Metamaterials
To the exciting, fast growing, and fully challenging area — Metamaterials.
Preface

Metamaterial, first known as left-handed material (LHM) or negative refractive index material (NIM), has attracted attention in the scientific communities over the past 10 years. The concept of metamaterial has a much broader scope than that of LHM or NIM. Due in large part to metamaterials, the classical subject of electromagnetism and optics have experienced a number of new discoveries and advances in research. First proposed by Veselago theoretically in 1968 for a material whose electric permittivity and magnetic permeability are simultaneously negative, LHM possesses many new features such as negative refraction, backward wave propagation, reversed Doppler shift, and backward Cerenkov radiation. Research in LHM was stagnant for more than 30 years due to the lack of experimental verification. The first revolution dealing with LHM occurred in 1996 when Sir Pendry discovered the wire medium whose permittivity is negative, followed by the discovery of negative permeability by Sir Pendry et al. in 1999 and LHM by Smith et al. in 2001. Inspired by the experimental realization, LHM – also called negative refractive index material – has attracted growing attention in both theoretical exploration and experimental study. However, LHM has the unavoidable disadvantage of big loss and narrow bandwidth, and such disadvantages restrict the applications of LHM. Hence scientists seek other features of metamaterial beyond negative refraction.

The second revolution in metamaterials came in 2005 when the gradient refraction index medium was realized to bend electromagnetic waves, which was discovered by Smith et al. In 2006 the optical transformation was proposed to make invisible cloaks to control the propagation of electromagnetic waves using metamaterials was discovered by Pendry et al. Metamaterial has a much broader definition than LHM, which does not require the negative permittivity and/or negative permeability, and hence opens a completely new area. The soul of metamaterial is the ability to control electromagnetic waves. After the experimental realization of invisible cloaks in the microwave regime, an even larger explosion of interest has occurred in metamaterials and optical transformation. This resulted in many scientific papers and popular science articles being published in journals and magazines.

As a summary of these scientific papers, up to date nine books on metamaterials have been published since 2003. Below is a glance at such published books:


These books are all valuable contributions to metamaterials. However, such books have mainly focused on LHM, negative refraction, photonic band-gap structures, and numerical methods. Only the books published in 2009 mentioned a few discussions on invisible cloaks. The main purpose of this book is to address the recent progress in metamaterials. We will introduce the optical transformation theory, revealing invisible cloaks, concentrators, beam bending, and new type antennas. We will present a general effective medium theory on artificial metamaterials composed of periodic structures. We also propose a rapid design method for inhomogeneous metamaterials, which makes it easier to design cloaks. Finally, we present broadband and low-loss non-resonant metamaterials, along with optical metamaterials.

Chapter 1 by Tie Jun Cui, David R. Smith and Ruopeng Liu, discusses the broad concepts of metamaterials and briefly reviews recent progress in the field. Chapter 2 by Wei Xiang Jiang and Tie Jun Cui presents the concepts and principles of optical transformation and discusses optical transformation devices, such as electromagnetic cloaks, concentrators, wave bending and antennas. Chapter 3 by Ruopeng Liu, Tie Jun Cui, and David R. Smith builds a general effective medium theory for artificial metamaterials composed of periodic structures. The theory provides effective permittivity and permeability in the system level by considering mutual coupling and spatial dispersion. Chapter 4 by Jessie Yao Chin, Ruopeng Liu, Tie Jun Cui and David R. Smith describes a rapid design method for complicated inhomogeneous metamaterials based on the general effective medium theory and the optimal-searching approach. Chapter 5 by Ruopeng Liu, Qiang Cheng, Tie Jun Cui and David R. Smith presents broadband and low-loss metamaterials using non-resonant structures, including the unit-cell design, the physical principle, and the effective medium theory. In Chapter 6, Ruopeng Liu, Jessie Y. Chin, Chunlin Ji, Tie Jun Cui, and David R. Smith show the design of electromagnetic cloaks using metamaterials, the 2D measurement system, and the measurement results of electromagnetic cloaks.
Chapter 7 by Christos Argyropoulos, Yan Zhao, Efthymios Kallos, and Yang Hao provides the full-wave simulations of electromagnetic cloaks using the radially dependent dispersive FDTD method to reveal the physics of the cloaks. In Chapter 8 Yijun Feng analyzes image focusing, rotation and lateral shift, as well as image magnification with sub-wavelength resolutions through differently designed structures of compensated anisotropic metamaterials. Chapter 9 by Xunya Jiang, Zheng Liu, Zixian Liang, Peijun Yao, Xulin Lin and Huanyang Chen investigates the dynamical performance of metamaterial systems, including the causality limit, the dynamical process, the correlation (coherence) study in metamaterial and extending the cloaking frequency range. In Chapter 10, Xueqin Huang, Shiyi Xiao, Lei Zhou, Weijia Wen, C. T. Chan and Ping Sheng show that a metallic plate with subwavelength fractal-shaped slits supports surface plasmon polaritons with the plasmon frequency tuned efficiently by the geometry of the fractal structure. The chapter also derives effective medium models to describe tunable plasmonic metamaterials. Chapter 11 by H. Liu, Y. M. Liu, T. Li, S. M. Wang, S. N. Zhu and X. Zhang reveals the recent developments in magnetic plasmonics arising from the coupling effect in metamaterials. Chapter 12 by Jun Xu, Anil Kumar, Pratik Chaturvedi, Keng H. Hsu and Nicholas X. Fang presents the development of plasmonic optical antennas for light concentration and near-field enhancement. In Chapter 13, Le-Wei Li, Ya-Nan Li and Li Hu investigate the applications of metamaterials in the microwave and RF components using wideband and low-loss metamaterials, and introduce some fast numerical methods to simulate metamaterials. Chapter 14 by Qiang Cheng, Xin Mi Yang, Tie Jun Cui, Ruopeng Liu and David R. Smith presents the applications of metamaterials in the design of new microwave components, antennas, and experiments. Finally in Chapter 15, Xin Hu and Sailing He propose a left-handed transmission line of low pass and study its applications.

