The Genesis of Fluid Mechanics, 1640–1780

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THE GENESIS OF FLUID MECHANICS 1640–1780

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My profound gratitude to all, although I think that truly the ones who should be most grateful are the readers, as once the book arrives in their hands, my task concludes, and they become the proprietors of the text.

Contents

PART II

Abbreviations

Acta Erud.	*Acta Eruditorum* of Leipzig
Mém. Acad. Berlin	*Histoire de l'Académie des Sciences et Belles Lettres* of Berlin
Nouv. Mém. Acad. Berlin	*Nouvelles Mémoires* Histoire de l'Académie des Sciences et Belles Lettres *of Berlin*
Mém. Acad. Paris	*Mémoires de l'Académie Royale des Sciences* de Paris
Comm. acad. petrop.	*Commentarii academiæ scientarum petropolitanæ* of St. Petertsburg
Novi comm. acad petrop.	*Novi commentarii academiæ scientarum petropolitanæ* of St. Petertsburg
Phil. Trans.	*Philosophical Transations* of the Royal Society of London
Jour. Sav.	*Journal de Sçavans*

Note

All measurements and sizes are expressed in the original units, followed by their conversion to the International System. The appendix lists the conversion factors used. Mathematical formulas and notation have been transcribed to present-day symbols, adding the constants required to render the formulas dimensionally consistent. This particularly affects the acceleration of gravity expressed by its customary symbol g, and consideration of density by its absolute value, that is to say mass/volume.

Introduction

Prolegomenon

In what follows we have set out to analyse the genesis of fluid mechanics as a modern scientific discipline. Two prior and interrelated questions must first be raised, however: what do we understand as the basic core of fluid mechanics, and when was this core developed?

As seen today, fluid mechanics is a science of great complexity and diversity. Yet, the field derived from a common source, and evolved into a succession of disciplines, each one with an increasing degree of specialization. This hierarchy is headed by fluid dynamics and fluid statics, understood as applied to liquids and gases, which in turn gave rise to subsonic, transonic and supersonic disciplines. Although they all sprang from the same common theoretical basis, each discipline has specific problems determined by the nature of the fluid, the dominating phenomena, the motion undergone and the boundary conditions. Looking back, we find that the basic hypotheses that mathematically regulate fluid mechanics, the structure and formulation of its fundamental equations, and the problems of its applications were all forged in the second half of the seventeenth and first half of the eighteenth centuries, to the extent that the process can be considered in many respects complete by the end of the 1750s. Key concepts in fluid mechanics—such as turbulence, boundary layer, discontinuity surfaces, viscosity and thermodynamic processes—were introduced in the course of the nineteenth and twentieth centuries.

We choose the decade of the 1750s as the jumping-off point for fluid mechanics, because a specific body of theory was built up by then, which (together with some experimental studies) provided the groundwork—despite a number of residual problems—for the modern scientific theory of fluids. This point was reached at the end of a century-long process in which the basic concepts gestated, were born, and evolved progressively with different theories and assumptions. It was only with the appearance of Isaac Newton's *Philosophiæ naturalis principia mathematica* in 1687, that the scientific nature of the discipline made its first appearance, for this work analyses for the first time the dynamics of fluids, basing itself on laws of nature of more general characteristics. Specifically, Newton introduces two basic premises: one concerning the definition of the constituents of the fluid, the other dealing with its behavior with respect to the general laws of dynamics as set out in the *Principia*. While it is true that before the appearance of this work, a large number of studies, some very worthy, existed on this subject, none of them provided the overall theoretical body of understanding supplied by the *Principia*. Nevertheless, despite a number of attempts over a period of 70 years, it was not until 1755 that Leonhard Euler was able to offer a definitive treatment.

While this process opened with Newton and closed with Euler, there were considerable differences in their fundamental assumptions about the nature of fluids. For Newton, a fluid like air was an aggregate of particles that respond individually to the laws of mechanics, in such a way that the force generated by the impact of a current against an object consisted of the sum of the effects of each individual impact. For Euler, by contrast, a fluid was a continuum, ideally separated into elemental domains capable of supporting forces and internal pressures, whose space–time evolution is regulated by the laws of dynamics, expressed by a set of differential equations. These two radically different concepts were established in 1687 and 1755, respectively, and they define the *termini* of the genesis of fluid mechanics. Before 1685, the concepts of fluid mechanics were still based on a simplistic (to our eyes) view of the physical universe; but after 1755 those concepts were replaced by the construction we use today.

The treatment of the development of fluid mechanics will deal with the dynamic aspects of fluids, liquids or gases subjected to forces, and limited by boundary conditions, as much as in their applications. Although static analyses are not dealt with as such, some aspects of fluid statics are looked at where this is needed to provide a more comprehensive picture. For example, in his 1755 work, Euler united static and dynamic accounts in the same theory, so in this case we deal explicitly to the former. On the other hand, we understand dynamics as the attempt to explain and reduce the motion of fluids in terms of the forces applied and the boundary conditions, and so do not deal with hydraulics, understood as the practical use of energy intrinsic to the movement of water.

