FROM *SUMMETRIA* TO SYMMETRY: THE MAKING OF A REVOLUTIONARY SCIENTIFIC CONCEPT

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From *Summetria* to Symmetry: The Making of a Revolutionary Scientific Concept

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For Ruth Lorand

Preface

Many literary critics seem to think that an hypothesis about obscure and remote questions of history can be refuted by a simple demand for the production of more evidence than in fact exists. The demand is as easy to make as it is impossible to satisfy. But the true test of an hypothesis, if it cannot be shown to conflict with known truths, is the number of facts that it correlates and explains.

Francis M. Cornford [1914] 1934, 220.

It was in the autumn of 1997 that the research project leading to this publication began. One of us [GH], while a visiting fellow at the Center for Philosophy of Science (University of Pittsburgh), gave a talk entitled, "Proportions and Identity: The Aesthetic Aspect of Symmetry". The presentation focused on a confusion surrounding the concept of symmetry: it exhibits unity, yet it is often claimed to reveal a form of beauty, namely, harmony, which requires a variety of elements. In the audience was the co-author of this book [BRG] who responded with enthusiasm, seeking to extend the discussion of this issue to historical sources in earlier periods. A preliminary search of the literature persuaded us that the history of symmetry was rich in possibilities for new insights into the making of concepts. John Roche's brief essay (1987), in which he sketched the broad outlines of the history of this concept, was particularly helpful, and led us to conclude that the subject was worthy of monographic treatment.

The received view is that symmetry is an innate concept that was always available to human thought. There is no doubt that we moderns perceive symmetrical elements in nature as well as in artifacts produced in virtually all cultures in all periods, but is it the case that the ancients noticed what we see? Contrary to a widely held expectation, the answer is negative, for no evidence has been adduced to support the claim that the ancients were alert to this concern; rather, it is a perspective imposed by modern historians and philosophers on their forebears. Indeed, as a matter of historical fact, prior to the mid-18th century the term, *symmetry*, does not occur in any of its modern senses. Moreover, there was no term or expression to connote the meaning of the modern concept of symmetry. Typically, this unsettling negative result leads to a request for the production of more evidence than in fact exists. And, as Cornford realized, the demand is as easy to make as it is impossible to satisfy.

Despite the lack of evidence in this period for the concept, we discern two coherent trajectories of the term, *symmetry*, which together constitute a fascinating tale which has not been told heretofore. We call the first path mathematical and the second, aesthetic. Thinkers such as Plato (427–347 BC), Euclid (*fl.* 300 BC), Archimedes (287–212 BC), and Isaac Barrow (1630–1677) contributed to the formulation of the concept in the mathematical path, while Vitruvius (1st century, BC), Leon Battista Alberti (1404–1472), Claude Perrault (1613–1688), and Montesquieu (1689–1755), are the principal players in the aesthetic domain. In the mathematical path the meaning remained stable for many centuries and then fell out of active use, but in the aesthetic path we find an intriguing set of developments. These issues will be discussed in Part I.

Beginning in the mid-18th century the term, *symmetry*, was used in scientific contexts in new ways, at first in rare instances. But during the Revolutionary and Napoleonic period (1789–1815) the pace quickened, for we find definitions and applications of the term in a wide variety of scientific disciplines, notably, natural history, mathematics, and physics. Most of these usages have not been considered in the secondary literature, let alone drawn together in a connected narrative. Indeed, we have not found any discussion in the secondary literature of the usages of *symmetry* in physics in the 18th and early 19th centuries. One of the main goals of this monograph is to fill this lacuna. In particular, we discuss in detail a radical new mathematical meaning for the term, *symmetry*, complete with a precise definition, introduced by Adrien-Marie Legendre (1752–1833) in 1794. This breakthrough has not been properly appreciated, and we recount it against the failed attempt of Immanuel Kant (1724–1804) to formulate a new concept which he called "incongruent counterparts". It is instructive to recognize that Kant, the foremost philosopher of the 18th century, struggled unsuccessfully to establish a new concept, connoting a sense akin to bilateral symmetry. Despite his failure to reach the goal he set for himself, Kant's essay (1768) contains many brilliant insights into mathematical structures shared by objects that were otherwise considered unrelated, e.g., snails and screws. These innovations will be analyzed in Part II.

In order to justify the claim that scientific usages of *symmetry* beginning in the mid-18th century were indeed novel, we decided to investigate earlier usages of the term. In so doing, we discovered that the traditions reported in Part I inform the innovations in Part II. For example, Perrault, the translator of Vitruvius's *De architectura* into French (1673), contrasted two meanings of symmetry, the ancient usage by Vitruvius and the French usage of his day. We suggest that this French usage in architecture, later called "respective symmetry", was part of the background for the invocation of the term, *symmetry*, by Gaspard Monge (1746–1818) in the description of a curved figure whose center of gravity he sought to determine (1788). One should bear in mind that Perrault's aesthetic usage expresses a value judgment, whereas Monge's scientific usage only functions as a descriptive term in mathematical physics. Monge's appeal to *symmetry* in a textbook on statics—addressed to students in the French naval academy—seems to be the first occurrence of the term in its modern bilateral sense in a treatise on physics, and it comes in a section dealing with the center of gravity of ships. We then realized that books on naval architecture in the 18th century, intended for practitioners of shipbuilding, have to be considered as providing precedents for this usage. Monge did not see the need for a definition of symmetry in the bilateral sense since his usage conformed to what was already in the relevant literature of naval architecture. But it is noteworthy that, after Legendre introduced his revolutionary definition in 1794, it became increasingly common to define symmetry in various scientific domains.

Aesthetic usages play an important role in Part I but not in Part II, where our interest shifts to early scientific usages of symmetry. We seek to trace the making of the concept in modern scientific discourse and argue that it mainly took place at the time of the French Revolution and, indeed, in France. In Europe from the 16th to the mid-18th century most instances of *symmetry* occurred in the context of architecture, a discipline in which aesthetics plays a prominent role, and the usages in this tradition are essential for understanding the novelty of the modern scientific concept.

The reader, however, may ask, as indeed our colleagues have done, How do you know that symmetry in the modern scientific sense was not used before the mid-18th century? It is true that initially we ourselves believed that symmetry was present from the very beginning (i.e., Greek antiquity). Indeed, a number of passages in scientific works before the mid-18th century were suggested to us as evidence for early usages of the modern concept. But when we examined these passages, they failed to provide any supporting evidence, and this cast doubt on our initial belief and the underlying methodology. We then decided to adopt a different approach: we began to look for the way the term, *symmetry*, was actually used, rather than identifying the concept of symmetry in passages where the term does not occur. In our view, the history of a concept cannot be entirely divorced from the words used to articulate it. Such a history should take into account the variety of meanings as the application of the concept changes over time—in different contexts, different problems arise and, in turn, different answers are given. In response to these concerns we adhere to a methodology in which we avoid anachronistic readings by paying careful attention to the relevant contexts and, for us, this means primarily the text surrounding an occurrence of *symmetry* and secondarily the network of near contemporary usages. Moreover, before claiming that any idea was new in the period from the mid-18thcentury to 1815, we have—at the very least—checked Diderot's *Encyclopédie* (1751–1765) which serves as a reliable guide to the state of knowledge in its day.

