Engineering Against Fracture
Engineering Against Fracture

Proceedings of the 1st Conference
Preface

Within the last thirty years there is a growing acknowledgement that prevention of catastrophic failures necessitates engagement of a large pool of expertise. Herein it is not excessive to seek advice from disciplines like materials science, structural engineering, mathematics, physics, reliability engineering and even economics.

Today’s engineering goals, independently of size; do not have the luxury of being outside a global perspective. Survival of the integrated markets and financial systems require a web of safe transportation, energy production and product manufacturing.

It is perhaps the first decade in engineering history that multidisciplinary approaching is not just an idea that needs to materialise but has matured beyond infancy. We can witness such transition by examining engineering job descriptions and postgraduate curricula.

The undertaking of organising a conference to reflect the above was not easy and definitely, not something that was brought to life without a lot of work and commitment. The 1st Conference of Engineering Against Fracture from its conceptual day until completion was designed in a way of underlying the need of bringing all the key players on a common ground that once properly cultivated can flourish. To achieve that the conference themes were numerous and despite their, in principle notional differences, it was apparent that the attendees established such common ground through argumentation. The reader can see this from the variety of research areas reflected by the works and keynote lecturers presented.

A booster to our endeavour was definitely the fact that the conference was also organised to honour the retirement of Prof. Theodore Kermanidis. A figure, which can easily receive the title of the forefather of our laboratory and department.

The editors of these proceedings are in debt to several people including the local and international organising committee and the postgraduate students of the Laboratory of Technology and Strength of Materials, University of Patras. Special thanks should be given to two very special ladies, E. Sotiropoulou and A. Koutsouliakou who without them the conference would have never been a success.

We would also like to acknowledge the help provided by the European Aeronautics Science Network in supporting the organisation of the conference and our sponsors the Technical Chamber of Greece, The University of Patras Research
Committee, the Office for Naval Research Global, the Ministry of Education and Religious Affairs of Greece and Set Point Technologies for financial support.

Special thanks should be given to Ms. Anneke Pot and Ms. Nathalie Jacobs from Springer for their patience and advice in publishing these proceedings.

Patras, Greece
May, 2008

Spiros Pantelakis
Chris Rodopoulos
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Ball-Burnishing and Roller-Burnishing to Improve Fatigue Performance of Structural Alloys

Lothar Wagner, Tomasz Ludian, and Manfred Wollmann

Abstract The HCF response to burnishing of a number of structural materials is compared and contrasted. It is shown that alloys which exhibit marked work-hardening during burnishing respond very beneficially with regard to HCF performance while others which show little work-hardening may even react with losses in HCF strength. Possible explanations for such behavior are outlined in terms of mean stress and environmental sensitivities of the fatigue strengths of the various materials and microstructures.

Keywords Ball-burnishing · Roller-burnishing · Work-hardening · Residual stresses · Fatigue crack nucleation · Fatigue crack propagation

1 Introduction

Mechanical surface treatments such as shot peening and burnishing are often applied to structural metallic materials to improve their HCF performance [1–4]. In general, this improvement can be derived from two contributing factors, namely surface strengthening by the induced high dislocation densities and residual compressive stresses. As opposed to shot peening, burnishing typically results in very smooth surfaces [5, 6]. The influence of burnishing on fatigue life is schematically described in Table 1.

While surface strengthening is able to enhance the resistance to fatigue crack nucleation, micro-crack propagation resistances are detrimentally affected owing to low residual ductility in the cold worked and strengthened surface layer. On the other hand, there is experimental evidence that residual compressive stresses can
Table 1  Effects of surface layer properties on the various stages on fatigue life (schematic)

<table>
<thead>
<tr>
<th>Surface layer properties</th>
<th>Crack nucleation</th>
<th>Micro-crack propagation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cold work</td>
<td>Retards</td>
<td>Accelerates</td>
</tr>
<tr>
<td>Residual compressive stresses</td>
<td>Minor effect</td>
<td>Retards</td>
</tr>
</tbody>
</table>