We hope that the contents presented in this book will reflect new avenues of research on metamaterials and give appropriate guidance to scientists, engineers, and graduate students in this exciting area, and also generate novel microwave/optical devices and structures.

Southeast University, China, May, 2009
Duke University, USA, May, 2009
Duke University, USA, May, 2009

Tie Jun Cui
David R. Smith
Ruopeng Liu
Acknowledgments

This book gathers the latest research results on metamaterials from several groups (in the order which appears in the book): Southeast University, China (TJC); Duke University, USA (DRS); Queen Mary, University of London, UK (Yang Hao); Nanjing University, China (Yijun Feng); Institute of Microsystem and Information Technology, China (Xunya Jiang); Fudan University, China (Lei Zhou); Nanjing University, China (Hui Liu); University of Illinois at Urbana-Champaign, USA (Nick Fang); National University of Singapore, Singapore (Le-Wei Li); and Zhejiang University, China (Sailing He). We would like to acknowledge all contributors in the above groups for their great contributions.

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Nanjing, China
Tie Jun Cui
Durham, NC, USA
David R. Smith
Durham, NC, USA
Ruopeng Liu
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### 8 Compensated Anisotropic Metamaterials: Manipulating Sub-wavelength Images

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List of Contributors

Christos Argyropoulos
Department of Electronic Engineering, Queen Mary, University of London, Mile End Road, London, E1 4NS, United Kingdom, e-mail: christos.a@elec.qmul.ac.uk

C. T. Chan
Department of Physics, Hong Kong University of Science and Technology, ClearWater Bay, Kowlong, Hong Kong, China, e-mail: phchan@ust.hk

Pratik Chaturvedi
Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, USA, e-mail: pchatur2@illinois.edu

Huanyang Chen
Department of Physics, Hong Kong University of Science and Technology, ClearWater Bay, Kowlong, Hong Kong, China and Department of Physics, Suzhou University, Suzhou 215006, China, e-mail: kenyou@ust.hk

Qiang Cheng
State Key Laboratory of Millimeter Waves, Department of Radio Engineering, Southeast University, Nanjing 210096, P. R. China, e-mail: qiangcheng@emfield.org

Jessie Y. Chin,
State Key Laboratory of Millimeter Waves, Department of Radio Engineering, Southeast University, Nanjing 210096, P. R. China, e-mail: jychin@seu.edu.cn

Tie Jun Cui
State Key Laboratory of Millimeter Waves, Department of Radio Engineering, Southeast University, Nanjing 210096, P. R. China, e-mail: tj cui@seu.edu.cn
Nicholas X. Fang  
Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, USA e-mail: nicfang@illinois.edu

Yijun Feng  
Department of Electronic Science and Engineering, Nanjing University, Nanjing, 210093, China, e-mail: yjfeng@nju.edu.cn

Yang Hao  
Department of Electronic Engineering, Queen Mary, University of London, Mile End Road, London, E1 4NS, United Kingdom, e-mail: yang.hao@elec.qmul.ac.uk

Sailing He  
Division of Electromagnetic Engineering, School of Electrical Engineering, Royal Institute of Technology, S-100 44 Stockholm, Sweden and Center for Optical and Electromagnetic Research, Zhejiang University, Hangzhou, 310058, China, e-mail: sailing@kth.se

Keng H. Hsu  
Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, e-mail: khsu5@uiuc.edu

Li Hu  
Department of Electrical and Computer Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260, e-mail: g0600165@nus.edu.sg

Xin Hu  
Center for Optical and Electromagnetic Research, Zhejiang University, Hangzhou, 310058, China, e-mail: xinhu@kth.se

Xueqin Huang  
Surface Physics Laboratory (State Key Laboratory) and Physics Department, Fudan University, Shanghai 200433, P. R. China, e-mail: xqhuang@fudan.edu.cn

Chunlin Ji  
Department of Statistical Science, Duke University, Durham, NC 27708, USA, e-mail: chunlin.ji@duke.edu

