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Structural Synthesis of Parallel Robots

Part 1: Methodology

By

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Parallel robotic manipulators can be considered a well-established option for many different applications of manipulation, machining, guiding, testing, control, tracking, haptic force feed-back, etc. A typical parallel robotic manipulator (PM) consists of a mobile platform connected to the base (fixed platform) by at least two kinematic chains called limbs. The mobile platform can achieve between one and three independent translations (T) and one to three independent rotations (R).

Parallel manipulators have been the subject of study of much robotic research during the last two decades. Early research on parallel manipulators has concentrated primarily on six degrees of freedom (DoFs) Gough-Stewart-type PMs introduced by Gough for a tire-testing device, and by Stewart for flight simulators. In the last decade, PMs with fewer than 6-DoFs attracted researchers' attention. Lower mobility PMs are suitable for many tasks requiring less than six DoFs.

The motion freedoms of the end-effector are usually coupled together due to the multi-loop kinematic structure of the parallel manipulator. Hence, motion planning and control of the end-effector for PMs usually become very complicated. With respect to serial manipulators, such mechanisms can offer advantages in terms of stiffness, accuracy, load-to-weight ratio, dynamic performances. Their disadvantages include a smaller workspace, complex command and lower dexterity due to a high motion coupling, and multiplicity of singularities inside their workspace. Uncoupled, fully-isotropic and maximally regular PMs can overcome these disadvantages.
Isotropy of a robotic manipulator is related to the condition number of its Jacobian matrix, which can be calculated as the ratio of the largest and the smallest singular values. A robotic manipulator is fully-isotropic if its Jacobian matrix is isotropic throughout the entire workspace, i.e., the condition number of the Jacobian matrix is equal to one. We know that the Jacobian matrix of a robotic manipulator is the matrix mapping (i) the actuated joint velocity space on the end-effector velocity space, and (ii) the static load on the end-effector and the actuated joint forces or torques. The isotropic design aims at ideal kinematic and dynamic performance of the manipulator.

We distinguish five types of PMs (i) maximally regular PMs, if the Jacobian $J$ is an identity matrix throughout the entire workspace, (ii) fully-isotropic PMs, if the Jacobian $J$ is a diagonal matrix with identical diagonal elements throughout the entire workspace, (iii) PMs with uncoupled motions if $J$ is a diagonal matrix with different diagonal elements, (iv) PMs with decoupled motions, if $J$ is a triangular matrix and (v) PMs with coupled motions if $J$ is neither a triangular nor a diagonal matrix. Maximally regular and fully-isotropic PMs give a one-to-one mapping between the actuated joint velocity space and the external velocity space.

The first solution for a fully-isotropic $T3$-type translational parallel robot was developed at the same time and independently by Carricato and Parenti-Castelli at University of Genoa, Kim and Tsai at University of California, Kong and Gosselin at University of Laval, and the author of this work at the French Institute of Advanced Mechanics. In 2002, the four groups published the first results of their works.

The general methods used for structural synthesis of parallel mechanisms can be divided into three approaches: the method based on displacement group theory, the methods based on screw algebra, and the method based on the theory of linear transformations. The method proposed in this book is based on the theory of linear transformations and the evolutionary morphology and allows us to obtain the structural solutions of decoupled, uncoupled, fully-isotropic and maximally regular PMs with two to six DoFs in a systematic way. The new formulæ for mobility, connectivity (spatiality), redundancy and overconstraint of PMs proposed recently by the author are integrated into the synthesis approach developed in this book.

Various solutions of $TaRb$-type PMs are known today. In this notation, $a=1,2,3$ indicates the number of independent translations and $b=1,2,3$ the number of independent rotations of the moving platform. The parallel robots actually proposed by the robot industry have coupled and decoupled motions and just some isotropic positions in their workspace. As far as we
are aware, this is the first book on robotics presenting solutions of uncoupled, fully-isotropic and maximally regular PMs.

Non-redundant/redundant, overconstrained/isostatic solutions of uncoupled and fully-isotropic/maximally regular PMs with elementary/complex limbs actuated by linear/rotary actuators with/without idle mobilities and two to six DoFs are present in a systematic approach of structural synthesis. These solutions are derived from families of PMs with decoupled motions. A serial kinematic chain is associated with each elementary limb and at least one closed loop is integrated in each complex limb.

The synthesis methodology and the solutions of PMs presented in this book represent the outcome of some recent research developed by the author in the last four years in the framework of the projects ROBEA-MAX (2002-2003) and ROBEA-MP2 (2004-2005) supported by the National Center for Scientific Research (CNRS). These results have been partially published by the author in the last two years. In these works the author has proposed the following for the first time in the literature:

a) new formulae for calculating the degree of mobility, the degree of connectivity/spatiality, the degree of redundancy and the number of over-constraints of parallel robotic manipulators that overcome the drawbacks of the classical Chebychev-Grübler-Kutzbach formulae,

b) a new approach to systematic innovation in engineering design called evolutionary morphology,

c) solutions to follow fully-isotropic parallel robots: parallel wrists with two and three degrees of mobility, planar parallel robots, parallel robots of types $T_2R_1$ and $T_3R_2$, parallel robots with Schönflies motions, hexapods with six degrees of freedom.