Table 2  Tensile properties of the various light alloys

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_{0.2}$ (MPa)</th>
<th>UTS (MPa)</th>
<th>UTS/$\sigma_{0.2}$</th>
<th>EL (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CuZn30</td>
<td>125</td>
<td>310</td>
<td>2.48</td>
<td>67</td>
</tr>
<tr>
<td>AISI 304 (1.4301)</td>
<td>270</td>
<td>660</td>
<td>2.44</td>
<td>82</td>
</tr>
<tr>
<td>AZ80</td>
<td>235</td>
<td>340</td>
<td>1.45</td>
<td>12</td>
</tr>
<tr>
<td>Ti-8.6Al, 10h 500°C</td>
<td>830</td>
<td>900</td>
<td>1.08</td>
<td>5</td>
</tr>
<tr>
<td>Ti-6Al-4 V, EQ, 24h 500°C</td>
<td>1,060</td>
<td>1,120</td>
<td>1.06</td>
<td>15</td>
</tr>
<tr>
<td>Ti-6Al-7Nb, D20/WQ, 24h 500°C</td>
<td>1,030</td>
<td>1,120</td>
<td>1.09</td>
<td>15</td>
</tr>
<tr>
<td>Ti-6Al-7Nb, D20/AC, 24h 500°C</td>
<td>920</td>
<td>995</td>
<td>1.08</td>
<td>14</td>
</tr>
<tr>
<td>Beta C, SHT</td>
<td>840</td>
<td>850</td>
<td>1.01</td>
<td>25</td>
</tr>
<tr>
<td>LCB, PS +0.5h 500°C</td>
<td>1,665</td>
<td>1,730</td>
<td>1.04</td>
<td>3</td>
</tr>
</tbody>
</table>

dramatically reduce the growth rate of surface microcracks while crack nucleation resistances are less affected [7–13]. Recently, it was shown that residual tensile stresses which must necessarily be present underneath the near-surface residual compressive stress field, can lead to subsurface fatigue crack nucleation and to losses in HCF strength relative to an electropolished reference [14, 15]. The present investigation is intended to highlight differences in the fatigue response of the various alloys to burnishing.

2  Experimental

The investigation was performed on a number of alloys which tensile properties are listed in Table 2.

Hour glass-shaped fatigue specimens were prepared with a minimum gage diameter of either Ø 4 or Ø 6 mm. In addition, specimens with a circumferential V60° notch and a notch root radius of 0.43 mm ($k_t = 2.5$) were machined (Fig. 1).

Ball-burnishing of the smooth specimens was performed by means of a conventional lathe using a device by which hard metal balls of Ø 3 or Ø 6 mm are hydrostatically pressed onto the rotating specimen surface. Roller burnishing of the notched specimens was performed by a roller element having V55° geometry and 0.35 mm tip radius. Due to roller-burnishing, the notch factor $k_t$ increased from 2.3 to 2.7. Electrolytically polished smooth and notched specimens were prepared to serve as reference. Electropolishing of the notched specimens decreased the
notch factor from 2.5 to 2.3. The exact \( k_t \) value was calculated for each specimen individually. Microhardness-depth profiles and residual stress-depth profiles as determined by the incremental hole drilling method were measured to characterize the process-induced changes in surface layer properties [16]. Fatigue tests were performed in rotating beam loading \( (R = -1) \) at frequencies of about \( 50 \text{s}^{-1} \). In addition, axial fatigue tests were done at stress ratios of \( R = -1 \) and \( R = 0.1 \) at frequencies of about \( 90 \text{s}^{-1} \). Some axial fatigue tests at \( R = -1 \) were performed also in vacuum.

### 3 Results and Discussion

Examples of the HCF performance of smooth and notched \( (k_t = 2.3) \) electropolished references are illustrated in Figs. 2 and 3. In addition to nominal stress amplitude \( \sigma_a \), the maximum stress amplitude at the notch root \( \sigma_a k_t \) of the notched specimens is plotted. On both AISI 304 (Fig. 2a) and CuZn30 (Fig. 2b), the notch factor \( k_t = 2.3 \) only slightly reduces HCF performance of the smooth \( (k_t = 1.0) \) specimens. Thus, the HCF strengths of the notched specimens in terms of \( \sigma_a k_t \) are much higher than those of the smooth specimens. On the contrary, results on notched specimens of various magnesium and titanium alloys have demonstrated marked losses in notched HCF performance relative to smooth specimens [17, 18]. Examples on the magnesium alloy AZ80 and the titanium alloy Ti-6Al-4V are illustrated in Fig. 3. Interestingly, the notched HCF strengths of both alloys AZ80 (Fig. 3a) and Ti-6Al-4V (Fig. 3b) in terms of \( \sigma_a k_t \) are the same as those of the smooth conditions.