Wei Xiang Jiang  
State Key Laboratory of Millimeter Waves, Department of Radio Engineering, Southeast University, Nanjing 210096, P. R. China, e-mail: wxjiang@emfield.org
Xunya Jiang
National Key-Lab of the Functional Material, Institute of Microsystem and Information Technology, CAS, Shanghai 200050, Peoples Republic of China, e-mail: xyjiang@mail.sim.ac.cn

Efthymios Kallos
Department of Electronic Engineering, Queen Mary, University of London, Mile End Road, London, E1 4NS, United Kingdom, e-mail: themos.kallos@elec.qmul.ac.uk

Anil Kumar
Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, e-mail: anilk2@illinois.edu

Le-Wei Li
Department of Electrical and Computer Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260, e-mail: lwli@nus.edu.sg

T. Li
Department of Physics, Nanjing University, Nanjing 210093, Peoples Republic of China, e-mail: liuhui@nju.edu.cn

Ya-Nan Li
Department of Electrical and Computer Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260, e-mail: bennyhappy@gmail.com

Zixian Liang
National Key-Lab of the Functional Material, Institute of Microsystem and Information Technology, CAS, Shanghai 200050, Peoples Republic of China, e-mail: zxliang@mail.sim.ac.cn

Xulin Lin
National Key-Lab of the Functional Material, Institute of Microsystem and Information Technology, CAS, Shanghai 200050, Peoples Republic of China, e-mail: xllin@mail.sim.ac.cn

H. Liu
Department of Physics, Nanjing University, Nanjing 210093, Peoples Republic of China, e-mail: liuhui@nju.edu.cn
Ruopeng Liu  
Center for Metamaterials and Integrated Plasmonics, Department of Electrical and Computer Engineering, Duke University, Box 90291, Durham, North Carolina 27708, USA, e-mail: ruopeng.liu@duke.edu

Y. M. Liu  
Nanoscale Science and Engineering Center, University of California, 5130 Etcheverry Hall, Berkeley, California 94720-1740, USA, e-mail: ymliu@berkeley.edu

Zheng Liu  
National Key-Lab of the Functional Material, Institute of Microsystem and Information Technology, CAS, Shanghai 200050, Peoples Republic of China, e-mail: liuzheng@mail.sim.ac.cn

Hui Feng Ma  
State Key Laboratory of Millimeter Waves, Department of Radio Engineering, Southeast University, Nanjing 210096, Peoples Republic of China, e-mail: hfma@emfield.org

Ping Sheng  
Department of Physics, Hong Kong University of Science and Technology, ClearWater Bay, Kowlong, Hong Kong, China, e-mail: sheng@ust.hk

David R. Smith  
Center for Metamaterials and Integrated Plasmonics, Department of Electrical and Computer Engineering, Duke University, Box 90291, Durham, North Carolina 27708, USA, e-mail: drsmith@duke.edu

S. M. Wang  
Department of Physics, Nanjing University, Nanjing 210093, Peoples Republic of China, e-mail: smwang@nju.edu.cn

Weijia Wen  
Department of Physics, Hong Kong University of Science and Technology, ClearWater Bay, Kowlong, Hong Kong, China, e-mail: phwen@ust.hk

Shiyi Xiao  
Surface Physics Laboratory (State Key Laboratory) and Physics Department, Fudan University, Shanghai 200433, P. R. China, e-mail: xsy@fudan.edu.cn

Jun Xu  
Department of Mechanical Science and Engineering, University of Illinois at Urbana-Champaign, e-mail: jebxu@illinois.edu
Xin Mi Yang  
State Key Laboratory of Millimeter Waves, Department of Radio Engineering,  
Southeast University, Nanjing 210096, People Republic of China, e-mail: xmyang@emfield.org

Peijun Yao  
National Key-Lab of the Functional Material, Institute of Microsystem and  
Information Technology, CAS, Shanghai 200050, Peoples Republic of China,  
e-mail: pjyao@mail.sim.ac.cn

X. Zhang  
Nanoscale Science and Engineering Center, University of California, 5130  
Etcheverry Hall, Berkeley, California 94720-1740, USA and Materials Sciences  
Division, Lawrence Berkeley National Laboratory, 1 Cycletron Road, Berkeley,  
CA 94720, USA, e-mail: xiang@berkeley.edu

Yan Zhao  
Department of Electronic Engineering, Queen Mary, University of London, Mile  
End Road, London, E1 4NS, United Kingdom, e-mail: yan.zhao@elec.qmul.ac.uk

Lei Zhou  
Surface Physics Laboratory (State Key Laboratory) and Physics Department, Fudan  
University, Shanghai 200433, P. R. China, e-mail: phzhou@fudan.edu.cn

S. N. Zhu  
Department of Physics, Nanjing University, Nanjing 210093, Peoples Republic  
of China, e-mail: snzhu@nju.edu.cn
Chapter 1
Introduction to Metamaterials

Tie Jun Cui, Ruopeng Liu and David R. Smith

Abstract There have been increasing interests in metamaterials in the past 10 years in the scientific communities. However, metamaterials are sometimes regarded as left-handed materials or negative refractive index materials by a lot of people including researchers. In fact, the rapid development in this exciting area has shown that metamaterials are far beyond left-handed materials. In this chapter, we will clarify what metamaterial is and report the recent progress on metamaterials. We also summarize the important issues for the development and future of metamaterials, including the optical transformation, effective medium theory for periodic structures, broadband and low-loss metamaterials, rapid design of metamaterials, and potential applications. The impact of computational electromagnetics on metamaterials is briefly discussed.