Although Newton is treated as the founder of the discipline, we have not neglected the contribution of several pre-Newtonian works, tracing the development of the discipline from Evangelista Torricelli's work in 1644, through Christiaan Huygens and Edmé Mariotte. Among the post-Newtonians, the Bernoulli family (Jakob, Johann and Daniel), Benjamin Robins, Alexis Claude Clairaut, Jean Le Rond d'Alembert and Leonhard Euler play crucial roles, especially Euler's 1755 work which brings the development to completion. In the 20 years after this, we witness a burst of activity in applied and experimental fields, in the work of Jean Charles Borda, Charles Bossut, Pierre Bouguer and Jorge Juan y Santacilia, so that the developments we are concerned with effectively come to a head only in the 1770s.

Some clarifying points are worth developing before we begin. In particular, we need to say something about fluids from the point of view of theory, experiments and applications; we need to examine the distinction between hydraulics and hydrodynamics; and we need to raise the question of the relation between mechanics in general and fluid mechanics.

Although it is common to think of science as a theoretical–experimental enterprise, problems arise when the question of applied science or of the application of science is introduced. For our part, we see a connection between both aspects, which we have called 'applications'. We understand these to be the attempt to use available knowledge, be it theoretical or experimental, for modelling, explaining and reducing scientifically the behavior of machines, be they real or imaginary devices, but without these studies implying or intending their subsequent construction, which is a matter for technology.[1] In a strict definition of science, one might be tempted to exclude these applications. But the advantages obtained in the theoretical analysis of any machine are a source of knowledge, obliging us to adjust the theories to a specific reality, something which is neither easy nor immediate. This is not the same as experiment, which has the great advantage of granting us freedom for creating an ideal space in order to render a specific effect. We treat the applications as forming part the science itself, on a different plane from experiments, but not constituting a separate group. Applications follow on from theory and experiments, and are continued by the actual construction of machines. This comes under the heading of technology, which is another subject, implying contributions of a different nature. Applications have another important aspect: they are the expression of a social need and through them society acts on science. They form another, additional nexus connecting the conceptual and social aspects of science.

The relation between hydrodynamics and hydraulics comes under the ambit of interrelations between practical and theoretical worlds, if the former is understood as coming under the aegis of the theoretical world and the latter under the practical. We must remember that hydraulics flourished from antiquity as an

[1] The problem of the technology is the production of technical entities, which is not exactly the same as engineering, although both partly coincide. The first step of the process starts by understanding the ideal object to be materialized. This is done according to the scientific knowledge available, which is sometimes very scanty or difficult to apply. In the second step this knowledge, together with materials provided by nature, allows these objects to be constructed, in a wide sense of the word, giving rise to the appearance of the technological sciences. In spite of the differences between science and technology, there is something both share: the scientific method. Therefore, the applications have to be treated scientifically.

artisan activity, and remained so in the seventeenth and eighteenth centuries,[2] its aim being to optimise the use, handling and distribution of water in watermills, waterwheels, canals, etc. It is natural that hydraulics posed problems which the sages of the time tried to study. However, hydraulics as then understood was not, nor did it become, a science in the modern definition. It remained a dignified artisan activity, which is what it had been since ancient times, with perhaps a few technological improvements. However, some confusion exists as to the terminology of the time, and until the term hydrodynamics appeared, the term hydraulics was often used in its place. D'Alembert tried to clarify the scope of both terms in the *Encyclopédie Méthodique*,[3] but the word hydraulics continued to be used frequently in engineering circles. Nevertheless, while in its debt in some respects, hydrodynamics does not proceed from hydraulics.

As regards mechanics, its evolution during those years deserves comment. The main aim of mechanics was the establishment of its general laws, allowing its problems to be reduced to mathematics. Several attempts were made to do this. One was Newton's, with the establishment of the laws or axioms of motion, which introduced forces and the concept of mass. Newtonian mechanics assumed that these masses were points, or able to be reduced to points, and that the forces acted between these mass points. Previously, Huygens had considered the underlying principle to be the conservation of what he called live force (*vis viva*). This was the sum of the products of the masses multiplied by the square of the velocity of all the particles of the system. Conservation was limited to certain conditions, such as elastic collisions and processes under the action of gravity. In contrast to Newton's laws, which deal with the interaction between individual particles, live force was extended to the entire fluid mass; which, together with the advance of infinitesimal calculus, made the live force theory very suitable for handling large groups of particles, the credit for this development going to Daniel Bernoulli. D'Alembert offered a different proposal, eliminating

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[2] The most important work on this subject *Architecture Hydraulique* de Bernard Forest de Belidor. The first edition appeared in 1737 and was published continuously until the nineteenth century.

[3] In the volume III of the Encyclopédie Méthodique (1785), under the term Hydraulique it is said: 'The part of Mechanics which contemplates the motion of fluids and which shows how water is channelled and how to raise them, as much for making the waters form a jet as for other uses. … The hydaulique deals not only with the water tubes and raising waters, but also the machines required to this end, but even more with the general laws of motion of fluid bodies. However, for quite a few years, mathematicians gave the name hydrodynamique to the general science of motion of fluids, and reserved the name hydraulique for the disciplines which, in particular, considered the motions of water: that is to say the art of channelling water, of raising it and handing it for the different requirements of daily life'. In the Hydrostatique talks of 'the part of mechanics which considers the equilibrium of fluid bodies, together with the bodies which are immersed in them. … The hydrostatique is often confused with the hydraulique, as the subject matter is similar, and several authors hardly separate them at all'.

the forces, and introducing conservation of the momentum in impact, along with a practical rule similar to virtual velocities. Subsequently, Maupertuis established the principle of minimum action. Whatever the principle was, the aim was the same: to obtain the laws of motion.

