

VISCOSITY OF LIQUIDS

Viscosity of Liquids

Theory, Estimation, Experiment, and Data

by

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Preface

The need for properties is ever increasing to make processes more economical. A good survey of the viscosity data, its critical evaluation and correlation would help design engineers, scientists and technologists in their areas of interest. This type of work assumes more importance as the amount of experimental work in collection and correlation of properties such as viscosity, thermal conductivity, heat capacities, etc has reduced drastically both at the industry, universities, and national laboratories.

One of the co-authors, Professor Viswanath, co-authored a book jointly with Dr. Natarajan “Data Book on the Viscosity of Liquids” in 1989 which mainly presented collected and evaluated liquid viscosity data from the literature. Although it is one of its kinds in the field, Prof. Viswanath recognized that the design engineers, scientists and technologists should have a better understanding of theories, experimental procedures, and operational aspects of viscometers. Also, rarely the data are readily available at the conditions that are necessary for design of the equipment or for other calculations. Therefore, the data must be interpolated or extrapolated using the existing literature data and using appropriate correlations or models. We have tried to address these issues in this book.

Although Prof. Viswanath had a vision of writing a book addressing the above issues, he never got to a point when he could sit for long hours, days, and months to undertake this challenge. During one of his visits to India, a former student and colleague, Dr. D. H. L. Prasad expressed his desire to work with Prof. Viswanath to bring this book to reality. Thus the journey began and started adding more collaborators to this effort.

Dr. Prasad was instrumental in putting together the framework for this book. His efforts were further supported by Dr. Dutt, Dr. Rani and Professor Ghosh.

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Tushar Ghosh thanks his wife Mahua for her patience and understanding during writing this book. He also thanks his son Atreyo and daughter Rochita for their understanding, patience and help in their own capacity during writing and proof reading the manuscript.

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Dutt wishes to place on record the encouragement and support received from his wife Anuradha and friends.

Yamuna Rani would like to thank Dr. D.H.L. Prasad for incorporating her participation into this venture, and also thanks Dr. M. Ramakrishna and Dr. K.V. Raghavan for permitting it. She also thanks the other co-authors for bearing with her and her family and friends for support.

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Chapter 1

INTRODUCTION

1.1 GENERAL INFORMATION

Viscosity is a fundamental characteristic property of all liquids. When a liquid flows, it has an internal resistance to flow. Viscosity is a measure of this resistance to flow or shear. Viscosity can also be termed as a drag force and is a measure of the frictional properties of the fluid. Viscosity is a function of temperature and pressure. Although the viscosities of both liquids and gases change with temperature and pressure, they affect the viscosity in a different manner. In this book, we will deal primarily with viscosity of liquids and its change as a function of temperature.

Viscosity is expressed in two distinct forms:

- a. Absolute or dynamic viscosity
- b. Kinematic viscosity

Dynamic viscosity is the tangential force per unit area required to slide one layer (*A*) against another layer (*B*) as shown in Figure 1.1 when the two layers are maintained at a unit distance. In Figure 1.1, force *F* causes layers *A* and *B* to slide at velocities v_1 and v_2 , respectively.

Since the viscosity of a fluid is defined as the measure of how resistive the fluid is to flow, in mathematical form, it can be described as:

$$\text{Shear stress} = \eta \text{ (Strain or shear rate)}$$

where η is the dynamic viscosity.

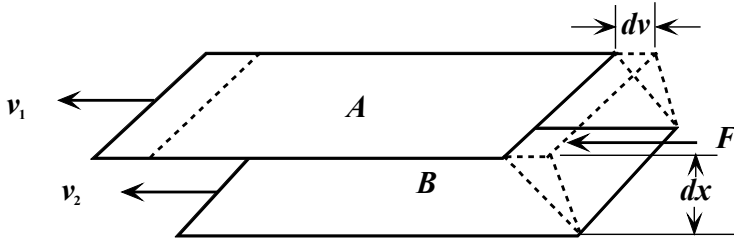


Figure 1.1. Simple shear of a liquid film.