Key words: Metamaterials, left-handed materials, negative refraction, optical transformation, invisible cloaks, general effective medium theory, spatial dispersion, rapid design of metamaterials, non-resonant metamaterials, broadband metamaterials, applications of metamaterials, computational electromagnetics.

1.1 What Is Metamaterial?

The term of metamaterial was synthesized by Rodger M. Walser, University of Texas at Austin, in 1999, which was originally defined as “Macroscopic composites having a synthetic, three-dimensional, periodic cellular architecture designed to produce an optimized combination, not available in nature, of two or more responses
to specific excitation” [52, 53]. Based on Wikipedia, the metamaterial is defined as “a material which gains its properties from its structure rather than directly from its composition” [53].

The above definitions reflect certain natures of metamaterial, but not all. Actually, **a metamaterial is a macroscopic composite of periodic or non-periodic structure, whose function is due to both the cellular architecture and the chemical composition.** If the metamaterial is regarded as an effective medium, there is an additional requirement that the cellular size is smaller than or equal to the sub-wavelength.\(^1\) In this book, the metamaterials are required to have sub-wavelength for the unit cell so that they can be described using the effective medium theory. Figure 1.1 shows two typical metamaterial structures in the microwave regime, in which Fig. 1.1(a) is a periodic structure that is equivalent to a homogeneous medium and Fig. 1.1(b) is a non-periodic structure that is equivalent to an inhomogeneous (gradient) medium. The microwave metamaterials are fabricated with printed circuit boards (PCB) by making different metal architectures on PCB. The properties of such metamaterials are mainly due to the cellular architecture, and also dependent

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\(^1\) Sometimes the photonic bandgap structure is also regarded as metamaterial whose cellular size is larger than sub-wavelength.
on the PCB substrates, which can be FR4, F4B, and Rodges. The dependence of metamaterial properties on the cellular architecture provides great flexibility to control metamaterials. One can create new materials which are unavailable in nature but can be realized in practice using metamaterial structures. This is the biggest advantage of metamaterials.

Usually, the material properties are characterized by an electric permittivity ($\varepsilon$) and a magnetic permeability ($\mu$). The thinnest material in nature is free space or air, whose permittivity is $\varepsilon_0$ and permeability is $\mu_0$. The relative permittivity and permeability of a material are defined as $\varepsilon_r = \varepsilon / \varepsilon_0$ and $\mu_r = \mu / \mu_0$, respectively, which define another important material parameter, the refractive index, as $n = \sqrt{\varepsilon_r \mu_r}$. In nature, most materials have the permeability $\mu_0$ and permittivity larger than $\varepsilon_0$. The metamaterial opens a door to realize all possible material properties by designing different cellular architectures and using different substrate materials. Figure 1.2 illustrates all possible properties of isotropic and lossless materials in the $\varepsilon$–$\mu$ domain. In Fig. 1.2, the first quadrant ($\varepsilon > 0$ and $\mu > 0$) represents right-handed materials (RHM), which support the forward propagating waves. From the Maxwell’s equations, the electric field $\mathbf{E}$, the magnetic field $\mathbf{H}$, and the wave vector $\mathbf{k}$ form a right-handed system. The second quadrant ($\varepsilon < 0$ and $\mu > 0$) denotes electric plasma, which support evanescent waves. The third quadrant ($\varepsilon < 0$ and $\mu < 0$) is the well-known left-handed materials (LHM), which was proposed by Veselago in 1968 [51], supporting the backward propagating waves. In LHM, the electric field $\mathbf{E}$, the magnetic field $\mathbf{H}$, and the wave vector $\mathbf{k}$ form a left-handed system. The fourth quadrant ($\varepsilon > 0$ and $\mu < 0$) represents magnetic plasma, which supports evanescent waves.

