Modern Fluid Dynamics
**Aims and Scope of the Series**

The purpose of this series is to focus on subjects in which fluid mechanics plays a fundamental role.

As well as the more traditional applications of aeronautics, hydraulics, heat and mass transfer etc., books will be published dealing with topics which are currently in a state of rapid development, such as turbulence, suspensions and multiphase fluids, super and hypersonic flows and numerical modeling techniques.

It is a widely held view that it is the interdisciplinary subjects that will receive intense scientific attention, bringing them to the forefront of technological advancement. Fluids have the ability to transport matter and its properties as well as to transmit force, therefore fluid mechanics is a subject that is particularly open to cross fertilization with other sciences and disciplines of engineering. The subject of fluid mechanics will be highly relevant in domains such as chemical, metallurgical, biological and ecological engineering. This series is particularly open to such new multidisciplinary domains.

The median level of presentation is the first year graduate student. Some texts are monographs defining the current state of a field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

For other titles published in this series, go to www.springer.com/series/5980
Clement Kleinstreuer

Modern Fluid Dynamics

Basic Theory and Selected Applications in Macro- and Micro-Fluidics
To my family,
Christin, Nicole, and Joshua
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Preface

This textbook covers essentials of traditional and modern fluid dynamics, i.e., the fundamentals of and basic applications in fluid mechanics and convection heat transfer with brief excursions into fluid-particle dynamics and solid mechanics. Specifically, it is suggested that the book can be used to enhance the knowledge base and skill level of engineering and physics students in macro-scale fluid mechanics (see Chaps. 1–5 and 10), followed by an introductory excursion into micro-scale fluid dynamics (see Chaps. 6 to 9). These ten chapters are rather self-contained, i.e., most of the material of Chaps. 1–10 (or selectively just certain chapters) could be taught in one course, based on the students’ background. Typically, serious seniors and first-year graduate students form a receptive audience (see sample syllabus). Such as target group of students would have had prerequisites in thermodynamics, fluid mechanics and solid mechanics, where Part A would be a welcomed refresher. While introductory fluid mechanics books present the material in progressive order, i.e., employing an inductive approach from the simple to the more difficult, the present text adopts more of a deductive approach. Indeed, understanding the derivation of the basic equations and then formulating the system-specific equations with suitable boundary conditions are two key steps for proper problem solutions.

The book reviews in more depth the essentials of fluid mechanics and stresses the fundamentals via detailed derivations, illustrative examples and applications covering traditional and modern topics. Similar to learning a language, frequent repetition of the essentials is employed as a pedagogical tool. Understanding of the fundamentals and independent application skills are the main learning objectives. For students to gain confidence and independence, an instructor may want to be less of a “sage on the stage” but more of a “guide on the side”. Specifically, “white-board performances”, tutorial presentations of specific topics in Chaps. 4–10 and associated journal articles by students are highly recommended.
The need for the proposed text evolved primarily out of industrial demands and post-graduate expectations. Clearly, industry and government recognized that undergraduate fluid mechanics education had to change measurably due to the availability of powerful software which runs on PCs and because of the shift towards more complicated and interdisciplinary tasks, tomorrow’s engineers are facing (see NAS “The Engineers of 2020” at http://national-academics.org). Also, an increasing number of engineering firms recruit only MS and Ph.D. holders having given up on BS engineers being able to follow technical directions, let alone to build mathematical models and consequently analyze and improve/design devices related to fluid dynamics, i.e., here: fluid flow, heat transfer, and fluid–particle/fluid–structure interactions. In the academic environment, a fine knowledge base and solid skill levels in modern fluid dynamics are important for any success in emerging departmental programs and for new thesis/dissertation requirements responding to future educational needs. Such application areas include microfluidics, mixture flows, fluid–structure interactions, biofluid dynamics, thermal flows, and fluid-particle flows. Building on courses in thermodynamics, fluid mechanics and solid mechanics as prerequisites as well as on a junior-level math background, a differential approach is most insightful to teach the fundamentals in fluid mechanics, to explain traditional and modern applications on an intermediate level, and to provide sufficient physical insight to understand results, providing a basis for extended homework assignments, challenging course projects, and virtual design tasks.

