IUTAM Symposium on One Hundred Years of Boundary Layer Research
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The fundamental questions arising in mechanics are: Why?, How?, and How much?
The aim of this series is to provide lucid accounts written by authoritative researchers
giving vision and insight in answering these questions on the subject of mechanics as it
relates to solids.

The scope of the series covers the entire spectrum of solid mechanics. Thus it includes
the foundation of mechanics; variational formulations; computational mechanics;
statics, kinematics and dynamics of rigid and elastic bodies; vibrations of solids and
structures; dynamical systems and chaos; the theories of elasticity, plasticity and
viscoelasticity; composite materials; rods, beams, shells and membranes; structural
control and stability; soils, rocks and geomechanics; fracture, tribology; experimental
mechanics; biomechanics and machine design.

The median level of presentation is the first year graduate student. Some texts are
monographs defining the current state of the field; others are accessible to final year
undergraduates; but essentially the emphasis is on readability and clarity.

For a list of related mechanics titles, see final pages.
IUTAM Symposium on
One Hundred Years of Boundary Layer Research
Proceedings of the IUTAM Symposium held at DLR-Göttingen, Germany, August 12–14, 2004

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## CONTENTS

**Preface** ix

### Session 1: Classification, Definition and Mathematics of Boundary Layers

<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prandtl’s Boundary Layer Concept and the Work in Göttingen</td>
<td>1</td>
</tr>
<tr>
<td>G.E.A. Meier</td>
<td></td>
</tr>
<tr>
<td>The Full Lifespan of the Boundary-Layer and Mixing-Length Concepts</td>
<td>19</td>
</tr>
<tr>
<td>P.R. Spalart</td>
<td></td>
</tr>
<tr>
<td>Rational Basis of the Interactive Boundary Layer Theory</td>
<td>29</td>
</tr>
<tr>
<td>J. Cousteix, J. Mauss</td>
<td></td>
</tr>
<tr>
<td>Symmetry Methods in Turbulent Boundary Layer Theory</td>
<td>39</td>
</tr>
<tr>
<td>M. Oberlack, G. Khujadze</td>
<td></td>
</tr>
<tr>
<td>Viscous/Inviscid Interaction Procedures for Compressible Aerodynamic Flow Simulations</td>
<td>49</td>
</tr>
<tr>
<td>M. Hafez, E. Wahba</td>
<td></td>
</tr>
</tbody>
</table>

### Session 2: Instability of Boundary Layers and Transition

<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Application of Optimal Control to Boundary Layer Flow</td>
<td>59</td>
</tr>
<tr>
<td>D.S. Henningson, A. Hanifi</td>
<td></td>
</tr>
<tr>
<td>Leading-Edge Boundary Layer Flow (Prandtl’s Vision, Current Developments and Future Perspectives)</td>
<td>73</td>
</tr>
<tr>
<td>V. Theofilis, A.V. Fedorov, S.S. Collis</td>
<td></td>
</tr>
<tr>
<td>Application of Transient Growth Theory to Bypass Transition</td>
<td>83</td>
</tr>
<tr>
<td>E. Reshotko, A. Tumin</td>
<td></td>
</tr>
<tr>
<td>Routes of Boundary-Layer Transition</td>
<td>95</td>
</tr>
<tr>
<td>V.S. Kachanov</td>
<td></td>
</tr>
<tr>
<td>Instabilities in Boundary-Layer Flows and their Role in Engineering</td>
<td>105</td>
</tr>
<tr>
<td>J.D. Crouch</td>
<td></td>
</tr>
<tr>
<td>In-Flight Investigations of Tollmien-Schlichting Waves</td>
<td>115</td>
</tr>
<tr>
<td>A. Seitz, K.-H. Horstmann</td>
<td></td>
</tr>
<tr>
<td>The Influence of Roughness on Boundary Layer Stability</td>
<td>125</td>
</tr>
<tr>
<td>M. Gaster</td>
<td></td>
</tr>
</tbody>
</table>
Boundary-Layer Instability in Transonic Range of Velocities, with Emphasis on Upstream Advancing Wave Packets
O.S. Ryzhov, E.V. Bogdanova-Ryzhova

Laminar-Turbulent-Laminar Transition Cycles
R. Narasimha

Session 3: Boundary Layers Control

A Century of Active Control of Boundary Layer Separation: A Personal View
I.J. Wygnanski

Boundary Layer Separation Control by Manipulation of Shear Layer Reattachment
P.R. Viswanath

Stability, Transition, and Control of Three-Dimensional Boundary Layers on Swept Wings
W. Saric, H. Reed

Transition to Turbulence in 3-D Boundary Layers on a Rotating Disk (Triad Resonance)
T.C. Corke, E.H. Matlis

Control and Identification of Turbulent Boundary Layer Separation
A. Seifert, L. Pack Melton