The various solutions proposed by the author belong to a modular family called Isoglide$^N$-$T_aR_b$ with $a+b=n$ with $2 \leq n \leq 6$, $a=1,2,3$ and $b=1,2,3$. The mobile platform of these robots can have any combination of $n$ independent translations ($T$) and rotations ($R$). The Isoglide$^n$-$T_aR_b$ modular family was developed by the author and his research team at the French Institute of Advanced Mechanics (IFMA) in Clermont-Ferrand.

This work is organized in two parts published in two distinct books. Part 1 presents the methodology proposed for structural synthesis and Part 2 the various topologies of parallel robots systematically generated by the structural synthesis approach. The originality of this work resides in combining the new formulae for mobility connectivity, redundancy and overconstraints, and the evolutionary morphology in a unified approach of structural synthesis giving interesting innovative solutions for parallel robots.

Part 1 is organized in ten chapters. The first chapter is intended to introduce the main concepts, definitions and components of the mechanical robotic system. Chapter 2 reviews the contributions in mobility calculation
systematized in the so called Chebychev-Grübler-Kutzbach mobility formulae. The drawbacks and the limitations of these formulae are discussed, and the new formulae for mobility, connectivity, redundancy and overconstraint are demonstrated via an original approach based on the theory of linear transformations. These formulae are applied in chapter 3 for the structural analysis of parallel robots with simple and complex limbs. The new formulae are also applied to calculate the mobility and other structural parameters of single and multi-loop mechanisms that do not obey the classical Chebychev-Grübler-Kutzbach formulae, such as the mechanisms proposed by De Roberval, Sarrus, Bennett, Bricard and other so called “paradoxical mechanisms”. We have shown that these mechanisms completely obey the definitions, the theorems and the formulae proposed in the previous chapter. There is no reason to continue to consider them as “paradoxical”. Chapter 4 presents the main models and performance indices used in parallel robots. We emphasize the Jacobian matrix, which is the main issue in defining robot kinematics, singularities and performance indices. New kinetostatic performance indices are introduced in this section to define the motion decoupling and input-output propensity in parallel robots. Structural parameters introduced in the second chapter are integrated in the structural synthesis approach founded on the evolutionary morphology (EM) presented in chapter 5. The main paradigms of EM are presented in a closed relation with the biological background of morphological approaches and the synthetic theory of evolution. The main difference between the evolutionary algorithms and the EM are also discussed. The evolutionary algorithms are methods for solving optimization-oriented problems, and are not suited to solving conceptual design-oriented problems. They always start from a given initial population of solutions and do not solve the problem of creating these solutions.

The first stage in structural synthesis of parallel robots is the generation of the kinematic chains called limbs used to give some constrained or unconstrained motion to the moving platform. The constrained motion of the mobile platform is obtained by using limbs with less than six degrees of connectivity. The various solutions of simple and complex limbs with two to six degrees of connectivity are systematically generated by the structural synthesis approach and presented in chapters 6-10. We focus on the solutions with a unique basis of the operational velocity space that are useful for generating various topologies of decoupled, uncoupled, fully-isotropic and maximally regular parallel robots presented in Part 2. Limbs with multiple bases of the operational velocity space and redundant limbs are also presented in these chapters. These limb solutions are systematized with respect to various combinations of independent motions of the distal link. They are defined by symbolic notations and illustrated in about 250 figures
containing more than 1500 structural diagrams. Special attention was paid to graphic quality of structural diagrams to ensure a clear correspondence between the symbolic and graphic notation of joints and the relative position of their axes. The graphic illustration of these solutions is associated with the author’s conviction that a good structural diagram really “is worth a thousand words”, especially when you are trying to disseminate the result of the structural synthesis of kinematic chains. The kinematic chains presented in chapters 6-10 are useful as innovative solutions of limbs in parallel, serial and hybrid robots. In fact, serial and hybrid robots may be considered as a particular case of parallel robots with only one limb which can be a simple, complex or hybrid kinematic chain. Many serial robots actually combine closed loops in their kinematic structure.

The various types of kinematic chains generated in chapters 6-10 are combined in Part 2 to set up innovative solutions of parallel robots with two to six degrees of mobility and various sets of independent motions of the moving platform.

Part 2 is organized in 16 chapters presenting the following: translational PMs T2-type (chapter 1), T1R1-type PMs with screw motion (chapter 2), other T1R1-type PMs (chapter 3), R2-type spherical parallel wrists (chapter 4), translational PMs T3-type (chapter 5), T2R1-type PMs with planar motion (chapter 6), other T2R1-type PMs (chapter 7), T1R2-type PMs (chapter 8), R3-type spherical parallel wrists (chapter 9), T3R1-type PMs with Schönflies motion (chapter 10), other T3R1-type PMs (chapter 11), T2R2-type PMs (chapter 12), T1R3-type PMs (chapter 13), T3R2-type PMs (chapter 14), T2R3-type PMs (chapter 15) and PMs with six degrees of freedom (chapter 16).

Parallel robots with coupled, decoupled and uncoupled motions, along with fully-isotropic and maximally regular solutions, are systematically presented in each chapter. Innovative solutions of overconstrained or non-overconstrained, non-redundant or redundantly actuated parallel robots with simple or complex limbs actuated by linear or rotating motors are set up in Part 2 by applying the structural synthesis methodology presented in this first part. The writing of part 2 is still in progress and will soon be finalized.