Pedagogical elements include a consistent 50/50 physics-mathematics approach when introducing new material, illustrating concepts, showing flow visualizations, and solving problems. The problem solution format follows strictly: System Sketch, Assumptions, and Concept/Approach – before starting the solution phase which consists of symbolic math model development (App. A), numerical solution, graphs, and comments on “physical insight”. After some illustrative examples, most solved text examples have the same level of difficulty as suggested assignments and/or exam problems. The ultimate goals are that the more serious student can solve basic fluid dynamics problems independently, can provide physical insight, and can suggest, via a course project, system design improvements.
The proposed textbook is divided into three parts, i.e., a review of essentials of fluid mechanics and convection heat transfer (Part A) as well as traditional (Part B) and modern fluid dynamics applications (Part C). In Part A, the same key topics are discussed as in the voluminous leading texts (i.e., White, Fox et al., Munson et al., Streeter et al., Crowe et al., Cengle & Cimbala, etc.); but, stripped of superfluous material and presented in a concise streamlined form with a different pedagogical approach. In a nutshell, quality of education stressing the fundamentals is more important than providing high quantities of material trying to address everything.

Chapter 1 starts off with brief comments on “fluid mechanics” in light of classical vs. modern physics and proceeds with a discussion of the basic concepts. For example, the amazing thermal properties of “nanofluids”; i.e., very dilute nanoparticle suspensions in liquids, are discussed in Sect. 1.4 in conjunction with the properties of more traditional fluids. Derivations of the conservation laws are so important that three approaches are featured, i.e., integral, transformation to differential, and representative-elementary-volume (Chap. 2). On the other hand, tedious derivations are relegated to App. C in order to maintain text fluidity. Each section of Chap. 2 contains illustrative examples to strengthen the student’s understanding and problem-solving skills. Appendix A provides a brief summary of analytical methods as well as an overview of basic approximation techniques. Chapter 3 continues to present typical 1st-year case studies in fluid mechanics; however, some 2nd-level fluids material appears already in terms of exact/approximate solutions to the Navier–Stokes equations as well as solutions to scalar transport equations. The concept of entropy generation in internal thermal flow systems for waste minimization is discussed as well.

Part B is a basic discourse focusing especially on practical pipe flows as well as boundary-layer flows. Specifically, applications to the bifurcation and slit flows as well as laminar or turbulent pipe flow, lubrication and compartmental system analysis are presented in Chap. 4, while Chap. 5 deals with boundary-layer and thin-film flows, including coating as well as drag computations.

Part C introduces some modern fluid dynamics applications for which the fundamentals presented in the previous chapters plus App. A form necessary prerequisites. Specifically, Chap. 6 discusses
simple two-phase flow cases, stressing power-law fluids and homogeneous mixture flows, previously the domain of only chemical engineers. Chapter 7 is very important. It deals with fluid flow in microsystems, forming an integral part of nanotechnology, which is rapidly penetrating many branches of industry, academia, and human health. After an overview of microfluidic systems given in the Introduction, Sect. 7.2 reviews basic modeling equations and necessary submodels. Then, in Sects. 7.3 to 7.5 key applications of microfluidics are analyzed, i.e., electrokinetic flows in microchannels, nanofluid flow in microchannels, and convective heat transfer with entropy generation in microchannels. Chapter 8 deals with fluid–structure interaction (FSI) applications for which a brief solid-mechanics review may be useful (Sect. 8.2). Clearly, fluid flows interacting with structural elements occur frequently in nature as well as in industrial and medical applications. The two-way coupling is a true multiphysics phenomenon, ultimately requiring fully coupled FSI solvers. Thus, young engineers should have had an exposure to the fundamentals of FSI before using such multiphysics software for R&D work. Chapter 9 deals with biofluid dynamics, i.e., stressing its unique transport processes and focusing on the three major applications of blood flow in arteries, air-particle flow in lung airways, and tissue heat transfer. An overview of CFD tools and solved examples with flow visualizations are given in Chap. 10, stressing computer simulations of internal and external flow examples.