If σ is shear stress and $\dot{\epsilon}$ is strain rate, then the expression becomes:

$$\sigma = \eta \dot{\epsilon} \quad (1.1)$$

The strain rate is generally expressed as

$$\dot{\epsilon} = \frac{1}{x} \frac{dx}{dt} = \frac{v}{x} \quad (1.2)$$

where x is the length, t is the time, and $\frac{dx}{dt}$ is the velocity v . Therefore, the dynamic viscosity can be written as

$$\eta = \sigma \frac{x}{v} \quad (1.3)$$

Kinematic viscosity requires knowledge of density of the liquid (ρ) at that temperature and pressure and is defined as

$$\nu = \frac{\eta}{\rho}. \quad (1.4)$$

1.2 VISCOSITY UNITS AND CONVERSION

Common units for viscosity are poise (P), Stokes (St), Saybolt Universal Seconds (SSU) and degree Engler.

Centipoise (cP) is the most convenient unit to report absolute viscosity of liquids. It is 1/100 of a Poise. (The viscosity unit Poiseuille, in short Poise was named after French physician, Jean Louis Poiseuille (1799 - 1869)).

In the SI System (*Système International d'Unités*) the dynamic viscosity units are $\text{N}\cdot\text{s}/\text{m}^2$, $\text{Pa}\cdot\text{s}$ or $\text{kg}/\text{m}\cdot\text{s}$ where N is Newton and Pa is Pascal, and,

$$1 \text{ Pa}\cdot\text{s} = 1 \text{ N}\cdot\text{s}/\text{m}^2 = 1 \text{ kg}/\text{m}\cdot\text{s}$$

The dynamic viscosity is often expressed in the metric system of units called CGS (centimeter-gram-second) system as $\text{g}/\text{cm}\cdot\text{s}$, $\text{dyne}\cdot\text{s}/\text{cm}^2$ or poise (P) where,

$$1 \text{ poise} = \text{dyne}\cdot\text{s}/\text{cm}^2 = \text{g}/\text{cm}\cdot\text{s} = 1/10 \text{ Pa}\cdot\text{s}$$

In British system of units, the dynamic viscosity is expressed in $\text{lb}/\text{ft}\cdot\text{s}$ or $\text{lbf}\cdot\text{s}/\text{ft}^2$. The conversion factors between various dynamic viscosity units are given in Table 1.1 and that for kinematic viscosity units are listed in Table 1.2.

For the SI system, kinematic viscosity is reported using Stokes (St) or Saybolt Second Universal (SSU) units. The kinematic viscosity is expressed as m^2/s or Stokes (St), where $1 \text{ St} = 10^{-4} \text{ m}^2/\text{s}$. Stokes is a large unit, and it is usually divided by 100 to give the unit called **Centistokes** (*cSt*).

$$1 \text{ St} = 100 \text{ cSt.}$$

$$1 \text{ cSt} = 10^{-6} \text{ m}^2/\text{s}$$

The specific gravity of water at 20.2°C (68.4°F) is one, and therefore the kinematic viscosity of water at 20.2°C is 1.0 cSt .

Saybolt Universal Seconds (SUS) is defined as the efflux time in Saybolt Universal Seconds (SUS) required for 60 milliliters of a petroleum product to flow through the calibrated orifice of a Saybolt Universal viscometer, under a fixed temperature, as prescribed by test method ASTM D 88¹. This is also called the SSU number (Seconds Saybolt Universal) or SSF number (Saybolt Seconds Furol).

Degree Engler is used in Great Britain for measuring kinematic viscosity. The Engler scale is based on comparing a flow of a fluid being tested to the flow of another fluid, mainly water. Viscosity in Engler degrees is the ratio of the time of flow of 200 cm^3 of the fluid whose viscosity is being measured to the time of flow of 200 cm^3 of water at the same temperature (usually 20°C but sometimes 50°C or 100°C) in a standardized Engler viscosity meter.

Table 1.1. Conversion factors between various dynamic viscosity units.

