the universe of all sets in the genre.

The OR operation shown above is the inclusive OR, it must be distinguished from the exclusive OR labelled XOR. This means set A or B, but not both. The XOR operation is not used in this text. Other operations in the literature are NAND and NOR, neither of which are needed here.

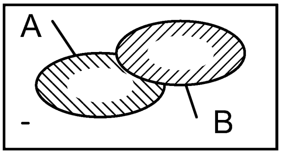

Figure 1.2. Venn diagram showing fuzzy intersection.

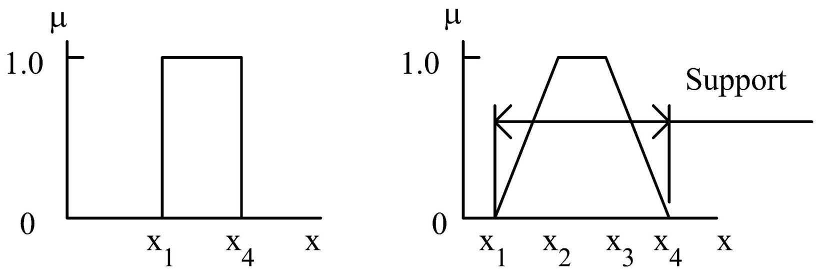

Figure 1.3. Comparison of typical set membership patterns. (a) Classical set. (b) Fuzzy set.

1.2. Fuzzy Logic (FL)

1.2.1. FUZZY SETS AND VENN DIAGRAMS

The sets in CL all have precise boundaries. In FL this requirement is relaxed and therefore the set boundaries in this case are imprecise. The sets on a FL Venn diagram therefore appear as shading, illustrating boundary zones. The imprecise shaded zone defining the boundaries of the sets are the zones in which partial membership, μ $(0 < \mu < 1.0)$ of the sets by the elements is attributed. A fuzzy Venn diagram is illustrated in Figure 1.2

The pattern of membership values of elements of sets is often conveniently displayed on a diagram. Figure 1.3 compares membership functions in the CL and FL cases, where the elements are members of a *continuum*. Values of the elements are set off along the abscissa and membership values of the elements are set off along the ordinate. In Figure 1.3(a), for $0 < x < x_1$, the membership value is zero. For $x_1 < x < x_4$ the membership value is unity, whilst for $x > x_4$ the membership value is again zero. Figure 1.3(a) defines a CL set. In the case of Figure 1.3(b), for $0 < x < x_1$ the membership value is zero, whilst for $x_1 < x < x_2$ it rises from zero to unity. For $x_2 < x < x_3$ the membership value is unity and for $x_3 < x < x_4$ the membership declines again from unity to zero. Beyond x_4 the membership value is again zero.

1.2.2. FUZZY SET SHAPES

The geometry of fuzzy set shapes may take on a variety of forms, but is subject to the requirement that any element must not have more than one membership value of a particular set. Simple possible shapes are illustrated in Figure 1.4. The singleton

Figure 1.4. Typical fuzzy set membership function shapes.

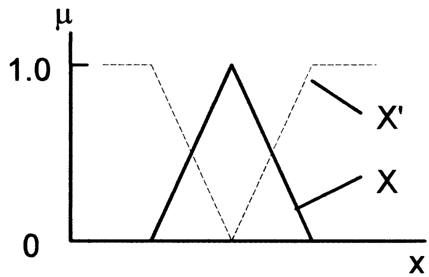

Figure 1.5. A fuzzy set and its complement.

is a special case of a set with only one element, which has unity membership value. Fuzzy sets may comprise continuous or discrete distributions of members. It is sometimes convenient in applications to approximate a continuous distribution of elements by a discrete distribution. The triangular form of membership function is the most common, and will be extensively used in this work.

1.2.3. THE COMPLEMENTARY FL SET

If X is a fuzzy set, then the complimentary fuzzy set is defined as $X' = 1 - X$. The set and its complement are illustrated in Figure 1.5.

1.2.4. UNIVERSE OF DISCOURSE (UD)

The elements of a universe are usually distributed amongst a number of sets and in FL they are often members of more than one set. They generally have fractional membership values of the several sets. The fuzzy sets are defined on a universe of discourse of categories of a particular attribute. For example, Figure 1.6 shows sets on a UD of speed in which the categories are varying degrees of "fast".

1.3. Measurement Scales

Although in engineering practice most elements are described in terms of numerical continua, there are various types of scales that are used. UDs can be described on several different scales, each with it own application. Four types are described below:

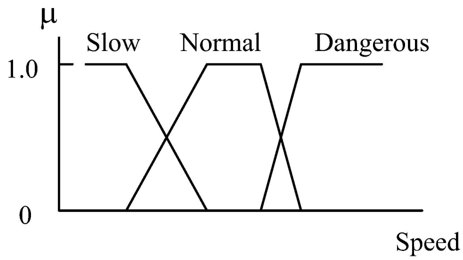

Figure 1.6. The universe of discourse of speed.