Session 4: Turbulent Boundary Layers

The Near-Wall Structures of the Turbulent Boundary Layer
J. Jiménez, G. Kawahara

Turbulence in Supersonic and Hypersonic Boundary Layers
A.J. Smits, M.P. Martin

The Role of Skin-Friction Measurements in Boundary Layers with Variable Pressure Gradients
H.-H. Fernholz

The Mean Velocity Distribution near the Peak of the Reynolds Shear Stress, Extending also to the Buffer Region
K.R. Sreenivasan, A. Bershadskii

Session 5: Numerical Treatment and Boundary Layer Modelling

Turbulence Modelling for Boundary-Layer Calculations
W. Rodi
## Contents

<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instability and Transition in Boundary Layers: Direct Numerical Simulations</td>
<td>257</td>
</tr>
<tr>
<td>H. F. Fasel</td>
<td></td>
</tr>
<tr>
<td>Wall Modeling for Large-Eddy Simulation of Turbulent Boundary Layers</td>
<td>269</td>
</tr>
<tr>
<td>P. Moin, M. Wang</td>
<td></td>
</tr>
<tr>
<td>Revisiting the Turbulent Scale Equation</td>
<td>279</td>
</tr>
<tr>
<td>F. R. Menter, Y. Egorov</td>
<td></td>
</tr>
<tr>
<td>Industrial and Biomedical Applications</td>
<td>291</td>
</tr>
<tr>
<td>F. Smith, N. Ovenden, R. Purvis</td>
<td></td>
</tr>
<tr>
<td>Analysis and Control of Boundary Layers: A Linear System Perspective</td>
<td>301</td>
</tr>
<tr>
<td>J. Kim, J. Lim</td>
<td></td>
</tr>
<tr>
<td>The Development (and Suppression) of very Short-Scale Instabilities in Mixed Forced-Free Convection Boundary Layers</td>
<td>313</td>
</tr>
<tr>
<td>P.W. Duck, J.P. Denier, J. Li</td>
<td></td>
</tr>
<tr>
<td>Computational Studies of Boundary-Layer Disturbance Development</td>
<td>325</td>
</tr>
<tr>
<td>C. Davies</td>
<td></td>
</tr>
</tbody>
</table>

### Session 6: Special Effects in Boundary Layers

<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hypersonic Real-Gas Effects on Transition</td>
<td>335</td>
</tr>
<tr>
<td>H.G. Hornung</td>
<td></td>
</tr>
<tr>
<td>Stabilization of Hypersonic Boundary Layer by Microstructural Porous Coating</td>
<td>345</td>
</tr>
<tr>
<td>A.A. Maslov</td>
<td></td>
</tr>
<tr>
<td>The Asymptotic Structure of High-Reynolds Number Boundary Layers</td>
<td>355</td>
</tr>
<tr>
<td>P.A. Monkewitz, H.M. Nagib</td>
<td></td>
</tr>
<tr>
<td>Instabilities near the Attachment-Line of a Swept Wing in Compressible Flow</td>
<td>363</td>
</tr>
<tr>
<td>J. Sesterhenn, R. Friedrich</td>
<td></td>
</tr>
<tr>
<td>Structure Formation in Marginally Separated Aerodynamic and Related Boundary Layer Flows</td>
<td>373</td>
</tr>
<tr>
<td>A. Kluwick, St. Braun</td>
<td></td>
</tr>
<tr>
<td>High Reynolds Number Turbulent Boundary Layers Subjected to Various Pressure-Gradient Conditions</td>
<td>383</td>
</tr>
<tr>
<td>H. M. Nagib, Chr. Christophorou, P. A. Monkewitz</td>
<td></td>
</tr>
<tr>
<td>Analysis of Adverse Pressure Gradient Thermal Turbulent Boundary Layers and Consequence on Turbulence Modeling</td>
<td>395</td>
</tr>
<tr>
<td>T. Daris, H. Bézard</td>
<td></td>
</tr>
<tr>
<td>Title</td>
<td>Page</td>
</tr>
<tr>
<td>----------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>The Significance of Turbulent Eddies for the Mixing in Boundary Layers</td>
<td>405</td>
</tr>
<tr>
<td>C.J. Kähler</td>
<td></td>
</tr>
<tr>
<td>Unstable Periodic Motion in Plane Couette System: The Skeleton of Turbulence</td>
<td>415</td>
</tr>
<tr>
<td>G. Kawahara, S. Kida, M. Nagata</td>
<td></td>
</tr>
<tr>
<td>Some Classic Thermal Boundary Layer Concepts Reconsidered (and their Relation to Compressible Couette Flow)</td>
<td>425</td>
</tr>
<tr>
<td>B.W. van Oudheusden</td>
<td></td>
</tr>
<tr>
<td>Vorticity in Flow Fields (in Relation to Prandtl’s Work and Subsequent Developments)</td>
<td>435</td>
</tr>
<tr>
<td>T. Kambe</td>
<td></td>
</tr>
</tbody>
</table>