As all books, this text relied on numerous sources as well as contributions provided by the author’s colleagues, research associates, former graduate students and the new MAE589K-course participants at NC State. Special thanks go to Mrs. Joyce Sorensen and Mrs. Joanne Self for expertly typing the first draft of the manuscript. Seiji Nair generated the system sketches and figures, while Christopher Basciano provided the computer simulations of Sects. 10.3 to 10.5. Dr. Jie Li then helped checking the content of all chapters after he generated result graphs, obtained the cited references, generated the index, and formatted the text. The critical comments and helpful suggestions provided by the expert reviewers Alex Alexeev (Georgia Tech, GA), Gad Hetsroni (Technion, Israel), and Alexander Mitsos (MIT, MA) are gratefully acknowledged as well. Many thanks for their support go also to the editorial staff at Springer Verlag, especially
the Publishing Editor Nathalie Jacobs, to the professionals in the ME
Department at Stanford University and in the Engineering Library.

A Solutions Manual, authored by Dr. Jie Li, is available for
instructors adopting the textbook. For technical correspondence, please
contact the author via e-mail ck@eos.ncsu.edu or fax 919.515.7968.

Raleigh, NC, 2009

Clement Kleinstreuer
NC State University, MAE Dept.  C. Kleinstreuer  
Spring 2009  BR4160; by appointment  
Library Reserve for MAE589K Website  ck@eos.ncsu.edu (any time)

**MAE 589K “Modern Fluid Dynamics”**  
(Tu & Th 13:30-14:45 in BR 3218)

Prerequisites: MAE 301, 308, 310, 314 (or equivalent); also: math and computer skills, including use of software (e.g., Matlab or Mathcad or MAPLE, and desirable: COMSOL, etc.)


Objectives: To strengthen the background in fluid dynamics (implying fluid mechanics plus heat transfer) and to provide an introduction to modern academic/industrial fluid dynamics topics. Report writing and in-class presentations are key preparations for GR School and the job market.

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1.1 Definitions and Concepts  
1.2 Conservation Laws  
1.3 Basic Fluid Dynamics Applications | • Review Chaps. 1–4  
• Solve Book Examples and Problems *independently*  
• HW Sets #1 and #2  
• White Board presentations |
| 7   | 2. Modern Fluid Dynamics Topics  
2.1 Film Drawing and Surface Coating  
2.2 Dilute Fluid-Particle Suspensions  
2.3 Microfluidics  
2.4 Fluid–Structure Interactions  
2.5 Biofluid Mechanics | • Study Chaps. 5–10  
• Solve selected Book Examples and Problems  
• White Board presentations  
• HW Set #3  
• Journal Article presentations |
| 3   | 3. Modern Fluid Dynamics Projects  
3.1 Math Modeling and Computer Simulation  
3.2 Nanofluid Flow in Microchannels  
3.3 Microfluidics and Medical Devices | • Revisit Chaps. 7–10  
• Course Project outlines  
• Course Project presentations |

Grading Policy: Three HW Sets plus two Tests: 70%; Presentations and Course Project: 30%
Part A

Fluid Dynamics Essentials
“Fluid dynamics” implies fluid flow and associated forces described by vector equations, while convective heat transfer and species mass transfer are described by scalar transport equations. Specifically, this chapter reiterates some basic definitions and continuum mechanics concepts with an emphasis on how to describe standard fluid flow phenomena. Readers are encouraged to occasionally jump ahead to specific sections of Chaps. 2 and 3. After refreshing his/her knowledge base, the student should solve the assigned Homework Problems independently (see Sect. 1.5) in conjunction with Appendix A (see Table 1.1 for acquiring good study habits).

It should be noted that the material of Part A is an extension of the introductory chapters of the author’s “Biofluid Dynamics” text (CRC Press, Taylor & Francis Group, 2006; with permission).

1.1 Approaches, Definitions and Concepts

A sound understanding of the physics of fluid flow with mass and heat transfer, i.e., transport phenomena, as well as statics/dynamics, stress–strain theory and a mastery of basic solution techniques are important prerequisites for studying, applying and improving engineering systems. As always, the objective is to learn to develop mathematical models; here, establish approximate representations of actual transport phenomena in terms of differential or integral equations. The (analytical or numerical) solutions to the describing
equations should produce testable predictions and allow for the analysis of system variations, leading to a deeper understanding and possibly to new or improved engineering procedures or devices. Fortunately, most systems are governed by continuum mechanics laws. Notable exceptions are certain micro- and nano-scale processes, which require modifications of the classical boundary conditions (see Sect. 7.4) or even molecular models solved via statistical mechanics or molecular dynamics simulations.