Fig. 1.2 All possible properties of isotropic materials in the $\varepsilon$–$\mu$ domain.
In Fig. 1.2, most natural materials only occur at certain discrete points on the line \( \mu = \mu_0 \) and \( \varepsilon \geq \varepsilon_0 \), and seldom natural electric plasma and magnetic plasma occur in very small parts in the second and fourth quadrants. Most of material properties have to be realized using metamaterials, even for RHM. In a long period, metamaterials, LHM, negative-refractive index materials (NIM), double negative materials (DNG), and backward-wave materials have been regarded as the same terms. However, they actually represent different meanings. Metamaterials have much broader scope than LHM, as shown in Fig. 1.2. In the \( \varepsilon-\mu \) domain, there are several special lines and points indicating special material properties. For example, the point \( \mu = -\mu_0 \) and \( \varepsilon = -\varepsilon_0 \) represents an anti-air in the LHM region, which will produce a perfect lens; the point \( \mu = 0 \) and \( \varepsilon = 0 \) represents a nihility, which can yield a perfect tunneling effect; the line \( \mu = \varepsilon \) in both RHM and LHM regions represents impedance-matching materials, which have perfect impedance matching with air, resulting no reflections. Also, the vicinity of \( \mu = 0 \) is called as \( \mu \)-near zero (MNZ) material, and the vicinity of \( \varepsilon = 0 \) is called as \( \varepsilon \)-near zero (ENZ) material, which has special properties.

Actually, metamaterials have much more features beyond those shown in Fig. 1.2. Metamaterials can be designed as weakly and highly anisotropic, depending on different requirements. The flexibility to design various material properties together with the optical transformation makes it possible to control electromagnetic waves at will using metamaterials.

### 1.2 From Left-Handed Material to Invisible Cloak: A Brief History

Metamaterials were first known as LHM or NIM. Although the concept of LHM was proposed by Veselago in 1968 [51], the negative refraction and backward-wave media had been discovered theoretically much earlier [50, 32, 31, 44]. The earliest publication on negative refraction was in lecture notes of Prof. Mandelshtam from Moscow University [32]. Then the Sommerfeld radiation condition in backward-wave media was studied by Malyuzhinets in 1951 [31], who showed that the phase velocity of waves pointed from infinity to the source. Furthermore in 1957, Sivukhin investigated materials with negative parameters and noticed the backward-wave property [44]. Except the theoretical study, backward-wave structures had been developed for the design of microwave tubes in 1960s [19, 1, 42], and the negative refraction was discovered even in periodical media [41]. A significant progress was made by Veselago in 1968, who proposed a systematic analysis of electromagnetic behaviors in materials with negative permittivity and permeability, and named the term of LHM [51]. Besides the negative refraction and backward-wave propagation, Veselago also showed some new features like reversed Doppler shift and backward Cerenkov radiation in LHM.

However, due to the non-existence in nature and lack of experimental verification, LHM had not attracted attention over 30 years in the scientific community.

\(^2\) The Russian version was published in 1967.
The first revolution on LHM or metamaterials occurred in 1996, when Pendry et al. realized the artificially electric plasma using the wire medium whose permittivity is negative [36]. Then Pendry and his coworkers discovered the artificially magnetic plasma whose permeability is negative in 1999 [35]. In this work, the well-known split-ring resonators (SRR) are used to achieve the magnetic response. The first artificial LHM was made by Smith et al. in 2001 using the combination of wires and SRRs [40]. In this famous experiment, the negative refraction phenomenon was verified. Inspired by the experimental realization, LHM has attracted growing attention in both theoretical exploration and experimental study [45, 34, 46, 3, 17, 12, 13], including the exciting discovery of perfect lens and super lens [34]. However, LHM has unavoidable disadvantages of big loss and narrow bandwidth, and such disadvantages restrict the applications of LHM.

In the meantime, an alternative representation of LHM was presented by three groups (Eleftheriades, Oliner, and Caloz-Itoh), almost simultaneously in June 2002, using the transmission-line (TL) approach [20, 33, 4, 21, 5, 6]. It is well known that a conventional TL is composed of distributed series inductance \( L \) and shunt capacitance \( C \), which can be shown equivalent to a one-dimensional (1D) RHM. As a dual model of the conventional TL, a new TL consisting of series capacitance and shunt inductance will support backward waves and hence can represent LHM. In the realization of left-handed (LH) TL, the distributed components have been used. Since the series capacitance is always accompanied by an inductance and the shunt inductance is accompanied by a capacitance, a general composite right-left-handed (CRLH) TL model has been proposed to represent RHM and LHM [6]. Based on the CRLH structures, a lot of microwave components and antennas have been proposed [6].

Despite the good properties, LHM suffers from the large loss and narrow bandwidth. Hence scientists seek other features of metamaterial beyond the negative refraction. Then the second revolution on metamaterial came in 2005 when the gradient refractive index medium was realized to bend electromagnetic waves [47], and in 2006 when the optical transformation was proposed to make invisible cloaks and to control the propagation of electromagnetic waves using metamaterials [37, 25]. From then on, metamaterials have a much broader meaning than LHM, which do not require the negative permittivity and/or negative permeability, and hence open a completely new area. After the experimental realization of invisible cloak in the microwave regime [39, 28], a large explosion of interest has been paid to metamaterials and the optical transformation, and many scientific papers have been published in journals, which are not cited here.