We recognize, of course, that the scientific concept of symmetry continued to develop throughout the 19th century and beyond. Indeed, symmetry considerations have taken on an ever greater role in many disciplines, and it would probably take a team of scholars to do justice to this central aspect of science in the 19th and 20th centuries. We leave this task to our successors.

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Contents

Chapter 1
Introduction

1.1 Setting the Problem: the Historical Context of Symmetry

There are dramatic moments in the history of science when the making of a new concept determines a whole way of thinking with the result that it obliterates the old modes to such an extent that it is almost impossible to retrieve them. Indeed, it takes a great effort of imagination and a convincing display of evidence to isolate such moments and cast them into relief. And when one succeeds in imagining such times when the successful concept was not yet available, the old mode of thinking may seem quaint and inconsequential in light of the power of the modern concept. We submit that such a moment took place in 1794 when Adrien-Marie Legendre (1752–1833) put forward a new definition of symmetry in a textbook on the elements of geometry.

Our objectives in this book are twofold: (1) to describe in great detail the story leading to this moment in 1794, and (2) to clarify the nature of this revolutionary scientific concept which has so powerfully fixed our gaze that we cannot see otherwise. Specifically, we will demonstrate that the concept of symmetry, as it is currently applied in many scientific domains, is entirely different from what was meant by the term, *symmetry*, in ancient and medieval times up to the early modern period. At the core of our argument is the claim that the scientific concept of symmetry as we know it today is in fact modern—it is a 19th-century concept.

We regard it as an important task for the historian and philosopher of science to liberate scientific concepts from the fetters of necessity, that is, to warn against turning concepts from historical contingencies into philosophical necessities. Like any other scientific concept, symmetry has a history: it has had various applications and different usages through the ages; it evolved for a long period of time and some of its usages have had a distinct inception and a time when they fell out of use. In this study we limit the historical discussion to the period from the inception of the term, *symmetry*, in Greek antiquity till an entirely new meaning was assigned to it in the late 18th century. We then place the novel meaning in its original context, specifically, the Revolutionary and Napoleonic era in France. This is a self-contained study; we will see that the period under consideration works as a unit. That is, in this period the various usages are closely related and evolved from the original meanings of the term, *symmetry*, in antiquity. We will argue that Legendre's

G. Hon, B.R. Goldstein, *From* Summetria *to Symmetry: The Making of a Revolutionary Scientific Concept*,

definition in Book V, Proposition 23, of his *Éléments de géométrie* (1794), breaks with previous traditions and, in fact, marks the watershed in the history of the scientific concept of symmetry.

> Two equal solid angles which are formed (by the same plane angles) but in the inverse order will be called *angles equal by symmetry*, or simply *symmetrical angles*.[1]

This definition of symmetry differs sharply from any previous usage of *symmetry* and in many respects it is revolutionary. Our principal claim is that this new definition was revolutionary in its implications for scientific advances after 1794.

Symmetry (Greek: *summetria*) had one basic meaning in Greek antiquity: proportionality. Its usage can be distinguished by the contexts in which it was invoked: (1) in a mathematical context it means that two quantities share a common measure (i.e., they are commensurable), and (2) in an evaluative context (e.g., appraising the beautiful), it means well proportioned. We explore both contexts and show that they constitute two different backgrounds for two different paths in the evolution of the concept. The coherence of these two trajectories corresponds to two distinct senses of the concept of symmetry: (1) a relation between two entities, and (2) a property of a unified whole, respectively. This distinction was maintained throughout our period—from antiquity to the early 19th century—even though in each category the meaning underwent substantial changes. Euclid (*fl.* 300 BC) and Vitruvius (1st century, BC) represent these two senses in antiquity, respectively, while Legendre and Sylvestre François Lacroix (1765–1843) stand for these senses in the modern period. Thus, in logical terms, we discern continuity in the distinction between the sense of relation and that of property.

Symmetry in its current scientific usage refers either to a mathematico-logical relation or to an intrinsic property of a mathematical entity which under certain classes of transformations, such as rotation, reflection, inversion, or other abstract operations, leaves something unchanged—invariant. When an invariant property is maintained, it is the subject of group theory—a mathematical theory which explores, systematizes, and formalizes features that are preserved under the transformation. To be sure, the aesthetic sense of symmetry can in fact be described mathematically,

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[1] Legendre [1794] 1817, 155: "Ainsi les deux angles solides dont il s'agit, qui sont formés par trois angles plans égaux chacun à chacun, mais disposés dans un ordre inverse, s'appelleront *angles égaux par symmétrie*, ou simplement *angles symmétriques*." Throughout the book, where published translations are quoted in the text, our convention is first to cite the bibliographic reference for that translation, followed by a reference to the original source. Otherwise, the first reference is to the source on which we have depended. In quotations we only use italics where they appear in the original texts. We also use italics for transliterations of Greek words in a Latin source, as well as for foreign words in an English text. Moreover, in quotations, we have maintained the spelling and capitalization as they appear in our sources, e.g., in Ch. 4, n. 100, below, Gerard (tr.) 1759, 280: "In all complex objects there must be a sort of counterballance [*sic*],. . .." We note further that up to the beginning of the 19th century the spelling in French of *symétrique* and *symétrie* had not yet been fixed, that is, one finds in the literature instances of both single and double 'm' (sometimes by the same author). We have also adopted the following convention: the term, *symmetry*, is in italics, as distinct from the concept, symmetry, which is not.

but the essential point is that in modern scientific usage symmetry is mathematical with no aesthetic component.