Many solutions for parallel robots obtained through this systematic approach of structural synthesis are presented here for the first time in the literature. The author had to make a difficult and challenging choice between protecting these solutions through patents, and releasing them directly into the public domain. The second option was adopted by publishing them in various recent scientific publications and mainly in this book. In this way, the author hopes to contribute to a rapid and widespread implementation of these solutions in future industrial products.
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List of abbreviations and notations

\(a_i\) – axial offset of the \(i\)th-link
\(A (1, 2, ..., n)\) and \(A (1, 2, ..., n)\) – simple open kinematic chain and its links
\(A (\theta = 1, 2, 3, 4, ... \theta)\) simple open kinematic chain with link \(I_\theta\) fixed to base \(\theta\)
\(A (l_\theta = 0, 2, ... n_\theta = l_\theta)\) – simple closed kinematic chain with link \(I_\theta\) fixed to base \(\theta\)
\(A = A_1 \cdot A_2 \ (1, 2, ..., n_1 = n_2, ... - 2, 1)\) simple open kinematic chain obtained by serial concatenation of two simple chains \(A_1\) and \(A_2\)
\(AI\) – artificial intelligence
\(b_i\) – length of the \(i\)th-link
\(B \cdot A_1 \cdot A_2 \ (1, 2, ..., n_{AB} = n_2, ... - 2, 1)\) single-loop kinematic chain obtained by connecting the distal links \((I_1 = I_2\) and \(n_1 = n_2)\) of two simple open kinematic chains \(A_1 (1, 2, ..., n_1)\) and \(A_2 (1, 2, ..., n_2)\)
\(c\) – condition number
\(C \cdot A_1 \cdot A_2 \cdot A_3\) parallel robot with \(k\) simple limbs \(A_i (i = 1, 2, ..., k)\)
\(C\) – cylindrical pair
\(CNRS\) – Centre National de la Recherche Scientifique (National Center for Scientific Research)
\(CGK\) – Chebychev-Grübler-Kutzbach
\(\gamma_i\) – instantaneous degree of motion coupling
\(\gamma\) – global degree of motion coupling
\(d\) – linear displacement
\(D \cdot A_1 \cdot A_2 \cdot ... \cdot A_{k_1} \cdot E_1 \cdot E_2 \cdot ... \cdot E_{k_2}\) parallel robot with \(k_1\) simple limbs \(A_i (i = 1, 2, ..., k_1)\) and \(k_2\) complex limbs \(E_i (i = 1, 2, ..., k_2)\)
\(DoF\) – degree-of-freedom
\(DPS\) – design parameters
\(e_A\) and \(e_{A_1}\) – link of \(A_1\)-limb \((e = 1, 2, 3, ..., n)\)
\(e_B\) and \(e_{A_2}\) – link of \(A_2\)-limb \((e = 1, 2, 3, ..., n)\)
\(e_C\) and \(e_{A_3}\) – link of \(A_3\)-limb \((e = 1, 2, 3, ..., n)\)
\(e_D\) and \(e_{A_4}\) – link of \(A_4\)-limb \((e = 1, 2, 3, ..., n)\)
\(E = (e_1, ..., e_n)\) – set of constituent elements
\(EA\) – evolutionary algorithm
\(EM\) – evolutionary morphology
\(ES\) – Evolution strategies
\(F\) – linear transformation
\(f\) – degree of mobility of a joint in the general motion space with six dimensions
$F \leftarrow G_1G_2...G_k$ general notation for the kinematic chain associated to a
serial, parallel or hybrid robot with $k$ simple and/or complex limbs
$G_i (i=1, 2,...,k)$

FRs – functional requirements
FR TIMS - Fédération de Recherche Technologies de l’Information et
de la Mobilité, de la Sûreté

GA – genetic algorithm
$G_i (1_Gr2_Gr...n_Gr)$ the kinematic chain associated to the $i^{th}$ limb
(i=1,2,...,k).

$H$ – characteristic point of the distal link/end-effector
$H$- helical pair

IFMA - Institut Français de Mécanique Avancée (French Institute of
Advanced Mechanics)

INRIA - Institut National de Recherche en Informatique et en Automa-
tique (The French National Institute for Research in Computer
Science and Control)

IRCCyN - Institut de Recherche en Communications et Cybernétique de
Nantes

$I_{n,n}$ - $n \times n$ identity matrix

$J$ - Jacobian matrix
$J^t$ - transpose of the Jacobian matrix
$J^i$ – inverse of the Jacobian matrix
$J_S$ - direct kinematics matrix/Jacobian
$J_P$ - inverse kinematics matrix/Jacobian

$k$ - total number of limbs of the parallel manipulator
$k_1$ - number of simple limbs of a parallel manipulator
$k_2$ - number of complex limbs of a parallel manipulator

KP - kinematic pair
KC - kinematic chain

$Q$ - generic notation for a kinematic chain or a mechanism

LaMI - Laboratoire de Mécanique et Ingénieries (Mechanical Engineer-
ing Research Group)

LASMEA - Laboratoire des Sciences et Matériaux pour l'Electronique,
et d'Automatique (Laboratory of Sciences and Materials for ELECT-
ronic, and of Automatic)

LIRMM - Laboratoire d'Informatique, de Robotique et de Microélec-
tronique de Montpellier (Montpellier Laboratory of Computer Sci-
ence, Robotics, and Microelectronics)

$L_c$ - characteristic length

$m$ - total number of links (mechanism elements) including the fixed base

$m_Q$ - number of joints in kinematic chain $Q$
$M$ - mobility
$M_Q$ - mobility of kinematic chain/mechanism $Q$
$iM$ - instantaneous mobility
$iM_Q$ - instantaneous mobility of kinematic chain/mechanism $Q$
$N$ - number of overconstraints
$n$ - number of kinematic elements (links) excepting the fixed link
$O_t = (o_1,...,o_n)$ - set of morphological operators applicable at each
generation $t$

$O_0x_0y_0z_0$ - reference frame
$O_n$ - position vector of point $O_n$ in fixed reference frame
$p$ - number of joints
$p_P$ - number of passive joints
$p$ - operational velocity vector
$P$ - prismatic joint/pair
$Pa$ - parallelogram loop
$Pa_t$ - telescopic parallelogram loop
$Pn2$ - planar close loop with two degrees of mobility
$Pn4$ - planar close loop with three degrees of mobility
$PM$ - parallel manipulator
$q$ - number of independent closed loops
$q$ - joint velocity vector
$q_i$ - finite displacement in the $i$th actuated joint
$r$ - number of joint parameters that lost their independence
$r_Q$ - number of joint parameters that lost their independence in kinematic
chain/mechanism $B$
$r_i$ - number of joint parameters that lost their independence in the closed
loops integrated in the complex limbs of a parallel robot
$r_i^{Gi}$ - number of joint parameters that lost their independence in the
closed loops which may exist in $i$th limb $G_i$ of a parallel robot
$R$ - revolute joint/pair
$R$ - actuated revolute joint
$Rb$ - rhombus loop
$R_F$ - the range of linear transformation $F$
$R_Q$ - vector space of operational velocities of the kinematic
chain/mechanism $Q$
$R_{a/b}^Q$ - vector space of relative velocities between links $a$ and $b$ in
kinematic chain/mechanism $Q$
$R_{a/b}^Q$ - base of the vector space $R_{a/b}^Q$
$\mathbf{K}$ - reactivity
$S_{a,b}^Q$ - connectivity between links $a$ and $b$ in a kinematic chain/mechanism $Q$

$S_Q$ - connectivity of kinematic chain/mechanism $Q$

$S_t^Q$ - instantaneous connectivity of kinematic chain/mechanism $Q$

$\mathcal{S}$ - semangularity

$T$ - structural redundancy

$T_i = (\tau_1, \ldots, \tau_n)$ - set of evolution criteria from generation $t$ to generation $t+1$

$U$ - vector space of joint velocities

$U_Q$ - vector space of joint velocities of kinematic chain/mechanism $Q$

$v$ - translational velocity vector

$\dot{w}$ - general velocity vector of the moving platform

$w_1, w_2, \ldots, w_6$ - general coordinates

$W$ - vector space of external velocities

$W_Q$ - vector space of external velocities of kinematic chain/mechanism $Q$

$x_P, y_P, z_P$ - coordinates of point $P$ in the reference frame

$\alpha, \beta, \delta, \phi$ - rotation angles

$\dot{\alpha}, \dot{\beta}, \dot{\delta}, \dot{\phi}$ - time derivatives of the rotation angles

$\Delta q$ - vector of the infinitesimal motions of joints

$\Delta \mathbf{w}$ - vector of infinitesimal motions of the end-effector

$t : t \rightarrow \{\text{true}, \text{false}\}$ - termination criterion for evolutionary morphology

$\nu_1$ - conditioning index

$\nu_2$ - minimum conditioning index

$\nu_3$ and $\nu_4$ - manipulability indexes

$\xi_i$ - square root of the eigenvalues of $(J^T J)^{-1}$

$\lambda$ - scalar / proportionality factor

$\mu$ - vector of the operational forces

$\tau$ - vector of the joint forces

$\psi$ - velocity transmission factor

$\omega$ - angular velocity

$\Sigma_t = (\sigma_1, \ldots, \sigma_n)$ - set of solutions at each generation $t$

$\Phi = (\phi_1, \ldots, \phi_n)$ - set of design objectives

$0$ - fixed base of a kinematic chain/mechanism

$l_1-2-\ldots-n$ - links (elements) of a kinematic chain/mechanism

$l_{Q1}-2_{Q}-\ldots-n_Q$ - links (elements) of kinematic chain/mechanism
1 Introduction

This section introduces the terminology and presents the main objectives of the book. The family of parallel robots is placed in the general field of robots and the structural synthesis in the general context of robotics.

1.1 Robot

The origin of the term *robot* can be found in a science fiction play written by Karel Čapek (1920). This play titled R.U.R. was published in Czech in 1920 as an abbreviation of *Rozum's Universal Robots*, a collective drama in a comic prologue and three acts first performed in National Theatre in Prague on January 25, 1921. We note that in Czech “rozum” means wisdom and “robota” means servitude/forced labour/drudgery. Since the word robot was apparently coined by the author's brother, cubist painter and writer Joseph Čapek, the term is used to refer to a wide range of man-made things from devices to programs that are used to do automatically and autonomously various tasks by replacing humans.