**Poster-Presentation**

<table>
<thead>
<tr>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>An Experimental Investigation of the Brinkman Layer Thickness at a Fluid-Porous Interface</td>
<td>445</td>
</tr>
<tr>
<td>A. Goharzadeh, A. Saidi, D. Wang, W. Merzkirch, A. Khalili</td>
<td></td>
</tr>
<tr>
<td>Experimental Investigations of Separating Boundary-Layer Flow from Circular Cylinder at Reynolds Numbers from $10^3$ up to $10^5$ (Three-dimensional Vortex Flow of a Circular Cylinder)</td>
<td>455</td>
</tr>
<tr>
<td>B. Gölling</td>
<td></td>
</tr>
<tr>
<td>Scale-Separation in Boundary Layer Theory and Statistical Theory of Turbulence</td>
<td>463</td>
</tr>
<tr>
<td>T. Tatsumi</td>
<td></td>
</tr>
<tr>
<td>On Boundary Layer Control in Two-Dimensional Transonic Wind Tunnel Testing</td>
<td>473</td>
</tr>
<tr>
<td>B. Rasuo</td>
<td></td>
</tr>
<tr>
<td>Theory of Boundary Layer Instability: Particle or Wave?</td>
<td>483</td>
</tr>
<tr>
<td>K.-Kh. Tan</td>
<td></td>
</tr>
</tbody>
</table>
Prandtl’s famous lecture with the title “Über Flüssigkeitsbewegung bei sehr kleiner Reibung” was presented on August 12, 1904 at the Third Internationalen Mathematischen Kongress in Heidelberg, Germany. This lecture invented the phrase “Boundary Layer” (Grenzschicht). The paper was written during Prandtl’s first academic position at the University of Hanover. The reception of the academic world to this remarkable paper was at first lukewarm. But Felix Klein, the famous mathematician in Göttingen, immediately realized the importance of Prandtl’s idea and offered him an academic position in Göttingen. There Prandtl became the founder of modern aerodynamics. He was a professor of applied mechanics at the Göttingen University from 1904 until his death on August 15, 1953. In 1925 he became Director of the Kaiser Wilhelm Institute for Fluid Mechanics. He developed many further ideas in aerodynamics, such as flow separation, base drag and airfoil theory, especially the law of the wall for turbulent boundary layers and the instability of boundary layers en route to turbulence.

During the fifty years that Prandtl was in the Göttingen Research Center, he made important contributions to gas dynamics, especially supersonic flow theory. All experimental techniques and measurement techniques of fluid mechanics attracted his strong interest. Very early he contributed much to the development of wind tunnels and other aerodynamic facilities. He invented the soap-film analogy for the torsion of noncircular material sections; even in the fields of meteorology, aeroelasticity, tribology and plasticity his basic ideas are still in use. Aside from the boundary layer and the boundary layer equations for which Prandtl rightly occupies an immortal place, his name lives through the Prandtl number, Prandtl’s momentum transport theory and the mixing length, the Prandtl-Kolmogorov formula in turbulence closure, the Prandtl-Lettau equation for eddy viscosity, the Prandtl-Karman law of the wall, Prandtl’s lifting line theory, Prandtl’s minimum induced drag, the Prandtl-Meyer expansion, the Prandtl-Glauert rule, and so forth. The string of young men he mentored is nothing short of remarkable. Among them we easily recognize Ackert, Betz, Blasius, Flachsbart, Karman, Nikuradse, Schiller, Schlichting, Tietjens, Tollmien and Wieselsberger. The list could, of course, be larger.
Preface

The hundredth anniversary of Prandtl’s invention was the first reason for us to apply for an IUTAM Symposium “One Hundred Years of Boundary Layer Research”. The other reason was to summarize the progress in the field by inviting the best known specialists for related contributions. The overwhelming response led to the many interesting lectures and contributions collected in these proceedings.

We thank F. Smith, R. Narasimha, H. Hornung, T. Kambe, I. Wygnanski, A. Roshko, P. Huerre, E. Reshotko, K. R. Sreenivasan for the revision of the manuscripts and helpful advice.

We especially appreciate Dr. Hans-Joachim Heinemann’s organisation of the meeting and his work managing the edition of the proceedings, without which the task would have been impossible. Monika Hannemann provided our internet presentation, Oliver Fries was responsible for finances, Helga Feine, Catrin Rosenstock and Monika Hannemann managed the conference office, and Karin Hartwig assisted in the preparation of the symposium.

All the technical organization and support was provided by the Institute of Aerodynamics and Flow Technology, DLR Göttingen, directed by Prof. Dr. Andreas Dillmann. We appreciate this support very much.

The Editors and the Managing-Editor are very grateful to Mrs. Anneke Pot, Senior Assistant to the Publisher, and Springer, Dordrecht, The Netherlands, for the excellent support and help in publishing this book.

It is our hope that the readers of this book will find it as pleasant as we do and discover new views on boundary layers and the related research which flows from Ludwig Prandtl’s work in 1904.