Multiply by		To						
From	Poiseuille (Pa.s)	Poise (dyne.s/cm ² = g/cm.s)	centipoise	kg/m.h	lb _f .s/ft ²	lb/ft.s	lb/ft.h	
Poiseuille (Pa.s)	1	10	10 ³	3.63×10 ³	2.09×10 ⁻²	0.672	2.42 10 ³	
Poise (dyne.s/cm ² = g/cm.s)	0.1	1	100	360	2.09×10 ⁻³	6.72×10 ⁻²	242	
centipoise	0.001	0.01	1	3.6	2.9×10 ⁻⁵	6.72×10 ⁻⁴	2.42	
kg/m.h	2.78×10 ⁻⁴	2.78×10 ⁻³	2.78×10 ⁻¹	1	5.8×10 ⁻⁶	1.87×10 ⁻⁴	0.672	
lb _f .s/ft ²	47.9	479	4.79×10 ⁴	1.72×10 ⁵	1	32.2	1.16×10 ⁵	
lb/ft.s	1.49	14.9	1.49×10 ³	5.36×10 ³	3.11×10 ⁻²	1	3.6×10 ³	
lb/ft.h	4.13×10 ⁻⁴	4.13×10 ⁻³	0.413	1.49	6.62×10 ⁻⁶	2.78×10 ⁻⁴	1	

Table 1.2. Conversion factors between various kinematic viscosity units.

Multiply by	To					
	Stokes	CentiStokes	m ² /s	m ² /h	ft ² /s	ft ² /h
Stokes	1	100	1.00×10^{-4}	3.60×10^{-1}	1.076×10^{-3}	3.875969
CentiStokes	0.01	1	1.00×10^{-6}	3.60×10^{-3}	1.08×10^{-5}	0.03876
m ² /s	1.00×10^4	1.00×10^6	1	3.60×10^3	1.08×10^1	3.88×10^4
m ² /h	2.78	2.78×10^2	2.78×10^{-4}	1	2.99×10^{-3}	1.08×10^1
ft ² /s	929.0	9.29×10^4	9.29×10^{-2}	3.34×10^2	1	3.60×10^3
ft ² /h	0.258	25.8	2.58×10^{-5}	9.28×10^{-2}	2.78×10^{-4}	1

1.3 FLUID FLOW AND VISCOSITY

Liquid viscosities are needed by process engineers for quality control, while design engineers need the property for fixing the optimum conditions for the chemical processes and operations as well as for the determination of the important dimensionless groups like the Reynolds number and Prandtl number. Liquid viscosity is also important in the calculation of the power requirements for the unit operations such as mixing, pipeline design, pump characteristics, atomization (liquid droplets), storage, injection, and transportation.

The flow characteristics of liquids are mainly dependent on the viscosity and are broadly divided into three categories:

- a) Newtonian
- b) Time independent Non-Newtonian, and
- c) Time dependent Non-Newtonian.

When the viscosity of a liquid remains constant and is independent of the applied shear stress, such a liquid is termed a Newtonian liquid.

In the case of the non-Newtonian liquids, viscosity depends on the applied shear force and time. For time independent non-Newtonian fluid, when the shear rate is varied, the shear stress does not vary proportionally and is shown in Figure 1.2. The most common types of time independent non-Newtonian liquids include pseudoplastic (This type of fluid displays a decreasing viscosity with an increasing shear rate and sometimes called shear-thinning), dilatant (This type of fluid display increasing viscosity with an increase in shear rate and is also called shear-thickening), and Bingham plastic (A certain amount of force must be applied to the fluid before any flow is induced). Bingham plastic is somewhat idealized representation of many real materials, for which the rate of shear is zero if the shearing stress is less than or equal to a yield stress e_0 . Otherwise, it is directly proportional to the shearing stress in excess of the yield stress.

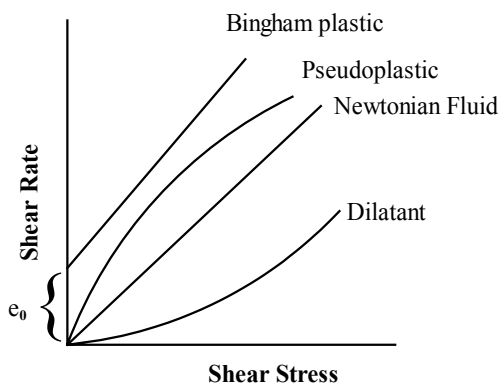


Figure 1.2. Various types of fluids based on viscosity.