Human-made analogs of living systems could be found long ago in Antiquity despite the recent usage of the word robot. The pigeon conceived by the Greek mathematician Archytas of Tarentum in 425 BC and chronicled by Favorinus and by Aulus Gellius in his Attic Nights could be considered the first “flying robot” and the south-seeking chariot, possibly used by the Chinese Yellow Emperor Huan Ti, the first “field robot”. Other legendary solutions such as the “walking machines” called wooden ox and gliding horse of Chu-ko Liang were invented to transport army food supplies across rough terrain during the period of China’s Three Kingdoms in 209 AC (Yan 1999). The automatic machines (Al-Jazary 1973) constructed by Al-Jazari (1136-1206), the designs of mechanical knight (Rosheim 2006) and flying machines (Da Vinci) of Leonardo da Vinci (1452-1519), the mechanical duck and the flute player – the first automatons (Balpe and Drye 1994) constructed by Jacques de Vaucanson (1709-1782) and the androids (the draughtsman, the musicians and the writer) constructed in 1774 by Pierre Jacquet-Droz (1771-1790) and his
son Henri-Louis (1753-1791) could also be considered as precursors of the current robots from the pre-digital era.

Hundreds of robot definitions are known today and hundreds of millions of web sites could be referenced today by a simple search on the word robot. According to the American Heritage Dictionary a robot is a mechanical device that is capable of performing a variety of often complex human tasks on command or by being programmed in advance. Other definitions of the word robot proposed by on-line dictionaries are systematized in Table 1.1.

Robots can be found today in the manufacturing industry, agricultural, military and domestic applications, space exploration, medicine, education, information and communication technologies, entertainment, etc.

We can see that the word robot is mainly used to refer to a wide range of mechanical devices or mechanisms, the common feature of which is that they are all capable of movement and can be used to perform physical tasks. Robots take on many different forms, ranging from humanoid, which mimic the human form and mode of movement, to industrial, whose appearance is dictated by the function they are to perform. Robots can be categorized as robotic manipulators, wheeled robots, legged robots, swimming robots, flying robots, androids and self reconfigurable robots which can conform themselves to a given task. This book focuses on the parallel robotic manipulators which are the counterparts of the serial robots.

We also note that the term robot has recently morphed to also refer to “bots”, which are automated programs used in several online functions in computer science and artificial intelligence. The term has also entered into the informal language to define certain forms of human behaviour or even a traffic light in South Africa.
Table 1.1. Various definitions of the term robot

<table>
<thead>
<tr>
<th>Dictionary</th>
<th>Definition</th>
</tr>
</thead>
</table>
| Encarta® World English Dictionary, North American Edition¹ | 1. Programmable machine for performing tasks: a mechanical device that can be programmed to carry out instructions and perform complicated tasks usually done by people.  
2. Imaginary machine: a machine that resembles a human in appearance and can function like a human, especially in science fiction.  
3. Person like a machine: somebody who works or behaves mechanically and emotionlessly.  
| Compact Oxford English Dictionary²            | A machine capable of carrying out a complex series of actions automatically, especially one programmable by a computer.                        |
| Merriam-Webster’s Online Dictionary, 10th Edition³ | 1a. A machine that looks like a human being and performs various complex acts (as walking or talking) of a human being; also: a similar but fictional machine whose lack of capacity for human emotions is often emphasized.  
1b. An efficient insensitive person who functions automatically.  
2. A device that automatically performs complicated often repetitive tasks.  
3. A mechanism guided by automatic controls. |
| Cambridge International Dictionary of English⁴ | 1. A machine used to perform jobs automatically, which is controlled by a computer.  
2. Disapproving someone who does things in a very quick and effective way but never shows their emotions.  
3. A traffic light in South Africa. |
| Cambridge Dictionary of American English⁵     | A mechanical device that works automatically or by computer control.                                                                        |
| Columbia Encyclopedia, Sixth Edition⁶         | Mechanical device designed to perform the work generally done by a human being.                                                            |

²http://www.AskOxford.com  
³http://www.m-w.com/  
⁴http://dictionary.cambridge.org/  
⁵⁶http://dictionary.cambridge.org/
### Table 1.1. (cont.)

<table>
<thead>
<tr>
<th>Dictionary</th>
<th>Definition</th>
</tr>
</thead>
</table>
| The American Heritage® Dictionary of the English Language<sup>7</sup> | 1. A mechanical device that sometimes resembles a human and is capable of performing a variety of often complex human tasks on command or by being programmed in advance.  
2. A machine or device that operates automatically or by remote control.  
3. A person who works mechanically without original thought, especially one who responds automatically to the commands of others. |
| The Wordsmyth English Dictionary-Thesaurus<sup>8</sup> | 1. A mechanical device, esp. one controlled electronically, that can perform physical tasks of a human being, in industrial assembly or the like.  
2. A person who moves or acts mechanically, without thought or feeling. |
| Infoplease Dictionary<sup>9</sup> | 1. A machine that resembles a human and does mechanical, routine tasks on command.  
2. A person who acts and responds in a mechanical, routine manner, usually subject to another's will; automaton.  
3. Any machine or mechanical device that operates automatically with humanlike skill. |
| Dictionary.com<sup>10</sup>       | 1. A mechanical device that sometimes resembles a human and is capable of performing a variety of often complex human tasks on command or by being programmed in advance.  
2. A machine or device that operates automatically or by remote control.  
3. A person who works mechanically without original thought, especially one who responds automatically to the commands of others. |
| Wiktionary<sup>11</sup>           | 1. A machine that resembles humans in shape or scope of function.  
2. A machine that operates automatically.  
3. A machine controlled by a fundamentally ingrained computer. |