The effect of notches on the performance of the various alloys is summarized in Table 3 where the fatigue notch sensitivity is expressed as:

\[
q = \frac{k_f - 1}{k_t - 1}, \quad 0 \leq q \leq 1 \quad \text{and} \quad k_f = \frac{\sigma_{a10^7\text{smooth}}}{\sigma_{a10^7\text{notched}}}. 
\]
Obviously, with the exception of CuZn30 and AISI 304, all other tested alloys are fully (100%) notch sensitive with regard to HCF performance. As seen from Table 1, both CuZn30 and AISI 304 exhibit marked work-hardening capabilities ($\text{UTS}/\sigma_{0.2} \geq 2$) which may explain their very low fatigue notch sensitivities.
Ball-Burnishing and Roller-Burnishing to Improve Fatigue Performance

Table 3  Notch factor $k_t$, fatigue notch factor $k_f$ and fatigue notch sensitivity $q$ of the various alloys (condition EP)

<table>
<thead>
<tr>
<th></th>
<th>$k_t$</th>
<th>$k_f$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CuZn30</td>
<td>2.3</td>
<td>1.28</td>
<td>0.25</td>
</tr>
<tr>
<td>AISI 304</td>
<td>2.3</td>
<td>1.26</td>
<td>0.20</td>
</tr>
<tr>
<td>AZ80</td>
<td>2.3</td>
<td>2.3</td>
<td>1</td>
</tr>
<tr>
<td>Ti-6Al-4 V EQ, 24 h 500°C</td>
<td>2.3</td>
<td>2.3</td>
<td>1</td>
</tr>
<tr>
<td>Beta C, SHT</td>
<td>2.3</td>
<td>2.3</td>
<td>1</td>
</tr>
<tr>
<td>LCB, PS +0.5 h 500°C</td>
<td>2.3</td>
<td>2.3</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 4  Compressive residual-stress depth profile after ball-burnishing (BB) and shot peening (SP) in Beta C, SHT

An example of the compressive residual-stress depth profile after ball-burnishing of Beta C is illustrated in Fig. 4 where in addition, results after shot peening are also shown.

Ball-burnishing as opposed to shot peening led to deeper penetration depths of residual stresses, however, maximum residual stresses after shot peening were higher than after ball-burnishing.

Both alloys AISI 304 and AZ80 respond very beneficially to ball burnishing (Fig. 5). The enhancements in HCF performance are even superior to those typically observed after optimum shot peening. Similar to results on shot peened HCF specimens, subsurface fatigue crack nucleation was also observed in ball-burnished alloys.
HCF specimens while all electropolished specimens failed by surface crack nucleation. Examples of fatigue crack nucleation sites are illustrated in Fig. 6.

From axial HCF tests on ball-burnished specimens it is known that the depths of crack nucleation sites are about the same for a given burnishing pressure. Similar results were reported on shot peened conditions where the depths of subsurface crack nucleation were directly related to the Almen intensity applied. These results clearly indicate that the residual tensile stresses balancing the near-surface compressive stress field (Fig. 4) are not homogeneously distributed over the remaining cross-section [19, 20]. Instead, residual tensile peak stresses must exist at some distances below the compressive stress field. This situation is schematically demonstrated in Fig. 7.

Due to the presence of process-induced residual tensile stresses (Fig. 7), materials with high tensile mean stress sensitivities may respond quite critically to ball-burnishing or shot peening. This can be derived from results on shot peened conditions of Ti-6Al-7Nb [21]. Materials with normal mean stress sensitivity in fatigue as observed on water-quenched duplex microstructures (Fig. 8) also exhibit significant improvements in the HCF performance due to shot peening (Fig. 9). The

![Fig. 6](image)  
**Fig. 6** HCF crack nucleation sites in Ti-6Al-4 V

![Fig. 7](image)  
**Fig. 7** Residual stress-depth profile after ball-burnishing (schematic)
latter result is explained by the fact that in these microstructures, the stress amplitude at the endurance limit is hardly reduced by tensile mean stresses.

On the contrary, materials with anomalous mean stress sensitivity in fatigue as found on air cooled duplex microstructures of the same alloy (Fig. 10) exhibit hardly any improvement in HCF performance after shot peening (Fig. 11). The latter is explained by the fact that in these microstructures, an increase in tensile mean stress does not increase the maximum stress at endurance (Fig. 10), i.e., the corresponding stress amplitude at low tensile mean stresses is markedly reduced. Similar results are reported on duplex microstructures in Ti-6Al-4 V [22]. From optical microscopy under polarized light [23] and more recently from EBSD measurements [24] it can be
argued that these differences in mean stress sensitivity of (α + β) alloys are related to micro-textures being present in air-cooled conditions.

Since subsurface cracks must nucleate under quasi-vacuum conditions, the environmental sensitivity must be taken into account in order to understand the air HCF performance after ball-burnishing or shot peening. Examples of fatigue performance in air and vacuum of two contrasting alloys are shown in Fig. 12. As seen in Fig. 12a, the HCF strength of Ti-8.6Al in vacuum is much higher than the corresponding value in air this indicating high environmental sensitivity of this age-hardened α-alloy. Similar high environmental sensitivities were observed in (α + β)-titanium alloys such as Ti-6Al-4 V and Ti-6Al-7Nb.