1.3 Optical Transformation and Control of Electromagnetic Waves

Metamaterials have great advantage to realize the required material parameters, including those unavailable in nature, by adjusting the cellular architecture and its chemical composition. However, there remain two questions: What is the requirement for the metamaterial parameters? How to choose the metamaterial parameters?
In some cases, we can get the solutions easily using the known theory and geometrical optics, such as the perfect/super lens [34] and gradient refractive index lens [47]. In general cases to control electromagnetic waves, it is not so easy before the appearance of the optical transformation.

The optical transformation [37], or the transformation optics, is one of the most important methods in the development of metamaterials, which is going to have further impact on the future metamaterials. Based on the principle of form invariance in Maxwell’s equations, the optical transformation is derived rigorously in closed forms. The optical transformation builds up a bridge between the device functions and material properties. For example, if one wants to make a spherical invisible cloak which can hide the objects inside, one can obtain the material parameters using the optical transformation as [37]

\[
\varepsilon_r = \mu_r = \frac{b}{b-a} \frac{(r-a)^2}{r^2},
\]

\[
\varepsilon_\theta = \mu_\theta = \frac{b}{b-a},
\]

\[
\varepsilon_\phi = \mu_\phi = \frac{b}{b-a},
\]

in which \(a \leq r \leq b\), and \(a\) and \(b\) are radii of the inner and outer spherical surfaces of the cloak, respectively. Apparently, the spherical cloak is required to be biaxially anisotropic in the spherical coordinate system, and the three components of the permittivity and permeability have to satisfy Eqs. (1.1)–(1.3). From these equations, we also notice that the cloak is made of impedance-matching materials to eliminate any reflections.

Hence, the optical transformation enables the direct manipulation of electromagnetic waves and offers a powerful tool to design novel and complicated devices. Besides the invisible cloak, the optical transformation has also been used to generate electromagnetic concentrators which can focus electromagnetic waves into a small enclosed region [38, 22]; electromagnetic field and polarization rotators which can rotate electromagnetic fields [7]; wave-shape transformers which can transform cylindrical waves to plane waves to make high-gain antennas [23, 26]; electromagnetic-wave bending structures to bend the waves to desired directions [24, 18]; and other optical transformation devices. Chapter 2 gives more detailed discussions on this topic.

1.4 Homogenization of Artificial Particles and Effective Medium Theory

1.4.1 General Description

The optical transformation sets up a bridge between the device functions and material parameters. In order to realize the device functions, however, there is another
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Gap between the material parameters and the realistic metamaterial structures. As we mentioned earlier, metamaterials are composed of periodic or non-periodic structures with sub-wavelength unit cells. Now we must ask two questions: Is a periodic structure with sub-wavelength unit cell equivalent to a homogenous effective medium? How to determine the effective medium parameters once the periodic structure is known? The solutions to such questions are key points for the homogenization of artificial particles and effective medium theory.

In periodically structured metamaterials, the artificial inclusions with sub-wavelength size replace the atoms and molecules of conventional materials. Since the scale of these inclusions is much smaller than the wavelength, a homogenized description in the macroscope is valid [48]. In the homogenization of the periodically structured metamaterials, the macroscopic electromagnetic fields are determined by averaging the local fields. The detailed definition of the averaged fields can be found in Refs. [48] and [27]. From the averaged fields, the averaged permittivity and permeability are further defined [48, 27]. The above field-averaging method can be applied to homogenize any periodic structures with arbitrarily shaped sub-wavelength unit cells.

However, the averaged permittivity and permeability only represent the particle responses, or local properties, and hence cannot be used as the effective material parameters of the effective medium. In fact, the averaged permittivity and permeability are similar to the constitutive parameters in the Drude–Lorentz medium models [43], which are derived in the static and quasi-static limits. Apparently, the Drude–Lorentz medium models are inaccurate in microwave and higher frequencies. A scattering (S) parameter retrieval method has been shown an accurate approach to obtain the effective constitutive parameters [49]. This is a numerical- or experimental-based approach, in which the S parameters (reflection and transmission coefficients) of the periodic structure are used to derive the effective permittivity and permeability. Figure 1.3 shows the constitutive parameters (permeability) of a periodic structure whose inclusion is SRR, obtained from the Drude–Lorentz model and the S-parameter retrieval.

From Fig. 1.3, there exists a significant difference of the permeability between the two methods. Although the S-parameter retrieval method can provide accurate material parameters, it is numerically or experimentally oriented and hence is difficult to be used in the design of metamaterials. The Drude–Lorentz medium models have simply closed-form expressions, but are inaccurate since only the particle responses are involved. The mutual coupling among different unit cells, or the spatial dispersion, is not considered. In order to set up a relationship between the particle responses and the macroscopical system behaviors for artificial metamaterials composed of periodic structures, a general theory of effective media has been proposed [27]. Based on the general theory, the effective permittivity and permeability were derived using the discrete Maxwell’s equations in the macroscale, which have closed forms [27]. The excellent agreements between theoretical predictions and S-parameter retrieval results indicate the accuracy of the general theory. For details refer to Chapter 3.
Fig. 1.3 The constitutive parameters (permeability) of a periodic structure whose inclusion is SRR. (a) From the Drude–Lorentz model under the static and quasi-static limits (above). (b) From the S-parameter retrieval under full-wave simulations (below).