Clearly, transport phenomena, i.e., mass, momentum and heat transfer, form a subset of mechanics which is part of classical (or Newtonian) physics (see Fig. 1.1). Physics is the mother of all hard-core sciences, engineering and technology. The hope is that one day advancements towards a “universal theory” will unify classical with modern physics, i.e., resulting in a fundamental equation from which all visible/detectable phenomena can be derived and described.

Fig. 1.1 Subsets of Physics and the quest for a Unifying Theory In any case, staying with Newtonian physics, the continuum mechanics assumption, basic definitions, equation derivation methods and problem solving goals are briefly reviewed next – in reverse order.
**Approaches to Problem Solving** Traditionally, the answer to a given problem is obtained by copying from available sources suitable equations, needed correlations (or submodels), and boundary conditions with their appropriate solution procedures. This is called “matching” and may result in a good first-step learning experience.

**Table 1.1** Suggestions for students interested in understanding fluid mechanics and hence obtaining a good grade

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<th>1. Review topics:</th>
<th>Math Background (see App. A)</th>
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<td><strong>Eng. Sciences (Prerequisites)</strong></td>
<td><strong>Algebra, Vector Analysis &amp; Taylor Series Expansion</strong></td>
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<tr>
<td>• Problem Solution FORMAT: System Sketch, Assumptions, Approach/Concepts; Solution, Properties, Results; Graphing Analysis, &amp; Comments</td>
<td>• Calculus &amp; Functional including Graphing</td>
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<tr>
<td>• Differential Force, Energy &amp; Mass Balances (i.e., free-body diagram, control volume analysis, etc.)</td>
<td>• Surface &amp; Volume Integrals</td>
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<tr>
<td>• Symbolic Math Analyses, where # of Unknowns $\cong$ # of Equations</td>
<td>• Differential Equations subject to Boundary Conditions</td>
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<th>2. Preparation</th>
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<td>• Study Book Chapters, Lecture Notes, and Problem Assignments</td>
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<td>• Learn from solved Book Examples, Lecture Demos, and Review Problem Solutions (work independently!)</td>
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<tr>
<td>• Practice graphing of results and drawing of velocity or temperature profiles and streamlines</td>
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<tr>
<td>• Ask questions (in-class, after class, office, email)</td>
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<td>• Perform “Special Assignments” in-class, such as White-board Performance, lead in small-group work, etc.</td>
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<tr>
<td>• Solve Old Test Problems with your group</td>
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<tr>
<td>• Solve test-caliber questions &amp; problems: well-paced and INDEPENDENTLY</td>
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<th>3. Participation</th>
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<td>• Enrich your knowledge base and sharpen your communication skills via Presentations</td>
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<tr>
<td>• Understand some Fluid Mechanics Topics in more depth from exploring Flow Visualizations as well as doing Computer Project Work, and Report Writing.</td>
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However, it should be augmented later on by more independent work, e.g., deriving governing equations, obtaining data sets, plotting and visualizing results, improving basic submodels, finding new, interdisciplinary applications, exploring new concepts, interpreting observations in a more generalized form, or even pushing the envelope of existing solution techniques or theories. In any case, the triple pedagogical goals of advanced knowledge, skills, and design can be achieved only via independent practice, hard work, and creative thinking. To reach these lofty goals, a deductive or “top-down” approach is adopted, i.e., from-the-fundamental-to-the-specific, where the general transport phenomena are recognized and mathematically described, and then special cases are derived and solved. For the reader’s convenience and pedagogical reasons, specific (important) topics/definitions are several times repeated throughout the text.

While a good grade is a primary objective, a thorough understanding of the subject matter and mastery in solving engineering problems should be the main focus. Once that is achieved, a good grade comes as a natural reward (see Table 1.1).

**Derivation Approaches** There are basically four ways of obtaining specific transport equations reflecting the conservation laws. The points of departure for each of the four methods are either given (e.g., Boltzmann equation or Newton’s second law) or derived based on differential mass, momentum and energy balances for a representative elemental volume (REV).

(i) **Molecular Dynamics Approach**: Fluid properties and transport equations can be obtained from kinetic theory and the Boltzmann equation, respectively, employing statistical means. Alternatively, \( \sum \vec{F} = m \vec{a} \) is solved for each molecule using direct numerical integration (see Sect. 1.3).