As opposed to α- and (α + β)-titanium alloys, the HCF strengths of metastable β-titanium alloys such as Beta C or TIMETAL LCB are hardly affected by the air environment as shown in Fig. 12b. This result may be related to the absence of hydrogen embrittlement of the bcc β-phase.

Because of the absence of environmental effects, subsurface quasi-vacuum fatigue crack nucleation in titanium alloys belonging to the metastable β-alloy class is rather easy. Thus, both ball-burnishing and shot peening can lead to marked losses in HCF strength relative to the electropolished reference as demonstrated on TIMETAL LCB and Beta C in Fig. 13.
TIMETAL LCB (PS, 0.5 500°C)  Beta C (SHT)

Fig. 13  S-N curves of metastable β-titanium alloys in rotating beam loading

AISI 304  Beta C (SHT)

Fig. 14  S-N curves of the conditions BB and RB in rotating beam loading (R = −1) in air

How much the HCF strength in these alloys decreases was shown to depend on the degree of aging as well as on the intensities used in shot peening and burnishing. The influence of roller-burnishing on notched fatigue performance is illustrated in Fig. 14 where results on AISI 304 (Fig. 14a) are compared with those on Beta C (Fig. 14b). Unlike the effect of notches in the electropolished references (Figs. 2 and 3), roller-burnishing of notched HCF specimens [25] led to enhancements which indicate a full compensation of the notch factor k_t (compare Fig. 14 with Figs. 2 and 3).
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Shot Peening in High Strength Aluminium Alloys, Shot Peening (L. Wagner, ed.) Wiley-VCH, ICSP8, Garmisch-Partenkirchen, Germany, 547.
Dual Scale Fatigue Crack Monitoring Scheme Considering Random Material, Geometric and Load Characteristics

G.C. Sih

Abstract  Effectiveness of the existing fracture control methodology depends on the non-destructive evaluation of defects arising from repeated service of structures. Damage detection is by far being automatic since not all defects are dangerous. The correct diagnostics require not only a knowledge of the size of the prevailing defects but also their likely locations of occurrence from the initial design and/or past experience. What is not known is the time when the defects will become critical since the service conditions change as a rule and the material age in time. These inherent variables can affect the accuracy of non-destructive evaluation which in its strict sense implies certain cut-off scale of small defects that are assumed to be not harmful. Material microstructural effects have been known to affect the structural integrity. The obvious implication is multiscaling, the least consideration of which involves the dual scale of micro/macro. Hence, defect monitoring presents overwhelming difficulties because of the entanglement of multi-variables.

Starting with the basic quantities of the local force and displacement that can be detected from the commercially available transducers, continuous records can be made available for the local compliance \( C \) and its time rate history, say \( dC/dt \) or \( dC/dN \) in fatigue with \( N \) being the number of cycles. These data can be stored in microprocessors and made available for analyses. The conversion of the force and displacement records to damage by fatigue crack growth is the challenge of this work. Appeal is made to a dual scale micro/macro fatigue crack growth model that has the capability to delineate micro- and macro-cracking. The model makes use of three parameters \( \mu^*, d^* \) and \( \sigma^* \). They account for the interaction between the micro/macro effects in terms of the, respective, relative shear modulus \( \mu_{\text{micro}}/\mu_{\text{macro}} \), the micro-tip characteristic length \( d/d_0 \) and the crack surface tightness ratio \( \sigma_0/\sigma_\infty \) that controls the opening of the fatigue crack. These parameters are assumed to vary in a random fashion as the crack size increase with the repetition of loading.

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Random normal distribution of statistical values of $\mu^*$, $d^*$ and $\sigma^*$ is investigated for different mean $<\mu>$ and standard deviation $<\sigma>$. The amount of dispersion around the mean is examined for the crack length $a$ and crack growth rate $da/dN$ in addition to the compliance $C$ and its rate $dC/dN$. It is demonstrated that randomness of the material microstructure has an effect on the local compliance which will increase with crack growth in a stable fashion and then rises more quickly with the number of cycles. The characteristic is reminiscent of the compliance to crack length relation used for determining the energy release rate in fracture mechanics.

The ability to account for micro/macro damage transition is considered to be fundamental for assessing the meaning of the compliance data. Randomness of the material microstructure parameters provides the means for studying the dispersion of the results in terms of fatigue crack growth. The preliminary findings are encouraging and suggest several new directions of research for health monitoring.