1.4.2 A TL-Metamaterial Example

In order to show the role of spatial dispersion in the effective medium parameters, we consider a simple 1D TL metamaterial [11], as shown in Fig. 1.4(a). This is an infinite 1D periodic structure with series impedance $Z_s$ and shunt admittance $Y_p$, with the period of $\rho$. The unit cell of the periodic structure is chosen as a symmetrical T form, as illustrated in Fig. 1.4(b). The general forms of $Z_s$ and $Y_p$ are written as
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\[ Z_s = -i\omega L_s - 1/(i\omega C_s) \] and \[ Y_p = -i\omega C_p - 1/(i\omega L_p) \], in which \( L_s \) and \( C_s \) are series inductance and capacitance, and \( L_p \) and \( C_p \) are shunt inductance and capacitance. According to Eqs. (3.23a) and (3.23b) in Ref. [6], the particle-based permittivity and permeability are expressed as

\[ \mu = \mu(\omega) = \frac{1}{p}[L_s - 1/(\omega^2 C_s)], \quad (1.4) \]

\[ \varepsilon = \varepsilon(\omega) = \frac{1}{p}[C_p - 1/(\omega^2 L_p)]. \quad (1.5) \]

Apparently, the spatial dispersion was not considered in the above expressions.

To involve the spatial dispersion in the effective medium parameters, one starts from the Bloch theorem and circuit theory. After simple derivation, one obtains the dispersion equation as [11]

\[ \sin^2(\theta/2) = ZY/4, \quad (1.6) \]

and the wave impedance as

\[ Z_0 = V_n/I_n = \frac{1}{2}Z/\tan(\theta/2), \quad (1.7) \]

in which \( \theta = kp \), and \( Z = \omega L_s - 1/(\omega C_s) \) and \( Y = \omega C_p - 1/(\omega L_p) \) are real numbers. They can be either positive or negative. Next we discuss the dispersion equation with different values of \( Z \) and \( Y \).

If \( 0 \leq ZY \leq 4 \), then \( \theta \) will be a real number: \( \theta = \pm 2\arcsin(\sqrt{ZY}/2) \), and hence the TL metamaterial supports propagating modes. When \( Z \) and \( Y \) are both positive, \( \theta \) is positive, corresponding a forward propagating mode; when \( Z \) and \( Y \) are both negative, \( \theta \) is negative, representing a backward propagating mode. If \( ZY < 0 \), then \( \theta \) is a pure imaginary number: \( \theta = \pm i2\arcsinh(\sqrt{-ZY}/2) \), which corresponds to
pure plasma modes. When “−” is taken, it denotes an active metamaterial; when “+” is taken, it represents a passive metamaterial, which is considered here. If $ZY > 4$, then $\theta$ is a complex number: $\theta = \theta_R + i \theta_I = \pm \pi + i 2 \arccosh(\sqrt{ZY}/2)$. Hence the TL metamaterial supports resonant crystal bandgap modes.

From above discussions, $ZY = 0$ and $ZY = 4$ define boundaries of such three kinds of modes. When $ZY = 0$, one obtains two critical frequencies

$$\omega_1 = \min\{\omega_s, \omega_p\}, \quad \omega_2 = \max\{\omega_s, \omega_p\},$$

(1.8)
in which $\omega_s = 1/\sqrt{L_sC_s}$ and $\omega_p = 1/\sqrt{L_pC_p}$ are resonant frequencies of the series and shunt branches, respectively. When $ZY = 4$, one gets the other two critical frequencies

$$\omega_3 = \sqrt{\omega^2_c - \omega^2_d}, \quad \omega_4 = \sqrt{\omega^2_c + \omega^2_d},$$

(1.9)
in which $\omega^2_c = 2/(L_sC_p) + (\omega^2_s + \omega^2_p)/2$ and $\omega^2_d = \omega^4_c - \omega^2_c \omega^2_p$. The four critical frequencies satisfy $\omega_3 < \omega_1 < \omega_2 < \omega_4$. Hence the whole frequency regime is divided into five regions by the four critical frequencies, as shown in Fig. 1.5.

![Fig. 1.5](image)

**Fig. 1.5** Different wave modes of the general periodic structure.

In regions $\omega_3 < \omega < \omega_1$ and $\omega_2 < \omega < \omega_4$, we have $0 < ZY < 4$. Hence such two regions support the propagating modes: $k = \theta_p/p$. In the first region, both $Z$ and $Y$ are negative, and hence $\theta_p = -2 \arcsin(\sqrt{ZY}/2)$, corresponding to the backward propagating mode. In the second region, both $Z$ and $Y$ are positive, and hence $\theta_p = 2 \arcsin(\sqrt{ZY}/2)$, corresponding to the forward propagating mode. In such two regions, the wave impedance $Z_0 = Z/[2 \tan(\theta_p/2)]$ is always real and positive. Then the effective permittivity $\varepsilon_{\text{eff}}$ and permeability $\mu_{\text{eff}}$ of the TL metamaterial are easily derived from the wavenumber and wave impedance as
\[ \mu_{\text{eff}} = L_{\text{eff}} \theta_p / \tan(\theta_p/2), \quad \varepsilon_{\text{eff}} = C_{\text{eff}} \theta_p \tan(\theta_p/2). \] (1.10)