(i) Ordinal scales. The elements on this scale are placed in some sort of order of progression, for example, the stations on a railway line or items in a programme.
(ii) Nominal scale. The elements represent discrete categories or conceptual objects, for example, types of screw thread or types of non-ferrous metal.
(iii) Interval scale. Units or intervals of a given size marked out in sequence, for example, the intervals on a clock face.
(iv) Ratio scale. An interval scale with a zero point at some arbitrary or agreed value, for example, a tape measure division.

This offers a range of scales suitable for different applications. The essential feature of a scale is that it reflects a common theme amongst the elements which defines the UD.

1.4. Propositions

Logic processes are conducted in the form of propositions, the simplest of which comprises an antecedent (premise) and a conclusion, IF A THEN B, e.g., if that is a litre of water then it will weigh one kilogram (at sea level). CL asserts that the membership value is 1 or 0.

Consider a FL proposition: If the brake pedal is pressed, then the car will stop. Now there are various levels of brake pressure that may be applied and there are various levels of stopping from an emergency stop to gradual slowing down. Thus there are various categories of stopping and a CL conclusion is not possible. The resolution of this type of problem is the theme of the remainder of this text.

Compound propositions can take several different forms. One of the more important and frequently used forms is: IF A AND B THEN C, for example, if the distance of a hotel along a road is possibly between one and five kilometers and the distance to a garage is possibly between three and seven kilometers (both $+ve$) then the distance between them is somewhere between three and five miles. This case is illustrated in Figure 1.7.

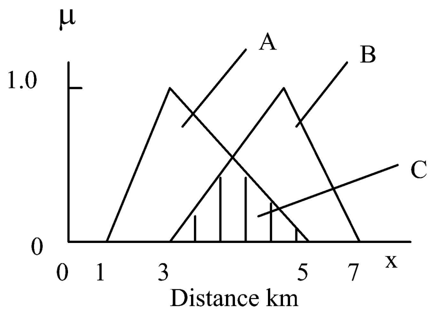

Figure 1.7. Illustration of fuzzy intersection.

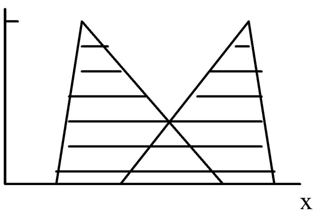

Figure 1.8. Illustration of fuzzy union.

Another important and often used case of a compound proposition is expressed as: IF A OR B THEN C. For example, if the wind is strong or very strong then it is unsafe for novice sailors to race. This case is illustrated in Figure 1.8.

1.5. Notation

In this work sets and subsets are labelled with capital letters. The elements of a set are written in lower case letters and contained in brackets { }. A fuzzy number is expressed as a fuzzy set. A fuzzy number may be represented in discrete or continuous form. The discrete form is of the type

$$A = [\mu_1//a_1 + \mu_2//a_2 + \mu_3//a_3 + \cdots]. \tag{1.1}$$

The elements with their membership values (μ_i) are enclosed in square brackets [ ]. The membership value is separated from its corresponding element (a_i) by the symbol //; this does not denote devision. The + sign within the [ ] brackets denotes continuation, it does not denote addition.

The *continuum* form of fuzzy number is expressed as

$$X = \int \mu(x)//x, \tag{1.2}$$

where $\int$ does not mean "integral of ...", but "continuous distribution of ...".

In the solution of problems it is often useful to represent fuzzy sets as piecewise continuous distributions. Discrete values are also used to represent continuous dis-

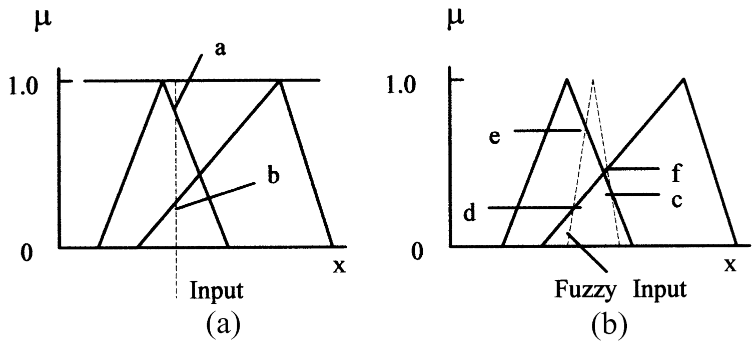

Figure 1.9. Assigning input data membership values. (a) Singleton input. (b) More general fuzzy input.

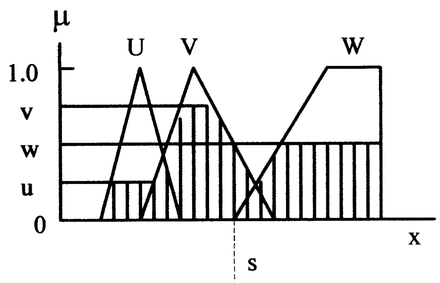

Figure 1.10. The union of several fuzzy subsets.

tributions. The volume of work in manipulating discrete sets may be reduced by considering principal sets, as defined in the Appendix.