Eugene Wigner (1902–1995), the doyen of the application of symmetry in physics, received the Nobel Prize in physics in 1963 for "his contributions to the theory of the atomic nucleus and the elementary particles, particularly through the discovery and application of fundamental symmetry principles."[2] He claimed that, in addition to the initial conditions of the system and the laws of nature which describe the dynamical development of the system, one has to consider the symmetry of the arrangement which imposes important constraints. The hierarchy of knowledge of the world progresses, according to Wigner, from events to laws of nature, and from laws to symmetry or invariance principles.[3] He stated that

> Symmetry and invariance considerations, and even conservation laws, undoubtedly played an important role in the thinking of physicists, such as Galileo and Newton, and probably even before them. However, these considerations were not thought to be particularly important and were articulated only rarely.[4]

Wigner projected his view deep into the past; he relates it not only to Galileo and Newton but also to scholars of earlier times who were concerned, according to Wigner, with symmetry and even with invariance. Wigner's historical gaze was fixated on his own view; he could not see otherwise. This anachronistic attitude, namely, since we think in a certain way, surely our predecessors did so as well, has consequences not only for the history of science, but for science too. Indeed, it distorts the history and endows scientific concepts with necessity that may shackle the free spirit of the mind.

In many respects we respond to the call issued by George Sarton (1884–1956), one of the pioneers of the modern history of science. In 1921, in an essay review of Francis M. Jaeger's *Lectures on the Principle of Symmetry and its Applications in All Natural Sciences*, Sarton challenged historians:

> It would be fascinating to retrace the development of the idea of symmetry from the Pythagorean days down to our time. Such a study would enable us to make a master section through the whole history of scientific thought and would provide us with an excellent touchstone to appreciate the relations of science and art at various times. This examination would be very comprehensive, for it would take us into almost every department of knowledge; it would attract us into the workshops of the craftsmen as well as into the laboratories of the scientists; it would oblige us even to make a pleasant excursion in the realm of Chinese philosophy and aesthetics. Professor Jaeger himself might be tempted to carry on these investigations. . ..[5]

To the best of our knowledge, Jaeger did not take up the challenge. Nor did Hermann Weyl (1885–1955) who, in his masterful work, *Symmetry*, connects three distinct domains with the concept of symmetry: (1) material artifacts, (2) natural

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[2] *Nobelstiftelsen* 1972, 1.

[3] Wigner [1964a] 1997a, 313.

[4] Wigner [1964b] 1997b, 297.

[5] Sarton 1921, 32–33.

phenomena, and (3) physical theories. Weyl convincingly showed that group theory is the underlying mathematical structure for symmetry in all three domains.[6] Yet this profound over-arching analysis fails to acknowledge the fact that the scientific concept of symmetry is a modern invention.

We take up Sarton's challenge and trace the development of the concept of symmetry from antiquity to the early modern era. Sarton imagined that this study would involve the entire range of scientific thought and thus provide a good basis for appreciating the relations of science and art through the ages. This indicates that Sarton held an anachronistic approach to symmetry, for he projected his understanding of symmetry in the early years of the 20th century deep into the past. Indeed, we will see that, in light of our findings, Sarton's expectations need to be modified substantially. As we demonstrate, before 1794 symmetry considerations affected only a few limited aspects of science, and even in those cases the usages differ from Sarton's expectation.

We respond then to Sarton with a twist. We do not examine Chinese philosophy; instead, we demonstrate that within the European tradition there is a break, a revolution: the scientific concept of symmetry is not simply the result of a process of development based on earlier usages either in aesthetic or mathematical contexts. Indeed, we emphasize that this modern concept is categorically different from the ancient concept of symmetry (for a detailed overview of our claim, see § 1.5).

Consider the following example: Ptolemy makes use of the term, *summetria*, in the *Almagest*, but he does not use it, as one might expect, in the sense of bilateral symmetry. Instead, he uses this Greek term in the sense of being appropriate, well proportioned, or well suited to the task. Early on in this influential book, Ptolemy discusses the sizes of chords in a circle and then constructs a table that facilitates various computations. He writes,

> Such, then, is the easiest way to undertake the calculation of the chords. But, as I said, in order that we may have the actual amounts of the chords readily available for every occasion, we shall set out tables [for that purpose] below. They will be arranged in sections of 45 lines to achieve a symmetrical [*summetron*] appearance.[7]

The sense is that a table of 45 lines fits appropriately on a standard papyrus role.[8] This meaning of symmetry has nothing whatsoever to do with what we have been accustomed to regard as symmetry—the expectation of a contemporary reader who projects the common understanding of symmetry onto practitioners in the past.

Similarly, a naïve reader might consider the usage of *symmetry* in Vitruvius's *De architectura* to refer to a bilateral, left and right relation. After all, many Greek temples and Roman villas exhibit such a property. The naïve reader typically interprets—without any hesitation—the appeal to symmetry in this treatise as pertaining to the bilateral structure of ancient edifices, but this reading of the text is

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[6] Weyl 1952, 6–8, 16–17, 28, 133–135.

[7] Toomer (tr.) 1984, 56; Ptolemy, *Almagest*, I.10; Heiberg (ed.) 1898–1903, 1: 47.

[8] Cf. Toomer (tr.) 1984, 56 n. 67.

simply wrong. We will show that, while symmetry is a central concept in Vitruvius's theory of architecture, it connotes the Greek idea of well proportioned (see § 3.2). Vitruvius tells us that symmetry

> is the appropriate agreement of the elements of the work itself, a correspondence [*responsus*], in any given part, of the separate parts to the entire figure as a whole. Just as in the human body there is a symmetric quality of eurhythmies [*symmetros est eurythmiae qualitas*] expressed in terms of the cubit, foot, palm, digit, and other small units, so it is in perfect works [of architecture].[9]

It is significant that correspondence (*responsus*) here refers to a relation of the parts of the structure to the whole, and not between parts on the left and similar parts on the right with respect to some axis, as the modern concept suggests. Vitruvius defines symmetry as a special kind of property of an edifice in which the parts and the whole are related in a way that is designed to attain some value, aesthetic or practical: "appropriate agreement" (*conveniens consensus*) is Vitruvius's formulation of this value (see § 3.2).

The changes we have discerned in the meaning of symmetry are much more radical than those that are typical of other key concepts in science. In the historical record of appeals to this concept, we find evidence for both evolution and revolution. In particular, the meaning of symmetry prior to 1794 is very different from its meaning after that date, that is, from this time on symmetry became a powerful concept in many scientific disciplines.[10] In other words, the concept did not undergo a refinement as one might say about other scientific concepts such as "force" or "error", but a major shift in the meaning of the term. In short, the modern sense of the concept of symmetry differs fundamentally from the meanings of symmetry from antiquity to early modern times. How and in what circumstances did this shift take place, and what were its immediate consequences? This historical account and the accompanying philosophical analysis are intended to answer these questions and to provide insight into the making of the scientific concept of symmetry.

1.2 The Perceptual Approach: Ill-Founded Expectations

What is the common expectation (which, in fact, is shared by most contemporary educated people) when attention is called to the symmetry of objects in the visual field? Based on a perceptual approach, this expectation was articulated by John Ruskin (1819–1900) and Ernst Mach (1838–1916) in the latter part of the 19th

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[9] Rowland et al. (trs.) 1999, 25 (slightly modified); Granger (ed. and tr.) [1931] 1962, 1: 26; Vitruvius, *De architectura*, I.2, 4.