Göttingen, August 2004

G.E.A. Meier and K.R. Sreenivasan
(Cochairmen)
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PRANDTL’S BOUNDARY LAYER CONCEPT
AND THE WORK IN GÖTTINGEN

A historical view on Prandtl’s scientific life

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Abstract: The invention of the “Boundary Layer” by Ludwig Prandtl goes back to his
famous lecture in August 8, 1904 with the title “Über Flüssigkeitsbewegung
bei sehr kleiner Reibung” which was held at the “III. International Mathema-
tischen Kongreß” in Heidelberg. These proceedings and the related IUTAM
Symposium celebrate the 100th anniversary of this event. The following his-
torical remarks will be a short record of Prandtl’s scientific life with emphasis
on his “Boundary Layer” work.

Key words: Ludwig Prandtl, history, scientific work, fluid mechanics, boundary layer.

1. PRANDTL’S EDUCATION AND HIS EARLY
PROFESSIONAL CAREER

Ludwig Prandtl was born February 4, 1875 in Freising, Bavaria. His fa-
thor was a professor at an agricultural school in Weihenstephan. He spent his
school years in Freising and lived later in Munich until 1894. After gradua-
tion from school he studied eight semesters of “Maschinentechnik” (me-
chanical engineering) at the Technical High School in Munich where he was
awarded the degree of a “Maschineningenieur” (mechanical engineer) in
1898. Professor August Föppl was his teacher in Technical Mechanics and
became his mentor later on. Prandtl spent an additional year in Föppl’s
laboratory for his dissertation at the University of Munich as a doctor of
philosophy, because the Technical High School was not allowed to provide a doctoral thesis in those days. His Dissertation with the title “Kipperscheinungen, ein Fall von instabilem elastischem Gleichgewicht” was the foundation of his scientific carrier.

In the beginning of the year 1900 he was affiliated as an engineer at the “Maschinenfabrik Augsburg-Nürnberg” (MAN) in Augsburg. There he was involved with work on diffusers for wood cutting machines. When designing for this company a device for sucking dust and splices, Prandtl noticed that the pressure recovery he expected from a divergent nozzle was not realized. Soon he detected the still famous rule that half the divergence angle of a diffuser may not be larger than about 7° in order to avoid separation of the decelerating flow. In those experiments his ideas of a special behavior of the near wall parts of the flow field have been born obviously. Already there, he was confronted with the phenomenon of flow separation and this consequently was the initiation of his interest in flow phenomena and the real reason of his invention of the boundary layer concept [1, 2]. Later as a professor at the University of Hanover he showed the compatibility of his boundary layer approximations with the Navier-Stokes Equations which led to a development of historical dimensions.

Fig. 1: Unsteady separation and the first closed loop tunnel.

Already in October 1901 Prandtl became a full professor of mechanics at the Technical High School of Hanover. There he built his first hand driven
water tunnel for his elucidating experiments (Fig. 1). The flow was seeded on the free surface for visualizing the separation and vortices. He spent only three years in this place and position, but he published several important papers and finally also the famous lecture at the “III. Internationaler Mathematischer Kongress in Heidelberg 1904” with the title “Über Flüssigkeitsbewegung bei sehr kleiner Reibung”, which publication nowadays is seen as the publication presenting the discovery of the boundary layer concept and as the beginning of the related research (Fig. 2).

An asymptotic approach to the full momentum equation.

This lecture in Heidelberg was also the reason for the famous mathematician Felix Klein, who was a professor of mathematics in the University of Göttingen, to offer Prandtl a university position in Göttingen as an Extra Ordinarius. Although Prandtl had to step back this way from a full professorship, he finally took the position to change into an environment with his own laboratory and to contact the famous scientists in the University of Göttingen [1,2,3,5].

2. THE EARLY WORK IN GÖTTINGEN

In 1905 Felix Klein also motivated the mathematician Carl Runge to come from Hanover to Göttingen, with the three later founding the “Institut
Für Angewandte Mathematik und Mechanik” and this became a very fruitful scientific environment for themselves and their students in the following years. Following the common enthusiasm about aeronautics together with Runge in 1907, Prandtl held his first seminar on aerodynamics in the University.

Prandtl directed in this institute the PhD works of Blasius, Boltze and Hiemenz covering boundary layer problems. Prandtl’s former own work in boundary layer theory has been continued with the thesis of Blasius in 1908 on laminar boundary layer development on a flat plate. Blasius solved Prandtl’s boundary layer equations in his PhD thesis for the flat plate successfully. Boltze solved in 1908 the laminar boundary layer for a body of revolution and Hiemenz in 1910 solved the laminar boundary layer for a cylinder in cross flow.

In 1906 the young Theodor von Karman from Hungary was asking Prandtl for a PhD opportunity and was promoted in 1908 to Göttingen with a topic in the field of elasticity. Later, in connection with the work of Hiemenz and Rubach, he invented the “Wirbelstraße” (vortex street). Already in April 1913 von Karman became a professor at the Technical High School of Aachen.