1.6. Fuzzification and Defuzzification

1.6.1. FUZZIFICATION

The operations in FL are performed in terms of fuzzy sets. In practice, the input data may also be in terms of fuzzy sets or a singleton (single element with a membership value of unity), which is infact a special type of fuzzy set. The input data needs to be assigned membership values of one or more fuzzy sets into which the UD has been partitioned. The membership values are found from the intersections of the data sets with the fuzzy sets of the UD. Figure 1.9(b) illustrates the graphical method of finding membership values in the case of a singleton (Figure 1.9(a)), and the more general fuzzy input (Figure 1.9(b)).

For the singleton in Figure 1.9(a), there are two intercepts, i.e., at *a* and *b*, which determine the membership values. Whilst for the fuzzy input in Figure 1.9(b) there are four intercepts at *c*, *d*, *e* and *f* which determine the membership values.

1.6.2. DEFUZZIFICATION

This means the reduction of the fuzzy set or subset to a singleton. The fuzzy set is usually the union of several subsets representing the conclusion of a fuzzy proposition. Normally, a fuzzy set cannot be represented by a singleton, therefore defuzzification can only be undertaken with the loss of information. The union of several subnormal (no membership value equal to unity) fuzzy subsets is illustrated in Figure 1.10 and s is the single element on the UD which is deemed to represent the union of the fuzzy subsets. Such a representation discards the span of the conclusion and the membership values of the subsets. But for calculations in design (for example) a specific value is required and provides the motivation for defuzzifying, but it is important not to loose sight of the whole solution.

There are several ways of finding a representative number. Two common ways are outlined below.

(i) Centroid method. This is probably the most frequently used method and as the name suggests, it involves finding the position of the centre of area of the subsets on the abscissa (s).

$$\text{Continuous distribution:} \quad s = \int_{x=0}^{x=\infty} x \, \mathrm{d}a \Big/ \int \mathrm{d}a, \tag{1.3}$$

$$\text{Discrete distribution:} \quad s = \sum_{i=1}^{i=n} x_i \delta A_i / \sum \delta A_i. \tag{1.4}$$

(ii) Weighted abscissa method. This is evaluated by taking the sum of the normalised weighting of each of the set principal values, $x_i\,(\max)$

$$s = \sum_{i=1}^{i=n} \mu_i x_i\,(\max) \Big/ \sum \mu_i. \tag{1.5}$$

In the trapezoidal shape of fuzzy set, it is the mid-support value that is used for x_i. The centroid and weighted abscissa methods generally give somewhat different values of the defuzzified representative number. It may be observed that given a representative number, it is generally not possible to recover the original fuzzy subsets.

1.7. Equivalent (Triangular) Fuzzy Number (EFN)

Given the union of two fuzzy sets, as illustrated in Figure 1.11, it is possible to find a symmetrical triangular fuzzy number that would produce the same fuzzy subsets by intersection with the partitioning fuzzy sets on the UD. This is called the Equivalent Fuzzy Number (EFN). There is less loss of information with this representation than with the defuzzified number. Given an EFN, it is possible to

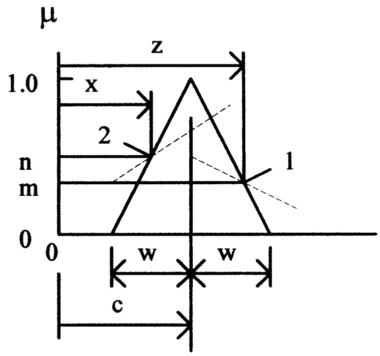

Figure 1.11. An Illustration of the equivalent fuzzy number.

recover two pairs of fuzzy subsets, one of which is the original fuzzy subset, but generally not with a defuzzified number.

Figure 1.11 shows a symmetrical EFN in which points 1 and 2 represent points of intersection of the EFN with the two different fuzzy sets on the UD. The membership values of the intersection points 1 and 2 are m and n, respectively, therefore from the membership functions of the fuzzy sets the values of z and x at the intersections may be obtained. Now by similar triangles,

$$(c - x)/(1 - n) = (c + w - z)/m = w/1.$$

Hence,

$$c + w - z = mw. \tag{1.6}$$

Therefore,

$$c - z = w(m - 1). \tag{1.7}$$

Also,

$$c - x = w(1 - n), \tag{1.8}$$

Subtracting Equation (1.7) from Equation (1.8),

$$z - x = w(2 - n - m).$$

Hence,

$$w = (z - x)/(2 - m - n). \tag{1.9}$$

Also from Equation (1.7),

$$c = x + w(1 - n). \tag{1.10}$$

Equations (1.9) and (1.10) determine the principal value (c) and the support of the EFN ($2w$). At the cross-over point $z = x$ and hence $w = 0$, the fuzzy number