<sup>7</sup>http://www.bartleby.com/61/s0.html  
<sup>8</sup>http://www.wordsmyth.net/  
<sup>9</sup>http://www.infoplease.com/dictionary.html  
<sup>10</sup>http://www.dictionary.com/  
<sup>11</sup>http://www.wiktionary.org
<table>
<thead>
<tr>
<th>Source</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rhymezone(^{12})</td>
<td>A mechanism that can move automatically.</td>
</tr>
<tr>
<td>LookWAYup</td>
<td></td>
</tr>
<tr>
<td>Translating Dictionary/Thesaurus(^{13})</td>
<td></td>
</tr>
<tr>
<td>WordNet 1.7</td>
<td></td>
</tr>
<tr>
<td>Vocabulary Helper(^{14})</td>
<td></td>
</tr>
<tr>
<td>Wikipedia, the Free Encyclopedia(^{15})</td>
<td>1. A robot is a type of device.</td>
</tr>
<tr>
<td></td>
<td>2. Robot can also refer to Internet bot, an automated software program used on the Internet.</td>
</tr>
<tr>
<td>AllWords.com Multi-Lingual Dictionary(^{16})</td>
<td>1. Especially in science-fiction: a machine that vaguely resembles a human being and which can be programmed to carry out tasks. Compare android.</td>
</tr>
<tr>
<td></td>
<td>2. An automatic machine that can be programmed to perform specific tasks.</td>
</tr>
<tr>
<td></td>
<td>3. Someone who works efficiently but who lacks human warmth or sensitivity.</td>
</tr>
<tr>
<td></td>
<td>4. An automatic traffic signal.</td>
</tr>
<tr>
<td>WebmasterWorld Webmaster and Search Engine Glossary(^{17})</td>
<td>Program that automatically does &quot;some action&quot; without user intervention. In the context of search engines, it usually refers to a program that mimics a browser to download web pages automatically. A spider is a type of robot. Some times referred to as Webbots.</td>
</tr>
<tr>
<td>Netlingo(^{18})</td>
<td>Traditionally, it's a device that can move and react to sensory input…</td>
</tr>
<tr>
<td></td>
<td>The term &quot;robot&quot; has morphed to also refer to bots, which are automated programs used in several online functions.</td>
</tr>
<tr>
<td>Webopedia(^{19})</td>
<td>1. A device that responds to sensory input.</td>
</tr>
<tr>
<td></td>
<td>2. A program that runs automatically without human intervention.</td>
</tr>
</tbody>
</table>

\(^{12}\)http://www.rhymezone.com/
\(^{13}\)http://lookwayup.com/free
\(^{14}\)http://poets.notredame.ac.jp/cgi-bin/wn
\(^{15}\)http://www.wikipedia.org
\(^{16}\)http://www.allwords.com/
\(^{17}\)http://www.webmasterworld.com/glossary/
\(^{18}\)http://www.netlingo.com/index.cfm
\(^{19}\)http://www.pcwebopaedia.com/
### Table 1.1. (cont.)

| CCI Computer[20] | 1. A mechanical device that performs a task that would otherwise be done by a human. Robots can be useful for jobs that are boring or dangerous for humans to perform. The simplest robots are capable only of repeating a programmed motion; the most sophisticated models can use sensors and artificial intelligence to distinguish between objects, understand natural language, and make decisions. Robots can be programmed or operated by remote control.  
2. A computer program that performs intelligent tasks such as retrieving World Wide Web documents and indexing references.  
3. A program that performs a programmed communication function such as automated email answering, responding to newsgroup message, or regulatory functions in IRC, graphical chat, and other online environments |
| Computer Telephony & Electronics Dictionary and Glossary[21] | 1. A device that responds to sensory input. They are often just called "Bots".  
2. A program that runs automatically without human intervention. Typically, a robot is endowed with some artificial intelligence so that it can react to different situations it may encounter. Two common types of robots are agents and spiders. |
| Internet Terms[22] | A robot is a program that is designed to automatically go out and explore the Internet for a specific purpose. Robots that record and index all of the contents of the network to create searchable databases are sometimes called spiders or Worms. WebCrawler and Lycos are popular examples of robots. |
| TECHNICAL[23] | Robot - Besides being a mechanical device used to mimic human form, usually to accomplish some repetitive task, this term refers to a computer program that scans Web pages and links. Like a similar spider program, robots are used to scan Web pages and index them. |

The science of robotics has undergone an explosive growth since the word *robotics* was coined by the writer Isaac Asimov in 1950. In his science fiction book “I, Robot” he presented the famous three laws of robotics: (i) a robot may not injure a human being, or, through inaction, allow a human being to come to harm; (ii) a robot must obey the orders given it by human beings except where such orders would conflict with the first law and (iii) a robot must protect its own existence as long as such protection does not conflict with the first or second law. According to the American Heritage Dictionary, robotics is the science or study of the technology associated with the design, fabrication, theory, and application of robots. This technology combines mechanical devices, actuators, sensors, controllers, software and computers.

Today robots get closer to humans and new issues rise in matters related to human robot interaction, in advances and experiences of robots and automation at home, at work, for education, as well as in other emerging areas. The next generation of robots will be capable of interacting with humans on a sophisticated level, making decisions, helping, caring and giving companionship.