The organised boundary layer and turbulence research started in 1909 with the PhD works of Hochschild, Rubach, Kröner, Nikuradse and Dönch. In this early work, one can see the beginning of Prandtl’s interest in turbulence research and flow control (Fig. 3). Later this research work was intensified by his contributions to the problem of the drag of a sphere.
In measuring the drag on spheres, scientists like Prandtl and Eiffel from Paris were very surprised about large differences in the drag coefficients measured in their wind tunnels. The contradiction in drag coefficients for spheres, which differed by 50%, finally could be explained by the different separation at different Reynolds numbers. It was Prandtl who explained these discrepancies with an “experimentum crucis” where he introduced for the first time a trip wire at the wall to change the state of the boundary layer from laminar to turbulent. Prandtl made this special experiment with the trip wire to demonstrate that also in case of lower Reynolds numbers, the drag figures of the supercritical regime could be achieved.
Using the trip wire with the wind tunnel set at a constant speed, the drag could be reduced considerably. It was once again the different separation location which led to this phenomenon. He clearly pointed out that due to the more downstream separation in case of a turbulent boundary layer, the pressure drag is reduced substantially. The test results could finally be understood by Prandtl’s boundary layer theory with the introduction of the critical Reynolds number for transition (Fig. 4) \[1,4\].

Inspired by the experiments in the habilitation thesis of W. Nusselt, Prandtl discovered in 1910 the analogy between heat convection and friction in fluid boundary layers. His idea was based on the analogy between the differential equations of heat convection and flow in the vicinity of the wall. In connection with his boundary layer theory, he solved some problems for laminar and turbulent flows on plates and through tubes. Later in 1928 he improved this simulation by introducing more precise properties of the turbulent flow. Honouring his work in this field, the ratio of cinematic viscosity and temperature conductivity was called the “Prandtl Number” later on. But, since Nusselt had used this ratio in his former work, Prandtl was never very accepting of this honour (Fig. 5).
Fig. 5: By the analogy of the equations of flow and heat convection, Prandtl established a mapping of heat exchange in flows over walls.

In the years after 1912, Carl Wieselsberger was one of the important scientists in Prandtl’s “Aerodynamische Versuchsanstalt (AVA)” (aerodynamic research establishment). Wieselsberger mainly conducted drag measurements for airships and airfoils (Fig. 6). Also the drag of sails was measured in the wind tunnel and the results have been compared with those of Gustave Eiffel from Paris, France.

Fig. 6: Comparable drag of a small disk and a streamlined body.

In 1920 Prandtl realized that the drag of a flat plate is closely related to the drag of a straight pipe by considering that only the flow field close to the wall (the boundary layer) is important for the friction effects. This also implies that the velocity distribution near the wall is determined only by the
law of friction. So he concluded that the flow velocity is proportional to the square root of wall distance as previously shown by Blasius for pipe flow.

This finally resulted in the law called “Universelles Wandgesetz” - the universal law of the wall. The theory for the friction on a flat plate by Blasius for the laminar case from 1908 and Prandtl’s own theory from 1921 for the turbulent case were verified for Reynolds Numbers close to a million by Liepmann and Dhawan in 1951 (Fig. 8).

By later experiments with high Reynolds numbers, Prandtl in parallel to von Karman came to the conclusion that by a logarithmic formulation introducing the shear stress velocity, a fully universal law for the velocity distributions near the wall could be achieved (Fig. 9).

With respect to the description of the fully developed turbulent flow Prandtl had introduced in 1924 the term “Mischungsweg” (mixing length). His idea was that fully developed turbulence is characterized by some characteristic length, after which the eddies loose their individuality. He mainly used this idea to understand the momentum exchange between the turbulent eddies and to explain the turbulent shear stress this way. The mixing length formulation for the turbulent shear stress which is in essence identical to the earlier formulation by Reynolds was independently invented by Prandtl in 1926. His formulation had the advantage of introducing the wall distance $y$ and a typical constant which later by von Karman was found to be $k=0.4$ (Fig. 7). The mixing length concept led to some useful theoretical considerations for the mixing of a free jet by W.Tollmien and some interpretations of the velocity profiles in ducts with rectangular cross sections by Dönch.

In 1926, Prandtl discovered on the basis of measurements of Nikuradse in rectangular and triangular ducts, turbulent secondary flows which had not been observed in the laminar case. Prandtl understood these phenomena as a consequence of the momentum exchange in the three dimensional turbulent flow. This was far from any possible theoretical treatment in those days. In contrast to the secondary flows in curved ducts he named these phenomena secondary flows of the second kind.

In 1907, Prandtl rejected an offer of the Technical High School of Stuttgart to become a full professor of Technical Mechanics; he preferred to stay in Göttingen to finish his plans for a “Modellversuchsanstalt” and to stay in the fruitful scientific environment of the Alma Mater there [3,5,6,7].