**Keywords** Micro/macro · Fatigue · Cracking · Random properties · Microstructure · Scale transition · Compliance · Health monitoring · Crack growth

## 1 Introduction

All structures are vulnerable to damage by aging or natural disasters such as earthquakes or tornadoes. Effective methods for inspection and detection of damage are needed to determine whether the structures are worthy of repair and restoration. When the decision entails the loss of life and property such as the collapse of tall buildings or the fracture of bridges, there is the relentless pressure to develop failure control methodologies that can minimize premature failure. It is now a recognized fact that the existing hardware of sensors and transducers must be implemented with the appropriate software. Threshold criterion is needed for developing damage detection models that can account for cumulative effects in terms of measurable parameters with physical meaning. Monitored signal must be supplemented with the correct diagnostic decision.

The work to follow will center on structural damage by fatigue. Accumulation of energy in a unit volume of material that is represented by the sum of the areas enclosed by the hystereses loops [1, 2]. These loops are formed by reversing the cyclic load on a uniaxial test specimen. The size and shape of each loop depend on the previous history. This is in fact the volume energy density function $dW/dV$, the sum of which when expressed in terms of their mean, can be related directly to the energy density range $\Delta S$ at an incremental distance $\Delta r$ from the location of failure or crack initiation. That is $\Delta r/\Delta N = (1/A)\Delta S$ [3, 4] where $A$ stands for the mean area of the hystereses loops. The more familiar form used in fatigue studies is $da/dN = (1/A)\Delta S$, where the crack length has replaced $r$ while $N$ is the number of cycles. A variety of related forms of $da/dN$ can be found since $\Delta S$ can be expressed in terms of parameters that depend on the material microstructure, crack geometry, loading and other factors. Arbitrariness of the form of $da/dN$ will be limited by starting from $da/dN = (1/A)\Delta S$ or the equivalent. While the use of the crack length
parameter appears to be a restriction, it can adopt a much broader physical meaning such as the mean characteristic length of a cluster of microcracks. What should be kept in mind is that the final structure instability is dominated by one or very few large cracks. Not all of the cracks are important. Many of them are in fact beneficial as they are self-arresting once the local stress concentrations are relieved. The spirit of fracture mechanics is to trace back from these few damaging cracks to the conditions at a lower scale level so as to determine the probabilities that would favor certain inhomogeneous microscopic conditions to trigger the formation of macrocracks. Not all cracks have the same probability to cause global instability. A sub-set of $\Delta S$ would be $\Delta K$ which has been used conventionally for treating cracks that propagate nearly straight.

Random normal distribution of the micro/macro material, geometric and loading parameters are invoked for the appropriate mean $<\mu>$ and the standard deviation $<\sigma>$. The former pertains to the average values of the variables $\mu^*(=\mu_{\text{micro}}/\mu_{\text{macro}})$, $d^*(=d/d_0)$ and $\sigma^*(=\sigma_0/\sigma_\infty)$ while the latter specifies the amount of dispersion around the mean. Simulated are the randomness of the internal material microstructure that in term would cause the microcrack tip to wonder irrespective of the macroscopic load symmetry or skew–symmetry effects. Such a behavior would show up as dispersion of the crack length versus number of cycle relation and transfer to the variation of the compliance as a function of the fatigue cycle. As expected, the dispersion is scale sensitive. It is also indicative of the distance from the crack tip. This is precisely the type of information required in monitoring physical damage.

Detection of crack-like defects should be distinguished from others since crack growth as a rule is accompanied by the release of other forms of energy at the different scale. Acoustic emission when tuned for detecting the nucleation of microcracks is not suitable for finding macrocracks or defect size smaller than the acoustic wave length. Electromagnetic waves are used to detect even smaller inhomogeneities. An a priori knowledge of the scale size with reference to the prevailing non-uniformity is therefore necessary. For large structures, the displacement gage is the basic measurement device for recording the change of distance between two points that may not indicate the presence of surface separation such as that of a crack unless the gages are placed very close to the crack mouth. In practice, the major effort is to find the location of the crack. Conversion of displacement and force history data to local compliance or stiffness can provide surveillance to a larger area but also indicate the tendency of cracks to reach instability. In essence, it is the change of displacement that is calibrated to measure force. This work carries one step further to the compliance, the change of which with the crack length has been recognized as the fundamental parameter for characterizing fracture [5].

2 Micro/Macro Fatigue Crack Growth Model

A modified version of the conventional macrocrack growth model for fatigue has been extended [6, 7] to account for the transition of cracking damage from the micro to macro or vice versa by replacing the $\Delta K$ for a macrocrack by $\Delta K_{\text{macro}}$ for
micro-/macro-cracking. Unless the cracks take sharp turns, it is adequate to use \( \Delta K_{\text{micro}}^{\text{macro}} \) instead of the energy density range \( \Delta S_{\text{micro}}^{\text{macro}} \). The dual scale fatigue model for cracks growing nearly straight can be written as

\[
\frac{da}{dN} = M_o (\Delta K_{\text{micro}}^{\text{macro}})^m
\]  

(1)

where \( M_o \) and \( m \) are assumed to be constants. Although Eq. (1) was not arrived by fundamental means, the procedure for deriving a differential equation to solve the crack length will be the same. For this purpose, it suffices to start with [6, 7]

\[
\Delta K_{\text{macro}} = \frac{6\sqrt{\pi}(1 - \nu_{\text{macro}})\mu_{\text{micro}}}{5d^{0.25}\mu_{\text{macro}}} \sqrt{c^2 - a^2} \left(1 - \frac{2\sigma_o}{\pi\sigma_\infty} \sin^{-1} \frac{a}{c}\right)(\sigma_{\text{max}} - \sigma_{\text{min}})
\]

(2)

The notations for \( d = c - a \) can be found in Fig. 1 where symmetry is assumed to prevail across the \( x_2 \)-axis. The cyclic restraining stress induced by the oscillating fatigue load \( \sigma_\infty(N) \) away from the crack is denoted by \( \sigma_o(N) \) as shown in Fig. 1. Anti-symmetry of the condition across the microcrack tip is guaranteed by the mixed boundary condition invoked as an expediency. The motivation for such a representation can be found partly in [8] and is beyond the scope of this discussion. In view of the fact that \( a \approx d \), an equivalent but approximate form of Eq. (2) can be used:

\[
\Delta K_{\text{macro}} = \frac{6\sqrt{\pi}(1 - \nu_{\text{macro}})\mu_{\text{micro}}}{5\mu_{\text{macro}}} d^{0.25} \sqrt{c + d} \left(1 - \frac{2\sigma_o}{\pi\sigma_\infty} \sin^{-1} \frac{a}{c}\right)(\sigma_{\text{max}} - \sigma_{\text{min}})
\]

(3)

The relation \( \sigma_{\text{max}} - \sigma_{\text{min}} = 2\sigma_a \) can be further applied to yield

\[
\Delta K_{\text{macro}} = \frac{12\sqrt{2}\pi(1 - \nu_{\text{macro}})\sigma_a}{5} \mu_{\text{micro}} d^{0.25} \left(1 - \frac{\sigma_o}{\sigma_\infty}\right)
\]

(4)

Take \( \nu_{\text{macro}} = 0.3 \). Equation (4) becomes

\[
\Delta K_{\text{macro}} \approx 4\sigma_a \sqrt{a} \frac{\mu_{\text{micro}}}{\mu_{\text{macro}}} d^{0.25} \left(1 - \frac{\sigma_o}{\sigma_\infty}\right)
\]

(5)

Fig. 1 Micro/macro crack with one-half symmetry
From the definition of the relations

\[ d^* = \frac{d}{d_o}, \mu^* = \frac{\mu_{\text{micro}}}{\mu_{\text{macro}}}, \sigma^* = \frac{\sigma_o}{\sigma_\infty} \] (6)

there results

\[ \Delta K_{\text{macro}}^{\text{micro}} \approx 4d_o^{0.25} \sigma_o \sqrt{a(N)} \mu^*(N) [d^*(N)]^{0.25} [1 - \sigma^*(N)] \] (7)

Note that Eq. (7) has the \( \sqrt{a} \) dependency. It can be substituted into Eq. (1) to obtain the governing differential equation solving for the half crack length \( a \). Since \( M_o \) and \( m \) are assumed to be constants which vary with the material, they will not affect the integration for finding \( a = a(N) \).

3 Determination of Crack Growth History

Equation (7) is unique; it reveals the interaction of the material microstructure, the microcrack tip reaction and tightness of the crack mouth due to the relative restrain of the material with reference to the applied load. The relation \( a = a(N) \) can be obtained by putting Eq. (7) into Eq. (1) to yield the governing differential equation:

\[ \frac{da}{dN} = M_o \left\{ 4d_o^{0.25} \sigma_o \sqrt{a(N)} \mu^*(N) [d^*(N)]^{0.25} [1 - \sigma^*(N)] \right\}^m \] (8)

Define a new parameter

\[ M = M_o \left[ 4d_o^{0.25} \sigma_o \right]^m \] (9)

such that Eq. (8) becomes

\[ \frac{da}{dN} = M \left\{ \mu^*(N) [d^*(N)]^{0.25} [1 - \sigma^*(N)] \sqrt{a(N)} \right\}^m \] (10)

The solution of Eq. (10) for \( a = a(N) \) depends on the value of \( m \). Numerical integration of Eq. (10) can always be made.

As an illustration, let \( m = 2 \) and \( \sigma^* \), \( d^* \) and \( \mu^* \) to be fixed for each \( N \) so that Eq. (10) can be integrated for arbitrarily small segments and written as

\[ \frac{da}{dN} = M (k_o)^2 a(N) = k a(N), \text{ for } m = 2 \] (11)

such that

\[ M = 16M_o (\sigma_o)^2 \sqrt{d_o} \] (12)

The contractions

\[ k = Mk_o^2 \text{ and } k_o = \mu^*(d^*)^{0.25} (1 - \sigma^*) \] (13)
have been made in Eq. (11) which can be integrated to render
\[
a(N) = a_0 e^{k(N-N_0)}, \text{ for } m = 2
\]
where \(a_0\) and \(N_0\) are the initial crack half length and corresponding number of cycles elapsed initially. The range of \(k_0\) or \(k\) in Eq. (13) can be established by assigning a material with known value of \(M_0\) since \(M\) can be obtained from the y-intercept extending from the straight line portion of the sigmoidal curve of \(da/dN\). Numerical values of Eq. (11) can thus be found.

In general, \(m\) may differ from 2. Equation (10) takes the form
\[
\frac{da}{dN} = M(k_0 \sqrt{a})^m
\]
Integration gives
\[
N - N_0 = \frac{2a_0}{(m-2)M(k_0 \sqrt{a_0})^m}[1 - \left(\frac{a_0}{a}\right)^{\frac{m}{2}} - 1], \ m \neq 2
\]
Solving for the half crack length, the results is
\[
a = \left[a_0^{\frac{1-m}{2}} + (1 - \frac{m}{2})Mk_0^m(N-N_0)\right]^{\frac{1}{1-\frac{m}{2}}}, \ m \neq 2
\]
Keep in mind that \(M\) depends also on the stress amplitude as shown by Eq. (9). The parameter \(k_0\) has been defined by the second of Eqs. (13). The oscillatory character of \(k_0\) depending on the randomness of the micro/macro parameters governing the stiffness, microcrack tip, restraining character of the material will be discussed subsequently.

4 Physical Model of Random Material Properties

For small cracks that are comparable with the characteristic length of the material microstructure, it is necessary to consider the inhomogeneity of the material since it will influence the crack growth behavior.

4.1 Generation of Random Numbers

The parameters \(k_0\) in the second of Eqs. (13) will be assumed to vary randomly with the number of fatigue cycles such that \(\sigma^* = \sigma^*(N), \mu^* = \mu^*(N)\) and \(d^* = d^*(N)\) take discrete values for different \(N\). Figure 2a–c show, respectively, the randomly
generated values of $\sigma^*$, $\mu^*$ and $d^*$ with the identical probability in their ranges [9] which are known from previous work [6, 7]. For example, consider $s^* = 0$, $d^* = 1$ and $\mu^* = 1 \sim 5$. Hence, the second of Eqs. (13) gives $1 \leq k_o \leq 5$. Moreover, $\sigma^* = 0.9$, $d^* = 10$ and $\mu^* = 1 \sim 5$ yield $0.178 \leq k_o \leq 0.889$ which establishes the range for generating the random values of $\sigma^*$, $\mu^*$ and $d^*$ in Fig. 2a–c.

Since the material will degrade with time or number of cycle $N$, it is anticipated that the range for $\sigma^*$, $\mu^*$ and $d^*$ will not be constant with $N$ but it is expected to decrease with $N$. The second of Eqs. (13) shows that $k_o$ will also decrease with $N$. Over a range of about two million cycles, six segments of $k_o$ can be considered such that each successive segment has decreasing amplitude as given in Table 1. Displayed in Fig. 3 are the oscillatory character of $k_o$ with $N$. Randomness variations of $\sigma^*$, $\mu^*$ and $d^*$ or $k_o$ may not have same probabilities. This requires the consideration of random normal distribution with different mean $<\mu>$ and standard deviation $<\sigma>$ in each of the six segments.