In this connection, a little more needs to be said on the relation between fluid mechanics and mechanics understood in a general sense. If, as we have noted, hydraulics followed its own path, fluid mechanics emerged as part of mechanics.[4] This is palpable in Newton, who dedicated Book II of his *Principia* to fluids, and the Book I to solids, but in both cases with the same title. This way of proceeding is also clear in Mariotte, who proposes some models of what we have called mechanics of jets. The mathematicians of the eighteenth century contributed greatly to mechanics, and they treated fluids as one of three types of bodies: solids, deformable bodies and fluids. The difficulty in treating fluids mathematically was enormous, and the help of differential analysis was absolutely decisive. Moreover, the process of mathematization in Euler and d'Alembert requires the definition of fluids as a continuum, in spite of the fact that they imagined them to be physically constituted by an aggregate of particles. It is interesting to note that in the second half of the 1700s, fluid mechanics, or rather the rational mechanics of fluids, was at the vanguard of theoretical mechanics.

The genesis of fluid dynamics: a summary

Let us begin with a brief summary of the entire process of the genesis and evolution of fluid mechanics in order to aid comprehension, and to place the rest of the work in context. It is a core thesis that this entire evolution took place along two main lines of activity: one dedicated to the effects that fluid current exercises upon a body immersed in it, and the other to dealing with how fluids discharge themselves through tubes or reservoirs. We have called the first 'the problem of resistance' and the second the 'problem of discharge'. Almost all the authors of the time treated these subjects separately, and set them in one or other

[4] In the Preface of the *Principia*, Newton says: 'In this sense *rational mechanics* will be the science, expressed in exact propositions and demonstrations, of the motions that result from any forces whatever and of the forces that are required for any motions whatever. The ancients studied this part of *mechanics* in terms of the *five powers* that relate to the manual arts. … But since we are concerned with natural philosophy rather than manual arts, and are writing about natural rather than manual power, we concentrate on aspects of gravity, levity, elastic forces, resistance of fluids, and forces of this sort, whether attractive or impulsive. And therefore our present work sets forth mathematical principles of natural philosophy. For the basic problem of philosophy seems to be to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces'. [p. 382]

of these two contexts, sometimes even in the same book. Both enjoyed considerable independence, and a set of theories existed in each one, with its experiments and of course with its applications. Their methodology and evolution are not strictly comparable, as there are considerable differences between them, which makes them all the more interesting. Both respond to two sets of different preoccupations: in the case of resistance, to navigation problems and to machines driven by fluids such as mills and waterwheels; in the case of discharge, to water distributions and to jet reaction machines. The first was more practical and of more immediate interest; the second more theoretical, but conceptually superior, as the subsequent history bears out.

In the main body of this work, we have separated both parts, dedicating the first part to the problem of resistance, and the second to that of discharge, and in each of these we have dealt with the theoretical, experimental and application aspects separately. This way of the presenting the facts emphasizes the coherence of each the two lines, but it breaks the general chronology of the work, so in this Introduction we shall describe the evolution of fluid mechanics as a whole, following a single chronological line, while making reference to the resistance or the discharge in each case, according to its type.

We establish the starting point of the development, and more specifically of the line denominated 'the discharge problem' in Torricelli, who, in 1644, announced the law known today by his name, and which says that the efflux velocity of a liquid contained in a recepticle through an orifice is that which a heavy body would acquire when falling from a height equal to the depth at which the orifice is found.[5] The law appeared in the work *De motu gravium* (*On the motion of heavy bodies*), which deals with the motion of projectiles, and was a result of his ballistic studies. He tells us that the clue that led him to this law is the commonly observed fact that when a reservoir discharges through a vertical tube acting as a spout, the water reaches a height very close to the level of the reservoir (Fig. I-1a). He observed that the jet ended up converted into droplets in its highest part, and he attributed this to its losing some height. It is interesting to note the empirical origin of his law, which is not so obvious, as the efflux of water is a different phenomenon from the fall of the heavy body that serves him as a reference. Should the orifice be in the lateral wall, he indicates that the trajectory followed would be a parabola, similar to the one a projectile would describe (Fig. I-1b). What Torricelli does, is to assimilate the water to a body in free fall. That is to say, he converts the liquids into an aggregate of solid bodies.

[5] Mathematically, it is expressed as $v_e = \sqrt{2gh}$. That is, the efflux velocity is proportional to the square root of the depth.

The law, apart from being the first one to refer to fluids in motion,[6] is of considerable interest for various reasons. First, it identifies fluids with solid bodies, in spite of their having such a different appearance and behaviour. Second, he proposes a mathematical relation for the first time. Third, he opens up an experimental field that would be a focal point for the next 50 years. In addition, it was the starting point of the discharge problem.

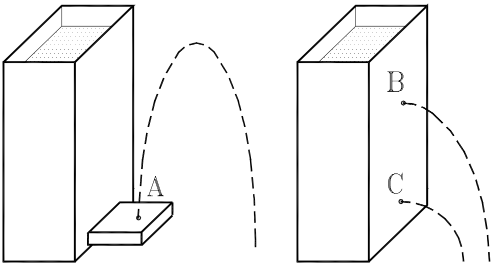

Fig. I-1. Torricelli springs

As regards the paternity of this law, Torricelli, who was in Italy when he wrote the aforementioned book, says that it was already previously known by Benedetto Castelli, his master. However, later authors, such as Daniel Bernoulli or Giovanni Poleni, say that some 3 years before (i.e., around 1641), Castelli had supposed that the outlet velocity was proportional to the depth, instead of its square root, as Torricelli had stated. It is difficult to know which account is correct; the fact is that the law was attributed to Torricelli by his contemporaries.