3. **THE “KAISER WILHELM INSTITUT FÜR AERODYNAMIK”**

When in Berlin 1910, the plans for the founding of the “Kaiser Wilhelm Gesellschaft” (KWG) became virulent, Felix Klein had the idea to propose a
“Kaiser Wilhelm Institute for Aerodynamics”. The purpose was mainly to keep Prandtl in Göttingen by providing him with an institute for all problems of aerodynamics and hydrodynamics. Prandtl himself later wrote a proposal for this research institute which was consisting of a “Kanal-Haus” with all kinds of test tubes and water test facilities for flow experiments, a machine house, a calibration chamber, shops and finally a flying station for measurement in open air. In recognition of Prandtl’s merits in sciences and especially in aerodynamics and hydrodynamics, this institute was granted by the “Kaiser Wilhelm Gesellschaft” in June 1913.

But in 1914 the First World War began and so the plans for the founding of the Kaiser Wilhelm Institute were postponed. Only the wind tunnel project, which was important for the aircraft industry could be completed in 1917. Also in these difficult times, Prandtl could only use about one third of the wind tunnel time for research purposes. Special reports, the so called “Technische Berichte”, dealt with problems of airfoil sections, drag of fans and coolers, and design of fuselage and propellers. In cooperation with Monk and Betz, Prandtl also made remarkable progress in his airfoil theory.

Fig. 6: Left: Prandtl studying turbulence. Right: Grid turbulence.
In August 1920, Prandtl was offered to become successor of his father in law August Föppl on a full chair for mechanics at the Technical High School in Munich. This was very attractive for him because many of his supporters in Göttingen like von Böttinger and Felix Klein faded away and the situation of the “Versuchsanstalt” was not very good.

So after this offer, a time of difficult negotiations started to keep Prandtl in Göttingen. His intention to switch from the more applied research in the “AVA” to a more scientific research in the frame of a fully developed “Kaiser Wilhelm Institute” and to get rid of the lectures at the university was a difficult problem in those days, since the financial situation of the government and the “Kaiser Wilhelm Gesellschaft” was poor. But finally, also with the help of his friends in the administration and in industry, he was granted a directorship in a “Kaiser Wilhelm Institute” and could keep his full professorship for Technical Physics in the University of Göttingen as well. The main reason that these negotiations came to a successful end was that the scientific community and also the administration realized that there was nobody else who could replace Prandtl at Göttingen in those days.

All the work of Prandtl in the years after the First World War was devoted to the aerodynamics of transport vehicles. Mainly, the aerodynamic problems of civil aircraft but also the drag and smoke emissions of railway steam engines and the drag of racing cars and automobiles were studied.
Prandtl could start in 1924 building his new institute which had a laboratory for gas dynamic experiments and later also a rotating laboratory which was designed for studies of atmospheric flows. The rotating laboratory was at first operated by the young Busemann studying the influence of Coriolis forces on the flows in an open water tank. Beside the scientific results, he got all information about dealing with sea sickness.

For the new “Kaiser-Wilhelm-Institut”, which was physically built in 1924, Prandtl named beside, gas dynamics and cavitation, mainly boundary layers, vortices, and viscous flows as the targets of research. Among the experimental facilities were two towing tanks for boundary layer and wake studies. The bigger one had a length of 13 meters.

In this way, two institutes existed since 1925 in parallel, as Prandtl was the director of the “Kaiser Wilhelm Institute für Strömungsforschung” and the AVA, which was in fact directed by the deputy director Albert Betz.

Already in 1924 Prandtl became honorary member of the London Mathematical society and in 1927 he was invited for the Wilbur Wright Memorial Lecture by the Royal Aeronautical Society. In those years, he also got honorary PhD’s from the Universities of Danzig and Zürich, Switzerland. Later he was honoured in the same way in Bukarest, Cambridge, Istanbul, Prag and Trondheim.

In the twenties, Prandtl’s work was devoted mainly to the problems of the origin of turbulence and the properties of turbulent flows (Fig. 6). The first studies of instability of laminar boundary layers had been conducted by Tietjens in his dissertation. In 1925, Prandtl published his results about the drag in pipes and the first ideas about his mixing length model for turbulent flows.
In the early twenties, Prandtl started intensive considerations about the origin of turbulence. He built a special tunnel about six meters long with a seeding possibility to observe the flow on the surface by floating particles. The intermittent vortices and waves he observed were not what he expected, because small amplitude distortions were considered to be stable in those days. Together with Tietjens, he found in theoretical considerations instability of the laminar flow with respect to small distortions. But these simplified theoretical considerations did not explain the stability of the boundary layer for small Reynolds numbers. From this experience he concluded that the understanding and quantitative treatment of turbulence was a futile task [1,3,5].

4. THE WORK OF PRANDTL IN THE THIRTIES

The reason why Prandtl was so important for the Research Centre in Göttingen was mainly due to his work in the field of boundary layers. By consideration that friction in flows with small viscosity is only important in the
vicinity of walls, the whole range of complex flow phenomena in vehicles and engines became transparent. Another field was airfoil theory which mainly, by the introduction of the induced drag, provided a foundation for all kind of airfoil designs. Since many other researchers and institutions were in those days doing successful research in this field, one can understand Prandtl’s idea to switch to new horizons in the newly built institute.