[10] We note that Copernicus in the 16th century (see § 5.2), Galileo in the 17th century (see § 5.3), and Linnaeus in the 18th century (see § 6.1), applied the term, *symmetry*, in different ways which, however, can all be traced to Vitruvius's usages. Moreover, in the second half of the 18th century Duhamel du Monceau and Monge invoked this term in a physical context where a whole (the horizontal section of the hull of a ship) is divided into equal and similar halves by a line or a plane, but neither of these authors offered a definition (see § 9.2).

century. Ruskin, the great Victorian critic of art and society, argued that we typically associate symmetry with the horizontal. In the essay, "The Lamp of Beauty," one of his *Seven Lamps of Architecture*, Ruskin points out that "evidently there is in symmetry a sense not merely of equality, but of balance: now a thing cannot be balanced by another on the top of it, though it may by one at the side of it."[11] Ruskin offered the following advice to the young architect when it comes to vertical considerations:

> get rid of equality; leave that to children and their card houses: the laws of nature and the reason of man are alike against it, in arts, as in politics. There is but one thoroughly ugly tower in Italy I know of, that is so because it is divided into vertical equal parts:—the tower of Pisa.[12]

Ruskin alludes to the common perception that we are sensitive to what is currently called bilateral symmetry, a relation that expresses the correspondence observed between two single elements in the whole assemblage with respect to some axis. Hence, symmetry in a building means that, e.g., the façade may comprise a door in the middle and windows of equal dimensions on either side of it, at the same height and at the same distance from the vertical line drawn through the middle of the door. The corresponding windows are usually identical in size and shape rather than mirror images of each other. For a human face to be symmetrical each one of a pair (e.g., eyes, ears) must be at the same distance from the middle and at the same level, but in this case the elements of the pair are mirror images of each other.[13]

Mach elaborated on this theme in his *Analysis of Sensations* of 1886. "It is well known," Mach observed, "that the symmetry of a landscape and of its reflexion in water is not felt. The portrait of a familiar personage, when turned upside down, is strange and puzzling to a person who does not recognise it intellectually."[14] According to Mach, this is comprehensible, "since the motor apparatus of the eye is asymmetrical with respect to a horizontal plane."[15] Mach took the pairs of letters 'd' and 'b' as well as 'q' and 'p' to be the two halves of a symmetrical figure about a vertical axis, while the pairs 'd' and 'q' as well as b' and 'p' are the two halves of

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[11] Ruskin [1880] 1989, 128. For the background to the perceptual approach, see § 4.2.

[12] Ruskin [1880] 1989, 129. This is the kind of sensitivity that Augustine expressed without elaboration. Unlike Ruskin, Augustine did not appeal to symmetry to refer to this sensitivity; rather, he invoked "agreement" [*conuenientia*]: see Ch. 4, n. 89, below.

[13] For another account, see Ruskin 1858, 174: "Symmetry or the balance of parts or masses in nearly equal opposition, is one of the conditions of treatment under the law of Repetition. For the opposition, in a symmetrical object, is of like things reflecting each other." See also pp. 173–175, 188–204.

[14] Cora M. Williams (tr.) [1897/1914] 1959, 113; Mach [1886] 1922, 94: "Daß die Symmetrie einer Landschaft und ihres Spiegelbildes im Wasser gar nicht empfunden wird, ist bekannt. Das von oben nach unten umgekehrte Portrait einer bekannten Persönlichkeit ist fremd und rätselhaft für jeden, der nicht durch intellektuelle Anhaltspunkte sie erkennt."

[15] Cora M. Williams (tr.) [1897/1914] 1959, 113; Mach [1886] 1922, 94: "Das ist auch verständlich, weil der motorische Augenapparat in bezug auf eine horizontale Ebene unsymmetrisch ist."

a symmetrical figure about a horizontal axis. He then observes: "the first two [pairs] are confounded [*Verwechslung*]; but confusion is only possible of things that excite in us the same or similar sensations."[16] Thus, Mach reaches the same conclusion that Ruskin had, namely, the dominance of symmetry with respect to a vertical plane over symmetry with respect to a horizontal plane. But Mach goes further with his explanation.

In an early lecture (originally given in 1871 and initially published in 1872), "On symmetry", Mach put forward the following definition:

> If ... we can divide an object by a plane into two halves so that each half, as seen in the reflecting plane of division, is a mirror image [*Spiegelbild*] of the other half, such an object is termed symmetrical, and the plane of division is called the plane of symmetry.[17]

This definition agrees with contemporary usage and may be considered a formulation of the perceptual approach. The crucial element in the definition is the determination of "the plane of symmetry"; without such a plane, the definition collapses. As we have seen, Mach indicates that there is often a confusion between 'b' and 'd' but not between 'b' and 'p'; in the former case the plane of symmetry is vertical while in the latter it is horizontal. Mathematically there is no difference between left and right symmetry and up and down symmetry—they both comply with the definition; but physiologically there is a marked difference and it has to do with the paired structure of human eyes. Mach asks, "Why do the two halves of a symmetrical figure about a vertical [plane] produce the same or similar sensations?" The answer Mach gives has to do with the fact that the visual organ itself is symmetrical with respect to a vertical plane: "the left eye is the mirror image [*Spiegelbild*] of the right [eye]. And the perception of light [*lichtempfindende*] by the retina of the left eye is a mirror image of the perception of light by the retina of the right [eye], in all its dispositions."[18] Mach, however, does not seem to realize that the explicit appeal to this left and right distinction in describing bilateral symmetry is a modern development.

Mach's discussion of the relation of the physiological (perceptual) to the physical (metrical) is very illuminating. He demonstrated an important distinction between these two modes:

> Symmetric geometrical figures are, owing to our symmetric physiological organization, very easily taken to be identical, whereas metrically and physically they are entirely

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[16] McCormack (tr.) [1894] 1986, 95; Mach [1872] 1910, 105–106: "Zwischen den ersteren tritt Verwechslung ein, was nur zwischen solchen Dingen möglich ist, welche gleiche oder ähnliche Empfindungen erregen."

[17] McCormack (tr.) [1894] 1986, 94 (slightly modified); Mach [1872] 1910, 104: "Wenn man nun einen Gegenstand durch eine Ebene so in zwei Hälften zerlegen kann, daß jede Hälfte das Spiegelbild der anderen in der spiegelnden Teilungsebene sein könnte, so nennt man diesen Gegenstand symmetrisch und die erwähnte Teilungsebene die Symmetrieebene."