The apparatus used for the experimental verification of this law was based on collecting water flowing out in the discharge during a certain measurable time interval, while the reservoir was kept full. Once the surface of the orifice was known, it was easy to calculate the outlet velocity of the water. A variant consisted in not filling the vessel during the process, and measuring the time it took to discharge completely, as by calculation this time was found to be double that in which an equal quantity of water would be discharged if the level of the vessel was maintained. Whichever procedure was used, the reality was that this apparently clear and simple experiment provided disparate results, a fact that caused enough headaches to stimulate an in-depth study of the phenomenon. Huygens and Mariotte in the newly founded Academy of Sciences of Paris,

[6] The hydrostatics laws are older. The first one is due to Archimedes in the third century BC, and others were found by Simon Stevin (1548–1620) and Blaise Pascal, who was a contemporary of Torricelli.

and Guglielmini and Poleni in Italy, are the only ones known to have published results, although many more must have tried.

Huygens performed experiments in Paris in 1668, and his results confirmed Torricelli's law as opposed to the supposition of Castelli.[7] However, in the following year he went back on what he said, and questioned the previous results. What happened was that in the experiment less water was collected than predicted, and it would appear that the same thing happened in the various attempts made by him and other experimenters. We now know that the discharge process is not purely kinematic, as Torricelli's law supposes, but dynamic, and all the water held in the recepticle intervenes in the phenomena; moreover, details such as the form of the vessel, the type of spout and the relation between the surface of the orifice and the surface of the reservoir play an important part. This was then unknown, and was only discovered with great difficulty. According to the sentiment of the times, these anomalies could be interpreted in two ways: either this velocity was not proportional to the square root of the depth; or the height, when considering the fall, was not that of the reservoir itself but a fraction of this. The experiments pointed towards this last alternative, which gave rise to the initiation of a process for revising the law, admitting the mathematical form of the square root, but changing the proportionality constant. Moreover, as the geometry of the apparatus influenced the results, the discrepancies among the measurements obtained by all and sundry were significant.

Huygens' work on motion in fluids was not just limited to discharge: he also devoted considerable effort to the effect undergone by bodies moving inside a fluid. Huygens, like Torricelli, was also a student of ballistics, and one of his preoccupations was to introduce the effect of the resistance of air into the motion of projectiles. If the trajectory followed by projectiles in the supposed absence of a resistant force was a parabola, when the resistance is included, the trajectory deviates from this geometric shape, and in addition, the determination of its path involves a considerable mathematical complication. Initially, it appears that Huygens estimated that the resistance encountered by the projectile was proportional to the velocity, but, after the experiments he made in 1669, he deduced that this was not so. Instead it was proportional to the square of the velocity. He introduced this law for the first time, and, with some nuances, it is still considered to hold. The 'problem of resistance' also started at this point: as we shall see, it is different in nature from that of the discharge, although it also originated in ballistics. An initial comparison between the two shows that the problem of resistance was dynamic in character, while discharge was rather more kinematic,

[7] Cf. Jean-Baptiste du Hamel, *Regiæ Scientarum Academia Historia Parisiis*, 1698.

even in spite of the fact that the concept of force was not yet clear.[8] In order to measure the resistance, Huygens assimilated this to the weight of a column of fluid whose base was the frontal surface of the body in question. For the reference height of the column, he took the height from which the moving body would be dropped, in order for it to acquire a velocity equal to its motion; thus it was that the resistance was measured as being equivalent to one or two, or several times this height. We have called this height the 'kinetic height', and, as we shall see, it will play a fundamental role in the study of fluids. This height is the same as the depth in Torricelli's law.[9]

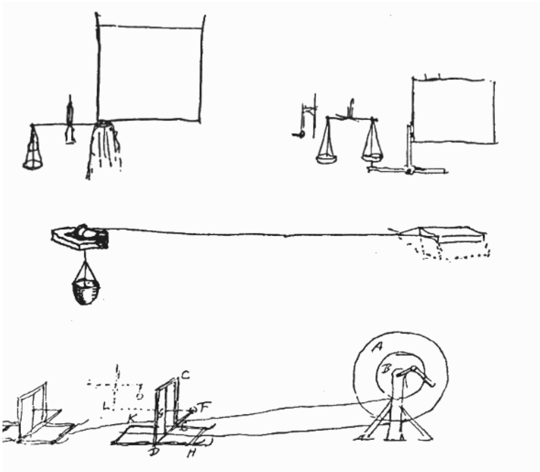

Fig. I-2. Huygens experiments

Huygens carried out experiments on resistance following three different procedures as detailed in Fig. I-2. The first consisted in measuring the force produced by a jet against a plate, which was an extension of the discharge phenomenon. The second was to measure the towing force of a small ship in a pond; a method which has been perpetuated up to the present in the towing tanks that began to be built in the middle of the eighteenth century. The third, which also makes use of the motion of the object in a fixed medium, consisted of small sleds transporting a plate attached by counterweights, in such a way that the

[8] To express it, they used terms such as impulse, impressions and thrusts and it would not be until Newton that the concept of force in a modern sense would enter physics, although it took some time to be admitted. However, the problem of force in Newton has to be dealt with cautious; see Westfall, *Force in Newton's Physics*, especially Chapter 7.