(ii) **Integral Approach**: Starting with the Reynolds Transport Theorem (RTT) for a fixed open control volume (Euler), specific transport equations in integral form can be obtained (see Sect. 2.2).
(iii) **Differential Approach:** Starting with 1-D balances over an REV and then expanding them to 3-D, the mass, momentum and energy transfer equations in differential form can be formulated. Alternatively, the RTT is transformed via the divergence theorem, where in the limit the field equations in differential form are obtained (see Sects. 2.3–2.5).

(iv) **Phenomenological Approach:** Starting with balance equations for an open system, i.e., a control volume, transport phenomena in complex flows are derived largely based on empirical correlations and dimensional analysis considerations. A very practical example is the description of transport phenomena with compartment models (see Sect. 4.4). These “compartments” are either well-mixed, i.e., transient lumped-parameter models without any spatial resolution, or they are transient with a one-dimensional resolution in the axial direction.

**Definitions** Elemental to transport phenomena is the description of fluid flow, i.e., the equation of motion, which is also called the momentum transfer equation. It is an application of Newton’s second law, \( \sum \mathbf{F}_{\text{ext}} = m \ddot{\mathbf{a}} \), which Newton postulated for the motion of a particle. For most engineering applications the equation of motion is nonlinear but independent of the mass and heat transfer equations, i.e., fluid properties are not measurably affected by changes in solute concentration and temperature. Hence, the major emphasis in Chap. 1 is on the description, solution and understanding of the physics of fluid flow. Here is a review of a few definitions:

- A **fluid** is an assemblage of gas or liquid molecules which deforms continuously, i.e., it **flows** under the application of a shear stress. Note, solids do not behave like that; but, what about borderline cases, i.e., the behavior of materials such as jelly, grain, sand, etc.?
- Key fluid properties are density \( \rho \), dynamic viscosity \( \mu \), species diffusivity \( D \), heat capacities \( c_p \) and \( c_v \), and thermal conductivity \( k \). In general, all six are temperature and species concentration dependent. Most important is the viscosity (see
also kinematic viscosity $\nu = \mu / \rho$) representing frictional (or drag) effects. Certain fluids, such as polymeric liquids, blood, food stuff, etc., are also shear-rate dependent and hence called *non-Newtonian fluids* (see Sect. 6.3).

- **Flows** can be categorized into:

<table>
<thead>
<tr>
<th>Internal flows</th>
<th>and</th>
<th>External flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Oil, air, water or steam in pipes and inside devices</td>
<td>- Air past vehicles, buildings and planes</td>
<td></td>
</tr>
<tr>
<td>- Blood in arteries/veins or air in lungs</td>
<td>- Water past pillars, submarines, etc.</td>
<td></td>
</tr>
<tr>
<td>- Water in rivers or canals</td>
<td>- Polymer coating on solid surfaces</td>
<td></td>
</tr>
</tbody>
</table>

- **Driving** forces for fluid-flow include gravity, pressure differentials or gradients, temperature gradients, surface tension, electromagnetic forces, etc.

- Any fluid-flow is described by its *velocity* and *pressure* fields. The velocity vector of a fluid element can be written in terms of its three scalar components:

$$\vec{v} = u \hat{i} + v \hat{j} + w \hat{k} \quad <\text{rectangular coordinates}> \quad (1.1a)$$

or

$$\vec{v} = v_r \hat{r} + v_\theta \hat{\theta} + v_z \hat{z} \quad <\text{cylindrical coordinates}> \quad (1.1b)$$

Its total time derivative is the fluid element acceleration (see App. A):

$$\frac{d\vec{v}}{dt} \approx \frac{D\vec{v}}{Dt} = \vec{a}_{\text{total}} = \vec{a}_{\text{local}} + \vec{a}_{\text{convective}} \quad (1.2)$$

where Eq. (1.2) is also known as Stokes, material or substantial time derivative.

- **Streamlines** for the visualization of flow fields are lines to which the local velocity vectors are tangential. For example, for steady 2-D flow:
\[
\frac{dy}{dx} = \frac{v}{u}
\]  

(1.3)

where the 2-D velocity components \( \mathbf{v} = (u, v, 0) \) have to be given to obtain, after integration, the streamline equation \( y(x) \).