[18] McCormack (tr.) (1894) 1986, 96 (slightly modified); Mach (1872) 1910, 107: "Das linke Auge ist das Spiegelbild des rechten, und namentlich ist die lichtempfindende Netzhaut des linke Auges in allen ihren organischen Einrichtungen ein Spiegelbild der rechten Netzhaut."

> different. A screw with its spiral winding to the right and one with its spiral winding to the left, two bodies rotating in contrary directions, etc., appear very much alike to the eye. But we are for this reason not permitted to regard them as geometrically or physically equivalent. Attention to this fact would avert many paradoxical questions. Think only of the trouble that such problems gave Kant![19]

Mach assumed that in 1768 Kant had at his disposal the same concept of symmetry as did Mach in 1905, and that Kant had been confused by not realizing the distinction between the physiological and the physical. But is this the case? Did Kant have access to a modern scientific concept of symmetry or, for that matter, the perceptual approach? We will argue that in fact Kant did not; rather, at that time he was groping unsuccessfully for this concept. He tried to link the left and right distinction with what he called *incongruent counterparts*—a line of research which he later abandoned, turning instead to metaphysics (see § 7.3). Thus, in spite of his historical sensitivity, Mach took symmetry for granted as giving him the conceptual tool for analyzing spatial orientation in all historical periods.

It appears, then, that neither Ruskin nor Mach consider symmetry a concept with a history. For them the perception of symmetry is innate, that is, they seem to refer to some faculty of the mind that functions in this way. Ruskin and Mach may well be right about such an innate faculty, but our concern is with the articulation of symmetry as a scientific concept. We will demonstrate that both of them appeal to a concept that was not enunciated before the end of the 18th century, with revolutionary developments in the 19th century. Moreover, our historical account will show that the term, *symmetry*, had a variety of meanings before it was attached to bilateral and rotational symmetries.

According to current non-technical usage, *symmetry* refers either to bilateral symmetry or rotational symmetry. Bilateral symmetry is ordinarily treated as mirror image of the left and right sides of an object or a complex of objects, and a figure has rotational symmetry around an axis if it is carried into itself successively by rotations about this axis. The implicit assumption of Ruskin, Mach, and the contemporary learned person is that these two features of symmetry have always been recognized since the dawn of history.[20] They are after all perceptual modes, directly applicable in any visual experience. Furthermore, as we have seen, a scientist of the stature of Wigner assumed that past thinkers, such as Galileo and Newton, and even those who came before them, had the concept of symmetry, and that they had also transformed

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[19] McCormack and Foulkes (trs.) 1976, 288 (slightly modified); Mach [1905/1926] 2002, 308: "*Symmetrische* geometrische Gebilde erscheinen uns vermöge unserer symmetrischen *physiologischsen Organisation* sehr leicht als *gleich*, während dieselben *metrisch* und *physisch* gänzlich *verschieden* sind. Eine rechts- und eine linksgewundene Schraube, zwei entgegengesetzt rotierende Körper u. s. w. sind für die Anschauung sehr ähnlich, wir dürfen sie aber deshalb nicht für geometrisch oder physisch gleichwertig halten. Beachtung dieses Umstandes möchte manche *paradoxe* Frage ausschalten. Man bedenke, was solche Fragen Kant zu schaffen gemacht haben." For a detailed analysis of Kant's discussion, see § 7.3.

[20] Cf. Allen 1879, 302: "... whenever we find an object artificially shaped into a symmetrical form, we conclude that its maker was some animal sufficiently resembling ourselves to deserve the name of man. We have thus, in fact, informally recognized the taste for symmetry as a real differentia of humanity."

it into a principle, relating it in an abstract way to invariance. A close examination of the history of "symmetry", the kind we undertake in this book, quickly reveals that these expectations are ill-founded.

We take seriously Mach's warning not to confuse the physiological or the psychological with the physical and so we exclude the former from our discussion, leaving such issues to others. We are concerned here with scientific concepts, specifically with the concept of symmetry and its background in relevant aspects of aesthetics. Moreover, just as we are not dealing with the psychology of perception, we are also not dealing with the psychology of making concepts. The reason that motivates this exclusion concerns our belief that scientific concepts (e.g., "electron") are distinct from concepts in natural linguistic usage (e.g., "chair"). In our view the two categories reflect different processes of inception and have different statuses. This distinction has a direct bearing on current debates concerned with the way concepts are made and deployed in scientific theories.[21] Our discussion of the making of the modern scientific concept of symmetry is designed, in part, to contribute to this general debate—How are new concepts made, and what role do they play in scientific revolutions?

1.3 Philosophical Perspectives

By and large, historians and philosophers of science have neglected the role of scientific concepts by concentrating principally on theories and their interrelations. We suggest that the *locus* of change in science should be sought in the elements involved in the course of constructing theories, rather than in the shift from one theory to another. The growth of science is an issue of much philosophical concern, and an essential part of this growth is the construction of theories which in turn is dependent on the determination of concepts—the building blocks of theories. In view of the dramatic, indeed revolutionary, changes in modern scientific theories on the one hand, and the exponential growth of science on the other, it became urgent to understand the cumulative nature of scientific knowledge which, in an apparent paradox, is based on change—often of a radical kind. Put bluntly, previous theories are declared inadequate or even utterly false, and yet science is cumulative. Is it then the case that in the scientific enterprise knowledge of dubious value—that is, knowledge that has been refuted—provides the foundation for subsequent theories? To approach scientific activity from this perspective is, however, absurd (despite attempts to the contrary). So what actually accumulates? What, then, is the nature of this change? What drives it, and on what is it based? Most importantly, can we grasp the change, this transition from one theory to the other, in rational terms? Convincing answers to this set of questions will undoubtedly increase our understanding of the nature of science and the paradox of its growth. The discussion of making concepts may help illuminate this paradoxical problem of scientific change.

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[21] See Andersen et al. 2006; cf. Arabatzis and Kindi 2008.

Scientific theories are subject to confirmation or refutation; by contrast, scientific concepts are either suitable or unsuitable for some purpose. A theory has many components, e.g., postulates, principles, definitions, laws, and concepts, which are set in logical relations that ultimately form an argument complete with presuppositions, rules of inference, and consequences. The elements of such an argument may be consistent, or in tension with one another, thereby weakening the theory or even undermining it altogether. Still, theory can accommodate tension, but a concept cannot, for in contrast to theories, concepts serve as units in the construction of theories. Speaking figuratively, a concept as a unit is rigid and thus brittle. Concepts correspond to objects, classes of objects, their properties and relations, all of which may be concrete or abstract. A concept is not an argument, and it does not have such a structure. Thus, logically, unlike a theory, a concept cannot be refuted; rather, it is the claim to existence which is refuted. The new concept may build on older ones, or it may break with the past; either way the path to the new concept does not depend on a refutation of the old concept, which may be declared unsuitable because it is in conflict with some empirical data or because its class is empty.