[9] Its mathematical expression was $h_c = v^2/2g$. Let us observe its identity with the Torricelli's law.

plate fell when the force produced by the moving air was superior to the value adjusted by the counterweights. As regards the last two methods, we can only say that his idea was clear, but that the capability of the instruments available to him was very rudimentary, although the methods were actually ahead of the times. Concerning the first of the three procedures, nowadays we know that the phenomenon of the impact of a jet against a plate differs from the resistance of a body submerged in a fluid. However, Huygens assimilated both in a single phenomenon, an idea that was a source of problems until Daniel Bernoulli separated them definitively in 1736. The jet experiment contributed to this identification, due to its closeness to the discharge. Diagrammatically, the matter was very simple: if the plate against which the jet strikes were to close the fluid outlet completely, the force on it would be the product of the static pressure times the outlet surface, i.e., the weight of a column of water whose height was the depth and area of the outlet. However, in the measurements made during the discharge, they found that the force was greater, and they could not find an explanation for this fact.

Huygens did not confine his experiments to jets of water, but dealt also with discharges of air jets on plates, using an interesting apparatus that later served Mariotte as a standard. Huygens ended by giving specific values to the resistance of a plate moving in air or water, and the fact that his values were high, according to our present knowledge, does not detract one jot from his merit, given his rudimentary measuring apparatus.

We have seen that Huygens approached the two major problems of fluids, resistance and discharge with different methods. A little later on we shall speak of Mariotte, who probably co-operated in the experiments with Huygens, and followed a very similar approach. However, for the sake of chronology, we shall go on to Domenico Guglielmini, who in 1683 carried out some experiments that are worth mentioning, and which were published in 1690 in his work *Aquarum fluentium mensura* (*The measurement of the motion of the waters*).

Guglielmini is considered as belonging to the so-called Italian School of Hydraulics, a denomination that we owe to René Dugas,[10] who traces its origins back to Galileo in the sixteenth century. The existence of the school was due to different factors, above all the Italian scientific flowering in the Renaissance, and the peculiar importance to Italy of solving practical problems of floods, lagoons and marshes. Guglielmini approached the problems of the motion of water in canals and its discharge from reservoirs. In order to study them he used an apparatus consisting of a barrow that discharged itself by successive regularly

[10] Carlo Maccagni also calls in that way in his article ‘Galileo, Castelli, Torricelli and others. The Italian School of Hydrodynamics in the 16th and 17th centuries’. *Hydraulics and Hydraulic Research. A Historical Review*.

spaced lateral orifices (Fig. I-3). The test procedure consisted in allowing the water to pass through an orifice, while maintaining the others sealed, and measuring the water flowing out in a specific time interval. Comparing the results obtained for the successive holes, he deduced the perfect proportionality of the outlet velocity with the square root of the height, as predicted by the Torricelli's law. However, he did not base himself on the former measurements for the absolute velocities, but made a new reference measurement whose result he took as standard, and which curiously enough did not coincide with any of the previous ones. Using this standard value and the square root law, Guglielmini presented a table of velocities as a function of the height. If we apply Torricelli's law to these, the height to be considered in the fall would be a quarter of the depth of water in the reservoir, a fact that Daniel Bernoulli made note of years later in his *Hydrodynamica*.

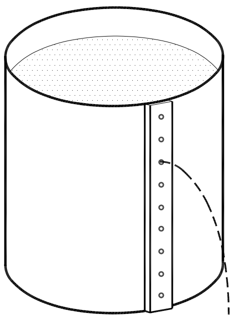

Fig. I-3. Guglielmini's bucket

In these very same years, Mariotte carried out several studies and experiments which he published in his *Traité de mouvement des eaux* (*Treatise of movement of the waters*) which appeared in 1686, 2 years after his death. As the title of the work indicates, it deals with several problems: sources, winds, etc., and among these those of discharge and resistance are to be found. In general, Mariotte views his work as a practical treatise, thus including numerous measurements and application rules. In particular, his experiments on the outlet of fluids were fairly well systematized. He successively analysed the law of flows, and later he measured them, calculating the effect of the outlet area. Like Guglielmini, he not only found that the square root law was satisfied, but also that the height did not correspond to the depth, a fact that, as we have said, was repeated in almost all the experiments.

He carried out several experiments regarding the measurement of resistance whose results were cited for a century. But apart from the experiments, Mariotte felt the need for a basic theory in order to handle the effects of the resistant forces. For this he assimilated these forces to the impact of what he called a 'jet' against a body, and he drew up five rules, defining this 'mechanics of jets'. He imagined one jet as a cylinder of fluid moving like a solid; and given that he supposed the fluid to be formed by an infinity of small corpuscles, the jet would be a set of particles moving together. The behaviour of a jet before an impact against a plate would differ from that of a solid body, but in essence it would be the sum of the individual impacts against the obstacle. From this he deduces, as Huygens had likewise done, that the effect of the impact of a jet against an obstacle is proportional to the square of its velocity. The argument he brings to bear would become classic: if the fluid goes twice as fast, then double the number of particles will impact at the same time, and with double the effect of each one, so that the total effect will be four times greater.

The interest of his theoretical proposals is complemented by the experiments. Even though his jets are a theorization of the phenomenon of the impact of the jet against a plate, he made the experiments with a plate totally submerged in a current, actually a river (Fig. I-4). Huygens, we remember, moved the models in fluids at rest. Mariotte did the opposite, these tests being the first of this type we know of. He interpreted the results in terms of his mechanics of jets, obtaining some standard values which he took as reference. If his tests are

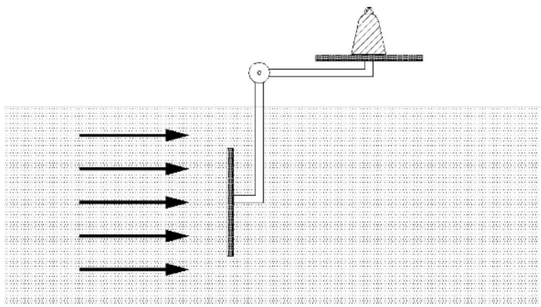

Fig. I-4. Force on a plate in a flow

interpreted under the form of the modern coefficient of resistance C_D,[11] we arrive at a value very close to 1, which is the equivalent of the weight of a column of liquid whose height is equal to what we call the kinetic height. Apart from the tests on the plate submerged in the current of a river, he also proposed another one for plates in air flows, although he presents the apparatus by itself without any reference whatsoever to a measurement. On comparing the values of resistance in the air and water, he established that the effects were proportional to the densities, another rule which also would persist. On the other hand, in the tests in water, this impinges on the plate perpendicularly, while in the case of air, it does so at an angle of 45°, thus breaking up the impinging force into two components.

Mariotte interprets the phenomena as percussions, and in this sense this appraisal is the first appearance of the 'impact theory' which we shall presently consider. The question why Mariotte did not follow the experimental method of Huygens is not easy to answer; but we conjecture he was influenced by the fact that the results of the experiment of the jet against a plate were quite uncertain, because he was familiar with water mills and experimented with them on the river Seine, as he mentions in his *Traité*. To sum up, Mariotte's contribution rightly merits the qualification of precursor that Dugas awards him.

To recapitulate, during these years, on the one hand we find that experiments with discharges are only partially reconciled with their theoretical predictions, and on the other hand that there appears to be a proportionality of the resistance with the square of the impinging velocity, and with the density of the medium. Besides (and this is an important matter) the need was already felt to establish a theoretical basis enabling these phenomena to be explained, a need that Mariotte had anticipated. It is in this context that the first edition of Newton's *Principia* saw the light. While Books I and III, respectively titled 'The Motion of Bodies' and 'The System of the World', are well known, Book II is not. It is also named 'The Motion of Bodies' although it deals with fluids, while the first deals with rigid bodies. The coincidence in the titles indicates that Newton put both solids and fluids under the same umbrella. However, Book I is structured as a closed set, characterised by an attempt to derive all motions from a few axioms or laws of motion. This is not the case in Book II, where he has to use several different additional hypotheses, which at the very least complement the others. In this respect, insufficiently justified suppositions abound, together with some small fudges and implausible constructions. In spite of this, Book II was of capital

[11] The definition of this coefficient is $C_D = F/(\frac{1}{2}\rho v^2 S)$, where F is the resistance force, ρ the fluid density, v the velocity and S the frontal surface. The meaning of this coefficient, as the times sense, was equivalent to the number of kinetic height the cylinder of equal weight that the resistance force would have. (cf. later Chapter 1, note 3).

importance throughout the eighteenth century, and almost all authors referred to it, some in order to follow it, others to refute it.

Book II deals with several different problems and the most significant one refers to the motion of projectiles. He dedicates the first sections to the dynamics of projectiles in dense media, that is, to bodies whose movement is resisted by forces depending on the velocity, be it in simple proportion, squared or a combination of both. Assuming the existence of this resistance and its mathematical form, the resulting problem is similar in nature to that dealt with in Book I, of which these sections are but an extension. We deduce from this that Newton arrives at the problem of resistance due to his interest in ballistics, as Huygens also did, and unlike Mariotte, who arrived at it via hydraulic machines.

Newton proposed a definition of a fluid as a body whose parts yield to any force impressed on it. However, he encounters difficulties when he tries to explain the different sources generating resistance. In terms of our present understanding, Newton considered the existence of a resistance produced by the forces of inertia, which he calls 'inertia of matter', and another resistance produced by viscous forces. His idea concerning the latter was a lot less clear, and he called it 'lubricity, tenacity or fluidity'. Although he established the definition of viscosity as a shear force proportional to the variation of the velocity, the range of viscous phenomena and their mechanisms lay outside his horizon. This explains why he gave them various names according to the aspects they presented.