In the new institute for “Strömungsforschung”, Prandtl gathered a lot of young students, who became famous researchers later on, like J. Ackeret, H. Blenk, A. Busemann, H. Goertler, H. Ludwieg, J. Nikuradze, K. Oswaltitsch, H. Schlichting, R. Seifert, W. Tollmien, O. Tietjens, W. Wuest, and others. Counting the number of the resulting PhD thesis’s and his own publications, about one quarter of Prandtl’s work was devoted to boundary layer and turbulence research.

Prandtl had understood in the twenties with his initial ideas from the beginning of the century the main properties of the laminar boundary layer, the reasons for separation and also the consequences for pressure drag. Additionally, he also found the possibility of reducing the pressure drag by shifting the separation point downstream by diminishing the area of separated flow. But in the thirties he was still excited about the problem of instability of the boundary layers and the route to turbulence (Fig. 6). Around 1930, Prandtl studied the influence of stabilizing effects on turbulence especially by curved surfaces and stratified fluids.

An important step to understand the mechanisms of instability was the asymptotic theory, which was put in final form by W. Tollmien. This theory for first time provided the stability limit for the flat plate accurately. Contributions in this field had been made by Prandtl and Tietjens before but also Lord Rayleigh and W. Heisenberg had contributed in this field. With Tollmien’s method, Schlichting and Pretsch solved the problem for other geometries, especially for curved walls. But Prandtl was always a little bit skeptical about this theory because the predicted instability waves could not be seen in his simple experiments. So Prandtl built a new water tunnel, better designed for studying laminar flow, but after his own words it was impossible to avoid all the distortions from the intake so that here and there a “turbulence herd” appeared. This indicates that Prandtl observed turbulent spots, which was later introduced in the literature by Emmons, Schubauer and Klebanoff.

It took another fifteen years until the end of the Second World War that Schubauer and Skramstad in the NBS under the supervision of H. L. Dryden conducted experiments in a tunnel with very low turbulence to prove the concept of Tollmien-Schlichting instability waves.

But also the mechanism of transition of the boundary layers and the persistent turbulence, which were not really understood until now, were still
Prandtl’s concern and he proposed a semi empirical approach to use the momentum equation of stationary boundary layers with an input of turbulent velocity distributions. With an additional empirical approach for the shear stress at the wall he could calculate the velocity profiles of the turbulent boundary layer.

In 1936, Prandtl built a new “Wall Roughness Tunnel” which was a wooden construction with a 6m long test section where the pressure gradient could be varied. Many interesting papers about turbulent boundary layers by famous authors like Ludwig, Schultz-Grunow, Wieghardt and Tillmann are originating from there. In this context for Prandtl, the work of Ludwieg and Tillmann was very helpful. They made the most accurate measurements of the shear stress in turbulent boundary layers in those days. This way, the universal “Law of the wall” which had been proposed by Prandtl and also von Karman in the days of considerations about Prandtl’s earlier power law hypothesis could be confirmed in a more precise way as by the early measurements in the thirty’s performed by Nikuradse (Fig. 9).

Nikuradse later mainly contributed under the supervision of Prandtl with some striking experiments on the influence of wall roughness on the drag in pipe flow. These were important data for the industry, especially chemical
engineering. These data are still in use today and have been extended to all kinds of flow geometries (Fig. 10). Based on Nikuradse's experiments, Prandtl and Schlichting published in 1934 a paper about the drag of plates with roughness. Schlichting worked with Prandtl until 1939 when he became a full professor in Braunschweig. In 1957, he followed Betz as director of the AVA in Göttingen.

But it was also in the thirties that Prandtl’s interest changed and the work in the “Kaiser Wilhelm Institute für Strömungsforschung” shifted to other fundamental problems which made use of his former research experiments in boundary layer flows. For instance together with H. Reichert he studied the influence of heat layers on the turbulent flow and he spent as well some activity in meteorology. Prandtl also wrote in those years a contribution to “Aerodynamic Theory” which was edited by W. F. Durand. In this book, Prandtl described all the work which had been done up to that time in Göttingen. The “Aerodynamic Theory” became standard literature in the field and was really the breakthrough for Prandtl’s ideas and his fame in the international community [3,8].

Fig. 10: Nikuradse's drag measurements for pipes.
5. PRANDTL’S WORK IN THE FORTIES

Even in the war in 1941 Prandtl built a small wind tunnel for the study of laminar to turbulent transition studies. Here the work of H. Reichardt and W. Tillmann about turbulence structure has to be mentioned. In 1945, Prandtl published two papers: One on the transport of turbulent energy and the other one on three dimensional boundary layers. The question where in the boundary layer turbulent energy is created and how it is propagated into the flow was still addressed by Prandtl and some co workers up to his death in 1956.