Recent research by the Japan Robotics Association (JPA), United Nations Economic Commission (UNEC) and the International Federation of Robotics (IFR) indicates that the nascent personal and service robotics market will exhibit exceptional near term growth. The service and personal robotics marketplaces together will equal the size of the industrial robotics market (the combination of manufacturing and bio-industrial) by 2005, and will be twice the size of the industrial robotics market by 2010, and almost 4X its size by 2025.

Many of the technologies required to build functional personal and service robots already exist, and markets for these products are already in place. Some of the most salient enabling technologies include advances in microprocessor technology, wireless technology, image processing, speech recognition, motion sensor technology, and embedded systems development tools.

Hundreds of universities worldwide have research programs in robotics and many are awarding degrees in robotics.

Various definitions of the word robotics are systematized in Table 1.2. We can see that they converge towards the integration of the design and the end use in the studies related to robotics. This book focuses on the conceptual design of parallel robots.
Table 1.2. Various definitions of the term robotics

<table>
<thead>
<tr>
<th>Dictionary</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Encarta® World English Dictionary, North American Edition¹</td>
<td>Design and use of robots: the science and technology relating to computer-controlled mechanical devices such as the automated tools commonly found on automobile assembly lines.</td>
</tr>
<tr>
<td>Compact Oxford English Dictionary²</td>
<td>The branch of technology concerned with the design, construction, and application of robots.</td>
</tr>
<tr>
<td>Merriam-Webster's Online Dictionary, 10th Edition³</td>
<td>Technology dealing with the design, construction, and operation of robots in automation.</td>
</tr>
<tr>
<td>Cambridge International Dictionary of English⁴</td>
<td>The science of making and using robots.</td>
</tr>
<tr>
<td>Cambridge Dictionary of American English⁵</td>
<td>The science of designing and operating robots.</td>
</tr>
<tr>
<td>Columbia Encyclopedia, Sixth Edition⁶</td>
<td>Science and technology of general purpose, programmable machine systems.</td>
</tr>
<tr>
<td>The American Heritage® Dictionary of the English Language⁷,¹⁰</td>
<td>The science or study of the technology associated with the design, fabrication, theory, and application of robots.</td>
</tr>
<tr>
<td>The Wordsmyth English Dictionary-Thesaurus⁸</td>
<td>The technology of designing and using robots for various, usually industrial tasks.</td>
</tr>
<tr>
<td>Infoplease Dictionary⁹</td>
<td>The use of computer-controlled robots to perform manual tasks, especially on an assembly line.</td>
</tr>
<tr>
<td>Wiktionary¹¹,¹⁵</td>
<td>The science and technology of robots, their design, manufacture, and application.</td>
</tr>
</tbody>
</table>

²http://www.AskOxford.com  
³http://www.m-w.com/  
⁴http://dictionary.cambridge.org/  
⁵http://dictionary.cambridge.org/  
⁶http://www.bartleby.com/61/s0.html  
⁷http://www.wordsmyth.net/  
⁸http://www.infoplease.com/dictionary.html  
⁹http://www.infoplease.com/dictionary.html  
¹⁰http://www.dictionary.com/  
¹¹http://www.wiktionary.org
<table>
<thead>
<tr>
<th>Table 1.2. (cont.)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rhymezone</strong>(^{12})</td>
</tr>
<tr>
<td><strong>LookWAYup Translating Dictionary/Thesaurus</strong>(^{13})</td>
</tr>
<tr>
<td><strong>WordNet 1.7 Vocabulary Helper</strong>(^{14})</td>
</tr>
<tr>
<td><strong>AllWords.com Multi-Lingual Dictionary</strong>(^{15})</td>
</tr>
<tr>
<td><strong>The On-line Medical Dictionary</strong>(^{17})</td>
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<tr>
<td><strong>Netlingo</strong>(^{18,19})</td>
</tr>
</tbody>
</table>

\(^{12}\)http://www.rhymezone.com/  
\(^{13}\)http://lookwayup.com/free  
\(^{14}\)http://poets.notredame.ac.jp/cgi-bin/wn  
\(^{15}\)http://www.wikipedia.org  
\(^{16}\)http://www.allwords.com/  
\(^{17}\)http://cancerweb.ncl.ac.uk/omd/  
\(^{18}\)http://www.netlingo.com/index.cfm  
\(^{19}\)http://www.pcwebopaedia.com/  
\(^{20}\)http://www.currents.net/resources/dictionary/index.html  
\(^{21}\)http://www.csgnetwork.com/glossary.html
1.3 Parallel Robot

Although the appearance and capabilities of robots vary vastly, all robots share the features of a mechanical, movable structure under some form of control. The structure of a robot is usually mostly mechanical and takes the form of a mechanism having as constituent elements the links connected by kinematic joints. Serial or parallel kinematic chains are concatenated in the robot mechanism. The serial kinematic chain is formed by links connected sequentially by joints. Links are connected in series as well as in parallel making one or more closed-loops in a parallel mechanism. The members of a mechanism may take various forms: bars, plates, platforms, cams, gears, sliders, sliding blocks, etc.

The mechanical architecture of parallel robots is based on parallel mechanisms in which a member called a mobile platform is connected to a reference element by at least two limbs that can be elementary (simple) or complex. The robot actuators are integrated in the limbs (also called legs) usually closed to the fixed member, also called the base or the fixed platform. The mobile platform positions the robot end-effector in space and could have between two and six degrees of freedom. Usually the number of actuators coincides with the degrees of freedom of the mobile platform, exceeding them only in the case of redundantly-actuated parallel robots.