Here, \( L_{\text{eff}} = Z/(2\omega p) \) and \( C_{\text{eff}} = 2/(Z\omega p) \) are effective inductance and capacitance, which can be either positive and negative. Apparently, the general circuit periodic structure is equivalent to an LHM in the frequency region \( \omega_3 < \omega < \omega_1 \), where both \( \varepsilon_{\text{eff}} \) and \( \mu_{\text{eff}} \) are negative and is equivalent to an RHM in the frequency region \( \omega_2 < \omega < \omega_4 \), where both \( \varepsilon_{\text{eff}} \) and \( \mu_{\text{eff}} \) are positive.

In the region \( \omega_1 < \omega < \omega_2 \), we have \( ZY < 0 \), corresponding to pure plasma modes: \( k = i\theta_l/p \). Here, \( \theta_l = 2\ln(\sqrt{-ZY/4 + \sqrt{ZY^2/4 + 1}}) \). In this case, the wave impedance is a pure imaginary number: \( Z_0 = -iZ/[2\tanh(\theta_l/2)] \). Hence one easily derives \( \varepsilon_{\text{eff}} \) and \( \mu_{\text{eff}} \) as

\[ \mu_{\text{eff}} = L_{\text{eff}} \theta_l / \tanh(\theta_l/2), \quad \varepsilon_{\text{eff}} = -C_{\text{eff}} \theta_l \tanh(\theta_l/2). \] (1.11)

When \( Z > 0 \), then \( \mu_{\text{eff}} > 0 \) and \( \varepsilon_{\text{eff}} < 0 \), representing an electric plasma; when \( Z < 0 \), then \( \mu_{\text{eff}} < 0 \) and \( \varepsilon_{\text{eff}} > 0 \), denoting a magnetic plasma.

In regions \( 0 < \omega < \omega_3 \) and \( \omega > \omega_4 \), we have \( ZY > 4 \). Hence such two regions support the resonant crystal bandgap modes: \( k = (\pi + i\theta_l)/p \), in which \( \theta_l = 2\ln(\sqrt{ZY/4 + \sqrt{ZY^2/4 - 1}}) \). The wave impedance is also a pure imaginary number: \( Z_0 = -iZ\tanh(\theta_l/2)/2 \). One easily obtains \( \varepsilon_{\text{eff}} \) and \( \mu_{\text{eff}} \) as

\[ \mu_{\text{eff}} = L_{\text{eff}}(\theta_l - i\pi) \tanh(\theta_l/2), \quad \varepsilon_{\text{eff}} = C_{\text{eff}}(-\theta_l + i\pi) / \tanh(\theta_l/2). \] (1.12)

Hence the general circuit periodic structure behaves like a crystal bandgap metamaterial in such two regions. When \( Z > 0 \), then \( \text{Re}\{\mu_{\text{eff}}\} > 0 \) and \( \text{Re}\{\varepsilon_{\text{eff}}\} < 0 \), and the metamaterial is electric–plasma type; when \( Z < 0 \), it is magnetic–plasma type. From Eq. (1.12), one of imaginary parts of the permittivity and permeability is positive (positive loss) and the other is negative (negative loss). They appear always in conjugate forms, representing the lossless nature of the original circuit structure.

In order to validate the proposed medium model for the TL metamaterial, we choose the circuit parameters arbitrarily as \( L_s = 20 \text{nH}, L_p = 5 \text{nH}, C_s = 2.5 \text{pF}, \) and \( C_p = 2 \text{pF} \). Then we get the four critical frequencies as \( f_1 = 0.71 \text{GHz}, f_2 = 1.59 \text{GHz}, f_3 = 0.49 \text{GHz}, \) and \( f_4 = 2.31 \text{GHz} \). In such a situation, the wavenumber and wave impedance versus frequencies are illustrated in Fig. 1.6. From the dispersion curve shown in Fig. 1.6(a), we clearly observe the crystal bandgap mode, backward propagating mode, pure plasma mode, forward propagating mode, and another crystal bandgap mode, which are exactly the same as predicted. We get similar conclusions to the wave impedance.

The effective permittivity and permeability of the TL metamaterial are demonstrated in Fig. 1.7(a) and (b), respectively. From Fig. 1.7, we observe the magnetic–plasma-type crystal bandgap metamaterial, LHM, the pure electric plasma, RHM, and the electric–plasma-type crystal bandgap metamaterial, which are exactly coincident to the earlier predictions. From Fig. 1.7, we also notice the conjugate imaginary parts of permittivity and permeability in the crystal bandgap regimes. Such a phenomenon has not been discovered in the earlier circuit-medium models [6].