![Fig. 2](image)

Randomly generated values [9] for micro/macro stiffness, geometric and material restraining stress

<table>
<thead>
<tr>
<th>Segment no.</th>
<th>Range of $N \times 10^6$</th>
<th>Range of $k_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.4 ~ 1.45</td>
<td>2.1 ~ 4.1</td>
</tr>
<tr>
<td>2</td>
<td>1.45 ~ 1.5</td>
<td>2.3 ~ 3.8</td>
</tr>
<tr>
<td>3</td>
<td>1.5 ~ 1.6</td>
<td>2.5 ~ 3.2</td>
</tr>
<tr>
<td>4</td>
<td>1.6 ~ 1.7</td>
<td>2.8 ~ 3.1</td>
</tr>
<tr>
<td>5</td>
<td>1.7 ~ 1.9</td>
<td>2.9 ~ 3.1</td>
</tr>
<tr>
<td>6</td>
<td>1.9 ~ 2.0</td>
<td>2.95 ~ 3.05</td>
</tr>
</tbody>
</table>
Table 2  Segmented range of N for normal distribution with different $\mu$ and $\sigma$

<table>
<thead>
<tr>
<th>Segment no.</th>
<th>Range of $N \times 10^6$</th>
<th>Mean $\mu$</th>
<th>Standard deviation $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.4 $\sim$ 1.45</td>
<td>3.0</td>
<td>0.8</td>
</tr>
<tr>
<td>2</td>
<td>1.45 $\sim$ 1.5</td>
<td>3.0</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>1.5 $\sim$ 1.6</td>
<td>3.0</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>1.6 $\sim$ 1.7</td>
<td>3.0</td>
<td>0.09</td>
</tr>
<tr>
<td>5</td>
<td>1.7 $\sim$ 1.9</td>
<td>3.0</td>
<td>0.05</td>
</tr>
<tr>
<td>6</td>
<td>1.9 $\sim$ 2.0</td>
<td>3.0</td>
<td>0.03</td>
</tr>
</tbody>
</table>

4.2  Normal Distribution with Mean $\mu$ and Standard Deviation $\sigma$

In order for the randomness of $\sigma^*$, $\mu^*$, and $d^*$ to have different probabilities, different mean $\mu$ and standard deviation $\sigma$ will be selected for the six segments as $N$ is increased while the range of the amplitude of $k_0$ will be kept as those in Table 1. To be specific, a fatigue life of two million cycles or $N = 2 \times 10^6$ will be considered. The crack initiation period is taken as 80% of the total fatigue life with the crack growth being the remaining 20% of the total life. For the purpose of discussion, numerical results will be found for $a_0 = 5$ mm, $N_0 = 1.4 \times 10^6$ while different $m$ and $M$ values may be considered.

Making use of the $\mu$ and $\sigma$ in Table 2, another set of $\sigma^*$, $\mu^*$, and $d^*$ will be found [9] with the same limits as those in Fig. 2a–c but their oscillations will differ. From the second of Eqs. (13), $k_0$ is also found to fluctuate with the mean and standard deviation in Table 2. The results are displayed in Fig. 4. The difference of the oscillations in Figs. 3 and 4 is obvious. The assumed trend of the material degradation combined with the geometric and crack tightness properties represented by $k_0$ is such that the amplitude of variations become decreasingly small in a random fashion as the fatigue life is approached. The specifics can be adjusted by altering the
limits of $\sigma^*$, $\mu^*$ and $d^*$ and the values of $<\mu>$ and $<\sigma>$. Much of this knowledge will depend on the interaction of the fatigue crack with the governing parameters that will change during the life of the structure.

4.3 Change of Local Compliance with Fatigue Crack Growth

Compliance or its reciprocal the stiffness reflects the integrity of a structure. The change of compliance is indicative of the presence of damage and/or deterioration of the material properties. The change of the local compliance $C$ in a specimen can be identified with the growth of a crack-like defect, the behavior of which has been used to develop the fracture control methodology for the inspection of aircrafts. The monolithic nature of the aircraft structure raises the sensitivity of the changes in the relation between $dC$ and $da$ where $a$ denotes the crack length. In fact, the rate $dC/da$ is known to increase progressively until it reaches a critical value when the onset of rapid fracture takes place under stress controlled situations. Strain or displacement controlled conditions are usually invoked in design to avoid catastrophic failure. The addition of redundant members in civil engineering structures is made for this purpose.

Consider a representative block of material that contains damage in the form of a line-crack of length $2a$. The particular shape of the defect or configuration of damage can be reflected by the $dC/da$ versus time or load cycle $N$ relation. Moreover, there is no loss in generality by assuming a line crack $2a$ for the sake of illustration. The object is therefore to seek the relation between $C = C(N)$ and $a = a(N)$ or $dC/da$ as a function of $N$ to $da/dN$. The local displacement $u = u(N)$ and force $P = P(N)$ are presumably monitored to reflect the effect of the crack. For a block of width $b$, height $h$ and thickness $B$, a relation between the equivalent strain $u/h$ and equivalent stress $P/bB$ can be written [10] as

$$\frac{u}{h} = \frac{1 - \nu^2_{macro}}{E_{macro}} \left(1 + \frac{2\pi a^2}{A_o}\right) \frac{P}{bB} \quad (18)$$