The paradigm of parallel robots is the hexapod-type robot, which has six degrees of freedom, but recently the machine industry has discovered the potential applications of lower-mobility parallel robots with just 2, 3, 4 or 5 degrees of freedom. Indeed, the study of this type of parallel manipulators is very important. They exhibit interesting features if compared to hexapods, such as simpler architecture, simpler control system, high-speed performance, low manufacturing and operating costs. Furthermore, for several parallel manipulators with decoupled or uncoupled motions the kinematic model can be easily solved to obtain algebraic expressions, which are well suited for an implementation in optimum design problems. Parallel mechanisms can be considered a well-established solution for many different applications of manipulation, machining, guiding, testing, control, etc.

1.4 Terminology

The terminology used in this book and defined in this section is mainly established in accordance with the terminology adopted by the International Federation for the Promotion of Mechanism and Machine Science
(IFToMM) and published in (Ionescu 2003). The main terms used in this book concerning kinematic pairs (joints), kinematic chains and robot kinematics are defined in Tables 1.3-1.5. They are completed by some complementary remarks, notations and symbols used in this book.

In the standard terminology, a **kinematic chain** is an assembly of links (mechanism elements) and joints, and a **mechanism** is a kinematic chain in which one of its links is taken as a “frame”. In this definition the “frame” is a mechanism element deemed to be fixed. In this book, we use the notion of **reference element** to define the “frame” element. The reference element can be fixed or may merely be deemed to be fixed with respect to other mobile elements. The **fixed base** is denoted in this book by \(0\). A mobile element in a kinematic chain \(Q\) is denoted by \(n_Q\) \((n=1, 2, \ldots)\). Two or more links connected together in the same link such that no relative motion can occur between them are considered as one link. The identity symbol “\(=\)” is used between the links to indicate that they are welded together in the same link. For example, the notation \(1_Q=0\) is used to indicate that the first link \(1_Q\) is connected to the fixed base by a common link. A kinematic chain \(Q\) is denoted by the sequence of its links. The notation \(Q(1_Q=0-2_Q-\ldots n_Q)\) indicates a kinematic chain in which the first member is fixed and the notation \(Q(1_Q-2_Q-\ldots n_Q)\) a kinematic chain with no fixed member.

We will use the notion of mechanism to qualify the whole mechanical system, and the notion of kinematic chain to qualify the sub-systems of a mechanism. So, in this book, the same assembly of links and joints \(Q_1\) is considered to be a kinematic chain when it is integrated as a sub-system in another assembly of links and joints \(Q_2\), and it is considered a mechanism when \(Q_1\) represents the whole system. The systematization, the definitions and the formulae presented in this book are also valuable for mechanisms and kinematic chains.

We use the term **mechanism element** or **link** to name a component (member) of a mechanism with any form (bar, gear, cam, etc.). In this text, unless otherwise stated, we consider all links to be rigid. We distinguish the following types of links:

a) **monary link** - a mechanism element connected in the kinematic chain by only one joint (a link which carries only one kinematic pairing element),

b) **binary link** - a mechanism element connected in the kinematic chain by two joints (a link connected to two other links),

c) **polinary link** - a mechanism element connected in the kinematic chain by more than two joints (ternary link - if the link is connected by three joints, quaternary link if the link is connected by four joints,...).
### Table 1.3. IFToMM terminology with respect to the kinematic pairs

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pairing element</td>
<td>Assembly of surfaces, lines or points of a solid body through which it may contact with another solid body.</td>
</tr>
<tr>
<td>Mechanism element</td>
<td>Solid body or fluid component of a mechanism.</td>
</tr>
<tr>
<td>Frame</td>
<td>Mechanism element deemed to be fixed.</td>
</tr>
<tr>
<td>Base</td>
<td>Body (link) of a robot that carries the first joint(s) of the kinematic chain of a manipulator or pedipulator.</td>
</tr>
<tr>
<td>Rigid body</td>
<td>Theoretical model of a solid body in which the distances between particles are considered to be constant, regardless of any forces acting upon the body.</td>
</tr>
<tr>
<td>Link</td>
<td>1. Mechanism element (component) carrying kinematic pairing elements.</td>
</tr>
<tr>
<td></td>
<td>2. Element of a linkage.</td>
</tr>
<tr>
<td>Limit position of a link</td>
<td>Position of a link for which a coordinate which describes its position relative to an adjacent link is a maximum or a minimum.</td>
</tr>
<tr>
<td>Input (driving) link</td>
<td>Link whereby motion and force are imparted to a mechanism.</td>
</tr>
<tr>
<td>Output (driven) link (follower)</td>
<td>Link from which required forces and motions are obtained.</td>
</tr>
<tr>
<td>Bar</td>
<td>Link that carries only revolute joints.</td>
</tr>
<tr>
<td>Crank</td>
<td>Link able to rotate completely about a fixed axis.</td>
</tr>
<tr>
<td>Rocker</td>
<td>Link that oscillates within a limited angle of rotation about a fixed axis.</td>
</tr>
<tr>
<td>Coupler (floating link)</td>
<td>Link that is not connected directly to the frame.</td>
</tr>
<tr>
<td>Slider</td>
<td>Link that forms a prismatic pair with one link and a revolute pair with another link.</td>
</tr>
</tbody>
</table>