IUTAM Symposium on Elementary Vortices and Coherent Structures: Significance in Turbulence Dynamics
Aims and Scope of the Series

The purpose of this series is to focus on subjects in which fluid mechanics plays a fundamental role.

As well as the more traditional applications of aeronautics, hydraulics, heat and mass transfer etc., books will be published dealing with topics which are currently in a state of rapid development, such as turbulence, suspensions and multiphase fluids, super and hypersonic flows and numerical modelling techniques.

It is a widely held view that it is the interdisciplinary subjects that will receive intense scientific attention, bringing them to the forefront of technological advancement. Fluids have the ability to transport matter and its properties as well as transmit force, therefore fluid mechanics is a subject that is particularly open to cross fertilisation with other sciences and disciplines of engineering. The subject of fluid mechanics will be highly relevant in domains such as chemical, metallurgical, biological and ecological engineering. This series is particularly open to such new multidisciplinary domains.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of a field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

For a list of related mechanics titles, see final pages.
IUTAM Symposium
on Elementary Vortices
and Coherent Structures:
Significance in Turbulence Dynamics

Proceedings of the IUTAM Symposium
held at Kyoto International Community House, Kyoto,
Japan, 26-28 October 2004

Edited by

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Springer
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This Symposium was held under the auspices of

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*The 21st Century COE Program for Formation and an International Center of Excellence in the Frontiers of Mathematics and Fostering of Researchers in Future Generations,* and

*The 21st Century COE Program for Center of Diversity and Universal-ity in Physics.*
Professor Isao Imai, 7. 10. 1914 – 24. 10. 2004
It was on 24 October 2004, two days before the opening of the present IUTAM Symposium, that Professor Isao Imai passed away in Tokyo on cardiac insufficiency at the age of ninety. He was involved in this Symposium as one of the supervisors of its mother project, Special Project Research on *New Developments in Turbulence Study by Means of Turbulent Elementary Vortices* supported by Ministry of Education, Culture, Sports, Science and Technology, and his presence to the Symposium was expected if his health permitted. In the Opening Session of the Symposium, a tribute to his memory has been paid by Professor Keith Moffatt, President of the IUTAM who participated in the Symposium as a member of the Scientific Committee, and a silent prayer to him has been offered by all attendants of the Session. In view of his great achievements in the fields of *Theoretical and Applied Mechanics* and devoted contribution to the IUTAM for many years, it has been decided to dedicate this IUTAM Symposium to the memory of the late Professor Isao Imai.

He was born in 1914 in Tairen of Manchuria, China, and has grown up in Kobe since his family came back to Japan afterward. His outstanding gift was already apparent from his boyhood as he finished his elementary and middle school courses each one year earlier and entered The First High School in Tokyo. Then he proceeded to Tokyo University, Faculty of Science, Department of Physics, and obtained there the degree of B. Sc. in 1936 at the age of 21. He started his academic career at Osaka University as a research associate to Prof. S. Tomotika, and after two years he came back to Tokyo University as a lecturer, then was promoted to an associate professor and nominated Professor of Physics in 1950. In Tokyo University, he achieved brilliant contributions in science, brought up a number of excellent scientists and extended his scientific influence to general people until his retirement in 1975. Then he continued his academic activity in Osaka University for three years and again came back to Tokyo to teach in Kogakuin University.

His scientific works have mostly been made in the fields of fluid mechanics and mathematical physics. When he started his research in 1936, fluid mechanics has been in a stage of innovation. The traditional hydrodynamics, dealing with the perfect fluid devoid of the compressibility and viscosity of the fluid, had already been in a deadlock and new fields such as high-speed airflows, boundary layers and turbulence have been attracting researchers’ interest. The representatives of his early works, that is, wing theory of arbitrary sections, transonic similarity of high-speed airflows and theory of slow viscous flows, are all excellent contributions to these new fields. These subjects concerning the real-fluid effects in fluid flows had already been dealt with by several researchers but not always satisfactorily, and his solutions derived using his unique methods
based on complex analysis are known to be the most general, systematic and complete. He was awarded the Asahi Culture Prize in 1951 and the Prize and Imperial Award of the Japan Academy in 1959. He was elected the Honorary Member of the American Aeronautical Society (now AIAA) in 1962.

The physical generality and the mathematical elegance of his approach have been displayed beyond the realm of fluid mechanics. Inspired by the progress in astro-physics and nuclear-fusion research after the War, great interests of researchers have been directed to the new field of magneto-hydrodynamics (MHD). He extended his activity to this field and, using a new concept of virtual fluid, expressed the MHD flows in terms of ordinary fluid mechanics and reformulated the framework of the MHD. Furthermore, his interest has been directed to electromagnetism itself and, defining the electro-magnetic field as a mechanical system satisfying the conservation laws of the momentum and the energy, he reformulated the theory of electromagnetism according to the fluid-mechanical concepts and methodology. As the result, several ambiguities and errors have been removed from the formulas of the electromagnetic forces acting on the solid bodies.

Another important extension of his approach has been made to the theory of hyper-function. Taking the reverse view of the conventional concept of fluid mechanics as a branch of applied mathematics, he considered the hyper-function of Sato as a vortex-layer in fluid mechanics and reformulated the theory of hyper-function. Such a change of the view-point seems to provide us with a clear image of the hyper-function and open a broad way of its application to science and technology.

His ability has also been displayed in administrative posts both domestic and international. He was appointed the president of the Physical Society of Japan, the president of the Japan Society of Fluid Mechanics, the vice-president of the International Union of Pure and Applied Physics (IUPAP), and the bureau member of the International Union of Theoretical and Applied Mechanics (IUTAM). He also held visiting professorship in several universities such as Maryland University, Marseilles University, Cornell University, and Aachen Technical University. In appreciation of his outstanding academic contributions, he was honored as the Person of Distinguished Services in Culture in 1979 and awarded the Order of Cultural Merit in 1988 and the First Order of Merit in 1992.

Looking back his brilliant scientific works for many years, we should recognize his clear line of thought on fluid physics. That is to consider the fluid as a mechanical system subject to physical conservation principles and to derive the physical laws of fluid flows by solving the governing mathematical equations, using new concepts and methods if necessary. Here the fluid flows are often expressed as the singularities of the equations and their physical laws take the form of the asymptotic similarity around the singularities. If such an approach
of fluid physics may be called mathematical physics of fluids, the outstanding feature of his works, the “clarity and generality”, seems to stem from his stoical adherence to mathematical physics of fluids.

Although he himself has not dealt with any particular problem of turbulence, he has always been encouraging the efforts by younger people to upgrade turbulence research to the stage of mathematical physics of fluids. Actually we may notice several papers in the present Symposium along this line of idea.

Now the subject of fluid physics is expanding from the classical Newtonian fluid to electromagnetic fluid, quantum fluid, reacting fluids, atmospheric and oceanic fluids, cosmic and planetary fluids and so on, and all of these new fluids are known to be associated with some kind of turbulent phenomena. It should be our pleasant task to take our steps of turbulence research forward to such new fields of mathematical physics of fluids.

Tomomasa Tatsumi
Emeritus Professor of Kyoto University
Elementary Vortices and Coherent Structures: Significance in Turbulence Dynamics, Kyoto, Japan, 26 – 28 October 2004
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Preface

By the elementary vortices, we mean those tubular swirling vortical structures with concentrated vorticity which are commonly observed in various kinds of turbulent flows as the smallest coherent motion. The elementary vortices play key roles in turbulence dynamics (e.g. enhancement of mixing, diffusion and resistance) and characterizes turbulence statistics (e.g. intermittency). Because of their dynamical importance, manipulation of elementary vortices and coherent structures is expected to be effective and useful in turbulence control as well as in construction of turbulence modeling. Besides its dynamical significance, the vortical structure is convenient in describing and understanding turbulence structure (e.g. skeleton representation with vortex axes), and the topological characterization by knottedness and crossing number. The vorticity equation has rich mathematical structures. The physics and mathematics of vortex reconnection and the finite-time singularity for both Euler and Navier-Stokes equations are related unresolved problems.

For the purpose of deepening our understanding on these subjects and searching new perspectives in theory, prediction, and control of turbulence the IUTAM Symposium entitled *Elementary Vortices and Coherent Structures — Significance in Turbulence Dynamics* was held during 26 – 28 October 2004, at Kyoto International Community House, Kyoto, Japan. There were eighty registered participants, representing eleven countries. The scientific program was composed of 5 sectional lectures of 50 minutes each, 16 regular lectures of 25 minutes each, and 21 posters with oral presentations of 3 minutes each. Most of these presentations are included in this proceedings. All the papers were refereed by all members of Scientific Committee and revised appropriately. They are divided into six groups: (A) Vortex dynamics, (B) Coherent structures, (C) Chaotic advection and mixing, (D) Statistical properties of turbulence, (E) Rotating and stratified turbulence, (F) Instability and transition, (G) Dynamics of thin vortices, (H) Finite-time singularity, and (I) Superfluid turbulence.

It was two days before this Symposium that Professor Isao Imai passed away at the age of ninety. It is really our deep grief that we lost such a distinguished and respectful scientist. He was one of the most influential scientists known worldwide in fluid mechanics and mathematical physics: high-speed flows, viscous flows, magnetohydrodynamics, theory of hyperfunctions, and so on. He was the member of IUTAM bureau in 1984 – 1992. At the opening of the Symposium a memorial address for him was given by Professor H.K. Moffatt, the President of IUTAM, and a silent prayer was offered by all participants. This proceedings is devoted to him with an obituary written by Professor T. Tatsumi, the former member of IUTAM bureau.
This Symposium would not have been performed successfully without devoted cooperation of many people. Professor T. Kambe, the representative of Japan in IUTAM, has encouraged and supported us continuously from the planning stage. The selection of papers for presentation in the Symposium as well as for publication in this proceedings was made based upon strict reviews by the Scientific Committee members. All the practical preparation of the Symposium, including the arrangement of the Symposium venue, the setup of the Symposium homepage, the raising of the financial support, and so on and so forth, was accomplished by the Local Organizing Committee members through their time-consuming efforts. All the miscellaneous tasks, occasionally being unexpected and confusing, were dealt with smoothly by Ms. Y. Shichida, the secretary general, with assistance of Ms. I. Goto. We would like to express our hearty gratitude to all of these people as well as all the participants of the Symposium who activated it.

Generous supports to the Symposium are gratefully acknowledged for IUTAM, the Commemorative Organization for the Japan World Exposition (‘70), Inoue Foundation for Science, and the 21st Century COE Programs in Kyoto University for Research and Education on Complex Functional Mechanical Systems, for Elucidation of the Active Geosphere (KAGI21), for Formation and an International Center of Excellence in the Frontiers of Mathematics and Fostering of Researchers in Future Generations, and for Center of Diversity and Universality in Physics.

Shigeo Kida

August 2005
Part A  Vortex dynamics
The spatial distribution of intense structures in isotropic turbulence is studied from numerical and experimental data. Box-counting of the intense vorticity and strain rate sets gives evidence of a strong clustering at intermediate scales, from which a possible fractal dimension can be defined. Algebraically distributed free intervals between intense velocity derivative from experimental time series confirms this self-similar clustering at larger Reynolds numbers, but without further specifying its dimensionality.

Keywords: Turbulence, clustering, box-counting, fractals, dimension

1. Introduction

Regions of high levels of dissipation and of vorticity in turbulent flows, such as vortex sheets and tubes, have been observed and characterized for a long time from numerical simulations. They result in highly non-Gaussian statistics of the velocity increments, which may depend both on the geometry of the individual structures, on their size distribution and on their spatial distribution at larger scales. Vortex tubes probably arise from stretched vortex layers formed at earlier time (Passot et al. 1995), and it is in those layers, and in the periphery of the vortex tubes, that high levels of energy dissipation are concentrated. Intense objects are often treated as being randomly distributed in space. With this assumption, Hatakeyama & Kambe (1997) obtained inertial range scaling from an assembly of random Burgers’ vortices. However, evidence
from numerics is inconsistent with a fully random distribution. Worms seem to accumulate in the interface between largely empty large-scale eddies (Jiménez et al. 1993), leading to an apparent inertial-scale clustering of intense vortices (Porter, Woodward & Pouquet 1997). Box-counting methods have been used by Moisy & Jiménez (2004) to further characterize this clustering.

Experimentally, spatial distributions can be inferred from waiting times between intense events recorded in one-point time series. The results of pressure measurements by Abry et al. (1994) showed algebraically-distributed waiting times, for inertial separations, between pressure drops marking large coherent vortices, suggesting self-similar clustering. Belin et al. (1996) and Mouri, Hori & Kawashima (2002) gave evidence, from one-point velocity time series, of the clustering of intense velocity gradients, which was shown to be self-similar by Camussi & Guj (1999) and Moisy (2000).

2. Box-counting of intense sets from numerical data

Examples of structures of intense vorticity and intense strain rates are shown in figure 1 (Moisy & Jiménez 2004). Here, a structure simply refers to a connected volume satisfying a thresholding criterion, $|\omega| \geq \tau \omega'$ or $|s| \geq \tau s'$, where the primes denote the rms values, with $\omega' = 2s' = \epsilon/\nu$ ($\epsilon$ is the energy dissipation rate and $\nu$ the kinematic viscosity). These structures have been extracted from numerical simulations of forced isotropic turbulence at $Re_\lambda = 168$ (Jiménez et al. 1993). The resolution is $512^3$ collocation points,
with periodic boundary conditions. The box size is $760\eta$ and the integral scale $L_0$ is around $1/4$ of the box size, providing a scale separation of $L_0/\eta \simeq 200$.

The vorticity structures essentially show ribbons for moderate thresholds, and long filamentary tubes (figure 1b) for higher ones. The biggest structures associated to a moderate vorticity threshold show patterns resulting from the interaction of an intense vortex tube with surrounding weaker tubes, as in figure 1a. Very large thresholds only show smaller tubes, probably parts of the larger ones observed at lower thresholds, but no sheets or ribbons. The situation is different for the strain rate structures, for which both moderate and large thresholds show essentially sheets or ribbons. For low thresholds, the selected objects show intricate sponge-like patterns (figure 1c), or assemblies of sheets and ribbons. Increasing the threshold results in structures more like isolated sheets or ribbons.

In order to characterize the distribution of these structures in space, we begin by applying the classical method of box-counting to the sets of points of intense vorticity and strain rate magnitude. The computational domain is divided into cubical boxes of side $r$, and the number $N(r)$ of boxes containing some point of the set is counted. In the case of a pure fractal set of dimension $D$, the number of boxes would follow a power law $N(r) \sim r^{-D}$. In real systems this relation only holds in a restricted range of scales between a large- and a small-scale cutoff.

Figure 2 shows box counts for the sets of points of high vorticity, $N_\omega(r)$, for different values of the threshold. Similar results are obtained for the box count of the strain rate sets. The curves approach $N_\omega(r) \sim r^{-3}$ as $r \to L$, in which

![Figure 2. Symbols: Number of boxes of size $r$ covering the vorticity sets, for the thresholds $|\omega| \geq 4, 8$ and $12 \omega'$. —, Box counts for sets of Poisson-distributed balls of the same total volume (data from the numerical simulation).](image)
case both the high-dissipation and the high-enstrophy sets look as a single solid object. At the small-scale end, $r \approx \eta$, the slopes also increase, reflecting the compactness of the objects at scales which are small enough for viscous effects to be important. For $r \ll \eta$ both sets should look as collections of small solid volumes, and one should expect the box counts to behave as $N(r) \approx r^{-3}$. For intermediate scales, $\eta \ll r \ll L$, the box counts show a continuous evolution with the threshold, and none of the curves displays a real power-law range.