Time dependent non-Newtonian fluids display a change in viscosity with time under conditions of constant shear rate. One type of fluid called Thixotropic undergoes a decrease in viscosity with time as shown in Figure 1.3a. The other type of time dependent non Newtonian fluid is called Rheopectic. The viscosity of rheopectic fluids increases with the time as it is sheared at a constant rate (See Figure 1.3b).

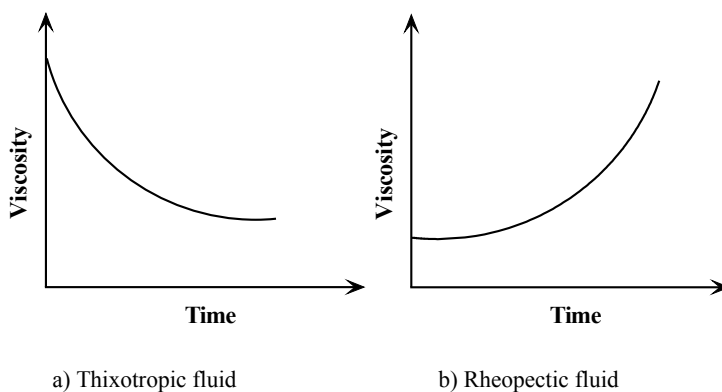


Figure 1.3. Change in Viscosity of time-dependent non-Newtonian fluids.

These as well as several other considerations, including attempts to understand interactions among different molecules, have resulted in a very large number of experimental, theoretical and analytical investigations in the areas related to viscosity. In this book, we will discuss both theory for estimation of viscosity and various measurement techniques.

A brief description of the material dealt in each chapter is given below.

Chapter 2 describes the general methodology of the experimentation and the techniques commonly used to measure viscosity with the essential details.

Chapter 3 is devoted to the description of the development of the theories of liquid viscosity.

Chapter 4 gives an account of the commonly used methods for the correlation and estimation of pure liquid viscosity.

Chapter 5 mentions the commonly used methods for the correlation and estimation of the viscosity of solutions and mixtures.

Chapter 6 provides the viscosity-temperature data for more than 1,000 liquids collected from literature sources. The data were evaluated by the authors and are provided in tabular form. The data have been correlated to three equations, and the goodness of fit is discussed.

REFERENCE

1. ASTM (American Society for Testing and Materials) D 88, *Standard Test Method for Saybolt Viscosity*.

Chapter 2

VISCOMETERS

The measurement of viscosity is of significant importance in both industry and academia. Accurate knowledge of viscosity is necessary for various industrial processes. Various theories that are developed for prediction or estimation of viscosity must be verified using experimental data. Instruments used to measure the viscosity of liquids can be broadly classified into seven categories:

1. Capillary viscometers
2. Orifice viscometers
3. High temperature high shear rate viscometers
4. Rotational viscometers
5. Falling ball viscometers
6. Vibrational viscometers
7. Ultrasonic viscometers

A number of viscometers are also available that combine features of two or three types of viscometers noted above, such as Friction tube, Norcross, Brookfield, Viscosity sensitive rotameter, and Continuous consistency viscometers. A number of instruments are also automated for continuous measurement of viscosity and for process control. Several apparatus named after pioneers in the subject as well manufactured by popular instrument manufacturers are available for each of the classes.

2.1 CAPILLARY VISCOMETERS

Capillary viscometers are most widely used for measuring viscosity of Newtonian liquids. They are simple in operation; require a small volume of sample liquid, temperature control is simple, and inexpensive. In capillary

viscometers, the volumetric flow rate of the liquid flowing through a fine bore (capillary) is measured, usually by noting the time required for a known volume of liquid to pass through two graduation marks. The liquid may flow through the capillary tube either under the influence of gravity (Gravity Type Viscometer) or an external force. In the instruments where an external force is applied, the liquid is forced through the capillary at a predetermined rate and the pressure drop across the capillary is measured. Capillary viscometers are capable of providing direct calculation of viscosity from the rate of flow, pressure and various dimensions of the instruments. However, most of the capillary viscometers must be first calibrated with one or more liquids of known viscosity to obtain “constants” for that particular viscometer.