- Forces acting on a fluid element can be split into \textit{normal} and \textit{tangential forces} leading to pressure and normal/shear stresses. Clearly, on any surface element:

\[
p \text{ or } \tau_{\text{normal}} = \frac{F_{\text{normal}}}{A_{\text{surface}}}
\]  

(1.4)

while

\[
\tau_{\text{shear}} = \frac{F_{\text{tangential}}}{A_{\text{surface}}}
\]  

(1.5)

As Stokes postulated, the stress can be viewed as a linear derivative, i.e., \( \tau \sim \nabla \mathbf{v} \) (see App. A), where relative motion of viscous fluid elements (or layers) generate a shear stress, \( \tau_{\text{shear}} \). In contrast, the total pressure sums up the mechanical (or thermodynamic) pressure, which is experienced when moving with the fluid (and therefore labeled “static” pressure and measured with a piezometer). The dynamic pressure is due to the fluid motion (i.e., \( \rho v^2/2 \)), and the hydrostatic pressure is due to gravity (i.e., \( \rho gh \)):

\[
p_{\text{total}} = p_{\text{static}} + p_{\text{dynamic}} + p_{\text{hydro–static}}
\]

\[
= p_{\text{static}} + \frac{\rho}{2} v^2 + \rho g z = \varphi
\]  

(1.6a, b)

where

\[
p_{\text{static}} + p_{\text{dynamic}} = p_{\text{stagnation}}
\]  

(1.7)

Recall for a stagnant fluid body (i.e., a reservoir), where \( h \) is the depth coordinate:

\[
p_{\text{hydro–static}} = p_0 + \rho gh
\]  

(1.8)
Clearly, the hydrostatic pressure due to the fluid weight appears in the momentum equation as a body force per unit volume, i.e., \( p \ddot{g} \) (see Example 1.1).

- **Dimensionless groups**, i.e., ratios of forces, fluxes, process or system parameters, indicate the importance of specific transport phenomena. For example, the Reynolds number is defined as (see Example 1.1):

\[
\text{Re}_L \equiv \frac{F_{\text{inertia}}}{F_{\text{viscous}}} := \frac{vL}{\nu}
\]  

(1.9)

where \( v \) is an average system velocity, \( L \) is a representative system “length” scale (e.g., the tube diameter \( D \)), and \( \nu \equiv \mu / \rho \) is the kinematic viscosity of the fluid. Other dimensionless groups with applications in engineering include the Womersley number and Strouhal number (both dealing with oscillatory/transient flows), the Euler number (pressure difference), the Weber number (surface tension), the Stokes number (particle dynamics), Schmidt number (diffusive mass transfer), Sherwood number (convective mass transfer) and the Nusselt number, the ratio of heat conduction to heat convection. The most common source, or derivation, of these numbers is the non-dimensionalization of partial differential equations describing the transport phenomena at hand as well as scale analysis (see Example 1.1).

**Example 1.1: Generation of Dimensionless Groups**

**(A) Scale Analysis**

As outlined in Sect. 2.4, the Navier–Stokes equation (see Eq. (2.22)) describes fluid element acceleration due to several forces per unit mass, i.e.,

\[
\mathbf{a}_{\text{total}} = \frac{\partial \mathbf{\ddot{v}}}{\partial t} + (\mathbf{\ddot{v}} \cdot \nabla) \mathbf{v} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{v} + \mathbf{g}
\]

where \( \mathbf{a}_{\text{total}} \) is the total acceleration, \( \mathbf{\ddot{v}} \) is the acceleration due to inertia, \( \nabla p \) is the pressure force, \( \nu \nabla^2 \mathbf{v} \) is the viscous force, and \( \mathbf{g} \) is the gravity force.
Now, by definition:

\[
\text{Re} = \frac{\text{inertial force}}{\text{viscous force}} := \frac{(\mathbf{v} \cdot \nabla)\mathbf{v}}{\nu \nabla^2 \mathbf{v}}
\]

Employing the scales \( \mathbf{v} \sim \mathbf{v} \) and \( \nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \sim \frac{1}{L} \)

where \( \mathbf{v} \) may be an average velocity and \( L \) a system-characteristic dimension, we obtain:

\[
\text{Re} = \frac{\left( \mathbf{v} \cdot \frac{1}{L} \right) \mathbf{v}}{\nu L^{-2} \mathbf{v}} = \frac{vL}{\nu}
\]