In order to interpret these box counts, it is of interest to compare them with box counts of sets with no clustering. If we consider a set of Poisson-distributed balls of radius $\delta$, the expected number of covering boxes is

$$N_0(r) = \left( \frac{L}{r} \right)^3 \left( 1 - \exp\left[ -(r + \delta)^3 / r_0^3 \right] \right),$$

where $r_0$ is the mean distance between the balls. Together with the box-counts of figure 1 are plotted the best fits given by equation (1), using the constraint that the actual sets and the Poisson sets have the same volume, i.e. $N_0(r) \approx N_\omega(r)$ for $r \rightarrow \eta$. Clearly the actual box counts are not described well by the assumption of Poisson-distributed balls. The actual number of covering boxes $N_\omega(r)$ is found to be significantly smaller than $N_0(r)$ for the central range $10\eta < r < 200\eta$, implying that the regions of high vorticity are concentrated on a smaller fraction of space than the random balls.

The clustering of the intense vorticity and strain rate sets for intermediate scales can be further characterized by introducing a local scaling exponent, defined as the logarithmic slope $D(r) = -\ln N(r)/dr$. Since one must recover the trivial exponent $D = 3$ at both large and small scales, one may expect the

\[ D_\omega, D_\epsilon \]

\[ |\omega|/|\epsilon| 

Figure 3. Minimum of the local slope of box counts as a function of the threshold, for the vorticity (■) and the strain rate (○) fields. The intersections of these curves with $D = 2$ (horizontal dashed line) indicate the thresholds above which sets of negative dimension are expected from experimental one-point measurements (see § 3).
minimum slope, \( D^* \), to provide a useful measure of the dimensionality of the clustering. In the ideal case of objects distributed in space with a fractal dimension \( D \), we should expect \( D^* \to D \) in the limit of very large scale separation \( \eta \ll r \ll L \). For finite scale separation, both the large- and the small-scale contamination tend to increase the observed minimum \( D^* \), which therefore only represents an upper bound for the possible fractal dimension.

This minimum slope \( D^* \) is plotted in figure 3 as a function of the threshold for the vorticity and the strain rate sets. Both \( D^*_\omega \) and \( D^*_s \) decrease as the threshold increases, and none of them shows a plateau on which to define a threshold-independent dimension. The sets associated to typical fluctuations, \( |\omega| \simeq \omega' \) and \( |s| \simeq s' \), have dimensions of about 2.5, suggesting that regions of typical dissipation and enstrophy levels are wrinkled sheets, in qualitative agreement with other indirect estimates (Sreenivasan 1991; Sreenivasan & Antonia 1997).

Since the vorticity and strain rate sets considered here are a collection of structures as those shown in figure 1, two contributions are expected for the box counts. For scales smaller or of the order of the structures size, the box counts essentially describe the geometry and the size distribution of the structures, while for larger scales the box counts is more sensitive to their spatial distribution. Since we are interested in the clustering of the intense structures, one may separate the latter contribution from the global box-counting, by replacing each structure by a single point located at its baricenter, and applying the box-counting method to the resulting point sets. This procedure is only valid in the limit of very large threshold, for which the mean distance between structures is expected to be much larger than the structure size.