The essential components of a capillary viscometer are

1. a liquid reservoir,
2. a capillary of known dimensions,
3. a provision for measuring and controlling the applied pressure,
4. a means of measuring the flow rate, and
5. a thermostat to maintain the required temperature.

Several types of capillary viscometers have been designed through variation of above components, and commercially available capillary viscometers can be classified into the following three categories based on their design.

1. Modified Ostwald viscometers,
2. Suspended-level viscometers, and
3. Reverse-flow viscometers.

Glass capillary viscometers are most convenient for the determination of the viscosity of Newtonian liquids. Often the driving force is the hydrostatic head of the test liquid itself. Kinematic viscosity is generally measured using these viscometers. The same principles can also be applied to measure the viscosity of Non-Newtonian liquids, however an external pressure will be necessary to make the Non-Newtonian liquid flow through the capillary. Glass capillary viscometers are low shear-stress instruments. Usually the shear stress ranges between 10 and 150 dyn/cm² if operated by gravity only and 10 to 500 dyn/cm² if an additional pressure is applied. The rate of shear in glass capillary viscometers ranges from 1 to 20,000 s⁻¹ (based on 200 to 800 s efflux time).

2.1.1 THEORY

The calculation of viscosity from the data measured using glass capillary viscometers is based on Poiseuille's equation. In this section, first the derivation of Poiseuille's equation is discussed and then various corrections made to this equation are explained.

Consider a cylindrical capillary of diameter a and length l , as shown in Fig. 2.1, with a pressure difference ΔP between the ends. P_1 and P_2 are pressures at two ends and the fluid is subjected to a force F .

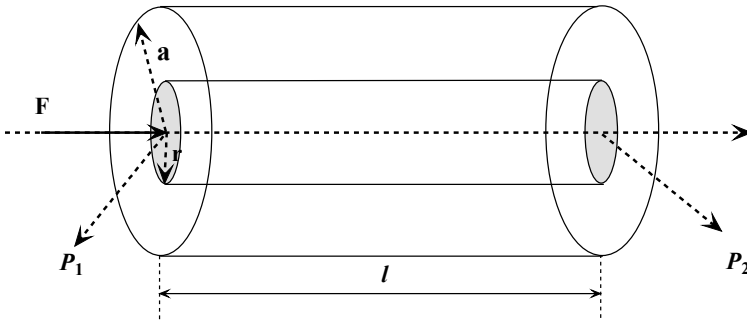


Figure 2.1. Derivation of Poiseuille's equation.

The following assumptions are made to derive Poiseuille's equation.

1. The flow is parallel to the axis of the tube everywhere, i.e., streamline flow is followed.
2. The flow is steady and there is no acceleration of the liquid at any point within the tube.
3. There is no slip at the wall, i.e., the liquid is stationary at the capillary wall.
4. The liquid is a Newtonian liquid.

Since the liquid is Newtonian, the following relationship holds.

$$\sigma = \eta \dot{\epsilon} = \eta \frac{dv}{dr} \quad (2.1)$$

In Eq. 2.1,

σ is shear stress,

$\dot{\epsilon}$ is strain rate,
 η is dynamic viscosity,
 v is the velocity, and
 r is any distance from the center of the capillary.

A force balance on the cylindrical element of length l and radius r coaxial with the capillary provides the following expression.

$$\sigma \times 2\pi r l = \Delta P \times \pi r^2 \quad (2.2)$$

Substitution of Eq. (2.1) into Eq. (2.2) yields

$$\frac{dv}{dr} = \frac{\Delta P}{2\eta l} r \quad (2.3)$$

Integration of Eq. (2.3), using the boundary condition that at the wall of the capillary $v(a) = 0$, gives

$$v = \frac{\Delta P(a^2 - r^2)}{4\eta l} \quad (2.4)$$

It may be noted from Eq. (2.4) that the velocity distribution across the capillary is parabolic. The volumetric flow rate through the capillary can be calculated by noting that in unit time between radii r and $r+dr$ the volume of liquid flowing is given by $2\pi r v dr$. The overall flow rate (Q cm³/s) can be obtained by integrating the following expression.