The making of new concepts can thus be distinguished from the generation of new theories, and this difference is reflected in their respective life histories: the story of concepts is different from the story of theories. A concept may be revolutionary because of its consequences but, even so, it may not be so perceived at the time because it may not threaten any theoretical structure. New concepts may point to difficulties in the old theories but, in our case, the new concept of symmetry, however revolutionary, did not require a change in the theory, namely, Euclidean geometry.

One way of studying how a new concept is made is to concentrate on its early applications, with the goal of characterizing those features of the concept that make it suitable for the (new) task at hand.[22] While the account of the usages and applications is mainly historical, characterizing features, that is, singling them out, requires philosophical considerations. We therefore add a philosophical perspective to our historical discussion for which purpose we appeal to a number of scholars from the 19th and 20th centuries to clarify philosophical issues involved in symmetry as a scientific concept. We analyze the nature of concepts in general and consider various aspects of the modern usage of symmetry as a scientific concept. We then argue that in many respects it is special among scientific concepts. This strengthens our historical claim that the scientific concept of symmetry is indeed modern, that is, it is not related to the symmetry of earlier times. The philosophical discussion leads us directly to the historiographical issue.

Consider the following proposition:

> The planet Mars moves on an elliptical orbit due to its mutual attraction with the Sun.

Now, “Mars” and “Sun” are proper names of individual entities, whereas “planet”, “orbit”, and “attraction” are scientific concepts. Some concepts refer to a class of

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[22] See, e.g., Goldstein and Hon 2005.

objects or entities (such as “planet”), whereas others (such as “attraction”) are abstract and depend on a set of properties and conditions and, indeed, on a theory. To be sure, the class, “planet”, is also abstract: a generalization or abstraction from specific instances. Still, the concept, “planet”, is different from the concept, “attraction”, in that classes (such as “planet”) are clusters of objects or entities that share a set of properties, whereas “attraction” is a relation between two objects (in this case, Mars and the Sun). The presence in the mind of concepts which connote classes of concrete objects and of concepts which connote abstract features makes it possible for us to think in general terms and conduct comparisons among individual entities and situations and, on this basis, we can assess whether or not they belong together in a single class or constitute a certain property. When we recognize some similarity among individuals, either as entities, properties, or relations, we form a conception of a phenomenon or a law. The question thus arises whether these concepts are present a priori in the mind. Are they innate? To take a few examples, are the concepts of “planet”, “orbit”, and “attraction” in the mind apart from any experience? We say, No. Concepts are made; they are constructed from experience, but differ from it. However, we do not intend to explore such processes; rather, we wish to shed light on the role of concepts as building blocks in the generation of theories.

We follow John Stuart Mill (1806–1873) and hold that the making of concepts—whatever their provenance—requires that the mind have the capacity for making generalizations, or as Mill puts it:

> When we form a set of phenomena into a class, that is, when we compare them with one another to ascertain in what they agree, some general conception is implied in this mental operation.[23]

There is no dispute that the mind has this capacity, but then the age-old question arises, where do concepts that allow for generalization come from? With a consistent empirical outlook and a strong belief in induction, Mill then remarks:

> It is not a law of our intellect, that, in comparing things with each other and taking note of their agreement, we merely recognise as realised in the outward world something that we already had in our minds.

And he continues,

> The conception originally found its way to us as the *result* of such a comparison. It was obtained (in metaphysical phrase) by *abstraction* from individual things. These things may be things which we perceived or thought of on former occasions, but they may also be the things which we are perceiving or thinking of on the very occasion.[24]

According to Mill, the concepts that facilitate the grouping together of facts to form some distinct phenomenon, “do not develop themselves from within, but are impressed upon the mind from without.” Indeed, “they are never obtained otherwise

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[23] Mill [1843] 1941, 425.

[24] *Ibid*.

than by way of comparison and abstraction." Thus, "we compare phenomena with each other to get the conception, and we then compare those and other phenomena with the conception."[25]

Although Mill's comment can be applied both to classes and to properties, he failed to distinguish explicitly between them. Classes are certainly not innate; they are clearly affected by cultural considerations. For example, consider the distinction between "cow" and "bull" vs. the general term, "cattle", that is, we may pay attention to some properties and ignore others. Put differently, the separation into classes depends on the property one chooses to consider. Certain assemblages of classes and properties may be taken to be more representative or typical than others and this has been widely confirmed by observational data in psychology.[26]

These empirical data render the classical theory of concepts too rigid and lacking in explanatory power with respect to the ways concepts in natural languages are introduced and applied. Essentially, the classical theory stipulates that concepts are lexical, they have a definitional structure that is composed of simpler concepts which express necessary and sufficient conditions for falling under the concept in question. The theory has the appeal of providing a unifying approach to all concepts, be they of daily language ("chair"), philosophical ("truth"), or scientific ("electron"). In recent decades the classical theory has come under strong criticism, mainly because of counterexamples which undermine any consensus on a definition, thereby making the application of a certain concept inconsistent. This leads us to suspect that concepts, after all, may not have a lexical structure.[27] Be that as it may, the standard approach is still to seek some definition, and this practice is applied to *symmetry* as well.

The current edition of the *Oxford English Dictionary* states that *symmetry*, among other things, is "a property by virtue of which something is effectively unchanged by a particular operation; an operation or set of operations that leaves something effectively unchanged; in *Physics*, a property that is conserved...." Hence, "symmetry operation [in] *Physics*, an operation or transformation that leaves something effectively unchanged."[28]

Is the recognition of such a property innate? We need not decide the issue; for our argument it is enough to acknowledge that in some cases it is probably innate, but in others it is not. No doubt, the mind has the innate capacity to recognize concepts in both ways, that is, by experience as well as by contemplation. We will argue that the scientific concept of symmetry—as distinct from concepts in natural language—is an example of a concept that is not innate, that is, we find no credible evidence to support the claim that this scientific concept is innately recognized. If it were so, we would expect to find it articulated in scientific texts of all periods

[25] Mill [1843] 1941, 427–428.

[26] For an overview of the various approaches to the study of concepts with many citations of the contemporary literature, see Margolis and Laurence 2006.

[27] Cf. Margolis and Laurence 2006, § 2.1.