Figure 4 shows the box counts \( N_b(r) \) for the set of baricentres of the intense vorticity structures for two values of the threshold. As before, the scaling \( N_b(r) \sim r^{-3} \) for large scales indicates the homogenous covering at large scales.
At small scales, $N_b(r)$ saturates at the total number of structures, as expected for a set of points. The cross-over between the small-scale plateau and the large-scale decrease occurs at the typical distance $r_0$ between structures, which depends on the threshold.

These box-counts may be compared to that of Poisson-distributed points, by taking $\delta = 0$ in equation (1). As for the global box-counting, the actual curves are well below the Poisson law, indicating that the points are concentrated in a smaller fraction of space than for the random set. This clustering fraction is maximum for scales in the inertial range, and takes values around 0.5. Similar results are obtained for the clustering of intense strain rate structures. One may conclude that the clustering effect shown in figure 2 is not only an effect of the intense vorticity field being concentrated into structures, but also that the structures themselves are concentrated into clusters.

3. **Clustering of intense events from experimental data**

An issue raised by the previous observations in the low Reynolds number numerical simulations is whether the clustering of intense regions is still present at higher Reynolds number, and whether a range of scales exists for which this clustering is self-similar.

The decrease of the dimension $D^*_{\alpha}$ as the threshold is increased in figure 3 has important consequences for experiments for which only one-point measurements are available. From those measurements, the clustering of intense regions may be characterized from the distribution of the free intervals between successive intense events (Belin et al. 1996; Moisy 2000; Mouri, Hori & Kawashima 2002). For a fractal set of points of dimension $0 < d < 1$ with self-similar clustering, the distribution of the free intervals $\Delta x$ decays as $\Delta x^{-1-d}$ (Feder 1988). However, for large enough thresholds, figure 3 shows that both the vorticity and the dissipation fields concentrate into sets of dimension $D < 2$. As a consequence, the corresponding sets defined from one-dimensional cuts, as obtained from one-point time series with the use of the Taylor’s hypothesis, should have a dimension $d = D - 2 < 0$, and are therefore almost surely empty. Only the presence of a small-scale cutoff, imposed by the Kolmogorov length scale or by the probe resolution, ensures that the one-dimensional sections are not empty.

Distributions of the free intervals $\Delta x$ between successive intense velocity derivatives have been computed from experimental time series. The data are from a low temperature helium experiment, in which a large range of microscale Reynolds numbers can be spanned in very controlled conditions, $Re_\lambda$ from 150 up to 2000 (Zocchi et al. 1994; Moisy, Tabeling & Willaime 1999). The flow takes place in a cylinder and is driven by two rotating disks equipped with blades, 20 cm in diameter and spaced 13 cm apart. Velocity measurements
Figure 5. Probability density functions of the free intervals between intense velocity derivative $|\partial_x u|$ from the experimental time series, for $Re_\lambda = 1300$. They have been computed using logarithmic bins to ensure an acceptable number of events in the bins corresponding to the highest intervals. Thresholds: $-\tau, 2(\partial_x u)^{\prime \prime}, \ldots, 5(\partial_x u)^{\prime \prime}; --, 8(\partial_x u)^{\prime \prime}$.

were carried out using a hot wire anemometer, and the Taylor hypothesis has been used to convert temporal fluctuations into spatial ones.

Figure 5 shows the probability density function $p(\Delta x/\eta)$ of the free intervals between intense longitudinal velocity derivative, $|\partial_x u| \geq \tau(\partial_x u)^{\prime}$, for 3 different values of the threshold $\tau$, for $Re_\lambda = 1300$. As before, the prime denotes the rms value, which is related to the mean energy dissipation rate using the assumption of isotropy, $(\partial_x u)^{\prime \prime 2} = 2s^{\prime \prime 2}/15 = \epsilon/15\nu$. Note that, since only the longitudinal component of the velocity can be measured in the experiment, the intense longitudinal velocity derivatives are expected to trace essentially the intense strain rate regions rather than the intense vorticity regions. With this approximation, the quantity $(\partial_x u)^{2}$ has been extensively used as a one-dimensional surrogate for the local energy dissipation rate $\epsilon(\mathbf{x})$ (Sreenivasan 1991).