$$Q = \int_0^a 2\pi r v dr \quad (2.5)$$

Substitution of Eq. 2.4 into Eq. 2.5 yields

$$Q = \int_0^a \frac{2\pi\Delta P}{4\eta l} r(a^2 - r^2) dr = \frac{2\pi\Delta P}{4\eta l} \int_0^a r(a^2 - r^2) dr \quad (2.6)$$

or,

$$Q = \frac{\pi \Delta P a^4}{8 \eta l} \quad (2.7)$$

This is known as Poiseuille's equation and is used for calculation of viscosity when using a capillary viscometer. For vertical tube arrangement which is the case for most of the capillary viscometer, the hydrostatic pressure, ρgh , depends on the height, h , of the liquid. Therefore, the pressure difference, Δp , in terms of hydrostatic pressure is given by

$$\Delta P = \rho gh \quad (2.8)$$

It should be noted that h is a function of time. Substitution of Eq. 2.8 into Eq. 2.7 and further rearrangement provides the expression for viscosity (η).

$$\eta = \frac{\pi g h a^4}{8 l V} \rho t \quad (2.9)$$

where,

$Q = V/t$ (V is the defined volume of the liquid dispensed during the experiment and t is the time required for this volume of liquid to flow between two graduation marks in a viscometer).

For a particular viscometer, Eq. 2.9 can be rewritten as

$$\eta = K \rho t \quad (2.10)$$

where K is a constant for a viscometer and is given by

$$K = \frac{\pi g h a^4}{8 l V} \quad (2.11)$$

Equation 2.10 can be used to obtain the kinematic viscosity

$$\nu = K t \quad (2.12)$$

where

$$v = \frac{\eta}{\rho} \quad (2.13)$$

A number of viscometers are designed based on Eq. 2.12. The equipment is calibrated for the value of K , which is obtained by using a liquid of known viscosity and density. Once the value of K is known, the viscosity of test liquid can be obtained by measuring the time required for a known volume of sample to flow between two graduation marks.

2.1.1.1 Kinetic Energy Corrections

A number of factors can influence the experiment and introduce errors in the measurements. To improve the accuracy of the measurement, various corrections are made to the experimentally determined data. Among these corrections, kinetic energy and end effect corrections are most significant. In most types of viscometers, a portion of the applied force is converted to kinetic energy that sets the liquid into motion. However, as Poiseuille's equation is strictly for flow of fluid with parabolic velocity profile, a correction to Poiseuille's equation is necessary to account for the pressure used in overcoming viscous resistance.

The work done due to the kinetic energy transferred to the liquid per unit time may be expressed as

$$W_{KE} = \int_0^a \frac{1}{2} v^2 2\pi r \rho v dr \quad (2.14)$$

Substitution of Eq. 2.4 into Eq. 2.14 yields

$$W_{KE} = \int_0^a \pi \rho \frac{\Delta P^3}{4^3 \eta^3 l^3} (a^2 - r^2)^3 r dr \quad (2.15)$$

Integration of Eq. 2.15 and subsequent substitution of Eq. 2.8 provides

$$W_{KE} = \frac{\rho Q^3}{\pi^2 a^4} \quad (2.16)$$

Therefore, the work done against the viscous force is given by

$$W_{vis} = PQ - \frac{\rho Q^3}{\pi^2 a^4} \quad (2.17)$$

The effective pressure difference, then, can be written as

$$\Delta P_{eff} = P - \frac{\rho Q^2}{\pi^2 a^4} \quad (2.18)$$

Several viscometers make this correction for kinetic energy by connecting a reservoir to both ends of the capillary. However, several researchers noted that still some correction is necessary. As noted by Dinsdale and Moore¹, the correction term takes the form:

$$\Delta P_{eff} = P - m \frac{\rho Q^2}{\pi^2 a^4} \quad (2.19)$$

where m is a constant which is determined experimentally.