After the Second World War, the “Kaiser Wilhelm Institute für Strömungsforschung” was transformed into the “Max Planck Institut für Strömungsforschung” (MPI) in Goettingen. In 1946, Prandtl retired from the directorship of the new MPI where Betz was his successor. After his retirement he had still a small group until 1951 where he studied the theory of tropical cyclones with E. Kleinschmidt. The main parts of the MPI were the two departments headed by Betz and Tollmien. In 1957 the AVA (Aerodynamische Versuchsanstalt) was established and the MPI-department of Betz was the core of the new AVA headed by Schlichting. Prandtl also gave up his chair in the University of Göttingen which was granted to Tollmien in 1947. Under Prandtl’s direction and by his initiative, 85 PhD theses have been conducted in the years from 1905 to 1947 at the University of Göttingen [6]. About 30 of these publications are devoted to problems of boundary layer and turbulence.

6. BOUNDARY LAYER WORK AFTER PRANDTL

The “Max Planck Institut für Strömungsforschung” was Prandtl’s scientific home for his last years and was always devoted to research on boundary layers and turbulence. Under Tollmien who followed Prandtl in 1956 as a director, the work in boundary layer instability, intermittency and turbulent structures was promoted in many doctoral theses. Also the work of Reichardt, Herbeck and Tillmann was directed on the structure and statistics of intermittent and turbulent flows. The work of Eckelmann and his co-workers with a newly built oil channel for extremely low Reynolds Numbers contributed to the ideas about the structure of sublayer instabilities and intermittency development.

In the seventies, pipe flow experiments found the locations where fluctuation energy is mainly generated and how it is propagating from this well defined location of generation into the boundary layer: Downstream with flow velocity and perpendicular to the wall with shear stress velocity. So something like a certain propagation angle for turbulent energy propagation is
defined by the two velocities locally. This is similar to the Mach angle in acoustics, defined by the flow velocity and the velocity of sound [12]. It is interesting that Prandtl’s question about the turbulent energy propagation was answered with the help of the shear stress velocity, which he introduced for his logarithmic law of the wall.

With the same pipe flow tunnel, Dinckelacker made interesting experiments on the influence of riblets on the boundary layer and friction. He was able to reduce the drag of turbulent pipe flow by more than 10%.

Until the end of fluid mechanics research in the Max-Planck-Institute, when it’s last director E.-A. Müller retired in 1998, a lot of work was done in vortex dynamics, turbulence control and the structure of turbulent boundary layers.

The successor of the former AVA in Göttingen, the “DFVLR-Institute für Strömungsmechanik” was headed since 1957 by Schlichting and had with Becker, Ludwieg, Riegels, Rotta and many others an excellent team for boundary layer research in the many wind tunnels of the institute but also in numerical and theoretical research projects.

Later, the “DLR Institut für Aerodynamik und Strömungstechnik”, also did a lot of work on boundary layers. The mysterious transition scenarios and the mechanisms of instability have been a major target in the years of improved experimental and numerical methods. Many interesting results for boundary layer instability have been received by solving the Navier Stokes equations numerically and also by experiments, using new optical tools, which have confirmed these results. The main finding was that the well known Tollmien-Schlichting-waves and other new instability forms undergo higher order instability processes which lead to new special wave forms and vortices which finally disintegrate in chaotic interaction [10,11].

One can say that from the initiative of Ludwig Prandtl as a scientist and organizer, boundary layer research was connected to the research centre in Göttingen for over 100 years from its reception and that we are proud to have hosted the related IUTAM symposium for the celebration in Göttingen.

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THE FULL LIFESPAN OF THE BOUNDARY-LAYER AND MIXING-LENGTH CONCEPTS

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Abstract: Ludwig Prandtl’s most penetrating contributions are approximations to the dynamics of fluids. As such, they are liable to be superseded, at the time it becomes possible to solve the original equations analytically or, more probably, to routinely obtain numerical solutions so accurate they solve the problem without explicit use of the approximations. The engineering value of the theories is distinguished from their educational and intuitive value. The purpose here is to envision when and how this shift will happen for the boundary-layer and mixing-length concepts, with an aside on lifting-line theory, thus defining in some sense the lifespan of Prandtl’s ideas.

Key words: Boundary layer, CFD, grid, mixing length, logarithmic layer, turbulence model, lifting line

1. BOUNDARY-LAYER THEORY

Engineering increasingly relies on Computational Fluid Dynamics. Few CFD codes use the boundary-layer equations today. They tend to be special-purpose codes, applied to the repeatable topologies and nearly-attached flow typical of airplanes in cruise, as opposed to vehicles, houses, factories, and airplanes landing. Examples of viscous-inviscid coupling are Boeing’s Tranair full-aircraft code and Drela’s MSES (Multiple-Element Streamline Euler Solver) airfoil code. Cruise and slightly off-design conditions for an airliner are an excellent application; the lower computing cost relative to Navier-Stokes codes allows multi-point, multi-disciplinary optimisation.

In tribute to Dr. W.-H. Jou