For sufficiently large threshold, the pdfs show a clear power law decay over a significant range of scales, starting from the dissipative range, $\Delta x \approx 3\eta$, up to a large scale cutoff, of order of $10^3 - 10^4\eta$, that depends on the threshold. This algebraic decay confirms that the intense events do not appear randomly in space, but tend to form self-similar clusters with no characteristic scale. Beyond the large scale cutoff, the pdfs decay approximately exponentially, indicating statistically uncorrelated events at large scales. The poorly defined scaling law for moderate threshold probably originates from the increasing contribution from the exponential decay, that may contaminate intermediate scales.
The exponent $\mu$ of the power law decay, $p(\Delta x) \sim \Delta x^{-\mu}$, is plotted as a function of the threshold $\tau$ in figure 6a. It is found to slightly decrease from values larger than 1, and saturates toward approximately 1 for large threshold. This trend is consistent with the fractal dimension $D^*$ determined from the numerical simulations, that takes values less than 2 for large enough threshold (see figure 3). As a consequence, the law $p(\Delta x) \sim x^{-1-d}$ with $d = D^* - 2$ does not hold any more for $d < 0$, and the distributions collapse towards the single curve $p(\Delta x) \sim \Delta x^{-1}$ for sufficiently large threshold. Similar observations have been reported for the free intervals between intense scalar fronts in turbulent mixing (Moisy et al. 2000).

In figure 6b is plotted the exponent $\mu_\infty$, obtained in the limit $\tau \gg 1$, as a function of $Re_\lambda$, indicating that the power law $p(\Delta x) \sim \Delta x^{-1}$ is robust for sufficiently large Reynolds numbers, $Re_\lambda > 400$. This asymptotic exponent $\mu_\infty$ is found to increase from 0.5 to approximately 1 for $Re_\lambda < 400$. It must be noted that values of $\mu$ less than 1 for low Reynolds numbers can not be interpreted in the frame of the law $p(\Delta x) \sim x^{-1-d}$ for an exact fractal set of points of dimension $d$, and probably results from finite scaling effects.

These experimental distributions confirm that the intense events appear within self-similar clusters, and cannot be considered as randomly distributed. This is consistent with the clustering of the intense dissipation events observed in the low Reynolds numbers simulations, but the one-dimensional cut in the experiment does not allow to further characterize the dimensionality of this clustering.
4. Discussion and conclusion

Three-dimensional box-counting from numerical simulations, and pdf of free intervals from experiments, gave evidence that the intense regions in isotropic turbulence, in the form of vortex sheets or tubes, tend to form clusters of inertial range extent. The dynamics of formation of the small scale structures from the instability of stretched shear layers at larger scales is probably the reason for this phenomenon. One may speculate that, for large Reynolds numbers, this process may repeat at different scales, leading to the observed self-similar clustering.

It is important to note that algebraic distributions for free intervals are not a trivial consequence of the self-similarity of the velocity field itself. Orey (1970) rigorously established that level sets from a Gaussian process with a power-law spectrum, $E(k) \sim k^{-n}$ with $1 < n < 3$, lead to fractal set of point of dimension $d = (3 - n)/2$. In the case of the Kolmogorov spectrum, $n = 5/3$, this relation yields $d = 2/3$, and pdf of free intervals between iso-values of the velocity should decay as $p(\Delta x) \sim \Delta x^{-d-1} \sim \Delta x^{-5/3}$. Although turbulent velocity fluctuations are not Gaussian, the experimental results of Praskovsky et al. (1993) and Scotti, Meneveau & Saddoughi (1995) were in good agreement with Orey’s theorem. However, it is clear that a fractal velocity field does not imply that the derivative fields are also fractal, and thus provides no insight into the spatial distribution of intense structures. Orey’s theorem does not hold for the vorticity or dissipation fields, which have a spectrum $k^2 E(k) \sim k^{1/3}$.