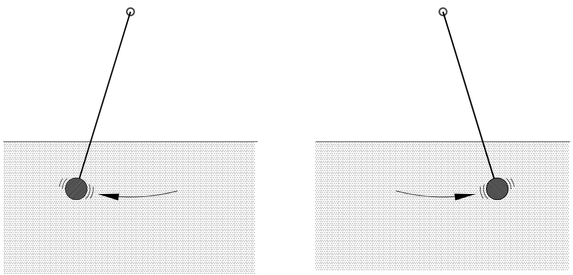

Fig. I-5. Pendulums in resistant media

Regarding resistance, and with the aim of determining its mathematical form by experiment, Newton proposed using pendulums oscillating in fluids (Fig. I-5). The idea was simple. The pendulum would maintain the amplitude of oscillation constant if there were no resistant forces. If these forces did exist, the amplitudes would be reduced with time, progressively diminishing until the pendulum came to rest, so that Newton could infer the mathematical laws of resistance from the observation and measurement of these diminished motions. Therefore, he introduced an extensive study of the motion of pendulums in resistant media, and arrived at the point of establishing a relation between the ratio of these reductions and the mathematical expression of the velocity in this law. Once the theory of the instrument had been established, he performed a set of experiments with pendulums of various sizes, making them oscillate in air, water and mercury. However, the conclusions he obtained did not indicate that the resistance was proportional to the square of the velocity, as was expected, and in consequence Newton tried to approach the problem by polynomials with different powers of velocity. What appeared more obvious to him was the proportionality of resistance to the cross section of the oscillating mass, and to a lesser degree to the density of the fluid. From the comments he makes, one can infer that the results of the experiment did not satisfy his expectations, but that in spite of these dubious results, he continued to think that the resistance was proportional to the square of velocity.

After the work with the pendulums, which comes within the framework of experimental physics, Newton directly tackled the case of bodies in fluid currents or vice versa, that is, moving bodies in a fluid at rest. He dedicated the most memorable sections of Book II to this problem. He begins with a differentiation of behavior among liquids or uncompressible or non-elastic fluids, as opposed to air, which he qualified as being compressible or elastic. Although he considered both classes to be aggregates of particles, he established a difference between them: in the air the particles are separated from each other and repel each other due to forces which he called ‘centrifugal’. These are inversely proportional to the separation existing among the particles; for liquids all particles are in contact, and are only united by the forces of lubricity. That is to say, the particles in the air are easily individualized, while this is not true for those of a liquid. The theories in each case are different, we ought to say qualitatively different. But there is more; in the second edition of the *Principia*, published in 1713, Newton rewrote the section referring to liquids almost completely, although he maintained the same methodology as in the first edition, which also obliges us to treat the topic in two different moments.

It is in the study of the motion of bodies in air that we consider fluid mechanics as becoming a truly modern scientific discipline, as it is there that Newton, in order to explain the phenomenon, introduces three basic hypothesis: one on the constitution of air, a second on how moving air and a body in its midst interact, and the third on the existence of a law regulating these interactions. The latter is the second law of dynamics. As we have said, he took a set of individual particles submitted to repelling forces for the air model, and for the interaction he adopted the mechanical model of impact. However, although the theory can be constructed with these three elements, the difficulty of calculation was almost insurmountable, as the motion of one particle of air affected all others to a certain extent, given the repulsion among them. In order to solve this difficulty, he simplified the air model by eliminating these repulsive forces, thereby rendering the particles immobile. He called the fluid resulting from this operation a 'rare medium', which, although it did not have the properties that he attributed to air, would behave in a very similar way to air. In this new medium, the phenomenon would be modeled as the motion of all the particles, completely independent from each other, that would rebound elastically in the impact against the body, or would remain immobile if the impact was not elastic. In either case, they would transmit a certain momentum to the body, whose variation with time would be the resistant force.

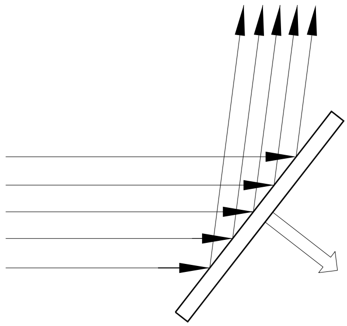

Fig. I-6. Impact theory

This model, which we have called the 'impact theory', was already sketched by Mariotte, although it acquires its authentic meaning in Newton. He supposes (Fig. I-6) that there is a deflection in the impact, depending on the angle of incidence. The corollary of the supposition is the existence of shadow zones where the fluid cannot impact. As we shall see, this theory will remain in force in fluid dynamics for more than a century. It will be criticized and doubted, but because of the lack of a better theory it will prove essential due to its instrumental function.

Newton applied this theory to a plate and a sphere. In the case of an elastic rebound, he obtained the values $C_D = 4$ and $C_D = 2$, respectively[12] as resistance coefficients which were reduced by half when dealing with a non-elastic impact.

[12] We quoted numerical values because the calculation of the actual magnitudes of these coefficients was one of the major milestones in the evolution of the problem of the resistance.

In addition, he made a short incursion into the case of a solid of minimum resistance.

When dealing with liquids, neither this model nor the impact theory was of use to him, as the particles are not only innumerable, but also they rub together, and when they impact with the object they transmit this action to the ones behind in a domino effect. Faced with this difficulty, Newton was forced to explain the phenomena using an alternative method. The model he used was the vertical discharge of fluid through an orifice, in whose current he had placed a resistant body (Fig. I-7). The method he followed in order to find the force acting on this body is based initially on finding the velocity of the discharging fluid as a function of the depth of the water in the reservoir. Afterwards, as a second step, he calculates the force upon the body also as a function of this height. Knowing both, and after eliminating the height parameter, the resistance is deduced as a function of the velocity. Obviously, this approach does not correspond with the impact theory, as there is no particle rebound. Likewise, we see that Newton made use of the discharge problem as an ancillary element, and although he used this methodology in the two first editions of the *Principia*, as we shall see, the solution he arrived at was different in each one.