[28] *Oxford English Dictionary* 2006: *Symmetry*.

and across all cultures in a consistent way. Here history may inform philosophical debates. Indeed, our historical study is in part a demonstration that such evidence is lacking.[29]

This point is central to the thesis of our book and therefore worthy of elaboration. We acknowledge a vast array of empirical data of ancient artifacts that, to the modern observer, seem to exhibit symmetrical properties. But we dispute the tacit assumption that, if we can discern such properties in objects made in the past, there must have been some process of abstraction by which the producers and consumers of these objects recognized these properties and articulated them in old texts. We argue that such evidence is lacking, that is, in contrast to the material culture, the written culture does not exhibit any articulation of the concept of symmetry as we know it today. Observe a perfectly bilateral symmetrical Greek temple and then consider the fact that Vitruvius does not mention this striking property of the edifice; this is an astounding omission. In other words, if Vitruvius had this concept, he had many opportunities to invoke it, but he did not. We distinguish then the capacity to recognize symmetry from the concept of symmetry and its explicit articulation. We have no objection to the claim that an artisan in antiquity had a vague idea of symmetry (without articulating it), coming from observation of, or from imitation of, nature, with little thought being given to it. Clearly, there is a difference between the making of a concept and assigning a term to it, on the one hand, and intuitively applying the concept without being aware of its application, on the other: a person may not know that he actually speaks prose, as Jean-Baptiste Molière (1622–1673) astutely remarked.[30]

The process of building a vocabulary of concepts is characteristic not only of natural language, but also of scientific terminology. In general, scientific concepts are related to carefully defined properties and many of these properties are not recognized by the untrained. Moreover, the properties that lead to classes in natural language are often not articulated by the ordinary person. There is no doubt that a large class of scientific concepts cannot be found in the practice of natural language. On the other hand, it is well known that a common term in natural language is often transformed into a precise scientific term for a specific concept. Consider the case of "force" in natural language vs. "force" in Newtonian mechanics or, for another

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[29] In his essay on the origin of the sense of symmetry, Allen (1879, 301) observes: "the love for symmetry among mankind is something that has grown and developed during the whole of historical and prehistoric time." He then poses the question, "what is the origin of the taste which we see thus displayed in every existing race of men?" But when Allen proceeds to reformulate this amorphous question in precise terms we realize that the subject of his inquiry is not symmetry. He asks (p. 302): "why did man first take to the two primordial elements of symmetry, the straight line, and the circle, or other regular curves?" In our view the problem Allen posed has nothing to do with the concept of symmetry as we know it today. The fact that (p. 304), "primitive man shared with all other animals an inherent tendency towards the construction of regular figures", does not impinge on the issue of symmetry as a scientific concept.

[30] Molière 1671, 43 (Acte II, scène iv): "MONSIEUR JOURDAIN.– Par ma foy, il y a plus de quarante ans que je dis de la Prose, sans que j'en sçeusse rien; & je vous suis le plus obligé du monde, de m'avoir apris cela."

example, note the difference between "error" in ordinary usage and "error" in scientific discourse. While there are words in all languages (of which we are aware) for 'left' and 'right' (probably referring, in the first instance, to hands or feet), this is not the same as articulating the *concept* of "left and right" as mirror image.[31]

The scientific concept of symmetry is, however, much wider than the limited perspective of the perceptual approach. There is general agreement that Weyl's set of lectures on symmetry (delivered at Princeton University in the early 1950s and published by its press in 1952) constitutes a milestone in the analysis and application of the concept of symmetry as a property. This profound treatment of the concept came after a century and a half of growing interest in, and usage of, the modern concept of symmetry, and it consists of two principal moves: methodological and metaphysical. On methodological grounds, Weyl proceeds from considerations of a vague concept to one that is precise, gradually reaching greater generality, and guided—as he puts it—"more by mathematical construction and abstraction than by the mirage of philosophy."[32] In this process of generalization, Weyl intentionally discards the aesthetic appeal of the concept and concentrates on precision, ultimately aiming at determining its unifying power. This approach coheres with the second move. Weyl's metaphysical claim is that mathematics underlies the concept of symmetry. He argues that mathematics is the common origin of symmetry in nature and in art: "the mathematical laws governing nature are the origin of symmetry in nature, the intuitive realization of the idea in the creative artist's mind its origin in art."[33] Weyl's book, *Symmetry*, is the crystallization of the idea that group theory, that is, a branch of mathematics, underlies symmetry in nature and art.

Weyl demonstrates persuasively that the concept of symmetry is an application of group theory. He begins with the notion of congruence, for it captures a structural feature of space: "any … congruent transformation … is a similarity or an automorphism.… It is evident", Weyl continues, "that the congruent transformations form a group, a subgroup of the group of automorphisms."[34] Congruence may, for example, be the result of an operation of rotation or of translation; in each such operation the point p of a certain space V which is occupied by some rigid body, is mapped onto a point p' in space V' such that the rigid body remains invariant under this operation.[35] Weyl then poses a question, "What has all this to do with symmetry?," to which he replies:

> It provides the adequate mathematical language to define it. Given a spatial configuration **F**, those automorphisms of space which leave **F** unchanged form a group Γ, and *this group describes exactly the symmetry possessed by* **F**.… The symmetry of any figure in space is described by a subgroup of that group.[36]

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[31] See, e.g., the discussions of Aristotle and Augustine in Ch. 4, nn. 8–14.

[32] Weyl 1952, 6.

[33] Weyl 1952, 6–8.

[34] Weyl 1952, 43.

[35] *Ibid*.

[36] Weyl 1952, 44–45.

This thorough analysis has been very influential in pointing to a link between the arts, nature, and the sciences, which is ultimately grounded in mathematics:

> From art, from biology, from crystallography and physics I finally turn to *mathematics*, which I must include all the more because the essential concepts, especially that of a group, were first developed from their applications in mathematics. . ..[37]

Thus, the modern concept of symmetry gains its strength not only from the fruitful applications of a mathematical property, but also from its ability to link a variety of domains, revealing common patterns in them. This observation, which Weyl stressed, made symmetry the powerful concept that it has become in domains well beyond mathematics and physics.

The key feature which Weyl stipulates for the definition of symmetry is the concept of a group, that is, symmetry is essentially a group theoretic concept. It is this defining feature that renders the over-arching analysis viable. Symmetry is a property of objects or elements that form a group and, to form this group, there must be a transformation with an invariance that satisfies four fundamental axioms: closure, associativity, identity, and inversion.[38] The invariance—what stays the same under the transformation—is the symmetrical property. "We found," concluded Weyl, "that objectivity means invariance with respect to the group of automorphisms."[39] This profound philosophical observation indicates that symmetry as a group of automorphisms may then be considered a heuristic principle in science:

> *Whenever you have to do with a structure-endowed entity Σ try to determine its group of automorphisms*, the group of those element-wise transformations which leave all structural relations undisturbed. You can expect to gain a deep insight into the constitution of Σ in this way.[40]

This "deep insight into the constitution of Σ" is in fact knowledge of an invariance, and it may be regarded as "objective reality".[41]

We follow Weyl's lead: the definition of the modern concept of symmetry must be formulated in group theoretic terms and involve a transformation with an invariance. In 1996, about half a century after Weyl, David J. Gross, one of the chief architects of the fundamental theory of the strong force—quantum chromodynamics—a Nobel Laureate in Physics (2004) and a notable advocate of string theory, reaffirmed this

[37] Weyl 1952, 135.

[38] Klein [1926] 1979, 315–316; Klein 1926, 335; Weyl 1952, 41–43. There is an important distinction between discrete (or finite) groups and continuous groups which has a considerable impact on the domains of application. See Weyl 1952, 106–107, 119–120.

[39] Weyl 1952, 132. The definition of invariance in the *Oxford English Dictionary* is: "The character of remaining unaltered after a linear transformation; the essential property of an invariant. Hence applied to a similar property with respect to any transformation or operation." See *Oxford English Dictionary* 2006: *Invariance*.

[40] Weyl 1952, 144.

[41] It should be noted that in these symmetry considerations there is no aesthetic component, be it qualitative or evaluative.

definition of Weyl. Gross reviewed the fundamental role of symmetry in physics and explicitly identified symmetry with invariance, arguing that symmetry provides structure and coherence to the laws of nature.[42]

In asking whether symmetry is identity, Marvin Chester develops a double perspective on symmetry which corresponds to our approach; the first belongs to the very reality of the object itself and the second takes the viewpoint of the observer. Although Chester does not clearly distinguish between the two approaches, we treat them separately. The former is a property of the system, it is objective and inherent in the world, while the latter requires an observer who alters his or her point of view. Chester calls the latter approach *altered scrutiny*, and we adopt his expression.

> Group theory is the mathematical formulation of internal consistency in the description of things. We *assume* that the system being observed has an intrinsic character independent of the observer's perspective. It's there. It possesses an objective reality. On this assumption—that it's there—how the system is perceived under altered scrutiny must be a matter of logic. Its appearance follows the logic of intrinsic sameness. ... The codification of that logic is a matter of group theory. And its success in portraying the physical world is what vindicates the assumption.[43]

Altered scrutiny is just another way of expressing symmetry that is inherent in nature and it is group theory which underlies the two perspectives and renders them equivalent.

Weyl analyzed a large number of applications of symmetry as property, but he did not systematically address symmetry as a feature of an argument and thus as an aspect of theory building.[44] Although he made a key claim, "all a priori statements in physics have their origin in symmetry,"[45] Weyl did not elaborate on the epistemological and methodological power of the concept of symmetry. Unlike any other scientific concept, symmetry, that is, the modern concept of symmetry, is applicable to issues of scientific method as well as to the content of science. Indeed, symmetry may be regarded as a unique scientific concept in that it embodies the very practice of scientific pursuit. For the purpose of this philosophical analysis we leave the domain of science, and move to its meta-level: the domain of methodology.

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[42] Gross 1996, 14256.

[43] Chester 2002, 111–112.

[44] Weyl refers very briefly to quantum mechanics in his *Symmetry*, explicitly informing the reader that he will "refrain from giving . . . a more precise account of this difficult subject." He then continues (1952, 135): "here symmetry once more has proved [to be] the clue to a field of great variety and importance." In this book Weyl did not report his own contributions, as an actor, to quantum mechanics which are based in part on appealing to symmetry arguments: see, e.g, Weyl 1928. In his study of classical groups, Weyl appeals to symmetry as a precisely defined technical term which does not function as an argument (see Weyl [1939] 1966) but, in his philosophical works on mathematics and physics, Weyl points out the well known use of symmetry arguments in developing probability theories (Weyl and Helmer [1927] 1949, 197). Thus, Weyl, the mathematical physicist, applied symmetry arguments in his scientific works, but Weyl, the philosopher-analyst, had little to say about them.

[45] Weyl 1952, 126.

Finding patterns is a fundamental activity in science which is generally followed by seeking some law governing them. What we call *phenomena* are in fact a recognition of some experiential pattern (be it perceptual or cognitive) to which one or another scientific method is applied in order to find a governing law and, in turn, apply it for predictive or explanatory purposes. The realization that symmetry may be applied as a key concept for attaining insight into scientific practice is at the core of the philosophical argument which considers symmetry a methodological concept.

To address this philosophical aspect, we refer to the analysis of Bas C. van Fraassen who discusses the group theoretic properties of symmetry from an epistemological perspective in his influential book, *Laws and Symmetry*.[46] In fact, van Fraassen appeals to Weyl's definition of symmetry and, in the section on "Symmetry and Invariance", he states that "symmetries are transformations (technically one-to-one functions which map onto their range) that leave all relevant structure intact—the result is always exactly like the original, in all *relevant* respects."[47] Van Fraassen classifies symmetry philosophically, rather than by the domain of its application. Thus, in contrast to Weyl who proceeds by analyzing the application of symmetry to artifacts, natural objects, and theoretical entities, van Fraassen seeks the general principle that governs these different applications.[48]

According to Chester, van Fraassen "shows us that the status of 'physical law' is conferred by symmetry—invariance under the transformations of nature."[49] Chester then asks, "but what is symmetry that it should underlie the very foundation of natural law?" This leads him to recast the problem in group theoretic terms. He writes,

> Alternatively put: why is group theory so effective in describing the physical world? The answer is that it codifies the basic axioms of the scientific enterprise. The logic of group theory is the logic of scientific inquiry so that the mathematics we use to describe nature is a carefully coded expression of our experience.[50]

Chester, following Ernst Cassirer (1874–1945), argues further: "the mathematical theory of symmetry—group theory—may transcend its problem-solving utility.... It has something to do with how we know; how we evaluate perception."[51] We are not concerned now with the content of science, say, some conservation law and its symmetry group. Nor are we concerned with Wigner's well-known conundrum: the effectiveness of mathematics in the natural sciences is unreasonable.[52] Rather, following Chester, we focus our attention on the very practice of the scientific

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[46] Van Fraassen 1989, 245–250, 266–267.

[47] Van Fraassen 1989, 243; on the group theoretic notion, see 266, 279 and 368 n. 7.

[48] Unlike van Fraassen, we address symmetry in specific scientific domains: physics, mathematics, architecture, natural history, etc.; we thus offer a close analysis of the relevant historical episodes.

[49] Chester 2002, 111.

[50] *Ibid.*

[51] Chester 2002, 112.

[52] Wigner [1959] 1979.