2.1.1.2 End Corrections

The converging and diverging streamlines at the entrance and exit of the capillary must be taken into consideration for accurate estimation of viscosity from capillary viscometers. Couette first suggested increasing the capillary length l by na to take into account the end effects. The Poiseuille's equation after correcting for kinetic energy and end effects can be written as

$$\eta = \frac{\pi a^4 P}{8Q(l+na)} - \frac{m \rho Q}{8\pi(l+na)} \quad (2.20)$$

The values of n varied between 0 and 1.2. Equation 2.20 can be written in terms of time as follows:

$$\eta = \frac{\pi a^4 Pt}{8V(l+na)} - \frac{m \rho V}{8\pi t(l+na)} \quad (2.21)$$

Equation 2.21 for relative viscometers may be written as

$$\eta = \alpha Pt - \beta \rho t \quad (2.22)$$

The constants α and β are determined from the experimental data using three or four liquids of known viscosity.

Equation 2.21 for gravity flow type viscometers becomes

$$\nu = At - \frac{B}{t} \quad (2.23)$$

The constants A and B are also determined from the experimental data using three or four liquids of known viscosity.

Descriptions of various capillary viscometers along with the experimental procedures are described below.

2.1.2 OSTWALD VISCOMETER

The most common design of gravity type viscometer is the U-tube type and best known as the Ostwald viscometer. It consists of two reservoir bulbs and a capillary tube as shown in Fig. 2.2.

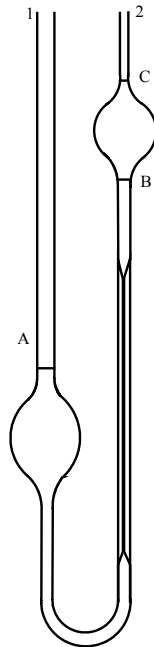


Figure 2.2. An Ostwald viscometer.

The efflux time (t) of a fixed volume of liquid under an exactly reproducible mean hydrostatic head is measured. The viscometer is filled with the liquid until the liquid level reaches the mark A . Usually a pipette is used to accurately measure the volume of liquid added to the viscometer. The viscometer is then placed inside a constant temperature bath to equilibrate the temperature of the test liquid with the bath temperature. The liquid is drawn through the side 2 of the U-tube using a suction and then the flow is timed between marks C and B . The viscosity is calculated using Eq. 2.12. The constant K is determined from the measurement of reference liquid such as water.

Ostwald type viscometers can cause significant error in the measurement if the viscometer is not vertical in alignment. If the distance between the two sides (1 and 2) of the viscometer is s , a tilt of θ from the vertical position can introduce a relative error in the hydrostatic head h by

$$\Delta h_{error} = 1 - \cos \theta \pm \left(\frac{s}{h} \right) \sin \theta \quad (2.24)$$

It follows from Eq. 2.24 that if s is $0.6h$, 1° deviation from vertical axis will introduce 1% error in the head.

Another source of error in Ostwald viscometer is the requirement to use exact volume of liquid for the reference liquid and the test liquid. This requirement becomes further problematic if the measurements are made at different temperatures. The accurate knowledge of density is necessary to adjust the volume at different test temperatures.

2.1.3 MODIFIED OSTWALD VISCOMETERS

Several modifications to the original design of Ostwald viscometer was made to address these issues and are discussed below. Modified Ostwald viscometers can be divided into two categories.

- a. Constant volume viscometer at filling temperature
 - a. Cannon-Fenske routine viscometer
 - b. Cannon Manning semi-micro viscometer
 - c. Pinkevitch viscometer
- b. Constant volume at the test temperatre
 - a. Zeitfuchs viscometer
 - b. SIL viscometer
 - c. BS/U-tube viscometer
 - d. BS/U-tube miniature viscometer

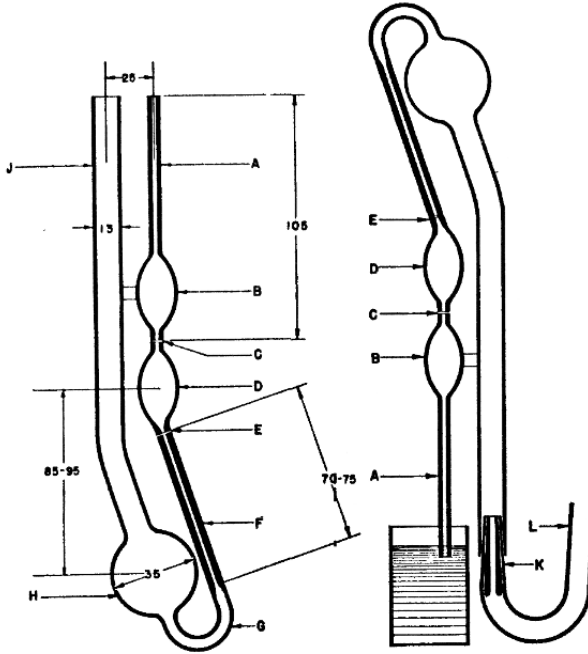
2.1.3.1 Cannon-Fenske Routine Viscometer

Ostwald viscometer is modified by Cannon and Fenske^{2,3} in such a manner that the upper bulb (Bulb *B*) and lower bulb (Bulb *D*) lie in the same vertical axis in order to reduce the error in the mean head caused by the deviation of the viscometer from the vertical position. A schematic drawing of a Cannon-Fenske Routine viscometer of Size 25 is shown in Fig. 2.3. The Cannon-Fenske routine viscometers are designed for measuring the kinematic viscosity of transparent Newtonian liquids in the range of 0.5 to 20,000 cSt (mm²/s). The Cannon-Fenske viscometers can also be used to measure shear stress versus shear rate relations that is useful to the study of non-Newtonian liquids, wax crystallization, and oil flow characteristics at low temperatures.

Cannon-Fenske Routine viscometer that is used to measure viscosity of transparent liquids is based on standard ASTM D-445 and D-446 and ISO 3104 and 3105 methods. The general procedure for using a Cannon-Fenske Viscometer is discussed below. Before any measurement, the viscometer must be cleaned using a suitable solvent or solvents. Although it is desirable to dry the viscometer by passing clean, dry, filtered air through the instrument to remove the final traces of solvents, filtered air may not be readily available in laboratories. In that case, the viscometer may be given a final rinse with acetone and then dried in an oven. The viscometer should be periodically cleaned with acid to remove trace deposits that might occur due to repeated use. One of the best acids for cleaning glasses is chromic acid. It is also advisable to filter the liquid sample to remove solid suspensions before filling the viscometer.

The viscometer shown in Figs. 2.3a and 2.3b depicts the arrangement convenient for filling the apparatus. The sample is drawn into the apparatus by inserting the tube *A* in the inverted position into the liquid (free from air bubbles) kept in a beaker and liquid is drawn applying suction to the arm as shown in Fig. 2.3a, through bulbs *B* and *D* up to the etched mark *E*. The viscometer is turned to its normal position, wiped carefully, inserted into a holder and placed in a thermostat. The viscometer is aligned vertically in the bath by means of a small plumb bob in tube *G*, if a self-aligning holder is not used. After reaching the equilibrium conditions at the required temperature, suction is applied to tube *A*, to bring the sample in to bulb *D* and allowed to reach a point slightly above mark *C*. The time required for the liquid meniscus to pass from the mark at *C* to mark *E* is recorded. The measurement should be repeated several times and the average time of all the runs should be used in the calculation of kinematic viscosity, which is obtained by multiplying the efflux time in seconds by the viscometer

constant. If the efflux time observed is less than 200s, the observation should be repeated with another viscometer with a smaller capillary.



(a) Dimensional sketch of size 25 (b) Method of filling

Figure 2.3. Cannon-Fenske Routine viscometer of Size 25 shown with the dimensions in millimeters and its filling procedure.

A single viscometer is not capable of measuring the viscosity over the entire range. The main limitation is the size of the capillary *F*. Table 2.1 lists the range for measuring viscosity using various size Cannon-Fenske Routine viscometers.

2.1.3.2 Cannon-Manning Semi-micro Viscometer

The Cannon-Manning Semi-Micro viscometer is a modified Ostwald type model requiring approximately 1 mL of the sample and is capable of measuring the kinematic viscosity of transparent Newtonian liquids in the same range of 0.4 to 20,000 cSt as that of Cannon-Fenske Routine viscometer⁴. The apparatus is shown in Fig. 2.4.