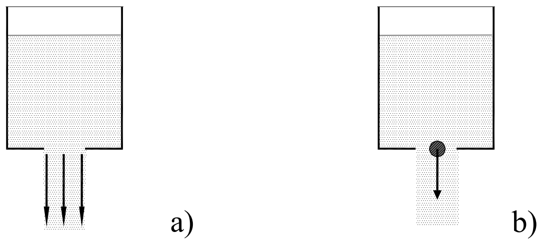

Fig. I-7. Discharge with a body inside the current

In the first edition he offers an argument from which he concludes that the discharge velocity corresponds to a fall height that is half that of the depth; that is to say the contrary of Torricelli's law. This was to some extent in tune with experimental results of the day, although it contradicted the experimental evidence of the vertical ascent of water in spring jets. The question was under the spotlight, and Newton must have convinced himself shortly afterwards of his error, as in 1690, in his personal copy of the *Principia* he made some notes from which we deduce that he had changed his mind. As regards the force exercised by the current on the sphere placed in it, he supposes it to be equal to the weight of a column of water whose height was that of the reservoir and had the same cross section as the sphere. If the submerged body were to have another geometrical form, but the same cross section, the weight of the column would be the same; that is to say, the resistance for a liquid movement is independent of the shape of the body, which did not occur for a body in air. The result he obtains for a sphere in a liquid is that the resistance coefficient $C_D = 2$, which is equal to that of the sphere in the air.

Although Newton deals with the discharge quite briefly, we find it has two opposing aspects. On the one hand was Torricelli's law, corroborated by the fact that vertical fountain discharges almost reach the level of the surface of the water in the reservoir. On the other hand, the results of the experiments indicated that when the reservoir is discharged by a lower orifice, and Torricelli's formula is introduced, the height of the discharge will oscillate around half the depth of the reservoir. The situation was contradictory, and at first Newton opted for the latter solution, which was included in the first edition of the *Principia*. Later on, as we have said, with the correction he wrote in his personal copy, the new coefficient of resistance was reduced to $C_D = 1$, that is, half the former, which furthermore partially agreed with the experimental values found by Mariotte. The solution to this contradiction will be given by Newton in the second edition.

To recap, we must underline two aspects of Newton: the new approach to the formalisation of the impact theory using the 'rare medium' instead of air, and the use of discharge phenomenon as an instrumental element for resolving the question in the case of liquids, where impact theory, strictly speaking, is not used. There is therefore a crossing over of the two major lines of work, resistance and discharge, although this crossover is only partial.

We now briefly leave Newton, but we will return to him when we consider the second edition of the *Principia*. In the two decades following *Principia*'s publication (1690–1710), a group of mathematicians—geometricians as they were known at the time—and another group of hydraulic mechanics entered the stage. The preoccupations of the former swung between what we could call pure applications, such as the discussions of the minimum resistance solid, and other more practical aspects such as applications to naval theory. Compared to them, the mechanics focused on the study of hydraulically driven machines, that is to say waterwheels with paddles.

The characteristic common to the mathematicians was the acceptance and extension of the impact theory even to liquids, without anyone questioning of its physical reality. Among them Jakob Bernoulli stood out, having used differential calculus to analyse the effects of liquid flow on two-dimensional shapes of ships. His conclusions went partly unnoticed, but for many years they were not surpassed, as he obtained the maximum advantage from an analysis of this type.

At the same time he extended his studies to sails, which, inflated by the wind, formed so-called sailing curves (*velarias*), related to the catenaries, which were also in fashion then. As regards the solid of minimum resistance, Newton offered a solution in the *Principia*, although almost without any justification. Its obscure nature caused Nicolas Fatio de Duillier, Guillaume Antoine de l'Hôpital and Johann Bernoulli to enter into the fray with new solutions in the final years of the century. All the solutions were purely mathematical; therefore we must consider the contributions they made to the study of fluids as being purely theoretical with little or no practical application. Proof of this is that they limited themselves to relative solutions between one shape and another, without including any values of the resistant forces.

Analysts of hydraulic machines, like Philippe de la Hire and Antoine Parent, constitute the opposite extreme. In 1704 and 1705, they studied the behavior of fluid-driven machines. They based their experiments on impact theory, with a proportionality coefficient derived from Mariotte's experiments, rounding it up to the nearest unit. To complement their reasoning, they introduced considerations concerning the mechanical efficiency of mechanical systems.

The application of fluid mechanics to ships and to hydraulic machines responded to important social needs. It is not necessary to go into detail regarding the importance of the navy at this time, but by the eighteenth century, maritime power had become one of the defining elements of a nation's military strength, and its most relevant exponent was the ship of the line. The ship of the line, at its most elemental, is a machine which balances the effects of the wind in the sails with that of the water on the hull. It is the epitome of the 'fluid machine'. As regards other machines, it is nothing new to say that windmills and waterwheels had been almost the only sources of mechanical power from ancient times. The growing urbanization of the eighteenth century required a method of distributing water, that is to say, a source of driving power and a raising device, which were the waterwheel and the water-raising pump.

Thus, we arrive at 1713, a year during which we find ourselves once again with Newton, and his second edition of the *Principia*. No new theory had appeared since the first edition, only, as we have seen, some mathematical and practical applications. Newton's new contributions were important, as he practically rewrote the part referring to motion of liquids. We remember that, where we had left him, his findings contradicted Torricelli's law, even when they complied with the discharge experiments. In order to deal with this, he modified the discharge process that had served as the basis of his method, introducing a theoretical construction, the cataract (Fig. I-8), with which he intended to reconcile both positions. In this type of discharge, a contraction of the stream is produced, which implies that the actual cross section of the outlets is smaller than the transit