Similarly, taking

\[
\frac{\text{local acceleration}}{\text{convective acceleration}} = \frac{\text{transient term}}{\text{inertia term}} = \frac{\partial \mathbf{v}}{\partial t} / (\mathbf{v} \cdot \nabla)\mathbf{v}
\]

we can write with system time scale \( T \) (e.g., cardiac cycle: \( T = 1 \text{s} \))

\[
\frac{v/T}{vL^{-1}v} = \frac{L}{vT} = \text{Str}
\]

which is the \textit{Strouhal number}. For example, when \( T >> 1 \), \( \text{Str} \rightarrow 0 \) and hence the process, or transport phenomenon, is quasi-steady.

\textbf{(B) Non-dimensionalization of Governing Equations}

Taking the transient boundary-layer equations (see Sect. 2.4, Eq. (2.22)) as an example,
we nondimensionalize each variable with suitable, constant reference quantities. Specifically, approach velocity $U_0$, plate length $\ell$, system time $T$, and atmospheric pressure $p_0$ are such quantities. Then,

$$
\hat{u} = \frac{u}{U_0}, \ \hat{v} = \frac{v}{U_0}; \ \hat{x} = \frac{x}{\ell}, \ \hat{y} = \frac{y}{\ell}; \ \hat{p} = \frac{p}{p_0} \text{ and } \hat{t} = \frac{t}{T}.
$$

Note: In Sect. 5.2 $\hat{y}$ is defined as $\hat{y} = y/\delta(x)$, where $\delta(x)$ is the varying boundary-layer thickness.

Inserting all variables, i.e., $u = \hat{u}U_0$, $t = \hat{t}T$, etc., into the governing equation yields

$$
\frac{\rho U_0}{T} \frac{\partial \hat{u}}{\partial \hat{t}} + \left[ \frac{\rho U_0^2}{\ell} \right] \left( \hat{u} \frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v} \frac{\partial \hat{u}}{\partial \hat{y}} \right) = \left[ \frac{p_0}{\ell} \right] \frac{\partial \hat{p}}{\partial \hat{x}} + \left[ \frac{\mu U_0}{\ell^2} \right] \frac{\partial^2 \hat{u}}{\partial \hat{y}^2}
$$

Dividing the entire equation by, say, $\left[ \frac{\rho U_0^2}{\ell} \right]$ generates:

$$
\frac{\ell}{\ell} \frac{\partial \hat{u}}{\partial \hat{t}} + \hat{u} \frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v} \frac{\partial \hat{u}}{\partial \hat{y}} = - \left[ \frac{p_0}{\rho U_0^2} \right] \frac{\partial \hat{p}}{\partial \hat{x}} + \left[ \frac{\mu}{\rho U_0 \ell} \right] \frac{\partial^2 \hat{u}}{\partial \hat{y}^2}
$$

Comments:

In a way three goals have been achieved:

- The governing equation is now dimensionless.
- The variables vary only between 0 and 1.
- The overall fluid flow behavior can be assessed by the magnitude of three groups, i.e., Str, Eu and Re numbers.
1.2 The Continuum Mechanics Assumption

Fundamental to the description of all transport phenomena are the conservation laws, concerning mass, momentum and energy, as well as their applications to continua. For example, Newton’s second law of motion holds for both molecular dynamics, i.e., interacting molecules, and continua, like air, water, plasma, and oils. Thus, solid structures and fluid flow fields are assumed to be continua as long as the local material properties can be defined as averages computed over material elements/volumes sufficiently large when compared to microscopic length scales of the solid or fluid, but small relative to the macroscopic structure. Variations in solid-structure or fluid-flow quantities can be obtained via differential equations. The continuum mechanics method is an effective tool to physically explain and mathematically describe various transport phenomena without detailed knowledge of their internal nano/micro structures. Specifically, fluids are treated as continuous media characterized by certain field quantities associated with the internal structure, such as density, temperature and velocity. In summary, continuum mechanics deals with three aspects:

- **Kinetics**, i.e., fluid element motion regardless of the cause
- **Dynamics**, i.e., the origin and impact of forces and fluxes generating fluid motion and waste heat, e.g., the stress tensor, heat flux vector, and entropy
- **Balance Principles**, i.e., the mass, momentum and energy conservation laws

Also, all flow properties are in local thermodynamic equilibrium, implying that the macroscopic quantities of the flow field can adjust swiftly to their surroundings. This local adjustment to varying conditions is rapidly achieved if the fluid has very small characteristic length and time scales of molecular collisions, when compared to the macroscopic flow variations.