For instance, a Gaussian process with power-law spectrum and random phase has sets of iso-derivatives that are randomly distributed. One may conclude that the clustering of intense structures with a distribution of free intervals as $p(\Delta x) \sim \Delta x^{-1}$ is not a trivial consequence of the Kolmogorov spectrum, but is a true intermittency effect, that reveals the hierarchical organization of the small scale structures in turbulence.

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References


MULTI MODES FOR THE VORTEX SHEET-TUBE TRANSFORMATION PROCESS AND VISCOELASTIC EFFECT

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Abstract

A process for formation of the stretched spiral vortex was investigated using the DNS data of homogeneous turbulence. It was shown that vortex tube was generated not by a rolling up of single vortex sheet but through an interaction of dual sheets. Depending on the alignment of vorticity vectors on vortex tube and vortex sheets which emanate from vortex tube, three modes of configuration were shown to exist. Frequency of appearance of three modes and its implication for turbulence generation was discussed in isotropic and sheared flows.

Keywords: Stretched spiral vortex, vortex sheet, isotropic turbulence, sheared turbulence

1. Introduction

Primary elements which constitute the turbulent flow field are vortex sheets and vortex tubes (e.g., Horiuti 2001). These two structures are not distinctively separable because vortex tube is generally formed along vortex sheet during the rolling up of the sheet. One of the notable models which induces energy cascade and subsequent energy dissipation is the stretched spiral vortex model (Lundgren 1982). This model comprises of the vortex tube and the sheets which wraps around the tube, and yields the energy spectrum obeying the $-5/3$ law. The aim of the present study is to reveal a process for formation of the stretched spiral vortex, and show the role of the occurrence of this formation process on turbulence generation.

2. A formation process of spiral vortex in isotropic turbulence

We utilized the DNS data for incompressible decaying/forced homogeneous isotropic turbulence, which were generated with $256^3$ and $512^3$ grid points.
In this section, we show the results obtained using the data in the decaying case with the $256^3$ grid points and at an instant when the Reynolds number based on the Taylor microscale, $R_\lambda \approx 88$. In the present study, vortex tube and vortex sheet were identified using the second-order invariant of the velocity gradient tensor, $Q$, and the eigenvalue of the $-(S_{ik}\Omega_{kj} + S_{jk}\Omega_{ki})$ term, $[-(S_{ik}\Omega_{kj} + S_{jk}\Omega_{ki})]_+$, respectively. $S_{ij}$ and $\Omega_{ij}$ denote the strain-rate and vorticity tensors, respectively. The eigenvalues were ordered so that the eigenvalue, the eigenvector of which is maximally aligned with the vorticity vector, is chosen as the $z$ component, the largest remaining eigenvalue as the $+$ component, the smallest one as the $-$ component (Horiuti 2001). The eigenvalues of $S_{ij}$ tensor are denoted as $\sigma_z$ and $\sigma_\pm$.

![Figure 1](image1.png) ![Figure 2](image2.png)

In an early stage, several flat sheets, the lateral extents of which were several times the integral scale, emerged in the flow field. With lapse of time, the formation of tube occurred along these extensive sheets. Figure 1 shows the side view of the isosurfaces of the $Q$ term, and the $[-(S_{ik}\Omega_{kj} + S_{jk}\Omega_{ki})]_+$ eigenvalue. It is seen in Fig. 1 that the vortex tube, which can be identified as a concentrated region of $Q$ and drawn using the black color in the figure, was formed and the vortex sheets, which were drawn using the white color, were stretched and entrained by the vortex tube, and spiraling around the tube. These tube and sheets formed a structure similar to that of the stretched spiral vortex model (Lundgren 1982). Figure 2 shows the front view of the distribution of the dissipation term superposed on the isosurfaces of the vortex sheet and tube. It can be seen that the dissipation takes large values along this stretched spiral