However, as the channel (or tube) size, typically indicated by the hydraulic diameter $D_h$, is reduced to the micro-scale, the surface-area-to-volume ratio becomes larger because $A/V \sim D_h^{-1}$. Thus, wall surface effects may become important; for example, wall roughness
and surface forces as well as discontinuities in fluid (mainly gas) velocity and temperature relative to the wall. When flow micro-conduits are short as in micro-scale cooling devices and MEMS, nonlinear entrance effects dominate, while for long microconduits viscous heating (for liquids) or compressibility (for gases) may become a factor (see Chap. 7). In such cases, the validity of the continuum mechanics assumption may have to be re-examined.

1.3 Fluid Flow Description

Any flow field can be described at either the microscopic or the macroscopic level. The *microscopic* or molecular models consider the position, velocity, and state of every molecule of a single fluid or multiple ‘fluids’ at all times. Averaging discrete-particle information (i.e., position, velocity, and state) over a local fluid volume yields macroscopic quantities, e.g., the velocity field \( \mathbf{v}(\mathbf{x}, t) \), at any location in the flow. The advantages of the molecular approach include general applicability, i.e., no need for submodels (e.g., for the stress tensor, heat flux, turbulence, wall conditions, etc.), and an absence of numerical instabilities (e.g., due to steep flow field gradients). However, considering myriads of molecules, atoms, and nanoparticles requires enormous computer resources, and hence only simple channel or stratified flows with a finite number of interacting molecules (assumed to be solid spheres) can be presently analyzed. For example, in a 1-mm cube there are about 34 billion water molecules (about a million air molecules at STP), which make molecular dynamics simulation prohibitive, but on the other hand, intuitively validates the continuum assumption (see Sect. 1.2).

Here, the overall goal is to find and analyze the interactions between *fluid forces*, e.g., pressure, gravity/buoyancy, drag/friction, inertia, etc., and *fluid motion*, i.e., the velocity vector field and pressure distribution from which everything else can be directly obtained or derived (see Fig. 1.2a, b). In turn, scalar transport equations, i.e., convection mass and heat transfer, can be solved based on the velocity field to obtain critical magnitudes and gradients (or fluxes) of species concentrations and temperatures.

In summary, *unbalanced surface/body forces and gradients cause motion in form of fluid translation, rotation, and/or deformation,*
while temperature or concentration gradients cause mainly heat or species-mass transfer. Note that flow visualization CDs plus web-based university sources provide fascinating videos of complex fluid flow, temperature and species concentration fields.

**(a) Cause-and-effect dynamics:**

\[
\begin{bmatrix}
\text{FORCES or} \\
\text{GRADIENTS}
\end{bmatrix} \quad \begin{cases}
\text{MOTION:} \\
\quad \text{Translation} \\
\quad \text{Deformation} \\
\quad \text{Rotation}
\end{cases}
\]

**(b) Kinematics of a 2-D fluid element (Lagrangian frame):**

\[
\begin{align*}
\text{At time } t + \Delta t: \\
P' \quad \text{Rotation} \quad \text{Deformation}
\end{align*}
\]

\[
\begin{align*}
\text{At time } t: \\
P \quad \text{Translation} \quad r(t) \\
r(t + \Delta t) \\
\end{align*}
\]

Notes:
- Translation \( \mathbf{\dot{v}}, \mathbf{a} \)
- Rotation \( \omega, \zeta \sim \nabla \times \mathbf{v} \)
- Deformation \( \varepsilon, \gamma \sim \nabla \mathbf{v} \)

*Fig. 1.2* Dynamics and kinematics of fluid flow: (a) force-motion interactions; and (b) 2-D fluid kinematics

Exact flow problem identification, especially in industrial settings, is one of the more important and sometimes the most difficult first task. After obtaining some basic information and reliable data, it helps to think and speculate about the physics of the fluid flow, asking: